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Research Article

Combined Model of Robust Data Envelopment Analysis with Complementary Slackness Condition for Efficiency Assessment

Amir Amini
PhD Student of Economic Development, Faculty of Economics and Management, Urmia University, Urmia, Iran

Yousef Mohammadzadeh*
Associate Professor, Department of Economics, Faculty of Economics and Management, Urmia University, Urmia, Iran

Alireza Alinezhad
Associate Professor, Faculty of Industrial and Mechanical Engineering, Qazvin Branch, Islamic Azad University, Qazvin, Iran
*Corresponding author: , *yo.mohammadzadeh@urmia.ac.ir*
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Abstract

Realizing the principle of performance improvement is one of the fundamental preconditions for organizations to survive in today's competitive and turbulent environment. For this purpose, proper establishment and implementation of a comprehensive and efficient performance assessment system to analyze the various dimensions of the organization will greatly contribute to its continuous improvement. Therefore, achieving a comprehensive evaluation system as a basis for making right decisions to pursuit and implement organizational goals and strategies has always been a major concern of managers. Data envelopment analysis is a nonparametric method that can evaluate decision making units with multiple inputs and outputs. In this paper, a combined model of robust data envelopment analysis with complementary slackness condition is considered for efficiency assessment. The combination of data envelopment analysis with other models can create strong models for efficiency assessment of decision making units. Besides, using combined models in uncertainty environment can make these models more practical. Therefore, use of robust data envelopment analysis in uncertainty environment can be very effective in efficiency evaluation applications. This study presents an integrated robust optimization and linear programming model to expand combined data envelopment analysis and complementary slackness condition in uncertainty environment. This can help to extend the use of this method over previous models. The In order to make the model practical, a numerical example has been solved. After that the results have been compared with the other models.

Keywords: Robust Data Envelopment Analysis, Complementary Slackness Condition, Robust Optimization Approach, Uncertainty, Efficiency

Introduction

In today's global and competitive economy, the survival and continuity of organizations is affected by their ability to compete and choose appropriate policies in response to environmental changes.Therefore, in order to make crucial and sensitive decisions, it is essential for managers to be aware of the correct functioning of all aspects of their organization (Ehrgott et al., 2018). Data envelopment analysis is one of the efficient tools for evaluating the efficiency of .decision-making units. Based on this non-parametric method, the efficiency of decision-making units can be well examined. The first model in the field of data envelopment analysis is the CCR[1] Model. Considering linear programming model, multiple inputs and outputs were used with appropriate weighting assigned to each of them to evaluate the efficiency of decision-making units in this model. Afterwards, Banker et al. (1984) proposed a new model called the BCC[2] model using the basic principles of data envelopment. The difference between these two models is in the type of returns to scale of production . In the CCR model, the returns to scale of production are considered constant , however in theBCC model the returns to scale of production are variable . Given the need for accurate methods for measuring efficiency, since data envelopment analysis is a suitable tool for measuring efficiency, researchers have proposed suggestions to make this method more efficient (Emrouznejad et al., 2016). One of these suggestions is to combine data envelopment analysis with other methods . One of the advantages of combining this method with other methods is that data envelopment analysis may declare a high percentage of decision-making units efficient in some cases, thus it not possible to make the right decision about the efficiency of companies . For this reason, combining data envelopment analysis with existingapproaches and models in the fields of statistics and operations research can be a suitable solution to prevent this problem. One of these new developments that is a new method in the field of measuring efficiency, is the combination of data envelopment analysis with strong complementary

