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Solutions of Diophantine equations in ordered fields

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Abstract. For each integer $n \geqslant 2$, solutions of the Diophantine equations

$$x_1^3 + x_2^3 + \cdots + x_{2n-1}^3 + x_{2n}^3 = y_1^3 + y_2^3 + \cdots + y_{2n-1}^3 + y_{2n}^3$$

and

$$x_1^4 + x_2^4 + \cdots + x_{2n-1}^4 + x_{2n}^4 = y_1^4 + y_2^4 + \cdots + y_{2n-1}^4 + y_{2n}^4$$

in an arbitrary ordered field are exhibited.

Keywords: Diophantine equations, ordered fields, computer algebra system.

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1. Introduction

In [3] two authors of the present paper have been inspired by a method due to Swinnerton-Dyer [4] (see also [2, p. 201]) to obtain rational solutions of the Diophantine equations

$$x^3 + y^3 + z^3 + r^3 = u^3 + v^3 + t^3 + s^3,$$
$$x^3 + y^3 + z^3 + r^3 + o^3 + q^3 = u^3 + v^3 + t^3 + s^3 + n^3 + p^3,$$

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$$x^4 + y^4 + z^4 + r^4 = u^4 + v^4 + t^4 + s^4,$$
$$x^4 + y^4 + z^4 + r^4 + o^4 + q^4 = u^4 + v^4 + t^4 + s^4 + n^4 + p^4.$$

In this work, we employ the same argument to get solutions of the Diophantine equations

$$x_1^3 + x_2^3 + \cdots + x_{2n-1}^3 + x_{2n}^3 = y_1^3 + y_2^3 + \cdots + y_{2n-1}^3 + y_{2n}^3$$

and

$$x_1^4 + x_2^4 + \cdots + x_{2n-1}^4 + x_{2n}^4 = y_1^4 + y_2^4 + \cdots + y_{2n-1}^4 + y_{2n}^4$$

in an ordered field, where n is an integer $\geqslant 2$. Additionally, by using `SymPy`, a `Python` library for symbolic mathematics (a computer algebra system), the aforementioned solutions are explicitly furnished in the case $n = 8$.

Throughout this work $\mathbb{K}$ will denote an arbitrary ordered field, which has necessarily characteristic zero [1, p. 32], that is, the mapping $n \in \mathbb{Z} \mapsto n \cdot 1 \in \mathbb{K}$ $(1 \in \mathbb{K})$ is a ring isomorphism from $\mathbb{Z}$ onto the subring $\mathbb{L} = \{n \cdot 1;\ n \in \mathbb{Z}\}$ of $\mathbb{K}$. For this reason, we shall write n in place of $n \cdot 1$. We shall also write $\mathbb{K}^* = \mathbb{K} \setminus \{0\}$.

2. Solutions of a cubic Diophantine equation

Regarding the equation

$$x_1^3 + x_2^3 + \cdots + x_{2n-1}^3 + x_{2n}^3 = y_1^3 + y_2^3 + \cdots + y_{2n-1}^3 + y_{2n}^3,$$

we shall establish the following

Theorem 2.1 Let n be an arbitrary integer $\geqslant 2$. Given non-zero elements $a_1, \ldots, a_n$, $b_1, \ldots, b_n$ of $\mathbb{K}$ such that $a_i > b_i$ for $i = 1, \ldots, n$, let us define, for each $k = 1, \ldots, n$,

$$\begin{aligned} x_{2k-1} &= a_k w + c_{2k-1}, \\ x_{2k} &= b_k w - c_{2k}, \\ y_{2k-1} &= a_k w + c_{2k}, \\ y_{2k} &= b_k w + c_{2k-1}, \end{aligned} \tag{1}$$

where $c_{2k-1} = \lambda(a_k^2 + b_k^2)$, $c_{2k} = \lambda(a_k^2 - b_k^2)$,

$$w = (a_1^2 - b_1^2)^3 + (a_2^2 - b_2^2)^3 + \cdots + (a_n^2 - b_n^2)^3,$$

and

$$\lambda = 6\big(a_1^2\, b_1^2\, (a_1 - b_1) + a_2^2\, b_2^2\, (a_2 - b_2) + \cdots + a_n^2\, b_n^2\, (a_n - b_n)\big). \tag{2}$$

Then $x_1, \ldots, x_{2n}, y_1, \ldots, y_{2n}$, defined in (1), is a solution in $\mathbb{K}$ of the Diophantine equation

$$x_1^3 + x_2^3 + \cdots + x_{2n-1}^3 + x_{2n}^3 = y_1^3 + y_2^3 + \cdots + y_{2n-1}^3 + y_{2n}^3. \tag{3}$$

Proof. We shall argue as in [3], where the cases $n = 2$ and $n = 3$ have been considered when $\mathbb{K} = \mathbb{Q}$. Indeed, for $w \in \mathbb{K}$, let us write

$$\begin{aligned}
&x_1 = a_1 w + c_1, \quad x_2 = b_1 w - c_2, \quad x_3 = a_2 w + c_3, \quad x_4 = b_2 w - c_4, \\
&\quad \ldots, \quad x_{2n-1} = a_n w + c_{2n-1}, \qquad x_{2n} = b_n w - c_{2n}, \\
&y_1 = a_1 w + c_2, \quad y_2 = b_1 w + c_1, \quad y_3 = a_2 w + c_4, \quad y_4 = b_2 w + c_3, \\
&\quad \ldots, \quad y_{2n-1} = a_n w + c_{2n}, \quad y_{2n} = b_n w + c_{2n-1},
\end{aligned}$$

where $c_1, \ldots, c_{2n}$ are elements of $\mathbb{K}$. Substituting the above-mentioned elements $x_1, \ldots, x_{2n}, y_1, \ldots, y_{2n}$ of $\mathbb{K}$ in the equation under consideration, we obtain a cubic equation in w in which the coefficient of w^3 is 0 and the coefficient of w^2 is

$$\begin{aligned}
3\Big(&a_1^2(c_1 - c_2) + a_2^2(c_3 - c_4) + \cdots + a_n^2(c_{2n-1} - c_{2n}) \\
&- b_1^2(c_1 + c_2) - b_2^2(c_3 + c_4) - \cdots - b_n^2(c_{2n-1} + c_{2n})\Big).
\end{aligned}$$

Thus, by taking

$$\begin{aligned}
c_1 &= a_1^2 + b_1^2, \\
c_2 &= a_1^2 - b_1^2, \\
c_3 &= a_2^2 + b_2^2, \\
c_4 &= a_2^2 - b_2^2, \\
&\vdots \\
c_{2n-1} &= a_n^2 + b_n^2, \\
c_{2n} &= a_n^2 - b_n^2,
\end{aligned}$$

the coefficient of w^2 will also be 0 and we arrive at the following linear equation in w:

$$\begin{aligned}
&3((c_1^2 - c_2^2)(a_1 - b_1) + (c_3^2 - c_4^2)(a_2 - b_2) + \cdots + (c_{2n-1}^2 - c_{2n}^2)(a_n - b_n))\, w \\
&= 2(c_2^3 + c_4^3 + \cdots + c_{2n}^3).
\end{aligned}$$

Moreover since, for each $k = 1, \ldots, n$,

$$c_{2k-1}^2 - c_{2k}^2 = (c_{2k-1} - c_{2k})(c_{2k-1} + c_{2k}) = 4a_k^2 b_k^2,$$

then

$$\begin{aligned}
&12(a_1^2 b_1^2(a_1 - b_1) + a_2^2 b_2^2(a_2 - b_2) + \cdots + a_n^2 b_n^2(a_n - b_n))w \\
&= 2((a_1^2 - b_1^2)^3 + (a_2^2 - b_2^2)^3 + \cdots + (a_n^2 - b_n^2)^3),
\end{aligned}$$

that is,

$$w = \frac{(a_1^2 - b_1^2)^3 + (a_2^2 - b_2^2)^3 + \cdots + (a_n^2 - b_n^2)^3}{\lambda}$$

(by hypothesis λ, defined in (2), is greater than 0). Finally, multiplying w and each c_i $(i = 1, \ldots, 2n)$ by the element $\lambda \in \mathbb{K}$, we obtain elements of $\mathbb{K}$ that we will still denote by w and c_i $(i = 1, \ldots, 2n)$ which, when substituted in (1), furnish the solution $x_1, \ldots, x_{2n}$, $y_1, \ldots, y_{2n}$ in $\mathbb{K}$ of the Diophantine equation (3). This completes the proof. ■

Remark 1 *Under the conditions of Theorem 2.1, $x_1, \ldots, x_{2n}, y_1, \ldots, y_{2n}$ belong to $\mathbb{L}$ if $a_1, \ldots, a_n, b_1, \ldots, b_n$ belong to $\mathbb{L}$.*

In view of Theorem 2.1, one can use `SymPy` to guarantee the validity of the following (see code at the appendix)

