Using Evolutionary Algorithms for Optimal Control and Lie Symmetry of Non-linear Fractional-Order Chaotic System of Criminally Active and Prisoner

Abstract

The purpose of this research is to present a non-linear model of mathematical fractional order for criminally active and prisoner system. Both genetic algorithm and particle swarm optimization algorithm were used to simulate optimal control. Modeling approach of the type of differential equation machine with fractional order derivatives was used. In the following, it was shown that the presented model has chaos, its order is of fractional order and it needs to be controlled. Genetic algorithm and particle swarm optimization algorithm were used for simulation and optimal control. It was shown that this model has a chaotic behavior; as a result, optimal control for this behavior was presented. The results of the genetic algorithm method are excellent. All the results obtained for the particle swarm optimization method show that this method is also very successful and the results are very close to the genetic algorithm method. Very low values of MSE and RMSE errors indicate that the simulation is effective and efficient. This article is the first article that performs nonlinear modeling of system criminally active and prisoner and optimally controls the chaos in the model. This type of modeling and optimal control has not been done so far. Also, software and algorithms have been used that are very fast, accurate and have the lowest possible error.

Keywords: criminally active, prisoner, optimal control, evolutionary algorithms.

1. Introduction

Criminal activities and issues related to prisoners are among the most important social, economic and security challenges of today's societies, which have profound effects on sustainable development, social stability and public welfare. The theoretical foundations in this field are based on criminological theories, social psychology and dynamic models of crime [1,2]. Crime is defined as behavior that violates social norms and formal laws of society and causes harm to individuals, property and public order. Criminal activities are influenced by several factors such as poverty, unemployment, social inequality, family structure, education and culture, and these factors interact in a complex way to cause the emergence and spread of crime [5,6,7]. Prisoners, as individuals who are held in detention centers for committing crimes, form part of this cycle of crime and punishment, and the management of this population also has its own challenges, including issues related to rehabilitation, prevention of further crimes and numerous social and economic costs.[3,4] In recent years, with the advancement of data science, mathematical modeling, and computational technologies, new approaches have been developed to analyze, predict, and control criminal activities and manage prison populations. One of the key topics in this area is the use of dynamic models and complex systems to simulate crime trends and prison population behavior [10,11]. These models often include various variables such as crime rates, levels of preventive activities, prison capacity, and the impact of social and economic policies. Using these models allows for the analysis of the interaction of different factors and helps policymakers make optimal decisions [8,9]. Along with modeling, optimization and optimal control are among the most important tools for reducing criminal activities and improving prison conditions [20,21]. Optimization means finding the best combination of policies and interventions that can reduce crime and minimize social, economic, and human costs. Classical optimization methods often have limitations in this field due to the complexity and multidimensionality of the problem. For this reason, metaheuristic algorithms such as genetic algorithms and particle swarm optimization have been introduced as new and powerful methods [24]. These algorithms, due to their global search capability and high flexibility, have been able to provide optimal and robust solutions for crime control and prisoner management. The use of genetic algorithms in this field has made it possible to design policies that, in addition to reducing crime rates, have positive effects on prisoner rehabilitation and reducing recidivism. By simulating the processes of natural selection, mating, and mutation, this algorithm allows discovering the best combination of interventions among thousands of options [26]. Also, the particle swarm optimization algorithm, modeled on the collective behavior of birds or fish, has a high convergence rate and is very suitable for optimization problems with continuous and complex variables. The application of PSO in the control of criminal activities helps to optimize intervention policies in a flexible and dynamic manner and to adjust them quickly in response to environmental or social changes.[23] The main advantages of using these algorithms in this area include the ability to search extensively and avoid getting stuck in local optima, the ability to deal with unstable and incomplete data, and the ability to solve multi-objective problems. For example, it is possible to simultaneously optimize crime rates, law enforcement costs, and social impacts of correctional programs [22]. In addition to reducing financial costs, these methods can help improve the quality of life of prisoners, reduce social disorders, and increase public safety. From a research perspective, several studies have shown that the use of dynamic models together with genetic algorithms and PSO in macro-social and judicial planning has improved the efficiency and effectiveness of crime and prisoner control policies [25]. Recent studies have examined different scenarios based on real crime and prison data, and their results demonstrate the ability of these algorithms to create an optimal balance between economic, social, and security goals. Also, combining these algorithms with machine learning models can improve the decision-making and forecasting process and accelerate the response to environmental changes. [13]. Finally, it can be said that the use of advanced optimization methods such as genetic algorithms and particle swarm optimization in the field of criminal activity control and prisoner management is an effective step towards reducing the social, economic, and human costs of crime. These approaches allow policymakers to design policies that are both financially cost-effective and promote social justice and public safety, and as a result, create a more sustainable and secure society [14,15]. Models of criminal activity and prison populations often exhibit nonlinear and chaotic behavior due to the inherent complexities of the influencing factors and the multifaceted interactions between social, economic, psychological, and legal variables. The chaotic nature of these models means that the system is highly sensitive to initial conditions and parameters; such that small changes in the inputs can lead to completely different and unpredictable results [18,19]. This feature causes the trend of criminal activity and changes in the number of prisoners to move in an unstable and complex manner over short or long time periods. Also, numerous positive and negative feedbacks, such as the impact of economic conditions on crime, and the impact of judicial policies on prison populations, increase the dynamic and chaotic dimensions of the model [16,17,18]. For this reason, accurate prediction and the design of efficient control policies in this area are difficult and require tools that can optimize in a nonlinear and chaotic space. In this regard, metaheuristic algorithms such as the genetic algorithm (GA) and particle swarm optimization (PSO) play an important role. The genetic algorithm, which is based on the natural processes of evolution and natural selection, is able to search the complex and multidimensional problem space extensively and move towards global optima by creating a population of possible solutions and updating them through operators such as selection, mating, and mutation [25,26,27]. This feature is especially important for chaotic crime and prisoner models that include multi-peak and nonlinear objective functions. With its deep exploration ability and high population diversity, the genetic algorithm reduces the probability of getting stuck in local optima and provides robust and balanced solutions for crime reduction and prison population control. The application of genetic algorithms (GA) and particle swarm optimization (PSO) in modeling criminal activity systems and prison populations has significant advantages due to the special characteristics of these algorithms and the inherent complexity of these systems [31,32,33]. First, both algorithms have the ability to search globally in complex and nonlinear problem spaces, which is crucial for modeling crime systems with chaotic and multidimensional behaviors. The genetic algorithm, with natural evolution simulation processes, is able to examine complex and multi-objective system structures in a multi-generational manner and discover the best combination of parameters and policies. This helps to form dynamic crime and prison models with higher accuracy and to consider various changes in social and economic conditions [28,29,30]. On the other hand, PSO, due to its high convergence speed and algorithm simplicity, has the ability to quickly find optimal points and is very suitable for models that require rapid updating and dynamic response. Second, both algorithms have the ability to deal with multi-objective problems; that is, they can simultaneously optimize for reducing crime rates, improving prisoner rehabilitation, and reducing economic and social costs. This capability is important in modeling crime systems, because these problems often have conflicting or multiple objectives that require careful balancing [37,38,39]. In the field of optimal control, the advantage of these algorithms is their high flexibility to find efficient intervention policies. Optimal control of criminal activities and prisoner management requires fine-tuning of multiple variables and rapid response to environmental changes. Genetic algorithms, due to their ability to diversify the solution population, avoid getting stuck in local optima and provide more creative solutions. PSO also provides dynamic and adaptive control due to its continuous updating of particle positions and ability to quickly adapt to new conditions [34 ,44,45]. The combination of these two algorithms (hybrid algorithms) allows us to benefit from both the extensive search capabilities of GA and the speed and accuracy of PSO, which is very valuable for complex and chaotic crime and prison systems. Ultimately, these methods allow managers and policymakers to optimize policies and interventions, in addition to reducing crime rates and improving prisoner conditions, minimizing direct and indirect social, economic, and human costs and achieving a safer and more sustainable society [35,36,37]. On the other hand, the particle swarm optimization algorithm is designed based on the collective behavior of living organisms such as birds or fish, and each particle in the search space seeks the best optimal position by moving and interacting with other particles [38,39,41] . Due to its simple structure, high convergence speed, and ability to handle continuous variables, this algorithm is very suitable for dynamic problems with a large number of parameters. In controlling crime and prison models, PSO can quickly reach practical optima and facilitate policy updates in variable and uncertain conditions. Combining the application of GA and PSO in criminal activity and prisoner population control models allows us to benefit from both the exploration power of GA and the convergence speed of PSO [42 ,45]. This hybrid combination can create a suitable balance between exploring the search space and exploiting the best regions found, which is very crucial for chaotic and nonlinear models. Finally, the application of these algorithms allows the design of optimal control policies that not only reduce the crime rate, but also minimize the economic, social, and human costs of crime and punishment [41,42,43]. These methods also have the ability to adapt and learn dynamically in the face of environmental and social changes, which is very important for complex and chaotic systems. Thus, the use of GA and PSO as optimization and control tools in models of criminal activities and prisoner populations provides an efficient and innovative solution for better management of these complex social phenomena [40,44].