[1] Charnes, Cooper and Rhodes
[2] Banker , Charnes and Cooper

slackness conditions . This new method was presented in the research of Sueyoshi and Sekitani (2007) and Sueyoshi and Sekitani (2009). By using the combination of data envelopment analysis and strong complementary slackness conditions, a method was created that does not require prior information in its studies. This is a very important factor that will be very useful in making this model more practical. Considering that this model is used in a definite conditions , using a model that can operate in uncertainty conditions and risk conditions can make this model more practical and bring this model closer to real conditions. As mentioned, data envelopment analysis is a nonparametric method and over time this method became more complete. In BCC model, whenever space and imperfect competition conditions impose restrictions on investment, it causes the unit to not operate at the optimal scale. For this reaso considering variable returns to scale can be a great help in the studies . We can also refer to the additive model that simultaneously consider reduction in inputs and increase in outputs (Charnes et al., 1995). This model has an important advantage over input-based and output-based models, because input-based models only consider reducing inputs and output-based models only pursue increasing outputs, while this model focuses on both inputs and outputsin the same time. Among the recent research . in data envelopment analysis, we can mention the research of Adler and Yazmezski(2010), in which it is proposed that by considering the reduction in the number of variables, this tool can increase the resolving power of the data envelopment analysis method and increase the accuracy of this method.

The use of DEA in the case of uncertainty can contribute to the application of this method. Research in this category has used DEA in cases where uncertain data are available, such as probabilistic and fuzzy data (Zahedi-Seresht et al., 2017). Among these studies, the research of Sengupta(1992), which initiated the study of DEA in the fuzzy state. Also, the research of Cooper et al. (2004), proposed theories based on considering probabilistic constraints in the model when the inputs and outputs used in DEA are in the probabilistic state. Other researches such as the research of Puri and Yadav (2013) in which fuzzy inputs with mixed efficiencies and the use of alpha cutoff in fuzzy DEA have been used for use in Indian banks to investigate the efficiency.

Some other studies have combined DEA with other models in operations research and statistic.This combination of models can make DEA models more efficient and powerful. Among these studies, the study by Stern et al. (2000) as well as the study by Sueyoshi and Goto (2009, 2012 & 2013) can be mentioned which combined data envelopment analysis with diagnostic analysis and other expansion and innovation in this field in Japanese industries and companies.

Moreover, the research of Zhou et al. (2012) combined data envelopment analysis and confidence space topic to evaluate the performance of manufacturing companies in different categories. Moutinho et al. (2017) combined data envelopment analysis and quantile regression and developed a hybrid model to select different assets.

The combination of data envelopment analysis with strong complementary slackness conditions was first p roposed by Sueyoshi & Sekitani (2007) and was continued in other forms in the research of Sueyoshi & Sekitani (2009) and Sueyoshi and Goto (2011). Although this useful method works in a situation that does not require prior data, it has some weaknesses including considering deterministic state which leads to a distance from real conditions . In the present study, an attempt has been made to apply this model in a non-deterministic state based on the robust data envelopment analysis approach in order to create a more practical model that is closer to real conditions.

Recently, the topic of robust optimization has been introduced which can be used as a complementary approach to model sensitivity analysis and probabilistic programming (Hladík, 2019).The basis for this paper is based on the research of Simar and Wilson (1998). In this research, self-starting techniques are used in the topic of data envelopment analysis. Also, in the research of Simar and Wilson (1999), and another research by Simar and Wilson (2000), the properties of probabilistic topics in data envelopment analysis have been studied. Furthermore, self-starting algorithms have been developed on data envelopment analysis that can measure the efficiency of decision-making units. In this approach, efficiency ratings are obtained with data envelopment analysis model . The permissible error rate in the data envelopment analysis estimates is obtained by the algorithm proposed in Simar and Wilson (1998). One of the problematic steps in this algorithm is finding the appropriate value of the smoothing parameters, and the next problem is the appropriate number of iterations to run this algorithm. The advantages of this algorithm, which is based on the optimization model include its high usability. In this proposed method, first the percentage of perturbation in the data used is examined, then by considering a specific approach, the efficiency estimate is obtained based on robust optimization (Amirkhan et al., 2018).

In the second part of this study, the theoretical foundations of the subject will be examined. The proposed research model, which uses a combination of robust data envelopment analysis with strong complementary slackness conditions to evaluate efficiency is presented in the third part. To demonstrate the applicability of the proposed model, a numerical example of the efficiency will be presented in the fourth part. Finally, the fifth part will discuss conclusions and suggestions for future research.