Example 2.2 If $a_i > b_i$ $(i = 1, \ldots, 8)$ are non-zero elements of $\mathbb{K}$,

$$\begin{aligned} x_1 = {} & a_1^7 + 3a_1^5b_1^2 - 6a_1^4b_1^3 + 9a_1^3b_1^4 + 6a_1^2a_2^3b_2^2 - 6a_1^2a_2^2b_2^3 + 6a_1^2a_3^3b_3^2 - 6a_1^2a_3^2b_3^3 + 6a_1^2a_4^3b_4^2 \\ & - 6a_1^2a_4^2b_4^3 + 6a_1^2a_5^3b_5^2 - 6a_1^2a_5^2b_5^3 + 6a_1^2a_6^3b_6^2 - 6a_1^2a_6^2b_6^3 + 6a_1^2a_7^3b_7^2 - 6a_1^2a_7^2b_7^3 \\ & + 6a_1^2a_8^3b_8^2 - 6a_1^2a_8^2b_8^3 - 6a_1^2b_1^5 + a_1a_2^6 - 3a_1a_2^4b_2^2 + 3a_1a_2^2b_2^4 + a_1a_3^6 - 3a_1a_3^4b_3^2 + 3a_1a_3^2b_3^4 + a_1a_4^6 \\ & - 3a_1a_4^4b_4^2 + 3a_1a_4^2b_4^4 + a_1a_5^6 - 3a_1a_5^4b_5^2 + 3a_1a_5^2b_5^4 + a_1a_6^6 - 3a_1a_6^4b_6^2 + 3a_1a_6^2b_6^4 + a_1a_7^6 - 3a_1a_7^4b_7^2 \\ & + 3a_1a_7^2b_7^4 + a_1a_8^6 - 3a_1a_8^4b_8^2 + 3a_1a_8^2b_8^4 - a_1b_1^6 - a_1b_2^6 - a_1b_3^6 - a_1b_4^6 - a_1b_5^6 - a_1b_6^6 - a_1b_7^6 - a_1b_8^6 \\ & + 6a_2^3b_1^2b_2^2 - 6a_2^2b_1^2b_2^3 + 6a_3^3b_1^2b_3^2 - 6a_3^2b_1^2b_3^3 + 6a_4^3b_1^2b_4^2 - 6a_4^2b_1^2b_4^3 + 6a_5^3b_1^2b_5^2 \\ & - 6a_5^2b_1^2b_5^3 + 6a_6^3b_1^2b_6^2 - 6a_6^2b_1^2b_6^3 + 6a_7^3b_1^2b_7^2 - 6a_7^2b_1^2b_7^3 + 6a_8^3b_1^2b_8^2 - 6a_8^2b_1^2b_8^3 \end{aligned}$$

$$\begin{aligned} x_2 = {} & a_1^6b_1 - 6a_1^5b_1^2 + 3a_1^4b_1^3 + 6a_1^3b_1^4 - 6a_1^2a_2^3b_2^2 + 6a_1^2a_2^2b_2^3 - 6a_1^2a_3^3b_3^2 + 6a_1^2a_3^2b_3^3 - 6a_1^2a_4^3b_4^2 \\ & + 6a_1^2a_4^2b_4^3 - 6a_1^2a_5^3b_5^2 + 6a_1^2a_5^2b_5^3 - 6a_1^2a_6^3b_6^2 + 6a_1^2a_6^2b_6^3 - 6a_1^2a_7^3b_7^2 + 6a_1^2a_7^2b_7^3 \\ & - 6a_1^2a_8^3b_8^2 + 6a_1^2a_8^2b_8^3 - 3a_1^2b_1^5 + a_2^6b_1 - 3a_2^4b_1b_2^2 + 6a_2^3b_1^2b_2^2 - 6a_2^2b_1^2b_2^3 + 3a_2^2b_1b_2^4 + a_3^6b_1 \\ & - 3a_3^4b_1b_3^2 + 6a_3^3b_1^2b_3^2 - 6a_3^2b_1^2b_3^3 + 3a_3^2b_1b_3^4 + a_4^6b_1 - 3a_4^4b_1b_4^2 + 6a_4^3b_1^2b_4^2 - 6a_4^2b_1^2b_4^3 \\ & + 3a_4^2b_1b_4^4 + a_5^6b_1 - 3a_5^4b_1b_5^2 + 6a_5^3b_1^2b_5^2 - 6a_5^2b_1^2b_5^3 + 3a_5^2b_1b_5^4 + a_6^6b_1 - 3a_6^4b_1b_6^2 + 6a_6^3b_1^2b_6^2 \\ & - 6a_6^2b_1^2b_6^3 + 3a_6^2b_1b_6^4 + a_7^6b_1 - 3a_7^4b_1b_7^2 + 6a_7^3b_1^2b_7^2 - 6a_7^2b_1^2b_7^3 + 3a_7^2b_1b_7^4 + a_8^6b_1 - 3a_8^4b_1b_8^2 \\ & + 6a_8^3b_1^2b_8^2 - 6a_8^2b_1^2b_8^3 + 3a_8^2b_1b_8^4 - b_1^7 - b_1b_2^6 - b_1b_3^6 - b_1b_4^6 - b_1b_5^6 - b_1b_6^6 - b_1b_7^6 - b_1b_8^6 \end{aligned}$$

$$\begin{aligned} x_3 = {} & a_1^6a_2 - 3a_1^4a_2b_1^2 + 6a_1^3a_2^2b_1^2 + 6a_1^3b_1^2b_2^2 - 6a_1^2a_2^2b_1^3 + 3a_1^2a_2b_1^4 - 6a_1^2b_1^3b_2^2 + a_2^7 + 3a_2^5b_2^2 \\ & - 6a_2^4b_2^3 + 9a_2^3b_2^4 + 6a_2^2a_3^3b_3^2 - 6a_2^2a_3^2b_3^3 + 6a_2^2a_4^3b_4^2 - 6a_2^2a_4^2b_4^3 + 6a_2^2a_5^3b_5^2 - 6a_2^2a_5^2b_5^3 \\ & + 6a_2^2a_6^3b_6^2 - 6a_2^2a_6^2b_6^3 + 6a_2^2a_7^3b_7^2 - 6a_2^2a_7^2b_7^3 + 6a_2^2a_8^3b_8^2 - 6a_2^2a_8^2b_8^3 - 6a_2^2b_2^5 + a_2a_3^6 \\ & - 3a_2a_3^4b_3^2 + 3a_2a_3^2b_3^4 + a_2a_4^6 - 3a_2a_4^4b_4^2 + 3a_2a_4^2b_4^4 + a_2a_5^6 - 3a_2a_5^4b_5^2 + 3a_2a_5^2b_5^4 + a_2a_6^6 - 3a_2a_6^4b_6^2 \\ & + 3a_2a_6^2b_6^4 + a_2a_7^6 - 3a_2a_7^4b_7^2 + 3a_2a_7^2b_7^4 + a_2a_8^6 - 3a_2a_8^4b_8^2 + 3a_2a_8^2b_8^4 - a_2b_1^6 - a_2b_2^6 - a_2b_3^6 - a_2b_4^6 \\ & - a_2b_5^6 - a_2b_6^6 - a_2b_7^6 - a_2b_8^6 + 6a_3^3b_2^2b_3^2 - 6a_3^2b_2^2b_3^3 + 6a_4^3b_2^2b_4^2 - 6a_4^2b_2^2b_4^3 + 6a_5^3b_2^2b_5^2 \\ & - 6a_5^2b_2^2b_5^3 + 6a_6^3b_2^2b_6^2 - 6a_6^2b_2^2b_6^3 + 6a_7^3b_2^2b_7^2 - 6a_7^2b_2^2b_7^3 + 6a_8^3b_2^2b_8^2 - 6a_8^2b_2^2b_8^3 \end{aligned}$$

$$\begin{aligned} x_4 = {} & a_1^6b_2 - 3a_1^4b_1^2b_2 - 6a_1^3a_2^2b_1^2 + 6a_1^3b_1^2b_2^2 + 6a_1^2a_2^2b_1^3 + 3a_1^2b_1^4b_2 - 6a_1^2b_1^3b_2^2 + a_2^6b_2 - 6a_2^5b_2^2 \\ & + 3a_2^4b_2^3 + 6a_2^3b_2^4 - 6a_2^2a_3^3b_3^2 + 6a_2^2a_3^2b_3^3 - 6a_2^2a_4^3b_4^2 + 6a_2^2a_4^2b_4^3 - 6a_2^2a_5^3b_5^2 + 6a_2^2a_5^2b_5^3 \\ & - 6a_2^2a_6^3b_6^2 + 6a_2^2a_6^2b_6^3 - 6a_2^2a_7^3b_7^2 + 6a_2^2a_7^2b_7^3 - 6a_2^2a_8^3b_8^2 + 6a_2^2a_8^2b_8^3 - 3a_2^2b_2^5 + a_3^6b_2 \\ & - 3a_3^4b_2b_3^2 + 6a_3^3b_2^2b_3^2 - 6a_3^2b_2^2b_3^3 + 3a_3^2b_2b_3^4 + a_4^6b_2 - 3a_4^4b_2b_4^2 + 6a_4^3b_2^2b_4^2 - 6a_4^2b_2^2b_4^3 \\ & + 3a_4^2b_2b_4^4 + a_5^6b_2 - 3a_5^4b_2b_5^2 + 6a_5^3b_2^2b_5^2 - 6a_5^2b_2^2b_5^3 + 3a_5^2b_2b_5^4 + a_6^6b_2 - 3a_6^4b_2b_6^2 + 6a_6^3b_2^2b_6^2 \\ & - 6a_6^2b_2^2b_6^3 + 3a_6^2b_2b_6^4 + a_7^6b_2 - 3a_7^4b_2b_7^2 + 6a_7^3b_2^2b_7^2 - 6a_7^2b_2^2b_7^3 + 3a_7^2b_2b_7^4 + a_8^6b_2 - 3a_8^4b_2b_8^2 \\ & + 6a_8^3b_2^2b_8^2 - 6a_8^2b_2^2b_8^3 + 3a_8^2b_2b_8^4 - b_1^6b_2 - b_2^7 - b_2b_3^6 - b_2b_4^6 - b_2b_5^6 - b_2b_6^6 - b_2b_7^6 - b_2b_8^6 \end{aligned}$$