1.2. Chaotic Fractional-Order Systems

This article investigates the parameters and conditions for which the fractional-order system could have chaotic behavior. In this section, two relevant theorem for fractional-order systems are stated [26,28]. The theorem is about proportional fractional-order systems.

Theorem 1.2.1. *In an autonomously system we have:*

$$\frac{d^{\alpha}x}{dt^{\alpha}} = Ax, \quad x(0) = x_0. \tag{1}$$

i) *By considering* $0 < \alpha < 1$ *and* $x \in \mathbb{R}^{n\times n}$ *, matrix* $A \in \mathbb{R}^{n\times n}$ *is asymptotically stable if and only if* $|arg(\lambda)| > \frac{\alpha\pi}{2}$ *is valid. In this equation,* λ *is the eigenvalue of matrixA In addition, this matrix is stable if and only If* $|arg(\lambda)| \geq \frac{\alpha\pi}{2}$.

ii) *The equilibrium point in fractional-order systems is calculated as in ordinary differential equations as below:*

$$\frac{d^{\alpha}x}{dt^{\alpha}} = f(x), \tag{2}$$

$$f(x) = 0. \tag{3}$$

In the equation above, we have $0 < \alpha < 1$ *and* $x \in \mathbb{R}^{n\times n}$*. The equilibrium point achieved by solving the equation is asymptotically stable if the calculated eigenvalue* λ *related to the Jacobian matrix* $J = \frac{df}{dt}$ *satisfies the following equation in equilibrium point [26,27]:*

$$|arg(\lambda)| > \frac{\alpha\pi}{2}\,. \tag{4}$$

Proof: See [26,28] for the proof.

Theorem.1.2.2. *The n-dimensional dynamic fractional-order system could be specified as follows[34]:*

$$\begin{gathered}\frac{d^{\alpha_1}x_1}{dt^{\alpha_1}} = a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n,\\ \frac{d^{\alpha_2}x_2}{dt^{\alpha_2}} = a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n,\\ \vdots\\ \frac{d^{\alpha_n}x_n}{dt^{\alpha_n}} = a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n.\end{gathered} \tag{5}$$

In the equation above, all α_i *coefficients have values between 0 and 1. It is assumed that M is the least common multiple of* u_i *that is expressed as* $\alpha = \frac{v_i}{u_i}$*. Here* $(u_i, v_i) = 1$ *and* $u_i, v_i \in \mathbb{Z}^+$*for* $i = 1,2,3, \dots, n$*.* $\Delta(\lambda)$ *Is described as below [35]:*

$$\Delta(\lambda) = \begin{pmatrix} \lambda^{M_{\alpha_1}} - a_{11} & -a_{12} & \cdots & -a_{1n} \\ -a_{21} & \lambda^{M_{\alpha_2}} - a_{22} & \cdots & -a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ -a_{n1} & -a_{n2} & \cdots & \lambda^{M_{\alpha_n}} - a_{nn} \end{pmatrix}. \tag{6}$$

The system response described in (5) is asymptotically stable if all roots (λ) of equation $det(\Delta(\lambda) = 0)$ *satisfy the condition:*

$$|arg(\lambda)| > \frac{\alpha\pi}{2}.$$

Denoting The matrix $\Delta(s)$ *is the characteristic matrix, and* $det(\Delta(s))$ *is the polynomial characteristic of the system (5).*

Proof: See [26,28] for the proof.

Definition. 1.2.3. The fractional-order system is considered as follows:

$$\frac{d^{\alpha_i}x_i}{dt^{\alpha_i}} = f_i(x_1, x_2, x_3, \dots, x_1), \quad i = 1,2,3, \dots, n. \tag{7}$$

In the equation above, all α_i coefficients have values between 0 and 1. The equilibrium point of the system (7) is acquired by solving the following Eq.[8]:

$$f_i(x_1, x_2, x_3, \dots, x_1) = 0, \quad i = 1,2,3, \dots, n. \tag{8}$$

It is assumed that $x_1^* = (x_1^*, x_2^*, x_3^*, \dots, x_n^*)$ is the equilibrium point of the system (7) meaning $f_i(x_1^*, x_2^*, x_3^*, \dots, x_n^*) = 0$. Considering the values for *i*, the equation below is defined to evaluate the stability of equilibrium point:

$$\varepsilon_i = x_i - x_i^*, \quad i = 1,2,3, \dots, n. \tag{9}$$

As the Caputo differentiation by a constant value is zero, we would conclude:

$$\frac{d^{\alpha_i}\varepsilon_i}{dt^{\alpha_i}} = f_i(x_1^* + \varepsilon_1, x_2^* + \varepsilon_2, \dots, x_n^* + \varepsilon_i), \quad i = 1,2,3, \dots, n. \tag{10}$$

If the second partial differentiation of function f_i around the equilibrium point x^* exists in the *n*-dimensional space of $\mathbb{R}^n$, the right-hand side of equation (10) could be rewritten as:

$$f_i(x_1^* + \varepsilon_1, x_2^* + \varepsilon_2, \dots, x_n^* + \varepsilon_i) = f_i(x_1^*, x_2^*, \dots, x_n^*) + \left[\frac{\partial f_i}{\partial x_1}\bigg|_{x^*} \frac{\partial f_i}{\partial x_2}\bigg|_{x^*} \cdots \frac{\partial f_i}{\partial x_n}\bigg|_{x^*}\right]\varepsilon + \overline{f_i}(\varepsilon). \tag{11}$$

In the equation above, $\varepsilon = [\varepsilon_1, \varepsilon_2, \dots, \varepsilon_n]^T$, and $\overline{f_i}(\varepsilon)$ consist of the higher-order terms of Taylor expansion that is neglected. In addition, it is assumed that we have $f_i(x_1^*, x_2^*, \dots, x_n^*) = 0$, for $i = 1,2,3, \dots, n$. As a result, we could conclude:

$$f_i(x_1^* + \varepsilon_1, x_2^* + \varepsilon_2, \dots, x_n^* + \varepsilon_i) \approx \left[\frac{\partial f_i}{\partial x_1}\bigg|_{x^*} \frac{\partial f_i}{\partial x_2}\bigg|_{x^*} \dots \frac{\partial f_i}{\partial x_n}\bigg|_{x^*} \right] \varepsilon + \overline{f_i}(\varepsilon). \tag{12}$$

Furthermore, we could assume the following equation:

$$\begin{bmatrix} \frac{d^{\alpha_1} x_1}{dt^{\alpha_1}} \\ \frac{d^{\alpha_2} x_2}{dt^{\alpha_2}} \\ \vdots \\ \frac{d^{\alpha_n} x_n}{dt^{\alpha_n}} \end{bmatrix} = J\varepsilon, \tag{13}$$

where we have $f = [f_1, f_2, \dots, f_n]^T$ and $J = \frac{\partial f}{\partial x}\Big|_{x^*}$.

It is assumed that *M* is the least common multiple of α_i that is defined as $\alpha_i = \frac{v_i}{u_i}, (u_i, v_i) = 1$, and $u_i, v_i \in \mathbb{Z}^+$ for $i = 1,2,3, \dots, n$. According to Theorem (1.2.1), if $|arg(\lambda)| > \frac{\alpha\pi}{2}$ for all λ calculated by the equation below, the equilibrium point $x = x^*$ of the system (7) is asymptotically stable [28]:

$$det(diag([\lambda^{M\alpha_1} \ \ \lambda^{M\alpha_2} \ \ \dots \ \ \lambda^{M\alpha_n}]) - J) = 0, \tag{14}$$

It should be noted that $diag([m_1 \ \ m_2 \ \ \dots \ \ m_n])$ is represented by a square $n \times n$ matrix as below:

$$diag([m_1 \ \ m_2 \ \ \dots \ \ m_n]) = \begin{bmatrix} m_1 & 0 & \cdots & 0 \\ 0 & m_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & m_n \end{bmatrix}. \tag{15}$$

1.2.4. The Required Conditions for the Presence of Chaos in Fractional-Order System

The saddle point is an equilibrium point in a three-dimensional integer-order system with at least one eigenvalue at the stable region (the left-hand part of the imaginary axis) and at least one eigenvalue in the unstable area (the right-hand part the imaginary axis). This saddle point is called saddle point of kind one if one of the eigenvalues is unstable and the others are stable, and if one eigenvalue is stable while two others are unstable, the saddle point is of kind two. The chaotic behavior in a chaotic system is demonstrated around a saddle point of kind two. The chaotic behavior could also be observed around a saddle point of the second kind in a three dimensional fractional-order system, just as the three-dimensional integer order one [38,39]. It is considered that the chaotic three-dimensional system of the form $\dot{x} = f(x)$ have chaotic attractors. It is also assumed that Ω is a set of equilibrium points of the system surrounded by a twisting. On the other hand, the $D^\alpha x = f(x)$ system with defined $D^\alpha \equiv \left(\frac{d^{\alpha_1}}{dt^{\alpha_1}}, \frac{d^{\alpha_2}}{dt^{\alpha_2}}, \frac{d^{\alpha_3}}{dt^{\alpha_3}} \right)$ and the system $\dot{x} = f(x)$ have equal equilibrium points. Therefore, the required condition for a fractional-order system of $D^\alpha x = f(x)$ to have chaotic attractor is stated as the following equation [40]:

$$\left(\frac{\pi}{2M}\right) - min|arg(\lambda_i)| \geq 0, \tag{16}$$

where λ_i are the roots of the equation below:

$$det([\lambda^{M\alpha_1} \ \ \lambda^{M\alpha_2} \ \ \dots \ \ \lambda^{M\alpha_n}]) - \frac{\partial f}{\partial x}\bigg|_{x-x^*} = 0, \quad \forall x^* \in \Omega. \tag{17}$$