Theoretical Foundation of Research

This section focuses on the theoretical foundations of the research in the form of data envelopment analysis, robust optimization approach, strong complementary slackness conditions and combining data envelopment analysis with complementary slackness conditions.

Data Envelopment Analysis

Data envelopment analysis is a nonparametric method that can evaluate a decision-making unit that has multiple inputs and outputs. The outputs and inputs used in DEA are unique to the system under consideration. There are no inputs in the traditional productivity analysis sense, but each DMU is assigned an equal "input" with a value of 1 (one) for all management options. In this case, the common input ensures that differences in outcomes (outputs) are not related to differences in inputs. Given the impact of multiple variables on performance in the system, it is important to identify and define each of them as input or output variables (Mehregan et al., 2006). This method has different models, two of the primary models are mentioned here.

1- The goal of CCR model in terms of input is to find a virtual decision-making unit that can produce the output Y_0 with minimum input. Model1 represents the CCR model in terms of input (Charnes, 1978):

$$
\begin{aligned}
&\min\ \theta_o \\
&s.t: \\
&\sum_{j=1}^{n} \lambda_j x_{ij} \le \theta_o x_{io}, \quad i = 1,\dots,m \\
&\sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{ro}, \quad r = 1,\dots,s \qquad\qquad (1) \\
&\lambda_j \ge 0 \quad , \quad j = 1,\dots,n \\
&\theta_o \quad free
\end{aligned}
$$

2- The BCC model in the nature of input was presented according to the CCR model. The set of possibilities for generating this model is obtained by removing the principle of ray infinity from the set of principles of data envelopment analysis. Model 2 represents the BCC model in the nature of input.

$$
\begin{aligned}
&\min \ \theta_o \\
&s.t: \\
&\sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta_o x_{io}, \quad i = 1,\ldots,m \\
&\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro}, \quad r = 1,\ldots,s \qquad (2) \\
&\sum_{j=1}^{n} \lambda_j = 1 \\
&\lambda_j \geq 0 \quad , \quad j = 1,\ldots,n \\
&\theta_o \quad free
\end{aligned}
$$

Performance assessment

Evaluating efficiency in multi-period models has attracted considerable attention among researchers. With numerous studies across various sectors, performance assessment is a fundamental aspect of all business and economics administration. Academic literature offers several methods to compare the performance of the models. Many scientific methods of efficiency analysis, namely ratio analysis, DEA, stochastic frontier etc. have been utilized for the evaluation of performance efficiency. Key figures or index figures are the output-to- input ratios (Krmac & Mansouri Kaleibar, 2022).

Robust Optimization Approach

Modeling approaches in operations research under uncertainty conditions have been investigated in models. In many modelings, uncertainty conditions have not been considered which leads to be far from the real conditions. Based on the presentation of a robust optimization structure in the research of Ben-Tal & Nemirovski (2000), and the research of Bertsimas and Sim (2004), a linear programming model was considered (model 3) (Bertsimas & Sim, 2004).

$$
\begin{aligned}
&\min c' x \\
&s.t: \\
&Ax \geq b \qquad (3) \\
&x \in X
\end{aligned}
$$

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In the robust optimization technique for models with uncertainty in the data, a special row i of the matrix A is considered, and J_i the set of coefficients in row i is under uncertainty. In the research of Ben Tal & Nemirovsky (2000) each $\tilde{a}_{ij}, j \in J_i$ is modeled as a bounded and symmetric random variable. Also, each a_{ij} is bounded by $[a_{ij} - \hat{a}_{ij}, a_{ij} + \hat{a}_{ij}]$ where $\hat{a}_{ij}$ is the size of estimation accuracy.