$$\begin{aligned}
x_5 = {} & a_1^6a_3 - 3a_1^4a_3b_1^2 + 6a_1^3a_3^2b_1^2 + 6a_1^3b_1^2b_3^2 - 6a_1^2a_3^2b_1^3 + 3a_1^2a_3b_1^4 - 6a_1^2b_1^3b_3^2 + a_2^6a_3 - 3a_2^4a_3b_2^2 \\
& + 6a_2^3a_3^2b_2^2 + 6a_2^3b_2^2b_3^2 - 6a_2^2a_3^2b_2^3 + 3a_2^2a_3b_2^4 - 6a_2^2b_2^3b_3^2 + a_3^7 + 3a_3^5b_3^2 - 6a_3^4b_3^3 + 9a_3^3b_3^4 \\
& + 6a_3^2a_4^3b_4^2 - 6a_3^2a_4^2b_4^3 + 6a_3^2a_5^3b_5^2 - 6a_3^2a_5^2b_5^3 + 6a_3^2a_6^3b_6^2 - 6a_3^2a_6^2b_6^3 + 6a_3^2a_7^3b_7^2 \\
& - 6a_3^2a_7^2b_7^3 + 6a_3^2a_8^3b_8^2 - 6a_3^2a_8^2b_8^3 - 6a_3^2b_3^5 + a_3a_4^6 - 3a_3a_4^4b_4^2 + 3a_3a_4^2b_4^4 + a_3a_5^6 - 3a_3a_5^4b_5^2 \\
& + 3a_3a_5^2b_5^4 + a_3a_6^6 - 3a_3a_6^4b_6^2 + 3a_3a_6^2b_6^4 + a_3a_7^6 - 3a_3a_7^4b_7^2 + 3a_3a_7^2b_7^4 + a_3a_8^6 - 3a_3a_8^4b_8^2 + 3a_3a_8^2b_8^4 \\
& - a_3b_1^6 - a_3b_2^6 - a_3b_3^6 - a_3b_4^6 - a_3b_5^6 - a_3b_6^6 - a_3b_7^6 - a_3b_8^6 + 6a_4^3b_3^2b_4^2 - 6a_4^2b_3^2b_4^3 + 6a_5^3b_3^2b_5^2 \\
& - 6a_5^2b_3^2b_5^3 + 6a_6^3b_3^2b_6^2 - 6a_6^2b_3^2b_6^3 + 6a_7^3b_3^2b_7^2 - 6a_7^2b_3^2b_7^3 + 6a_8^3b_3^2b_8^2 - 6a_8^2b_3^2b_8^3
\end{aligned}$$

$$\begin{aligned}
x_6 = {} & a_1^6b_3 - 3a_1^4b_1^2b_3 - 6a_1^3a_3^2b_1^2 + 6a_1^3b_1^2b_3^2 + 6a_1^2a_3^2b_1^3 + 3a_1^2b_1^4b_3 - 6a_1^2b_1^3b_3^2 + a_2^6b_3 - 3a_2^4b_2^2b_3 \\
& - 6a_2^3a_3^2b_2^2 + 6a_2^3b_2^2b_3^2 + 6a_2^2a_3^2b_2^3 + 3a_2^2b_2^4b_3 - 6a_2^2b_2^3b_3^2 + a_3^6b_3 - 6a_3^5b_3^2 + 3a_3^4b_3^3 + 6a_3^3b_3^4 \\
& - 6a_3^2a_4^3b_4^2 + 6a_3^2a_4^2b_4^3 - 6a_3^2a_5^3b_5^2 + 6a_3^2a_5^2b_5^3 - 6a_3^2a_6^3b_6^2 + 6a_3^2a_6^2b_6^3 - 6a_3^2a_7^3b_7^2 \\
& + 6a_3^2a_7^2b_7^3 - 6a_3^2a_8^3b_8^2 + 6a_3^2a_8^2b_8^3 - 3a_3^2b_3^5 + a_4^6b_3 - 3a_4^4b_3b_4^2 + 6a_4^3b_3^2b_4^2 - 6a_4^2b_3^2b_4^3 \\
& + 3a_4^2b_3b_4^4 + a_5^6b_3 - 3a_5^4b_3b_5^2 + 6a_5^3b_3^2b_5^2 - 6a_5^2b_3^2b_5^3 + 3a_5^2b_3b_5^4 + a_6^6b_3 - 3a_6^4b_3b_6^2 + 6a_6^3b_3^2b_6^2 \\
& - 6a_6^2b_3^2b_6^3 + 3a_6^2b_3b_6^4 + a_7^6b_3 - 3a_7^4b_3b_7^2 + 6a_7^3b_3^2b_7^2 - 6a_7^2b_3^2b_7^3 + 3a_7^2b_3b_7^4 + a_8^6b_3 - 3a_8^4b_3b_8^2 \\
& + 6a_8^3b_3^2b_8^2 - 6a_8^2b_3^2b_8^3 + 3a_8^2b_3b_8^4 - b_1^6b_3 - b_2^6b_3 - b_3^7 - b_3b_4^6 - b_3b_5^6 - b_3b_6^6 - b_3b_7^6 - b_3b_8^6
\end{aligned}$$

$$\begin{aligned}
x_7 = {} & a_1^6a_4 - 3a_1^4a_4b_1^2 + 6a_1^3a_4^2b_1^2 + 6a_1^3b_1^2b_4^2 - 6a_1^2a_4^2b_1^3 + 3a_1^2a_4b_1^4 - 6a_1^2b_1^3b_4^2 + a_2^6a_4 - 3a_2^4a_4b_2^2 \\
& + 6a_2^3a_4^2b_2^2 + 6a_2^3b_2^2b_4^2 - 6a_2^2a_4^2b_2^3 + 3a_2^2a_4b_2^4 - 6a_2^2b_2^3b_4^2 + a_3^6a_4 - 3a_3^4a_4b_3^2 + 6a_3^3a_4^2b_3^2 \\
& + 6a_3^3b_3^2b_4^2 - 6a_3^2a_4^2b_3^3 + 3a_3^2a_4b_3^4 - 6a_3^2b_3^3b_4^2 + a_4^7 + 3a_4^5b_4^2 - 6a_4^4b_4^3 + 9a_4^3b_4^4 + 6a_4^2a_5^3b_5^2 \\
& - 6a_4^2a_5^2b_5^3 + 6a_4^2a_6^3b_6^2 - 6a_4^2a_6^2b_6^3 + 6a_4^2a_7^3b_7^2 - 6a_4^2a_7^2b_7^3 + 6a_4^2a_8^3b_8^2 - 6a_4^2a_8^2b_8^3 - 6a_4^2b_4^5 \\
& + a_4a_5^6 - 3a_4a_5^4b_5^2 + 3a_4a_5^2b_5^4 + a_4a_6^6 - 3a_4a_6^4b_6^2 + 3a_4a_6^2b_6^4 + a_4a_7^6 - 3a_4a_7^4b_7^2 + 3a_4a_7^2b_7^4 + a_4a_8^6 \\
& - 3a_4a_8^4b_8^2 + 3a_4a_8^2b_8^4 - a_4b_1^6 - a_4b_2^6 - a_4b_3^6 - a_4b_4^6 - a_4b_5^6 - a_4b_6^6 - a_4b_7^6 - a_4b_8^6 + 6a_5^3b_4^2b_5^2 \\
& - 6a_5^2b_4^2b_5^3 + 6a_6^3b_4^2b_6^2 - 6a_6^2b_4^2b_6^3 + 6a_7^3b_4^2b_7^2 - 6a_7^2b_4^2b_7^3 + 6a_8^3b_4^2b_8^2 - 6a_8^2b_4^2b_8^3
\end{aligned}$$