The system's behavior around this point cannot tend to a chaotic attractor if the system has a stable equilibrium point, and the initial conditions related to the system do not lie inside the attracting region. In other words, the system cannot have a chaotic behavior for any initial condition, and some of the initial conditions do not actually represent chaotic behavior. In general, there is not a specific mathematical relation to the present attracting region. The condition of being chaotic for the fractional-order system of (7) could be stated as follows (by assuming$\alpha_1 = \alpha_2 = \alpha_3 = \alpha$, for more details see [8,51]):

$$\alpha \geq \frac{2}{\pi} min|arg(\lambda_i)|, \tag{18}$$

where λ_i are the eigenvalues of the Jacobin matrix that is defined as $\frac{\partial f}{\partial x}\Big|_{x-x^*} = 0$ for every $x^* \in \Omega$. The relation (18) states the necessary condition for chaos to occur in a fractional-order system [26,28]. Further detail could be found in. This relation could be used to acquire the minimum order of the system for which the chaotic behavior could occur.

1.3. Optimal control

By assuming a function called objective function, this technique aims to determine the control signal to optimizes an objective function. This method is applied in [7,8]. In the following section, further descriptions will be provided about this controlling scheme.

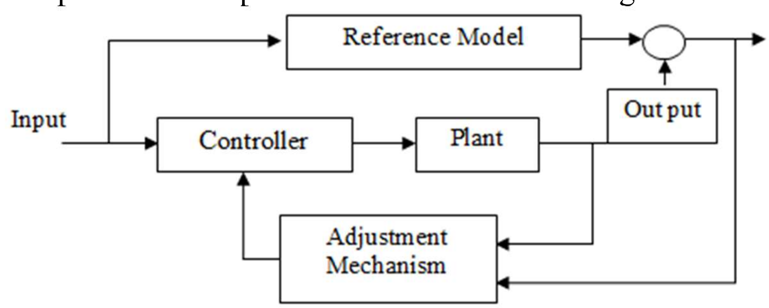

Fig 1. The diagram of controlling system by adaptive method [32,33]

2. Research method

In this chapter, we studied the optimal control problem of the fractional-order system of the criminally active and prisoner. We were ready to solve the specified fractional-order model by the particle swarm optimization and genetic algorithms.

2.1. Characteristics of the model understudy

We build and analyze a model of a dynamic system of criminally active, prisoner and recidivism using some parameters. In the models, they tried to find equations of the criminally active , prisoner and the factors affecting them. Accordingly, in this article, using the model related to crime in the coupled Eqs. (19), we examine the issue of optimal control of the criminally active and prisoner. So we will have [26]:

$$\begin{aligned} \dot{x} &= a_1 y - a_2 xy + a_3 z, \\ \dot{y} &= -a_1 y + a_2 xy - a_4 y + a_5 z, \\ \dot{z} &= a_4 y - (a_5 + a_3) z, \end{aligned} \tag{19}$$

Where The fundamental components of our model is: x: those who are not criminally active at a given time; y: those who are criminally active but have never been incarcerated; z:those who are incarcerated at a given time; a_1: Rate at which criminals discontinue criminal habits (desistance), a_2: Contagion parameter of criminal behavior, a_3: Rate at which incarcerated individuals are released and assimilate back into society, a_4: Rate at which criminals are incarcerated, a_5: Rate at which incarcerated are released and return to criminal life. It is clear that the system behavior is chaotic with parameters $a_1 = 0.4, a_2 = 0.9, a_3 = 0.1, a_4 = 0.5, a_5 = 0.6.$ and initial condition $x_0 = [0.3,1.5,0.5]$.

2.2. Fractional-Order system of the criminally active and prisoner

In this dissertation, we aim to control the fractional-order system of the criminally active and prisoner. Therefore, we consider a chaotic model with fractional-order derivatives based on the stability theorem related to fractional-order systems. Because modeling a system with fractional derivatives can show the system behavior better than ordinary derivatives. To find the lowest fractional-order for the system to be in the chaotic region, we put:

$$\alpha \geq \frac{2}{\pi} min|\arg(\lambda_i)|, \tag{20}$$

where for parameters [0.4,0.9,0.1,0.5,0.6] the system order is considered as [1, 0.99, 0.99]. Because for these parameters and specified order, the relation (20) is in work. Based on specified order, we show the chaotic system related to the growth of criminally active and prisoner with the differential equation of fractional-order as follows.

$$\begin{gathered} \dot{x}(t) = 0.4y - 0.9xy + 0.1z, \\ D_t^{0.99} y(t) = -0.4y + 0.9xy - 0.5y + 0.6z, \\ D_t^{0.99} y(t) = 0.5y - (0.5 + 0.1)z, \end{gathered} \tag{21}$$

2.3. Optimal control of system of the criminally active and prisoner

It is necessary first to determine the purpose of the control for optimal control of the system of the criminally active and prisoner. Here, our desired aim is to reach zero criminally active and prisoner. It is necessary to define a standard mathematical function based on the specified goal. It is feasible to represent the function by the following relation,

$$j = \int_0^{t_f} (x^2 + u^2) dt. \tag{22}$$

The physical meaning of the suggested standard function is that by selecting the appropriate control input, the criminally active and prisoner reach zero. In other words, the main task of the control is to optimally find the control signal so that it minimizes the standard function specified in (36). Now, the crime fractional-order model is regarded by considering the control variable as the following relation,

$$\begin{gathered} \dot{x}(t) = 0.4y - 0.9xy + 0.1z - u, \\ D_t^{0.99} y(t) = -0.4y + 0.9xy - 0.5y + 0.6z - 0.25u, \\ D_t^{0.99} y(t) = 0.5y - (0.5 + 0.1)z - 0.25, \end{gathered} \tag{23}$$

The model of the system and the objective function are specified, determining an optimal control method solves the problem. In this article, we used the particle swarm optimization algorithm and genetic algorithm methods to solve the problem. We present the results of each method.

3. Computational Results

3.1. Without control

In uncontrolled mode, in Figs. 2 and 3, we obtained the following results for three-mode variables that are not desirable:

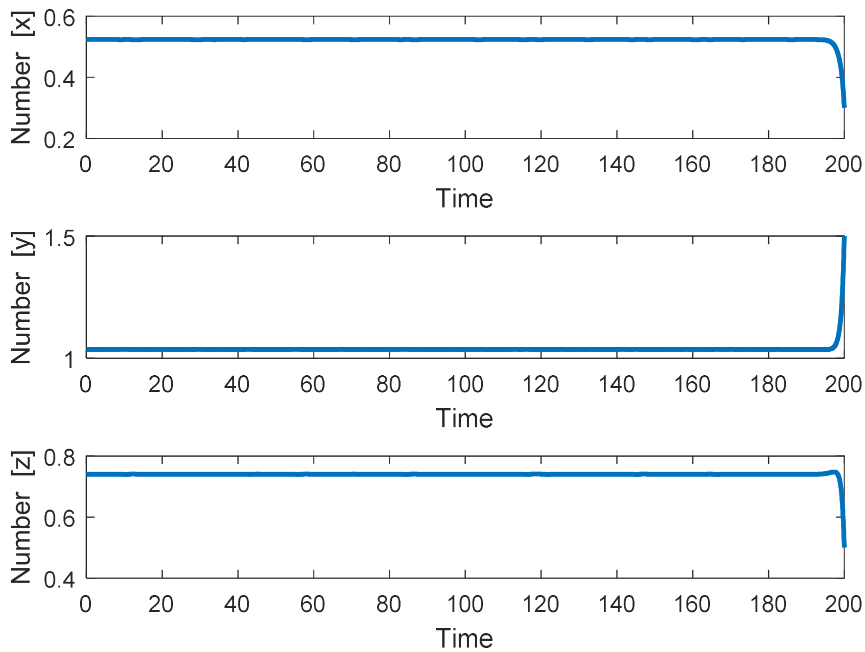

Fig 2.results for three-mode variables for while System order is considered as [1, 0.99, 0.99].