Robust data envelopment analysis Based on the Bundle Adjustment

In this approach J_i represents data with uncertainty in the i-th constraint. In this ,case model 3 is formulated as Model 4:

$$\begin{aligned} &\min c'x \\ &s.t: \\ &\tilde{a}_i x \geq b_i \quad \forall i \ , \ \forall \tilde{a}_i \in J_i \\ &x \in X \end{aligned} \tag{4}$$

In the above model, $\tilde{a}_i$ is the i-th vector of A'.To solve the model, Bertsimas and Sim (2003, 2004 & 2006), also Bertsimas et al. (2004) defined the deviation scale of a_{ij} as equation 5:

$$\eta_{ij} = \frac{\tilde{a}_{ij} - a_{ij}}{\hat{a}_{ij}} \quad \forall i, j \tag{5}$$

$\tilde{a}_{ij}$ and a_{ij} are uncertainty data and $\hat{a}_{ij}$ is the measure of the estimate accuracy also η_{ij} has a distribution that takes a value betwee n [-1, 1].The cumulative deviation scale for thei-th constraint can take any value between [-n, n] which must be bounded to equation 6:

$$\sum_{j=1}^{n} \eta_{ij} \leq \Gamma_i \quad \forall i \tag{6}$$

In the researches of Bertsimas and Sim (2003, 2004 & 2009) also Bertsimas et al. (2004), for each i constraint, a Γ_i parameter is introduced which is not necessarily an integer and can take a value between [0, n]. The role of this parameter is to balance the power of thintroduced method against a level of conservatism of the solution. It is also called a budget for the uncertainty of the i-th constraint. There are two cases for Γ_i:

First, there will be no protection against uncertainty if $\Gamma_i = o$.

Second: the i-th constraint is protected against uncertainty if $\Gamma_i = n$.

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Third: the decision maker can strike a balance between the level of constraint protection and the degree of conservatism of the solution *if* $\Gamma_i \in (o,n)$.

In this particular approach, the J_i set is defined as *equation 7:*

$$J_i = \{(\tilde{a}_{ij}) | \tilde{a}_{ij} = a_{ij} + \hat{a}_{ij}\eta_{ij}\} \quad \forall i \ , \ j, \eta \in Z \qquad (7)$$

Also Z is defined as:

$$Z = \{\eta | |\eta_{ij}| \leq ١ , \sum_{j=١}^{n} \eta_{ij} \leq \Gamma_i\} \ \forall i \qquad (8)$$

$$\sum_{j=١}^{n} \tilde{a}_{ij} x_j = \sum_{j=١}^{n} (a_{ij} + \hat{a}_{ij}\eta_{ij}) x_j = \sum_{j=١}^{n} a_{ij} x_j + \sum_{j=١}^{n} \hat{a}_{ij}\eta_{ij} x_j \quad \forall i$$

Now Model ٤ can be reformulated as Model ٩

$$\min c^{'} x$$

$$a_j^{'} x + \min_{\eta_i \in Z_i} \sum_{j=١}^{n} \hat{a}_{ij} x j \eta_{ij} \geq b_i \quad \forall i \qquad (9)$$

$$x \in X$$

Also For any given i, $\min_{\eta_i \in Z_i} \sum_{j=١}^{n} \hat{a}_{ij} x j \eta_{ij}$ will be equal to:

$$-\max \sum_{j=١}^{n} \hat{a}_{ij} |x_j| \eta_{ij}$$

$$s.t:$$

$$\sum_{j=١}^{n} \eta_{ij} \leq \Gamma_i \quad \forall i \qquad (10)$$

$$٠ \leq \eta_{ij} \leq ١ \quad \forall i, j$$

The dual model for model *10* is written as model *11:*

$$\min \Gamma_i p_i + \sum_{j \in J_i} q_{ij}$$

$$s.t:$$

$$p_i + q_{ij} \geq e a_{ij} |x_j^*| \quad \forall i, j \in J_i \qquad q_{ij} \geq ٠ \quad \forall i, j \in J_i \qquad (11)$$

$$p_i \geq ٠ \quad \forall i$$

The dual variables in this model are p_i, q_{ij} *.When the above model is* applied to *model 9, the* optimization formulation is based on the research of Bertsimas and Sim *(2003, 2004 & 2006) and* Bertsimas et al. (2004) it is written as *model 12.*