$$\begin{aligned}
x_8 = {} & a_1^6b_4 - 3a_1^4b_1^2b_4 - 6a_1^3a_4^2b_1^2 + 6a_1^3b_1^2b_4^2 + 6a_1^2a_4^2b_1^3 + 3a_1^2b_1^4b_4 - 6a_1^2b_1^3b_4^2 + a_2^6b_4 - 3a_2^4b_2^2b_4 \\
& - 6a_2^3a_4^2b_2^2 + 6a_2^3b_2^2b_4^2 + 6a_2^2a_4^2b_2^3 + 3a_2^2b_2^4b_4 - 6a_2^2b_2^3b_4^2 + a_3^6b_4 - 3a_3^4b_3^2b_4 - 6a_3^3a_4^2b_3^2 \\
& + 6a_3^3b_3^2b_4^2 + 6a_3^2a_4^2b_3^3 + 3a_3^2b_3^4b_4 - 6a_3^2b_3^3b_4^2 + a_4^6b_4 - 6a_4^5b_4^2 + 3a_4^4b_4^3 + 6a_4^3b_4^4 - 6a_4^2a_5^3b_5^2 \\
& + 6a_4^2a_5^2b_5^3 - 6a_4^2a_6^3b_6^2 + 6a_4^2a_6^2b_6^3 - 6a_4^2a_7^3b_7^2 + 6a_4^2a_7^2b_7^3 - 6a_4^2a_8^3b_8^2 + 6a_4^2a_8^2b_8^3 - 3a_4^2b_4^5 \\
& + a_5^6b_4 - 3a_5^4b_4b_5^2 + 6a_5^3b_4^2b_5^2 - 6a_5^2b_4^2b_5^3 + 3a_5^2b_4b_5^4 + a_6^6b_4 - 3a_6^4b_4b_6^2 + 6a_6^3b_4^2b_6^2 - 6a_6^2b_4^2b_6^3 \\
& + 3a_6^2b_4b_6^4 + a_7^6b_4 - 3a_7^4b_4b_7^2 + 6a_7^3b_4^2b_7^2 - 6a_7^2b_4^2b_7^3 + 3a_7^2b_4b_7^4 + a_8^6b_4 - 3a_8^4b_4b_8^2 + 6a_8^3b_4^2b_8^2 \\
& - 6a_8^2b_4^2b_8^3 + 3a_8^2b_4b_8^4 - b_1^6b_4 - b_2^6b_4 - b_3^6b_4 - b_4^7 - b_4b_5^6 - b_4b_6^6 - b_4b_7^6 - b_4b_8^6
\end{aligned}$$

$$\begin{aligned}
x_9 = {} & a_1^6a_5 - 3a_1^4a_5b_1^2 + 6a_1^3a_5^2b_1^2 + 6a_1^3b_1^2b_5^2 - 6a_1^2a_5^2b_1^3 + 3a_1^2a_5b_1^4 - 6a_1^2b_1^3b_5^2 + a_2^6a_5 - 3a_2^4a_5b_2^2 \\
& + 6a_2^3a_5^2b_2^2 + 6a_2^3b_2^2b_5^2 - 6a_2^2a_5^2b_2^3 + 3a_2^2a_5b_2^4 - 6a_2^2b_2^3b_5^2 + a_3^6a_5 - 3a_3^4a_5b_3^2 + 6a_3^3a_5^2b_3^2 \\
& + 6a_3^3b_3^2b_5^2 - 6a_3^2a_5^2b_3^3 + 3a_3^2a_5b_3^4 - 6a_3^2b_3^3b_5^2 + a_4^6a_5 - 3a_4^4a_5b_4^2 + 6a_4^3a_5^2b_4^2 + 6a_4^3b_4^2b_5^2 \\
& - 6a_4^2a_5^2b_4^3 + 3a_4^2a_5b_4^4 - 6a_4^2b_4^3b_5^2 + a_5^7 + 3a_5^5b_5^2 - 6a_5^4b_5^3 + 9a_5^3b_5^4 + 6a_5^2a_6^3b_6^2 - 6a_5^2a_6^2b_6^3 \\
& + 6a_5^2a_7^3b_7^2 - 6a_5^2a_7^2b_7^3 + 6a_5^2a_8^3b_8^2 - 6a_5^2a_8^2b_8^3 - 6a_5^2b_5^5 + a_5a_6^6 - 3a_5a_6^4b_6^2 + 3a_5a_6^2b_6^4 + a_5a_7^6 \\
& - 3a_5a_7^4b_7^2 + 3a_5a_7^2b_7^4 + a_5a_8^6 - 3a_5a_8^4b_8^2 + 3a_5a_8^2b_8^4 - a_5b_1^6 - a_5b_2^6 - a_5b_3^6 - a_5b_4^6 - a_5b_5^6 - a_5b_6^6 - a_5b_7^6 \\
& - a_5b_8^6 + 6a_6^3b_5^2b_6^2 - 6a_6^2b_5^2b_6^3 + 6a_7^3b_5^2b_7^2 - 6a_7^2b_5^2b_7^3 + 6a_8^3b_5^2b_8^2 - 6a_8^2b_5^2b_8^3
\end{aligned}$$

$$
\begin{aligned}
x_{10} = {} & a_1^6 b_5 - 3a_1^4 b_1^2 b_5 - 6a_1^3 a_5^2 b_1^2 + 6a_1^3 b_1^2 b_5^2 + 6a_1^2 a_5^2 b_1^3 + 3a_1^2 b_1^4 b_5 - 6a_1^2 b_1^3 b_5^2 + a_2^6 b_5 - 3a_2^4 b_2^2 b_5 \\
& - 6a_2^3 a_5^2 b_2^2 + 6a_2^3 b_2^2 b_5^2 + 6a_2^2 a_5^2 b_2^3 + 3a_2^2 b_2^4 b_5 - 6a_2^2 b_2^3 b_5^2 + a_3^6 b_5 - 3a_3^4 b_3^2 b_5 - 6a_3^3 a_5^2 b_3^2 \\
& + 6a_3^3 b_3^2 b_5^2 + 6a_3^2 a_5^2 b_3^3 + 3a_3^2 b_3^4 b_5 - 6a_3^2 b_3^3 b_5^2 + a_4^6 b_5 - 3a_4^4 b_4^2 b_5 - 6a_4^3 a_5^2 b_4^2 + 6a_4^3 b_4^2 b_5^2 \\
& + 6a_4^2 a_5^2 b_4^3 + 3a_4^2 b_4^4 b_5 - 6a_4^2 b_4^3 b_5^2 + a_5^6 b_5 - 6a_5^5 b_5^2 + 3a_5^4 b_5^3 + 6a_5^3 b_5^4 - 6a_5^2 a_6^3 b_6^2 + 6a_5^2 a_6^2 b_6^3 \\
& - 6a_5^2 a_7^3 b_7^2 + 6a_5^2 a_7^2 b_7^3 - 6a_5^2 a_8^3 b_8^2 + 6a_5^2 a_8^2 b_8^3 - 3a_5^2 b_5^5 + a_6^6 b_5 - 3a_6^4 b_5 b_6^2 + 6a_6^3 b_5^2 b_6^2 \\
& - 6a_6^2 b_5^2 b_6^3 + 3a_6^2 b_5 b_6^4 + a_7^6 b_5 - 3a_7^4 b_5 b_7^2 + 6a_7^3 b_5^2 b_7^2 - 6a_7^2 b_5^2 b_7^3 + 3a_7^2 b_5 b_7^4 + a_8^6 b_5 - 3a_8^4 b_5 b_8^2 \\
& + 6a_8^3 b_5^2 b_8^2 - 6a_8^2 b_5^2 b_8^3 + 3a_8^2 b_5 b_8^4 - b_1^6 b_5 - b_2^6 b_5 - b_3^6 b_5 - b_4^6 b_5 - b_5^7 - b_5 b_6^6 - b_5 b_7^6 - b_5 b_8^6
\end{aligned}
$$

$$
\begin{aligned}
x_{11} = {} & a_1^6 a_6 - 3a_1^4 a_6 b_1^2 + 6a_1^3 a_6^2 b_1^2 + 6a_1^3 b_1^2 b_6^2 - 6a_1^2 a_6^2 b_1^3 + 3a_1^2 a_6 b_1^4 - 6a_1^2 b_1^3 b_6^2 + a_2^6 a_6 - 3a_2^4 a_6 b_2^2 \\
& + 6a_2^3 a_6^2 b_2^2 + 6a_2^3 b_2^2 b_6^2 - 6a_2^2 a_6^2 b_2^3 + 3a_2^2 a_6 b_2^4 - 6a_2^2 b_2^3 b_6^2 + a_3^6 a_6 - 3a_3^4 a_6 b_3^2 + 6a_3^3 a_6^2 b_3^2 \\
& + 6a_3^3 b_3^2 b_6^2 - 6a_3^2 a_6^2 b_3^3 + 3a_3^2 a_6 b_3^4 - 6a_3^2 b_3^3 b_6^2 + a_4^6 a_6 - 3a_4^4 a_6 b_4^2 + 6a_4^3 a_6^2 b_4^2 + 6a_4^3 b_4^2 b_6^2 \\
& - 6a_4^2 a_6^2 b_4^3 + 3a_4^2 a_6 b_4^4 - 6a_4^2 b_4^3 b_6^2 + a_5^6 a_6 - 3a_5^4 a_6 b_5^2 + 6a_5^3 a_6^2 b_5^2 + 6a_5^3 b_5^2 b_6^2 - 6a_5^2 a_6^2 b_5^3 \\
& + 3a_5^2 a_6 b_5^4 - 6a_5^2 b_5^3 b_6^2 + a_6^7 + 3a_6^5 b_6^2 - 6a_6^4 b_6^3 + 9a_6^3 b_6^4 + 6a_6^2 a_7^3 b_7^2 - 6a_6^2 a_7^2 b_7^3 + 6a_6^2 a_8^3 b_8^2 \\
& - 6a_6^2 a_8^2 b_8^3 - 6a_6^2 b_6^5 + a_6 a_7^6 - 3a_6 a_7^4 b_7^2 + 3a_6 a_7^2 b_7^4 + a_6 a_8^6 - 3a_6 a_8^4 b_8^2 + 3a_6 a_8^2 b_8^4 - a_6 b_1^6 - a_6 b_2^6 - a_6 b_3^6 \\
& - a_6 b_4^6 - a_6 b_5^6 - a_6 b_6^6 - a_6 b_7^6 - a_6 b_8^6 + 6a_7^3 b_6^2 b_7^2 - 6a_7^2 b_6^2 b_7^3 + 6a_8^3 b_6^2 b_8^2 - 6a_8^2 b_6^2 b_8^3
\end{aligned}
$$