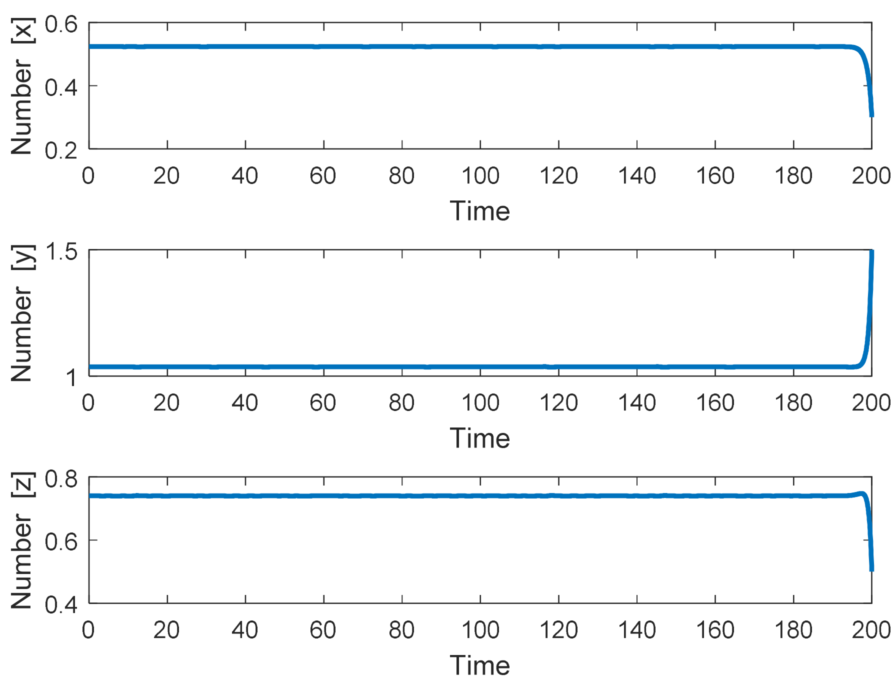

Fig 3.results for three-mode variables for while System order is considered as [1, 0.93, 0.93].

3.2. Results of genetic algorithm method

First, we consider the time of implementing the control input and obtain the following results. It is clear that the results are excellent as soon as the control input is applied (in Figs. 4 and 5, blue lines are for the uncontrolled method and red are for the controlled ones):

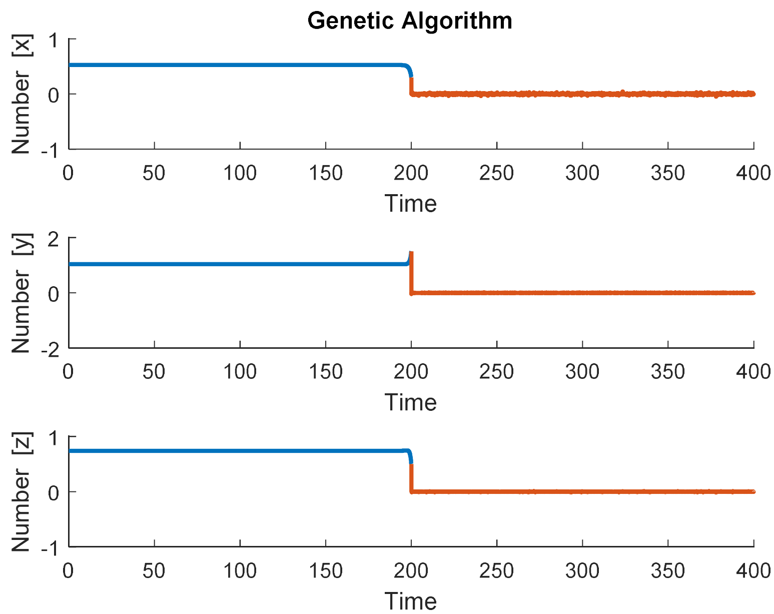

Fig 4.blue lines are for the uncontrolled method and red are for the controlled ones for while System order is considered as [1, 0.99, 0.99]

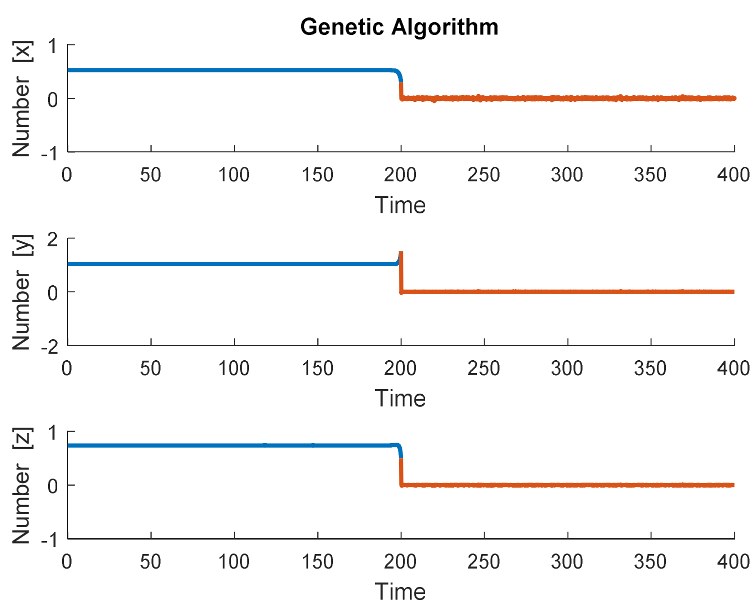

Fig 5.blue lines are for the uncontrolled method and red are for the controlled ones for while System order is considered as [1, 0.93, 0.93]

Again, in Figs. 6 and 7 we examine the results when the controller is in use from the beginning. It is easy to see that the answers are excellent from the start.

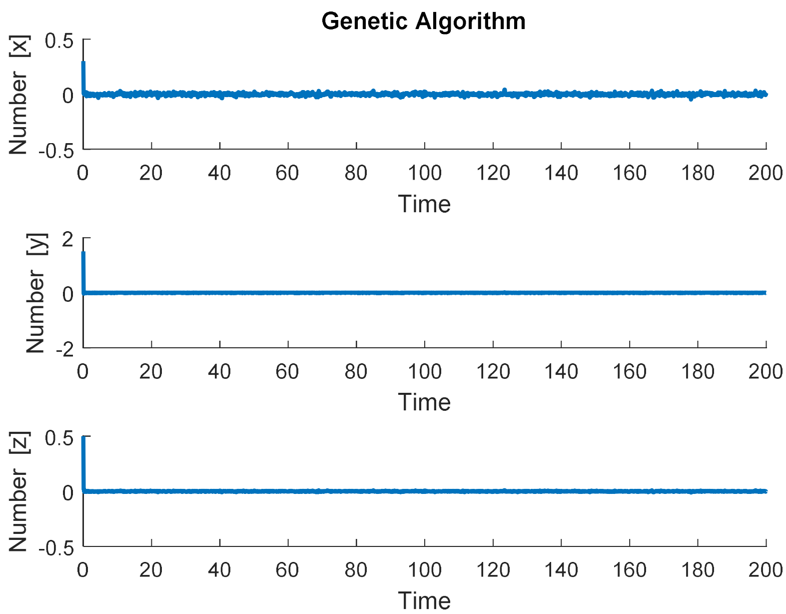

Fig 6.the results when the controller is in use from the beginning for while System order is considered as [1, 0.99, 0.99]

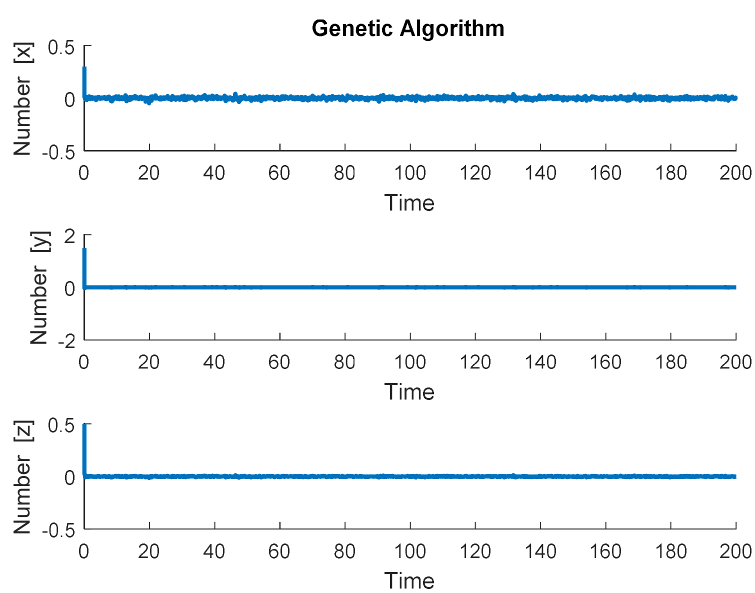

Fig 7.the results when the controller is in use from the beginning for while System order is considered as [1, 0.93, 0.93]

In Figs. 8 and 9, Changes in control input are as follows:

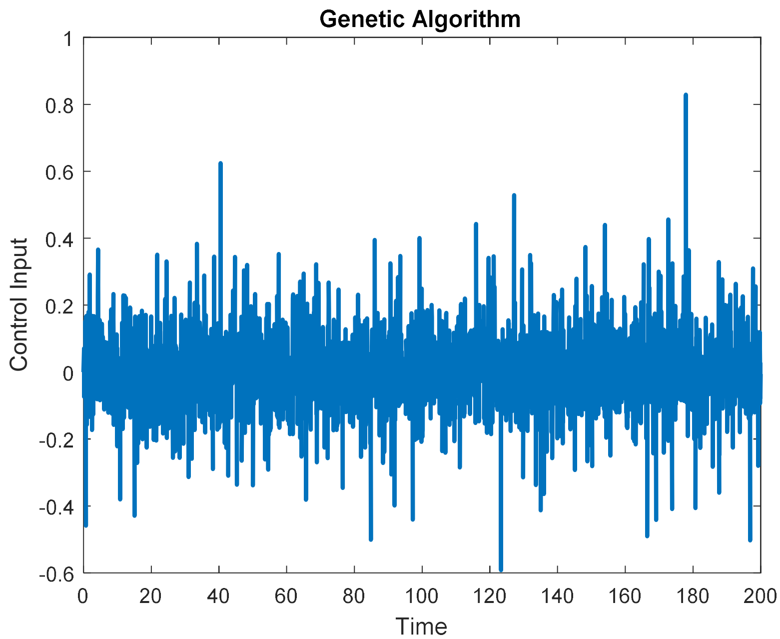

Fig 8. Changes in control input for while System order is considered as [1, 0.99, 0.99]

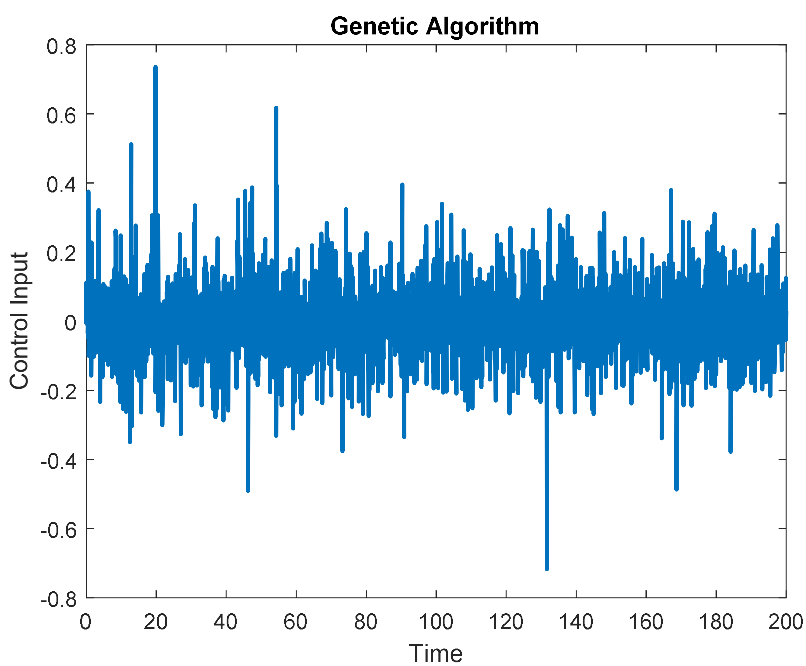

Fig 9. Changes in control input for while System order is considered as [1, 0.93, 0.93]

We saved an Excel file that contains the numeric values of the model variables and the control input (in full control mode). The following picture is only part of the Table 1.

Table 1.the numeric values of the model variables and the control input

Time	X	Y	Z	U	Time	X	Y	Z	U
0	0.3000	1.5000	0.5000	0	205.0000	0.0710	3.1731e-04	0.0077	-0.0130
5.0000	0.0015	4.4019e-04	4.2797e-04	0.0515	210.0000	0.0786	0.0044	0.0037	0.0010
10.0000	0.0106	0.0027	0.0027	0.0713	215.0000	0.0754	0.0022	0.0056	-0.1038
15.0000	0.0077	0.0018	0.0018	-0.0362	220.0000	0.0809	0.0052	0.0027	-0.0055
20.0000	0.0086	0.0023	0.0023	0.1486	225.0000	0.0780	0.0044	0.0037	-0.1177
25.0000	0.0102	0.0023	0.0023	-0.0070	230.0000	0.0685	0.0011	0.0091	0.0237
30.0000	0.0128	0.0032	0.0032	0.1127	235.0000	0.0669	0.0018	0.0098	0.0314
35.0000	0.0023	3.9178e-04	4.2137e-04	0.0579	240.0000	0.0556	0.0084	0.0155	0.0320
40.0000	0.0197	0.0050	0.0050	-0.1662	245.0000	0.0780	0.0041	0.0040	-0.1007
45.0000	4.0817e-04	3.2458e-04	2.9033e-04	-0.0042	250.0000	0.0702	2.4219e-04	0.0081	-0.0219
50.0000	0.0027	8.4078e-04	8.1092e-04	-0.0634	255.0000	0.0582	0.0065	0.0144	0.0909
55.0000	0.0038	9.5615e-04	9.5721e-04	-0.0095	260.0000	0.0740	0.0019	0.0061	0.0573
60.0000	0.0048	0.0011	0.0011	-0.0137	265.0000	0.0754	0.0030	0.0051	-0.0018
65.0000	0.0150	0.0040	0.0039	0.2948	270.0000	0.0797	0.0046	0.0033	-0.1748
70.0000	0.0018	4.8986e-04	4.8394e-04	-0.0250	275.0000	0.0675	0.0016	0.0096	-0.0161
75.0000	0.0011	5.4091e-04	4.9221e-04	-0.0671	280.0000	0.0757	0.0030	0.0051	-0.0029
80.0000	0.0014	3.0841e-04	3.1303e-04	-0.0102	285.0000	0.0657	0.0023	0.0103	0.2321
85.0000	0.0277	0.0065	0.0066	0.0304	290.0000	0.0758	0.0030	0.0051	-0.0983
90.0000	0.0047	0.0011	0.0011	0.0141	295.0000	0.0758	0.0028	0.0052	-0.0340
95.0000	0.0050	0.0011	0.0011	-0.0056	300.0000	0.0902	0.0101	0.0022	-0.0866
100.0000	0.0146	0.0038	0.0038	0.0733	305.0000	0.0775	0.0031	0.0044	-0.0820
105.0000	0.0066	0.0016	0.0016	0.0629	310.0000	0.0720	4.0637e-04	0.0073	0.0952
110.0000	0.0030	7.4476e-04	7.5225e-04	-0.0382	315.0000	0.0703	3.7212e-05	0.0080	0.0360
115.0000	0.0047	0.0011	0.0011	-0.0310	320.0000	0.0918	0.0117	0.0036	-0.0120
120.0000	0.0162	0.0042	0.0042	0.2840	325.0000	0.0702	4.8280e-04	0.0082	-0.0794
125.0000	0.0028	6.3692e-04	6.4951e-04	0.0260	330.0000	0.0740	0.0023	0.0059	0.2400
130.0000	0.0106	0.0025	0.0025	-0.0415	335.0000	0.0798	0.0050	0.0031	-0.0362
135.0000	0.0049	0.0012	0.0012	0.0094	340.0000	0.0687	0.0011	0.0090	0.0134
140.0000	0.0082	0.0020	0.0021	-0.0445	345.0000	0.0726	7.8538e-04	0.0070	0.0106
145.0000	0.0030	6.5503e-04	6.7603e-04	-0.0099	350.0000	0.0560	0.0076	0.0154	0.0377
150.0000	0.0019	4.5240e-04	4.6098e-04	-0.0074	355.0000	0.0687	8.0499e-04	0.0087	-0.0245
155.0000	0.0050	0.0012	0.0012	0.1417	360.0000	0.0719	0.0012	0.0069	0.0257
160.0000	0.0053	0.0011	0.0012	0.1741	365.0000	0.0751	0.0022	0.0057	-0.0518
165.0000	0.0145	0.0036	0.0037	-0.0931	370.0000	0.0807	0.0048	0.0027	0.0198
170.0000	0.0076	0.0017	0.0018	-0.0035	375.0000	0.0759	0.0030	0.0051	-0.0729
175.0000	5.8654e-04	3.4033e-04	3.0018e-04	-0.0959	380.0000	0.0721	9.1614e-04	0.0071	0.0225
180.0000	0.0028	6.3469e-04	6.4875e-04	-0.0284	385.0000	0.0718	8.8500e-04	0.0072	0.0124
185.0000	0.0021	5.4203e-04	5.3534e-04	-0.0130	390.0000	0.0727	0.0011	0.0069	-0.0095
190.0000	0.0064	0.0015	0.0015	0.0010	395.0000	0.0736	8.1218e-04	0.0065	0.0188

195.0000	0.0011	4.5470e-04	4.3061e-04	-0.1038	400.0000	0.0594	0.0060	0.0139	0.4018
200.0000	0.0075	0.0019	0.0019	-0.0055	-	-	-	-	-

In this control problem, the goal is to reduce the number of crime with its related costs , the variable z, to zero. For this reason, in Figs. 10 and 11 we draw a diagram for the approximation and error of the zero reference signals:

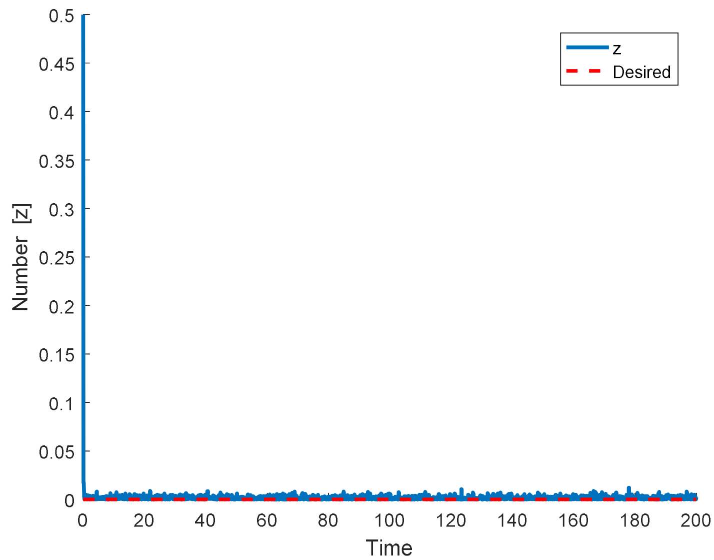

Fig 10.diagram for the approximation and error of the zero reference signal for while System order is considered as [1, 0.99, 0.99]

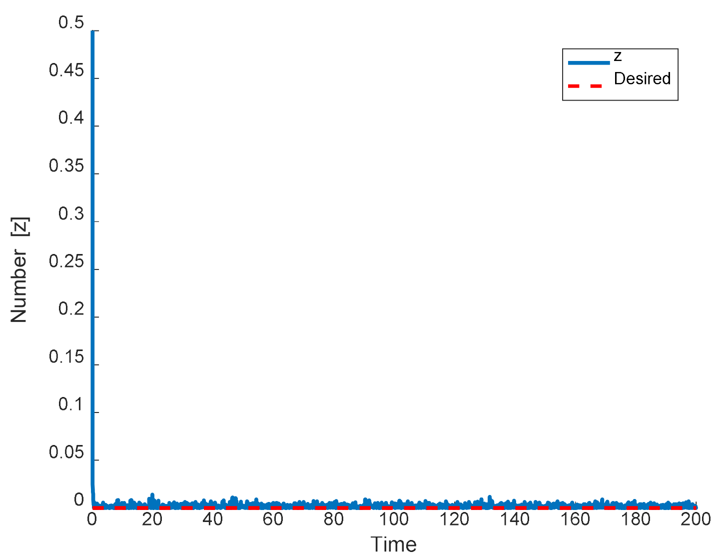

Fig 11.diagram for the approximation and error of the zero reference signal for while System order is considered as [1, 0.93, 0.93]

The MSE and RMSE specifications for error are on the table 2. We observe that their values are small. Consequently, the simulation is effective.

Table 2.The MSE and RMSE specifications for error

System order	MSE	RMSE
[1, 0.99, 0.99]	6.8194e-05	0.008258
[1, 0.93, 0.93]	6.9062e-05	0.0083104

3.3. Results of particle swarm optimization algorithm

We also repeated all the above steps for this method and observed that it is very successful. Moreover, in Figs 12 to 19, its results are very close to the genetic algorithm method.

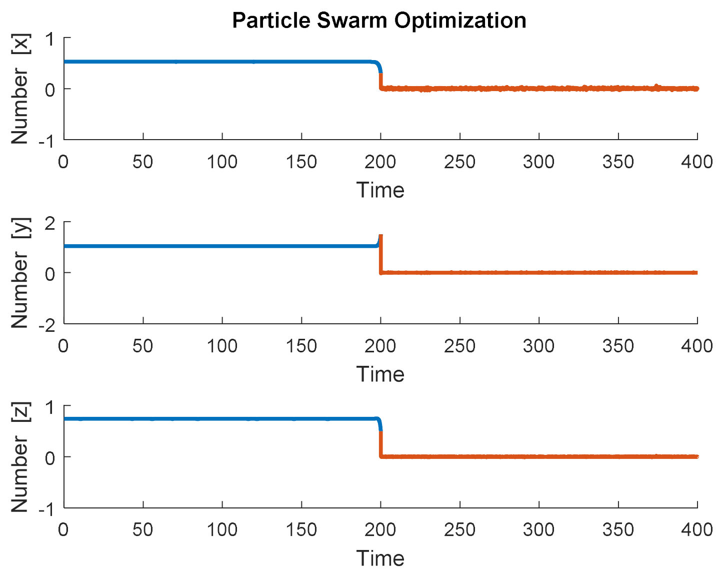

Fig 12.blue lines are for the uncontrolled method and red are for the controlled ones For while System order is considered as [1, 0.99, 0.99]

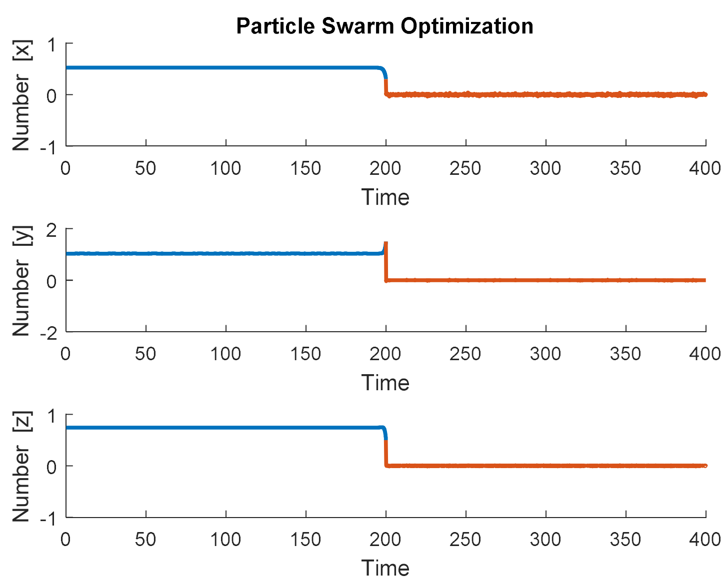

Fig 13.blue lines are for the uncontrolled method and red are for the controlled ones For while System order is considered as [1, 0.93, 0.93]

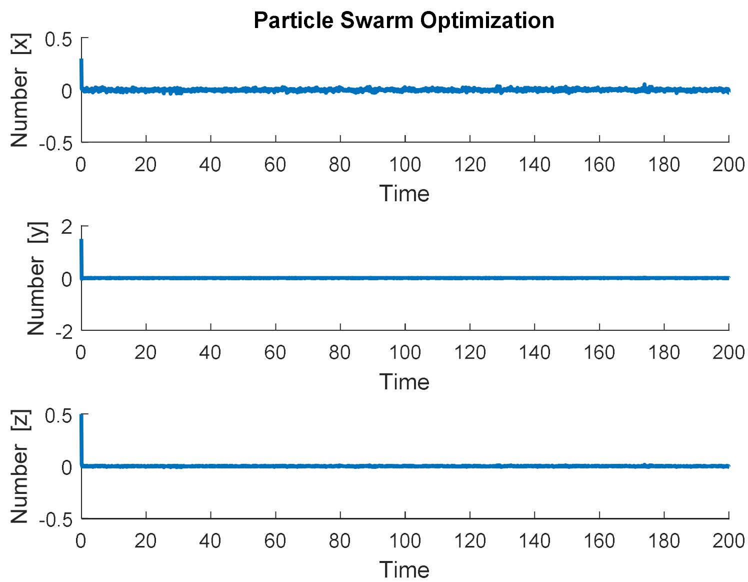

Fig14 .the results when the controller is in use from the beginning for while System order is considered as [1, 0.99, 0.99]

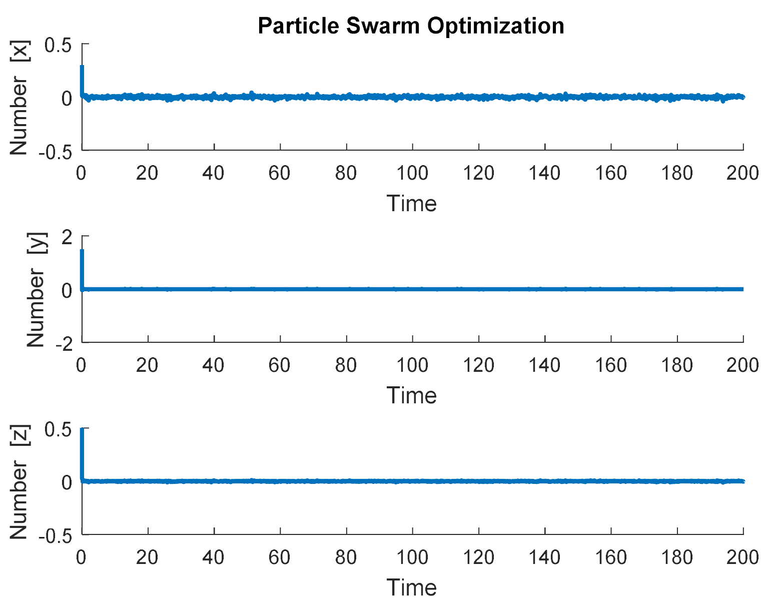

Fig 15.the results when the controller is in use from the beginning for while System order is considered as [1, 0.93, 0.93]