$$
\begin{aligned}
&\min c' x \\
&s.t: \\
&a_i' x - \Gamma_i p_i - \sum_{j \in J_i} q_{ij} \geq 0 && \forall i \\
&p_i + q_{ij} \geq e a_{ij} y_j && \forall i, j \qquad (12) \\
&-y_j \leq x_j \leq y_j && \forall j \\
&p_i \geq 0 && \forall i \\
&q_{ij} \geq 0 && \forall i, j \\
&x \in X
\end{aligned}
$$

Strong complementary slackness conditions

Strong complementary slackness conditions are the relationships that are formed between the primary and secondary models.The relationships that exist between the primary and secondary models of a programming model should be examined. The number of variables in the secondary model is equal to the number of constraints in the primary model, and the number of constraints in the secondary model is equal to the number of variables in the primary model. Strong complementary slackness conditions are a specific relationship between the constraints of the primary model and the (positive value) of the deficiency variables in the secondary model.

The strong complementary slackness condition states that if x^* is the optimal solution of the primary model and y^* is the optimal solution of the secondary model, also t^* is the optimal value of deficiency variables and w^* is the surplus variables of the secondary model, then equation 13 and 14 are established between the optimal solutions and the variables of each model:

$$x^* * w^* = o \qquad (13)$$

$$y^* * t^* = o \qquad (14)$$

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Combining data envelopment analysis with strong complementary slackness conditions

In the combined model of envelopment analysis, data with strong complementary slackness conditions x and y are respectively the input and output data which are considered for j=(1,...,n) decision-making units. Moreover, $u_r$is the weight given to the r-th output and v_i is the weightgiven to the i -th input. η is used to optimally maintain the strong complementarity shortage situation. Also, e=(1,1,...,1) is a unit vector and the sign of w specifies the type of returns to scale.

The model presented in the research of Sueyoshi and Sekitani (2007) is as equations (15):

$$
\begin{aligned}
&\max\ \eta \\
&s.t: \\
&\theta_o x_{io} - \sum_{j=١}^{n} \lambda_j x_{ij} \geq ٠ \qquad i = ١,\ldots,m \\
&\sum_{j=١}^{n} \lambda_j y_{rj} \geq y_{ro} \qquad r = ١,\ldots,s \\
&\sum_{j=١}^{n} \lambda_j = ١ \\
&\sum_{i=١}^{m} v_i x_{io} = ١ \\
&-\sum_{i=١}^{m} v_i x_{io} + \sum_{r=١}^{s} u_r y_{rj} + w \leq ٠ \qquad j = ١,..,n \\
&\theta_o = \sum_{r=١}^{s} u_r y_{ro} + w \\
&-\lambda_j - \sum_{i=١}^{m} v_i x_{ij} + \sum_{r=١}^{s} u_r y_{rj} + we^T \leq -e^T \qquad j = ١,..,n \qquad (15) \\
&v_i - \sum_{j=١}^{n} \lambda_j x_{ij} + \theta_o x_{io} \geq \eta \qquad i = ١,\ldots,m \\
&u_r + \sum_{j=١}^{n} \lambda_j y_{rj} \geq \eta e^T + y_{ro} \qquad r = ١,\ldots,s \\
&v_i \geq ٠ \quad i = ١,\ldots,m \\
&u_r \geq ٠ \quad r = ١,\ldots,s
\end{aligned}
$$

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$$
\begin{aligned}
&\lambda_{\mathrm{j}} \geq 0 \quad j=1,..,n \\
&\eta \geq 0 \\
&w \quad free \\
&\theta_o \quad free
\end{aligned}
$$