$$
\begin{aligned}
x_{12} = {} & a_1^6 b_6 - 3a_1^4 b_1^2 b_6 - 6a_1^3 a_6^2 b_1^2 + 6a_1^3 b_1^2 b_6^2 + 6a_1^2 a_6^2 b_1^3 + 3a_1^2 b_1^4 b_6 - 6a_1^2 b_1^3 b_6^2 + a_2^6 b_6 - 3a_2^4 b_2^2 b_6 \\
& - 6a_2^3 a_6^2 b_2^2 + 6a_2^3 b_2^2 b_6^2 + 6a_2^2 a_6^2 b_2^3 + 3a_2^2 b_2^4 b_6 - 6a_2^2 b_2^3 b_6^2 + a_3^6 b_6 - 3a_3^4 b_3^2 b_6 - 6a_3^3 a_6^2 b_3^2 \\
& + 6a_3^3 b_3^2 b_6^2 + 6a_3^2 a_6^2 b_3^3 + 3a_3^2 b_3^4 b_6 - 6a_3^2 b_3^3 b_6^2 + a_4^6 b_6 - 3a_4^4 b_4^2 b_6 - 6a_4^3 a_6^2 b_4^2 + 6a_4^3 b_4^2 b_6^2 \\
& + 6a_4^2 a_6^2 b_4^3 + 3a_4^2 b_4^4 b_6 - 6a_4^2 b_4^3 b_6^2 + a_5^6 b_6 - 3a_5^4 b_5^2 b_6 - 6a_5^3 a_6^2 b_5^2 + 6a_5^3 b_5^2 b_6^2 + 6a_5^2 a_6^2 b_5^3 \\
& + 3a_5^2 b_5^4 b_6 - 6a_5^2 b_5^3 b_6^2 + a_6^6 b_6 - 6a_6^5 b_6^2 + 3a_6^4 b_6^3 + 6a_6^3 b_6^4 - 6a_6^2 a_7^3 b_7^2 + 6a_6^2 a_7^2 b_7^3 - 6a_6^2 a_8^3 b_8^2 \\
& + 6a_6^2 a_8^2 b_8^3 - 3a_6^2 b_6^5 + a_7^6 b_6 - 3a_7^4 b_6 b_7^2 + 6a_7^3 b_6^2 b_7^2 - 6a_7^2 b_6^2 b_7^3 + 3a_7^2 b_6 b_7^4 + a_8^6 b_6 - 3a_8^4 b_6 b_8^2 \\
& + 6a_8^3 b_6^2 b_8^2 - 6a_8^2 b_6^2 b_8^3 + 3a_8^2 b_6 b_8^4 - b_1^6 b_6 - b_2^6 b_6 - b_3^6 b_6 - b_4^6 b_6 - b_5^6 b_6 - b_6^7 - b_6 b_7^6 - b_6 b_8^6
\end{aligned}
$$

$$
\begin{aligned}
x_{13} = {} & a_1^6 a_7 - 3a_1^4 a_7 b_1^2 + 6a_1^3 a_7^2 b_1^2 + 6a_1^3 b_1^2 b_7^2 - 6a_1^2 a_7^2 b_1^3 + 3a_1^2 a_7 b_1^4 - 6a_1^2 b_1^3 b_7^2 + a_2^6 a_7 - 3a_2^4 a_7 b_2^2 \\
& + 6a_2^3 a_7^2 b_2^2 + 6a_2^3 b_2^2 b_7^2 - 6a_2^2 a_7^2 b_2^3 + 3a_2^2 a_7 b_2^4 - 6a_2^2 b_2^3 b_7^2 + a_3^6 a_7 - 3a_3^4 a_7 b_3^2 + 6a_3^3 a_7^2 b_3^2 \\
& + 6a_3^3 b_3^2 b_7^2 - 6a_3^2 a_7^2 b_3^3 + 3a_3^2 a_7 b_3^4 - 6a_3^2 b_3^3 b_7^2 + a_4^6 a_7 - 3a_4^4 a_7 b_4^2 + 6a_4^3 a_7^2 b_4^2 + 6a_4^3 b_4^2 b_7^2 \\
& - 6a_4^2 a_7^2 b_4^3 + 3a_4^2 a_7 b_4^4 - 6a_4^2 b_4^3 b_7^2 + a_5^6 a_7 - 3a_5^4 a_7 b_5^2 + 6a_5^3 a_7^2 b_5^2 + 6a_5^3 b_5^2 b_7^2 - 6a_5^2 a_7^2 b_5^3 \\
& + 3a_5^2 a_7 b_5^4 - 6a_5^2 b_5^3 b_7^2 + a_6^6 a_7 - 3a_6^4 a_7 b_6^2 + 6a_6^3 a_7^2 b_6^2 + 6a_6^3 b_6^2 b_7^2 - 6a_6^2 a_7^2 b_6^3 + 3a_6^2 a_7 b_6^4 \\
& - 6a_6^2 b_6^3 b_7^2 + a_7^7 + 3a_7^5 b_7^2 - 6a_7^4 b_7^3 + 9a_7^3 b_7^4 + 6a_7^2 a_8^3 b_8^2 - 6a_7^2 a_8^2 b_8^3 - 6a_7^2 b_7^5 + a_7 a_8^6 - 3a_7 a_8^4 b_8^2 \\
& + 3a_7 a_8^2 b_8^4 - a_7 b_1^6 - a_7 b_2^6 - a_7 b_3^6 - a_7 b_4^6 - a_7 b_5^6 - a_7 b_6^6 - a_7 b_7^6 - a_7 b_8^6 + 6a_8^3 b_7^2 b_8^2 - 6a_8^2 b_7^2 b_8^3
\end{aligned}
$$

$$
\begin{aligned}
x_{14} = {} & a_1^6 b_7 - 3a_1^4 b_1^2 b_7 - 6a_1^3 a_7^2 b_1^2 + 6a_1^3 b_1^2 b_7^2 + 6a_1^2 a_7^2 b_1^3 + 3a_1^2 b_1^4 b_7 - 6a_1^2 b_1^3 b_7^2 + a_2^6 b_7 - 3a_2^4 b_2^2 b_7 \\
& - 6a_2^3 a_7^2 b_2^2 + 6a_2^3 b_2^2 b_7^2 + 6a_2^2 a_7^2 b_2^3 + 3a_2^2 b_2^4 b_7 - 6a_2^2 b_2^3 b_7^2 + a_3^6 b_7 - 3a_3^4 b_3^2 b_7 - 6a_3^3 a_7^2 b_3^2 \\
& + 6a_3^3 b_3^2 b_7^2 + 6a_3^2 a_7^2 b_3^3 + 3a_3^2 b_3^4 b_7 - 6a_3^2 b_3^3 b_7^2 + a_4^6 b_7 - 3a_4^4 b_4^2 b_7 - 6a_4^3 a_7^2 b_4^2 + 6a_4^3 b_4^2 b_7^2 \\
& + 6a_4^2 a_7^2 b_4^3 + 3a_4^2 b_4^4 b_7 - 6a_4^2 b_4^3 b_7^2 + a_5^6 b_7 - 3a_5^4 b_5^2 b_7 - 6a_5^3 a_7^2 b_5^2 + 6a_5^3 b_5^2 b_7^2 + 6a_5^2 a_7^2 b_5^3 \\
& + 3a_5^2 b_5^4 b_7 - 6a_5^2 b_5^3 b_7^2 + a_6^6 b_7 - 3a_6^4 b_6^2 b_7 - 6a_6^3 a_7^2 b_6^2 + 6a_6^3 b_6^2 b_7^2 + 6a_6^2 a_7^2 b_6^3 + 3a_6^2 b_6^4 b_7 \\
& - 6a_6^2 b_6^3 b_7^2 + a_7^6 b_7 - 6a_7^5 b_7^2 + 3a_7^4 b_7^3 + 6a_7^3 b_7^4 - 6a_7^2 a_8^3 b_8^2 + 6a_7^2 a_8^2 b_8^3 - 3a_7^2 b_7^5 + a_8^6 b_7 - 3a_8^4 b_7 b_8^2 \\
& + 6a_8^3 b_7^2 b_8^2 - 6a_8^2 b_7^2 b_8^3 + 3a_8^2 b_7 b_8^4 - b_1^6 b_7 - b_2^6 b_7 - b_3^6 b_7 - b_4^6 b_7 - b_5^6 b_7 - b_6^6 b_7 - b_7^7 - b_7 b_8^6
\end{aligned}
$$