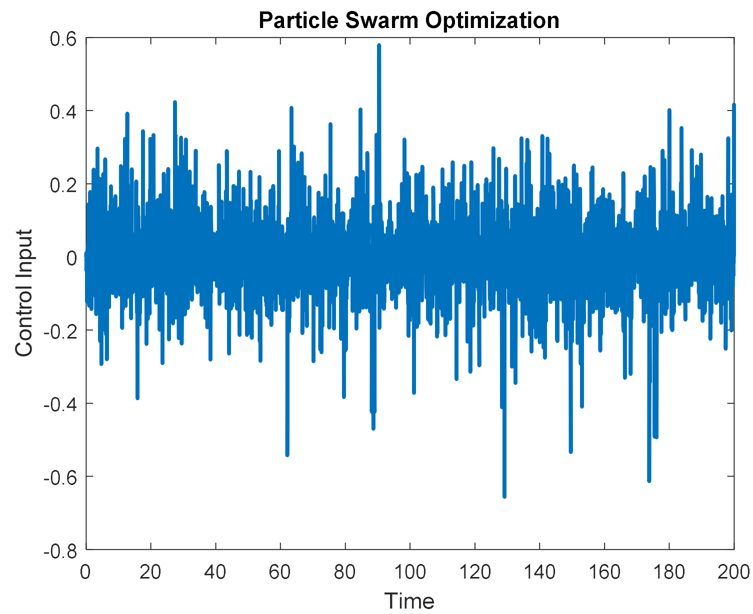

Fig 16. Changes in control input for while System order is considered as [1, 0.99, 0.99]

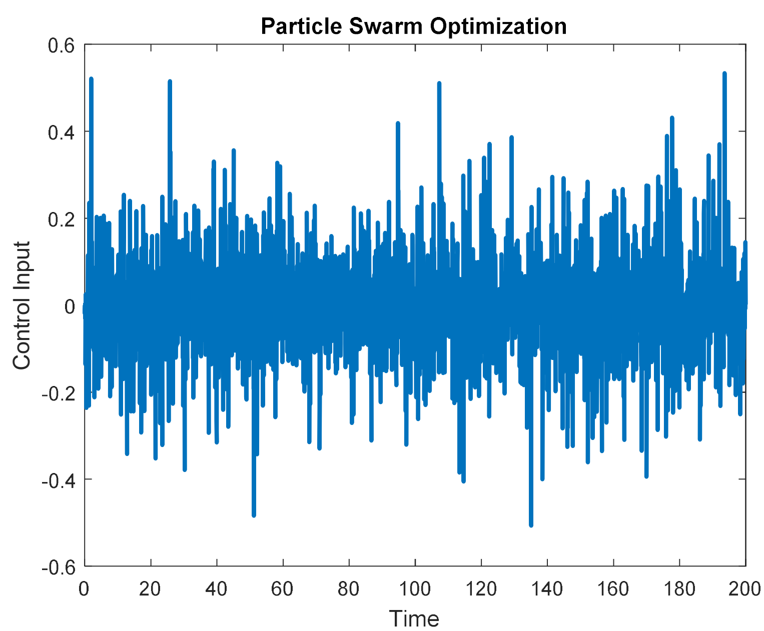

Fig 17. Changes in control input for while System order is considered as [1, 0.93, 0.93]

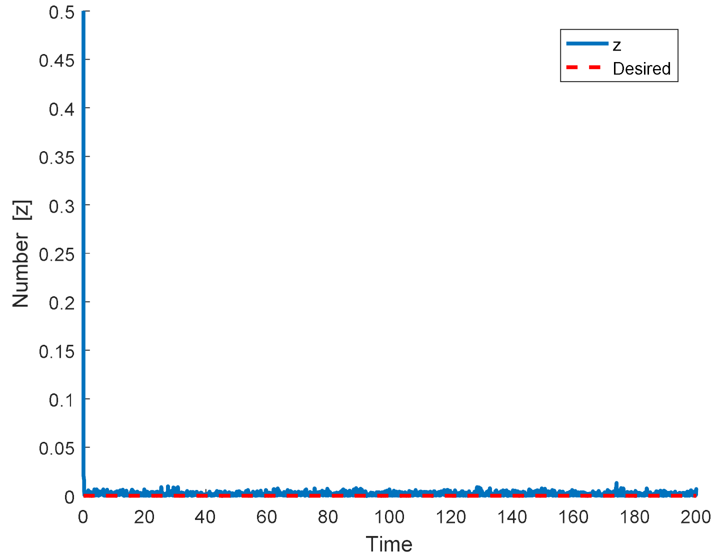

Fig 18.diagram for the approximation and error of the zero reference signals for while System order is considered as [1, 0.99, 0.99]

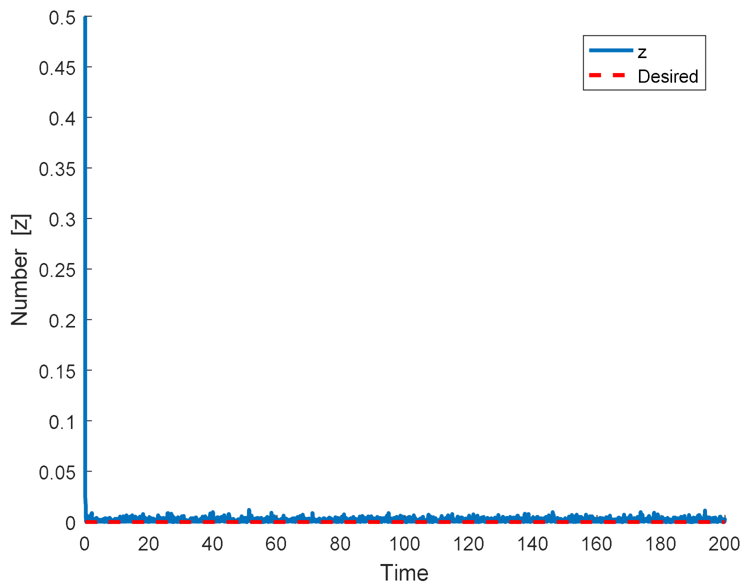

Fig 19. diagram for the approximation and error of the zero reference signals for while System order is considered as [1, 0.93, 0.93]

We observe that MSE and RMSE error values in table 3 are small. Consequently, the simulation is effective. The numeric values of the model variables and the control input (in full control mode). The following picture is only part of the Table 4.

Table 3.The MSE and RMSE specifications for error

System order	MSE	RMSE
[1, 0.99, 0.99]	6.8232e-05	0.0082603

[1, 0.93, 0.93]	6.8922e-05	0.0083019

Table 4.the numeric values of the model variables and the control input

Time	X	Y	Z	U	Time	X	Y	Z	U
0	0.3000	1.5000	0.5000	0	205.0000	0.2082	0.0026	0.0027	0.0997
5.0000	0.0045	0.0015	0.0015	-0.1646	210.0000	0.2266	0.0066	0.0068	0.0800
10.0000	0.0027	5.2272e-04	5.5937e-04	-0.0229	215.0000	0.2151	0.0016	9.8606e-05	-0.0859
15.0000	0.0028	4.1777e-04	4.6130e-04	0.0049	220.0000	0.2058	0.0039	0.0037	0.0023
20.0000	0.0106	0.0029	0.0029	-0.0112	225.0000	0.2233	0.0051	0.0051	-0.2572
25.0000	0.0044	9.2014e-04	9.7699e-04	-0.0336	230.0000	0.2108	0.0014	0.0011	-0.0029
30.0000	0.0033	7.1360e-04	7.6479e-04	-0.0165	235.0000	0.2232	0.0051	0.0050	-0.0966
35.0000	0.0013	5.8761e-04	5.6370e-04	-0.0839	240.0000	0.2110	0.0010	0.0015	8.6542e-05
40.0000	0.0332	0.0086	0.0087	-0.0287	245.0000	0.2202	0.0035	0.0034	0.0144
45.0000	0.0165	0.0037	0.0039	-0.0542	250.0000	0.2040	0.0045	0.0051	-0.0197
50.0000	0.0087	0.0027	0.0027	0.1749	255.0000	0.2248	0.0057	0.0059	0.0108
55.0000	0.0130	0.0038	0.0038	-0.1302	260.0000	0.2163	0.0016	0.0016	-0.2953
60.0000	0.0016	4.1929e-04	4.4063e-04	-0.0288	265.0000	0.2152	8.5603e-04	9.2926e-04	0.0086
65.0000	5.8052e-04	2.1394e-04	2.1224e-04	-0.0374	270.0000	0.2099	0.0017	0.0019	0.0017
70.0000	0.0066	0.0016	0.0016	-0.0353	275.0000	0.2131	1.2295e-05	3.8376e-04	-0.0197
75.0000	0.0045	0.0014	0.0014	0.0588	280.0000	0.2105	0.0017	9.5673e-04	-0.0253
80.0000	0.0105	0.0029	0.0029	-0.0073	285.0000	0.2181	0.0023	0.0024	-0.0423
85.0000	0.0024	7.7742e-04	7.6294e-04	0.0089	290.0000	0.2323	0.0093	0.0103	-0.0778
90.0000	8.3164e-04	1.8771e-04	2.0879e-04	-0.0935	295.0000	0.2075	0.0027	0.0035	0.1286
95.0000	0.0100	0.0021	0.0022	-0.0126	300.0000	0.2164	0.0013	0.0020	0.0057
100.0000	0.0028	7.2050e-04	7.3447e-04	-0.0400	305.0000	0.2157	0.0011	0.0013	-0.0361
105.0000	0.0091	0.0029	0.0029	0.1520	310.0000	0.2197	0.0032	0.0032	6.8514e-04
110.0000	0.0030	8.7406e-04	8.7128e-04	-0.0051	315.0000	0.2128	1.4426e-04	4.2155e-04	-0.0849
115.0000	0.0019	1.8370e-04	2.0725e-04	0.0761	320.0000	0.2313	0.0089	0.0095	0.0066
120.0000	0.0160	0.0050	0.0049	0.2533	325.0000	0.2132	1.8335e-04	7.4833e-05	-0.0221
125.0000	0.0077	0.0020	0.0020	0.0286	330.0000	0.2281	0.0075	0.0076	-0.0937
130.0000	0.0079	0.0022	0.0022	0.0413	335.0000	0.2150	7.7001e-04	8.2825e-04	5.8871e-04
135.0000	0.0065	0.0019	0.0019	0.0427	340.0000	0.2292	0.0077	0.0088	-0.0817
140.0000	0.0060	0.0016	0.0016	0.0067	345.0000	0.2156	0.0010	0.0012	0.0341
145.0000	0.0241	0.0066	0.0067	0.1505	350.0000	0.2072	0.0032	0.0032	0.0249
150.0000	0.0077	0.0021	0.0022	0.0567	355.0000	0.2108	9.9061e-04	0.0017	-0.0113
155.0000	0.0143	0.0040	0.0041	-0.1512	360.0000	0.2150	6.0774e-04	0.0010	0.0411
160.0000	0.0041	0.0013	0.0013	0.0955	365.0000	0.2127	4.0653e-04	2.5685e-04	-0.0253
165.0000	8.4085e-04	1.9388e-04	2.1074e-04	0.0530	370.0000	0.2109	0.0013	0.0014	0.0096
170.0000	0.0082	0.0028	0.0028	-0.3939	375.0000	0.2108	0.0014	0.0012	0.0531
175.0000	0.0039	0.0015	0.0014	-0.1568	380.0000	0.2254	0.0061	0.0062	-0.0086
180.0000	0.0062	0.0015	0.0016	0.0437	385.0000	0.2173	0.0021	0.0019	-0.0126
185.0000	3.9299e-04	4.9479e-04	4.6180e-04	0.1183	390.0000	0.2250	0.0064	0.0053	-0.0539