Research Methodology

The model obtained by combining data envelopment analysis with strong complementary *slackness* conditions in the uncertain state has the same principles as the definite model, with the difference that in th*e*new model the outputs, i.e. the vector*y, is* considered uncertain. Considering the uncertainty of the output data and the uncertainty of the new model and the changes made to it, the new proposed model with the bundle adjustment approach which was , mentioned in thetheoretical foundations of the research is written as *model 16.*

$$
\begin{aligned}
&\max \ \eta \\
&s.t: \\
&\theta_o x_{io} - \sum_{j=1}^{n} \lambda_{\mathrm{j}} \mathrm{x}_{\mathrm{i\,j}} \geq 0 \qquad i=1,...,m \\
&\sum_{j=1}^{n} \lambda_{\mathrm{j}} \mathrm{y}_{\mathrm{r\,j}} - \Gamma_{\mathrm{r}} \mathrm{p}_{\mathrm{r}} - \sum_{j=1}^{\mathrm{n}} \mathrm{q}_{\mathrm{rj}} - \mathrm{y}_{\mathrm{r\,o}} - \mathrm{q}_{\mathrm{r\,o}} \geq 0 \qquad r=1,...,s \\
&\sum_{j=1}^{n} \lambda_{\mathrm{j}} = 1 \\
&\sum_{i=1}^{m} v_i x_{io} = 1 \\
&-\sum_{i=1}^{m} v_i x_{io} + \sum_{r=1}^{s} u_r y_{rj} - \Gamma'_{\mathrm{j}} \mathrm{p}'_{\mathrm{j}} - \sum_{r=1}^{\mathrm{s}} \mathrm{q}'_{\mathrm{rj}} + w \leq 0 \qquad j=1,..,n \\
&\theta_o = \sum_{r=1}^{s} u_r y_{ro} - \Gamma'_{\mathrm{o}} \mathrm{p}'_{\mathrm{o}} - \sum_{r=1}^{\mathrm{s}} \mathrm{q}'_{\mathrm{rj}} + w \\
&-\lambda_{\mathrm{j}} - \sum_{i=1}^{m} v_i x_{ij} + \sum_{r=1}^{s} u_r y_{rj} - \Gamma'_{\mathrm{j}} \mathrm{p}'_{\mathrm{j}} - \sum_{r=1}^{\mathrm{s}} \mathrm{q}'_{\mathrm{rj}} + we^T \leq -\mathrm{e}^{\mathrm{T}} \qquad j=1,..,n \\
&v_i - \sum_{j=1}^{n} \lambda_{\mathrm{j}} \mathrm{x}_{\mathrm{i\,j}} + \theta_o x_{io} \geq \eta \qquad i=1,...,m
\end{aligned}
\tag{16}
$$

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$$u_r + \sum_{j=1}^{n} \lambda_j y_{rj} - \Gamma_r p_r - \sum_{j=1}^{n} q_{rj} - y_{ro} - q_{ro} \geq \eta e^T \qquad r = 1,\dots,s$$

$$p_r + q_{rj} \geq e y_{rj} t_j \qquad r = 1,\dots,s \;,\; j = 1,\dots,n$$

$$-t_j \leq \lambda_j \leq t_j \qquad j = 1,\dots,n$$

$$p'_j + q'_{rj} \geq e y_{rj} k_r \qquad r = 1,\dots,s \;,\; j = 1,\dots,n$$

$$-k_r \leq u_r \leq k_r \qquad r = 1,\dots,s$$

$$v_i \geq 0 \qquad i = 1,\dots,m$$

$$u_r \geq 0 \qquad r = 1,\dots,s$$

$$\lambda_j \geq 0 \qquad j = 1,..,n$$

$$p'_j \geq 0 \qquad j = 1,\dots,n$$

$$p_r \geq 0 \qquad r = 1,\dots,s$$

$$q'_{rj} \geq 0 \qquad r = 1,\dots,s \;,\; j = 1,\dots,n$$

$$q_{rj} \geq 0 \qquad r = 1,\dots,s \;,\; j = 1,\dots,n$$

$$\eta \geq 0$$

$$w \quad free$$

$$\theta_o \quad free$$

The value of Γ_j , Γ_r is obtained through e quation17, and the value of Φ is also obtained using the cumulative table of the Gaussian distribution.