$$
\begin{aligned}
x_{15} = {} & a_1^6a_8 - 3a_1^4a_8b_1^2 + 6a_1^3a_8^2b_1^2 + 6a_1^3b_1^2b_8^2 - 6a_1^2a_8^2b_1^3 + 3a_1^2a_8b_1^4 - 6a_1^2b_1^3b_8^2 + a_2^6a_8 - 3a_2^4a_8b_2^2 \\
& + 6a_2^3a_8^2b_2^2 + 6a_2^3b_2^2b_8^2 - 6a_2^2a_8^2b_2^3 + 3a_2^2a_8b_2^4 - 6a_2^2b_2^3b_8^2 + a_3^6a_8 - 3a_3^4a_8b_3^2 + 6a_3^3a_8^2b_3^2 \\
& + 6a_3^3b_3^2b_8^2 - 6a_3^2a_8^2b_3^3 + 3a_3^2a_8b_3^4 - 6a_3^2b_3^3b_8^2 + a_4^6a_8 - 3a_4^4a_8b_4^2 + 6a_4^3a_8^2b_4^2 + 6a_4^3b_4^2b_8^2 \\
& - 6a_4^2a_8^2b_4^3 + 3a_4^2a_8b_4^4 - 6a_4^2b_4^3b_8^2 + a_5^6a_8 - 3a_5^4a_8b_5^2 + 6a_5^3a_8^2b_5^2 + 6a_5^3b_5^2b_8^2 - 6a_5^2a_8^2b_5^3 \\
& + 3a_5^2a_8b_5^4 - 6a_5^2b_5^3b_8^2 + a_6^6a_8 - 3a_6^4a_8b_6^2 + 6a_6^3a_8^2b_6^2 + 6a_6^3b_6^2b_8^2 - 6a_6^2a_8^2b_6^3 + 3a_6^2a_8b_6^4 \\
& - 6a_6^2b_6^3b_8^2 + a_7^6a_8 - 3a_7^4a_8b_7^2 + 6a_7^3a_8^2b_7^2 + 6a_7^3b_7^2b_8^2 - 6a_7^2a_8^2b_7^3 + 3a_7^2a_8b_7^4 - 6a_7^2b_7^3b_8^2 + a_8^7 \\
& + 3a_8^5b_8^2 - 6a_8^4b_8^3 + 9a_8^3b_8^4 - 6a_8^2b_8^5 - a_8b_1^6 - a_8b_2^6 - a_8b_3^6 - a_8b_4^6 - a_8b_5^6 - a_8b_6^6 - a_8b_7^6 - a_8b_8^6
\end{aligned}
$$

$$
\begin{aligned}
x_{16} = {} & a_1^6b_8 - 3a_1^4b_1^2b_8 - 6a_1^3a_8^2b_1^2 + 6a_1^3b_1^2b_8^2 + 6a_1^2a_8^2b_1^3 + 3a_1^2b_1^4b_8 - 6a_1^2b_1^3b_8^2 + a_2^6b_8 - 3a_2^4b_2^2b_8 \\
& - 6a_2^3a_8^2b_2^2 + 6a_2^3b_2^2b_8^2 + 6a_2^2a_8^2b_2^3 + 3a_2^2b_2^4b_8 - 6a_2^2b_2^3b_8^2 + a_3^6b_8 - 3a_3^4b_3^2b_8 - 6a_3^3a_8^2b_3^2 \\
& + 6a_3^3b_3^2b_8^2 + 6a_3^2a_8^2b_3^3 + 3a_3^2b_3^4b_8 - 6a_3^2b_3^3b_8^2 + a_4^6b_8 - 3a_4^4b_4^2b_8 - 6a_4^3a_8^2b_4^2 + 6a_4^3b_4^2b_8^2 \\
& + 6a_4^2a_8^2b_4^3 + 3a_4^2b_4^4b_8 - 6a_4^2b_4^3b_8^2 + a_5^6b_8 - 3a_5^4b_5^2b_8 - 6a_5^3a_8^2b_5^2 + 6a_5^3b_5^2b_8^2 + 6a_5^2a_8^2b_5^3 \\
& + 3a_5^2b_5^4b_8 - 6a_5^2b_5^3b_8^2 + a_6^6b_8 - 3a_6^4b_6^2b_8 - 6a_6^3a_8^2b_6^2 + 6a_6^3b_6^2b_8^2 + 6a_6^2a_8^2b_6^3 + 3a_6^2b_6^4b_8 \\
& - 6a_6^2b_6^3b_8^2 + a_7^6b_8 - 3a_7^4b_7^2b_8 - 6a_7^3a_8^2b_7^2 + 6a_7^3b_7^2b_8^2 + 6a_7^2a_8^2b_7^3 + 3a_7^2b_7^4b_8 - 6a_7^2b_7^3b_8^2 + a_8^6b_8 \\
& - 6a_8^5b_8^2 + 3a_8^4b_8^3 + 6a_8^3b_8^4 - 3a_8^2b_8^5 - b_1^6b_8 - b_2^6b_8 - b_3^6b_8 - b_4^6b_8 - b_5^6b_8 - b_6^6b_8 - b_7^6b_8 - b_8^7
\end{aligned}
$$

$$
\begin{aligned}
y_1 = {} & a_1^7 + 3a_1^5b_1^2 - 6a_1^4b_1^3 - 3a_1^3b_1^4 + 6a_1^2a_2^3b_2^2 - 6a_1^2a_2^2b_2^3 + 6a_1^2a_3^3b_3^2 - 6a_1^2a_3^2b_3^3 + 6a_1^2a_4^3b_4^2 \\
& - 6a_1^2a_4^2b_4^3 + 6a_1^2a_5^3b_5^2 - 6a_1^2a_5^2b_5^3 + 6a_1^2a_6^3b_6^2 - 6a_1^2a_6^2b_6^3 + 6a_1^2a_7^3b_7^2 - 6a_1^2a_7^2b_7^3 \\
& + 6a_1^2a_8^3b_8^2 - 6a_1^2a_8^2b_8^3 + 6a_1^2b_1^5 + a_1a_2^6 - 3a_1a_2^4b_2^2 + 3a_1a_2^2b_2^4 + a_1a_3^6 - 3a_1a_3^4b_3^2 + 3a_1a_3^2b_3^4 + a_1a_4^6 \\
& - 3a_1a_4^4b_4^2 + 3a_1a_4^2b_4^4 + a_1a_5^6 - 3a_1a_5^4b_5^2 + 3a_1a_5^2b_5^4 + a_1a_6^6 - 3a_1a_6^4b_6^2 + 3a_1a_6^2b_6^4 + a_1a_7^6 - 3a_1a_7^4b_7^2 \\
& + 3a_1a_7^2b_7^4 + a_1a_8^6 - 3a_1a_8^4b_8^2 + 3a_1a_8^2b_8^4 - a_1b_1^6 - a_1b_2^6 - a_1b_3^6 - a_1b_4^6 - a_1b_5^6 - a_1b_6^6 - a_1b_7^6 - a_1b_8^6 \\
& - 6a_2^3b_1^2b_2^2 + 6a_2^2b_1^2b_2^3 - 6a_3^3b_1^2b_3^2 + 6a_3^2b_1^2b_3^3 - 6a_4^3b_1^2b_4^2 + 6a_4^2b_1^2b_4^3 - 6a_5^3b_1^2b_5^2 \\
& + 6a_5^2b_1^2b_5^3 - 6a_6^3b_1^2b_6^2 + 6a_6^2b_1^2b_6^3 - 6a_7^3b_1^2b_7^2 + 6a_7^2b_1^2b_7^3 - 6a_8^3b_1^2b_8^2 + 6a_8^2b_1^2b_8^3
\end{aligned}
$$

$$
\begin{aligned}
y_2 = {} & a_1^6b_1 + 6a_1^5b_1^2 - 9a_1^4b_1^3 + 6a_1^3b_1^4 + 6a_1^2a_2^3b_2^2 - 6a_1^2a_2^2b_2^3 + 6a_1^2a_3^3b_3^2 - 6a_1^2a_3^2b_3^3 + 6a_1^2a_4^3b_4^2 \\
& - 6a_1^2a_4^2b_4^3 + 6a_1^2a_5^3b_5^2 - 6a_1^2a_5^2b_5^3 + 6a_1^2a_6^3b_6^2 - 6a_1^2a_6^2b_6^3 + 6a_1^2a_7^3b_7^2 - 6a_1^2a_7^2b_7^3 \\
& + 6a_1^2a_8^3b_8^2 - 6a_1^2a_8^2b_8^3 - 3a_1^2b_1^5 + a_2^6b_1 - 3a_2^4b_1b_2^2 + 6a_2^3b_1^2b_2^2 - 6a_2^2b_1^2b_2^3 + 3a_2^2b_1b_2^4 + a_3^6b_1 \\
& - 3a_3^4b_1b_3^2 + 6a_3^3b_1^2b_3^2 - 6a_3^2b_1^2b_3^3 + 3a_3^2b_1b_3^4 + a_4^6b_1 - 3a_4^4b_1b_4^2 + 6a_4^3b_1^2b_4^2 - 6a_4^2b_1^2b_4^3 \\
& + 3a_4^2b_1b_4^4 + a_5^6b_1 - 3a_5^4b_1b_5^2 + 6a_5^3b_1^2b_5^2 - 6a_5^2b_1^2b_5^3 + 3a_5^2b_1b_5^4 + a_6^6b_1 - 3a_6^4b_1b_6^2 + 6a_6^3b_1^2b_6^2 \\
& - 6a_6^2b_1^2b_6^3 + 3a_6^2b_1b_6^4 + a_7^6b_1 - 3a_7^4b_1b_7^2 + 6a_7^3b_1^2b_7^2 - 6a_7^2b_1^2b_7^3 + 3a_7^2b_1b_7^4 + a_8^6b_1 - 3a_8^4b_1b_8^2 \\
& + 6a_8^3b_1^2b_8^2 - 6a_8^2b_1^2b_8^3 + 3a_8^2b_1b_8^4 - b_1^7 - b_1b_2^6 - b_1b_3^6 - b_1b_4^6 - b_1b_5^6 - b_1b_6^6 - b_1b_7^6 - b_1b_8^6
\end{aligned}
$$