190.0000	3.0504e-04	1.5835e-05	7.4937e-06	-0.0585	395.0000	0.2208	0.0035	0.0043	0.1216
195.0000	0.0059	0.0013	0.0014	-0.0156	400.0000	0.2086	0.0023	0.0027	0.0278
200.0000	0.0073	0.0020	0.0020	0.0013	-	-	-	-	-

3.4. Lie symmetry for criminally Active and prisoner model

In this section, we deal with the general procedure of Lie symmetry analysis for determining the symmetries for any system of nonlinear partial differential equation. Let us consider a general nonlinear system of n-th order partial differential equations (PDEs) in p independent variables [29,36]:

$$\mathrm{X} = x_1, \dots, x_p \epsilon \Re^p, \tag{23}$$

and q dependent variables viz.

$$\mathrm{U} = u(u^1, u^2, \dots, u^q) \epsilon \Re^q. \tag{24}$$

in the follo1wing form:

$$\Delta_\sigma\left(X, U^{(n)}\right) = 0, \quad \sigma = 1,2, \dots, l. \tag{25}$$

Where $U^{(n)}$ represents all the derivatives of u of all orders from 0 to n. We now consider a one parameter Lie group infinitesimal transformations acting on the both the independent and dependent variables of the system (25), given as:

$$\bar{x}^i = x^i + \varepsilon \xi_i(X, U) + O(\varepsilon^2), \quad i = 1,2, \dots, p, \tag{26}$$

$$\bar{u}^i = u^i + \varepsilon \eta_j(X, U) + O(\varepsilon^2), \quad j = 1,2, \dots, p \tag{27}$$

where $\varepsilon \ll 1$ is a small parameter of the transformation and ξ_i, η_i are the infinitesimals of the transformations for the independent and dependent variables respectively. The infinitesimal generator V associated with the above group of transformations can be written as [30,37]:

$$V = \sum_{i=1}^{p} \xi_i(X, U)\, \partial_{x^i} + \sum_{i=1}^{q} \eta_j(X, U)\, \partial_{u^j}. \tag{28}$$

The invariance under the infinitesimal transformations leads to the invariance conditions, which is given as:

$$Pr^{(n)} V[\Delta_\sigma\left(X, U^{(n)}\right)] = 0,, \quad \sigma = 1,2, \dots, l, \tag{29}$$

Where $Pr^{(n)}$ is called the nth order prolongation of the infinitesimal generator, given by:

$$Pr^{(n)} V = V + \sum_{k=1}^{q} \sum_j \eta_{\ k}^{\ j}\left(X, U^{(n)}\right) \partial_{u_j^k}, \tag{30}$$

Where $J = j_1, \dots, j_s$, $1 \ll j_s \ll p$, $1 \ll s \ll n$ and the sum is over all J's of order $0 < \#J \leq n$. If $\#J = s$, the coefficient $\eta_{\ k}^{\ j}$ of $\partial_{u_j^k}$ will only depend on s-th and lower order derivatives of **u**, and we have[31]:

$$\eta_{\ k}^{\ j}\left(X, U^{(n)}\right) = D_J\left(\eta_k - \sum_{i=1}^{p} \xi_i u_j^k\right) + \sum_{i=1}^{p} \xi_i \frac{\partial}{\partial_{u_{j,i}^k}} u_j^k\ , \tag{31}$$

Where: $\quad u_j^k = \frac{\partial u^k}{\partial x^i}$ and $u_{j,i}^k = \frac{\partial u_j^k}{\partial x^i}$.

In this section, techniques of Lie group analysis for the equation with Riemann-Liouville derivative have been investigated. Symmetries of this equation are obtained. We take into consideration a one-parameter Lie group of infinitesimal transformation:

$$\begin{aligned} x &\rightarrow x + \epsilon \xi^x(x, t, u), \\ t &\rightarrow t + \epsilon \xi^t(x, t, u), \\ u &\rightarrow x + \epsilon \xi^u(x, t, u), \end{aligned} \tag{32}$$

With a small parameter $\epsilon \leq 1$. The vector field associated with the above group of transformations can be written as

$$X = \xi^x(x,t,u)\frac{\partial}{\partial x} + \xi^t(x,t,u)\frac{\partial}{\partial t} + \eta\,(x,t,u)\frac{\partial}{\partial u}, \tag{33}$$

and the vector field will generate the symmetry group of equation. Thus, the Lie algebra of symmetries is spanned by the following vector fields:

$$X_1 = \frac{\partial}{\partial t}, \tag{34}$$
$$X_2 = \frac{\partial}{\partial x},$$
$$X_3 = -t\frac{\partial}{\partial t} + (u+1)\frac{\partial}{\partial u},$$

The one-parameter groups G_i generated by the $X_i(i = 1,2)$ are as follows:

$$G_1: \quad (x,t,u) \to (x, t+\epsilon, u),$$
$$G_2: \quad (x,t,u) \to (x+\epsilon, t, u), \tag{35}$$
$$G_3: \quad (x,t,u) \to (x, -te^{\epsilon}, (u+1)e^{\epsilon}),$$

We observe that G_1 is a time translation, $G_2$is space translation, while the group G_3 genuinely local group of transformation.

3.6. Invariant Transformation

Consider the list of infinitesimals of a symmetry group.

$$S := [\_\xi_x = 0, \_\xi_t = 1, \_\eta_u = 0] \quad (36)$$

In the input above we can also obtain infinitesimal generator:

$$G := f \to \frac{\partial}{\partial t} f \tag{37}$$

he transformation and its inverse, from the original variables {t, x, u(x , t)} to new coordinates, say {r, s, v(r, s)}, that reduces by one the number of independent variables of a Eq.(1) invariant under G above is obtained via:

$$\{r = x, v(r, s) = u(x, t)\}, \{x = r, u(x, t) = v(r, s)\} \tag{38}$$

Invariant Transformation(S, u(x, t), v(r, s), and jet notation) is:

$$\{r = x, v = u\}, \{u = v, x = r\} \quad (39)$$

4. Discussion and Conclusion

Chaotic behavior occurs in many engineering and natural systems. Although traditionally regarded as being irregular or unpredictable in nature and caused by random external influences, recent studies have shown that chaotic behavior is actually deterministic and is a typical characteristic of nonlinear systems. Chaos is undesirable in many engineering applications since it degrades the system performance and restricts the operating range of dynamic systems. Therefore, the problem of developing effective chaos control strategies has attracted significant interest over the years [40]. In this study, the model of the chaotic criminally active and prisoner system of fractional order was presented. Then the optimal control of this system was done by genetic algorithm and Particle swarm optimization algorithm. Also, Lie symmetry was presented for the proposed model. At the end, the optimal control of the proposed model for different states of data parameters was presented. All the results obtained for the particle swarm optimization method and genetic algorithm show that this methods has been very successful In addition we observe that error values are small. In the end simulation is effective.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work report in this paper.

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