$$\Gamma_j = 1 + \Phi^{-1}(1 - e_i)\sqrt{n} \qquad i = 1,\dots,m \;,\; j = 1,\dots,n \tag{17}$$

Finding and Results

This section presents a numerical example. In this example, four mineral processing companiesare considered and the aim is to examine the efficiency of these companies. The companies have two inputs and four outputs . The numbers used in this example are taken from theErtay et al. (2006). The input and output data of the four decision-making units are listed in table 1:

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Table 1.
Input and output data of decision-making units

DMUs	Inputs		Outputs			
	1	2	1	2	3	4
1	2039.56	6405	0.4697	0.013	0.0410	30.89
2	20411.22	5393	0.4380	0.0337	0.0484	31.34
3	20280.28	5294	0.4392	0.0308	0.0653	30.26
4	20053.20	4450	0.3776	0.0245	0.0638	28.03

The values of Γ_j, Γ_r are obtained by using the cumulative distribution table of the Gaussian distribution. In this example, given n= 4 (number of outputs) and considering e= 0.01, the value will be equal to 5.64. The results of solving the model considering e=0.01 are given in Table 2:

Table 2.
Results of solving the proposed model

Units	θ	η	λ_1	λ_2	λ_3	λ_4	v_1	v_2	u_1	u_2	u_3	u_4
Unit 1	0.9873	0.0124	0	0	0	1	0.00004	0	0.482	0.023	0.053	30.90
Unit 2	0.9824	0.0175	0	0	0	1	0.00004	0	0.455	0.051	0.065	31.35
Unit 3	0.9888	0.0111	0	0	0	1	0.00004	0	0.450	0.041	0.076	30.27
Unit 4	1	0	0	0	0.84	0	0	0.0002	0.377	0.024	0.063	28.03

The above model with two values of e=0.1 and e=0.01 has been compared with the two other models, namely the combined data envelopment analysis model and strong complementary slackness conditions and the BCC model. The results of comparisons are given in table 3:
According to table 3, it is understood that the efficiency will become more permissive by increasing e_i . Also, other results include the more lenient nature of the combined data envelopment analysis model and strong complementary slackness conditions in the certainty state compared to the steady state. It can also be noted that the efficiency evaluation results with the BCC model are close to the combined model proposed in this paper.

Table 3.
Comparison of the efficiency of the proposed model with other mode ls

DMUs	Robust-DEA-SCSC		DEA-SCSC	BCC
	e=0.01	e=0.1		
1	0.9873	0.9974	1	0.9873
2	0.9824	0.9925	0.999	0.9827
3	0.9888	0.9987	1	0.9899
4	1	1	1	1

Conclusion

In this study, a hybrid model of robust data envelopment analysis with strong complementaryslackness conditions for evaluating efficiency was examined. Given the need for new models in examining the efficiency of decision-making units, it is essential to present hybrid models based on data envelopment analysis that examine efficiencies in the case of uncertainty . In this study, first, explanations were given about the necessity of this research in the introduction, then the literature and research background were reviewed. The most important foundations of the subject were also examined, and each of the topics that have been used in this study in some way was discussed in separate titles. In the next section, a proposed model was presented. This new model presents robust data envelopment analysis in a hybrid mode with strong complementary slackness conditions, which can be used to evaluate the efficiency of decision-making units. After presenting the model, a numerical example was solved to examine the efficiency of the model. After solving the example, other models were compared with the presented model, and the results of these comparisons were included in the study. Considering the uncertainty conditions in this model, the proposed model can be used to examine the efficiency of decision-making units under uncertainty conditions. For future research, it is possible to examine other combined models in data envelopment analysis, such as combining data envelopment analysis with approaches discussed in operations research and statistics to achieve more efficient models. Also, by considering uncertainty conditions such as fuzzy, interval and probabilistic data, also in lack of access to reliable data it is possible to expand the above model and create new models.

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