$$
\begin{aligned}
y_3 = {} & a_1^6a_2 - 3a_1^4a_2b_1^2 + 6a_1^3a_2^2b_1^2 - 6a_1^3b_1^2b_2^2 - 6a_1^2a_2^2b_1^3 + 3a_1^2a_2b_1^4 + 6a_1^2b_1^3b_2^2 + a_2^7 + 3a_2^5b_2^2 \\
& - 6a_2^4b_2^3 - 3a_2^3b_2^4 + 6a_2^2a_3^3b_3^2 - 6a_2^2a_3^2b_3^3 + 6a_2^2a_4^3b_4^2 - 6a_2^2a_4^2b_4^3 + 6a_2^2a_5^3b_5^2 - 6a_2^2a_5^2b_5^3 \\
& + 6a_2^2a_6^3b_6^2 - 6a_2^2a_6^2b_6^3 + 6a_2^2a_7^3b_7^2 - 6a_2^2a_7^2b_7^3 + 6a_2^2a_8^3b_8^2 - 6a_2^2a_8^2b_8^3 + 6a_2^2b_2^5 + a_2a_3^6 \\
& - 3a_2a_3^4b_3^2 + 3a_2a_3^2b_3^4 + a_2a_4^6 - 3a_2a_4^4b_4^2 + 3a_2a_4^2b_4^4 + a_2a_5^6 - 3a_2a_5^4b_5^2 + 3a_2a_5^2b_5^4 + a_2a_6^6 - 3a_2a_6^4b_6^2 \\
& + 3a_2a_6^2b_6^4 + a_2a_7^6 - 3a_2a_7^4b_7^2 + 3a_2a_7^2b_7^4 + a_2a_8^6 - 3a_2a_8^4b_8^2 + 3a_2a_8^2b_8^4 - a_2b_1^6 - a_2b_2^6 - a_2b_3^6 - a_2b_4^6 \\
& - a_2b_5^6 - a_2b_6^6 - a_2b_7^6 - a_2b_8^6 - 6a_3^3b_2^2b_3^2 + 6a_3^2b_2^2b_3^3 - 6a_4^3b_2^2b_4^2 + 6a_4^2b_2^2b_4^3 - 6a_5^3b_2^2b_5^2 \\
& + 6a_5^2b_2^2b_5^3 - 6a_6^3b_2^2b_6^2 + 6a_6^2b_2^2b_6^3 - 6a_7^3b_2^2b_7^2 + 6a_7^2b_2^2b_7^3 - 6a_8^3b_2^2b_8^2 + 6a_8^2b_2^2b_8^3
\end{aligned}
$$

$$\begin{aligned}
y_4 = {} & a_1^6b_2 - 3a_1^4b_1^2b_2 + 6a_1^3a_2^2b_1^2 + 6a_1^3b_1^2b_2^2 - 6a_1^2a_2^2b_1^3 + 3a_1^2b_1^4b_2 - 6a_1^2b_1^3b_2^2 + a_2^6b_2 + 6a_2^5b_2^2 \\
& - 9a_2^4b_2^3 + 6a_2^3b_2^4 + 6a_2^2a_3^3b_3^2 - 6a_2^2a_3^2b_3^3 + 6a_2^2a_4^3b_4^2 - 6a_2^2a_4^2b_4^3 + 6a_2^2a_5^3b_5^2 - 6a_2^2a_5^2b_5^3 \\
& + 6a_2^2a_6^3b_6^2 - 6a_2^2a_6^2b_6^3 + 6a_2^2a_7^3b_7^2 - 6a_2^2a_7^2b_7^3 + 6a_2^2a_8^3b_8^2 - 6a_2^2a_8^2b_8^3 - 3a_2^2b_2^5 + a_3^6b_2 \\
& - 3a_3^4b_2b_3^2 + 6a_3^3b_2^2b_3^2 - 6a_3^2b_2^2b_3^3 + 3a_3^2b_2b_3^4 + a_4^6b_2 - 3a_4^4b_2b_4^2 + 6a_4^3b_2^2b_4^2 - 6a_4^2b_2^2b_4^3 \\
& + 3a_4^2b_2b_4^4 + a_5^6b_2 - 3a_5^4b_2b_5^2 + 6a_5^3b_2^2b_5^2 - 6a_5^2b_2^2b_5^3 + 3a_5^2b_2b_5^4 + a_6^6b_2 - 3a_6^4b_2b_6^2 + 6a_6^3b_2^2b_6^2 \\
& - 6a_6^2b_2^2b_6^3 + 3a_6^2b_2b_6^4 + a_7^6b_2 - 3a_7^4b_2b_7^2 + 6a_7^3b_2^2b_7^2 - 6a_7^2b_2^2b_7^3 + 3a_7^2b_2b_7^4 + a_8^6b_2 - 3a_8^4b_2b_8^2 \\
& + 6a_8^3b_2^2b_8^2 - 6a_8^2b_2^2b_8^3 + 3a_8^2b_2b_8^4 - b_1^6b_2 - b_2^7 - b_2b_3^6 - b_2b_4^6 - b_2b_5^6 - b_2b_6^6 - b_2b_7^6 - b_2b_8^6
\end{aligned}$$

$$\begin{aligned}
y_5 = {} & a_1^6a_3 - 3a_1^4a_3b_1^2 + 6a_1^3a_3^2b_1^2 - 6a_1^3b_1^2b_3^2 - 6a_1^2a_3^2b_1^3 + 3a_1^2a_3b_1^4 + 6a_1^2b_1^3b_3^2 + a_2^6a_3 - 3a_2^4a_3b_2^2 \\
& + 6a_2^3a_3^2b_2^2 - 6a_2^3b_2^2b_3^2 - 6a_2^2a_3^2b_2^3 + 3a_2^2a_3b_2^4 + 6a_2^2b_2^3b_3^2 + a_3^7 + 3a_3^5b_3^2 - 6a_3^4b_3^3 - 3a_3^3b_3^4 \\
& + 6a_3^2a_4^3b_4^2 - 6a_3^2a_4^2b_4^3 + 6a_3^2a_5^3b_5^2 - 6a_3^2a_5^2b_5^3 + 6a_3^2a_6^3b_6^2 - 6a_3^2a_6^2b_6^3 + 6a_3^2a_7^3b_7^2 \\
& - 6a_3^2a_7^2b_7^3 + 6a_3^2a_8^3b_8^2 - 6a_3^2a_8^2b_8^3 + 6a_3^2b_3^5 + a_3a_4^6 - 3a_3a_4^4b_4^2 + 3a_3a_4^2b_4^4 + a_3a_5^6 - 3a_3a_5^4b_5^2 \\
& + 3a_3a_5^2b_5^4 + a_3a_6^6 - 3a_3a_6^4b_6^2 + 3a_3a_6^2b_6^4 + a_3a_7^6 - 3a_3a_7^4b_7^2 + 3a_3a_7^2b_7^4 + a_3a_8^6 - 3a_3a_8^4b_8^2 + 3a_3a_8^2b_8^4 \\
& - a_3b_1^6 - a_3b_2^6 - a_3b_3^6 - a_3b_4^6 - a_3b_5^6 - a_3b_6^6 - a_3b_7^6 - a_3b_8^6 - 6a_4^3b_3^2b_4^2 + 6a_4^2b_3^2b_4^3 - 6a_5^3b_3^2b_5^2 \\
& + 6a_5^2b_3^2b_5^3 - 6a_6^3b_3^2b_6^2 + 6a_6^2b_3^2b_6^3 - 6a_7^3b_3^2b_7^2 + 6a_7^2b_3^2b_7^3 - 6a_8^3b_3^2b_8^2 + 6a_8^2b_3^2b_8^3
\end{aligned}$$

$$\begin{aligned}
y_6 = {} & a_1^6b_3 - 3a_1^4b_1^2b_3 + 6a_1^3a_3^2b_1^2 + 6a_1^3b_1^2b_3^2 - 6a_1^2a_3^2b_1^3 + 3a_1^2b_1^4b_3 - 6a_1^2b_1^3b_3^2 + a_2^6b_3 - 3a_2^4b_2^2b_3 \\
& + 6a_2^3a_3^2b_2^2 + 6a_2^3b_2^2b_3^2 - 6a_2^2a_3^2b_2^3 + 3a_2^2b_2^4b_3 - 6a_2^2b_2^3b_3^2 + a_3^6b_3 + 6a_3^5b_3^2 - 9a_3^4b_3^3 + 6a_3^3b_3^4 \\
& + 6a_3^2a_4^3b_4^2 - 6a_3^2a_4^2b_4^3 + 6a_3^2a_5^3b_5^2 - 6a_3^2a_5^2b_5^3 + 6a_3^2a_6^3b_6^2 - 6a_3^2a_6^2b_6^3 + 6a_3^2a_7^3b_7^2 \\
& - 6a_3^2a_7^2b_7^3 + 6a_3^2a_8^3b_8^2 - 6a_3^2a_8^2b_8^3 - 3a_3^2b_3^5 + a_4^6b_3 - 3a_4^4b_3b_4^2 + 6a_4^3b_3^2b_4^2 - 6a_4^2b_3^2b_4^3 \\
& + 3a_4^2b_3b_4^4 + a_5^6b_3 - 3a_5^4b_3b_5^2 + 6a_5^3b_3^2b_5^2 - 6a_5^2b_3^2b_5^3 + 3a_5^2b_3b_5^4 + a_6^6b_3 - 3a_6^4b_3b_6^2 + 6a_6^3b_3^2b_6^2 \\
& - 6a_6^2b_3^2b_6^3 + 3a_6^2b_3b_6^4 + a_7^6b_3 - 3a_7^4b_3b_7^2 + 6a_7^3b_3^2b_7^2 - 6a_7^2b_3^2b_7^3 + 3a_7^2b_3b_7^4 + a_8^6b_3 - 3a_8^4b_3b_8^2 \\
& + 6a_8^3b_3^2b_8^2 - 6a_8^2b_3^2b_8^3 + 3a_8^2b_3b_8^4 - b_1^6b_3 - b_2^6b_3 - b_3^7 - b_3b_4^6 - b_3b_5^6 - b_3b_6^6 - b_3b_7^6 - b_3b_8^6
\end{aligned}$$

$$\begin{aligned}
y_7 = {} & a_1^6a_4 - 3a_1^4a_4b_1^2 + 6a_1^3a_4^2b_1^2 - 6a_1^3b_1^2b_4^2 - 6a_1^2a_4^2b_1^3 + 3a_1^2a_4b_1^4 + 6a_1^2b_1^3b_4^2 + a_2^6a_4 - 3a_2^4a_4b_2^2 \\
& + 6a_2^3a_4^2b_2^2 - 6a_2^3b_2^2b_4^2 - 6a_2^2a_4^2b_2^3 + 3a_2^2a_4b_2^4 + 6a_2^2b_2^3b_4^2 + a_3^6a_4 - 3a_3^4a_4b_3^2 + 6a_3^3a_4^2b_3^2 \\
& - 6a_3^3b_3^2b_4^2 - 6a_3^2a_4^2b_3^3 + 3a_3^2a_4b_3^4 + 6a_3^2b_3^3b_4^2 + a_4^7 + 3a_4^5b_4^2 - 6a_4^4b_4^3 - 3a_4^3b_4^4 + 6a_4^2a_5^3b_5^2 \\
& - 6a_4^2a_5^2b_5^3 + 6a_4^2a_6^3b_6^2 - 6a_4^2a_6^2b_6^3 + 6a_4^2a_7^3b_7^2 - 6a_4^2a_7^2b_7^3 + 6a_4^2a_8^3b_8^2 - 6a_4^2a_8^2b_8^3 + 6a_4^2b_4^5 \\
& + a_4a_5^6 - 3a_4a_5^4b_5^2 + 3a_4a_5^2b_5^4 + a_4a_6^6 - 3a_4a_6^4b_6^2 + 3a_4a_6^2b_6^4 + a_4a_7^6 - 3a_4a_7^4b_7^2 + 3a_4a_7^2b_7^4 + a_4a_8^6 \\
& - 3a_4a_8^4b_8^2 + 3a_4a_8^2b_8^4 - a_4b_1^6 - a_4b_2^6 - a_4b_3^6 - a_4b_4^6 - a_4b_5^6 - a_4b_6^6 - a_4b_7^6 - a_4b_8^6 - 6a_5^3b_4^2b_5^2 \\
& + 6a_5^2b_4^2b_5^3 - 6a_6^3b_4^2b_6^2 + 6a_6^2b_4^2b_6^3 - 6a_7^3b_4^2b_7^2 + 6a_7^2b_4^2b_7^3 - 6a_8^3b_4^2b_8^2 + 6a_8^2b_4^2b_8^3
\end{aligned}$$

$$\begin{aligned}
y_8 = {} & a_1^6b_4 - 3a_1^4b_1^2b_4 + 6a_1^3a_4^2b_1^2 + 6a_1^3b_1^2b_4^2 - 6a_1^2a_4^2b_1^3 + 3a_1^2b_1^4b_4 - 6a_1^2b_1^3b_4^2 + a_2^6b_4 - 3a_2^4b_2^2b_4 \\
& + 6a_2^3a_4^2b_2^2 + 6a_2^3b_2^2b_4^2 - 6a_2^2a_4^2b_2^3 + 3a_2^2b_2^4b_4 - 6a_2^2b_2^3b_4^2 + a_3^6b_4 - 3a_3^4b_3^2b_4 + 6a_3^3a_4^2b_3^2 \\
& + 6a_3^3b_3^2b_4^2 - 6a_3^2a_4^2b_3^3 + 3a_3^2b_3^4b_4 - 6a_3^2b_3^3b_4^2 + a_4^6b_4 + 6a_4^5b_4^2 - 9a_4^4b_4^3 + 6a_4^3b_4^4 + 6a_4^2a_5^3b_5^2 \\
& - 6a_4^2a_5^2b_5^3 + 6a_4^2a_6^3b_6^2 - 6a_4^2a_6^2b_6^3 + 6a_4^2a_7^3b_7^2 - 6a_4^2a_7^2b_7^3 + 6a_4^2a_8^3b_8^2 - 6a_4^2a_8^2b_8^3 - 3a_4^2b_4^5 \\
& + a_5^6b_4 - 3a_5^4b_4b_5^2 + 6a_5^3b_4^2b_5^2 - 6a_5^2b_4^2b_5^3 + 3a_5^2b_4b_5^4 + a_6^6b_4 - 3a_6^4b_4b_6^2 + 6a_6^3b_4^2b_6^2 - 6a_6^2b_4^2b_6^3 \\
& + 3a_6^2b_4b_6^4 + a_7^6b_4 - 3a_7^4b_4b_7^2 + 6a_7^3b_4^2b_7^2 - 6a_7^2b_4^2b_7^3 + 3a_7^2b_4b_7^4 + a_8^6b_4 - 3a_8^4b_4b_8^2 + 6a_8^3b_4^2b_8^2 \\
& - 6a_8^2b_4^2b_8^3 + 3a_8^2b_4b_8^4 - b_1^6b_4 - b_2^6b_4 - b_3^6b_4 - b_4^7 - b_4b_5^6 - b_4b_6^6 - b_4b_7^6 - b_4b_8^6
\end{aligned}$$

$$
\begin{aligned}
y_9 = {} & a_1^6a_5 - 3a_1^4a_5b_1^2 + 6a_1^3a_5^2b_1^2 - 6a_1^3b_1^2b_5^2 - 6a_1^2a_5^2b_1^3 + 3a_1^2a_5b_1^4 + 6a_1^2b_1^3b_5^2 + a_2^6a_5 - 3a_2^4a_5b_2^2 \\
& + 6a_2^3a_5^2b_2^2 - 6a_2^3b_2^2b_5^2 - 6a_2^2a_5^2b_2^3 + 3a_2^2a_5b_2^4 + 6a_2^2b_2^3b_5^2 + a_3^6a_5 - 3a_3^4a_5b_3^2 + 6a_3^3a_5^2b_3^2 \\
& - 6a_3^3b_3^2b_5^2 - 6a_3^2a_5^2b_3^3 + 3a_3^2a_5b_3^4 + 6a_3^2b_3^3b_5^2 + a_4^6a_5 - 3a_4^4a_5b_4^2 + 6a_4^3a_5^2b_4^2 - 6a_4^3b_4^2b_5^2 \\
& - 6a_4^2a_5^2b_4^3 + 3a_4^2a_5b_4^4 + 6a_4^2b_4^3b_5^2 + a_5^7 + 3a_5^5b_5^2 - 6a_5^4b_5^3 - 3a_5^3b_5^4 + 6a_5^2a_6^3b_6^2 - 6a_5^2a_6^2b_6^3 \\
& + 6a_5^2a_7^3b_7^2 - 6a_5^2a_7^2b_7^3 + 6a_5^2a_8^3b_8^2 - 6a_5^2a_8^2b_8^3 + 6a_5^2b_5^5 + a_5a_6^6 - 3a_5a_6^4b_6^2 + 3a_5a_6^2b_6^4 + a_5a_7^6 \\
& - 3a_5a_7^4b_7^2 + 3a_5a_7^2b_7^4 + a_5a_8^6 - 3a_5a_8^4b_8^2 + 3a_5a_8^2b_8^4 - a_5b_1^6 - a_5b_2^6 - a_5b_3^6 - a_5b_4^6 - a_5b_5^6 - a_5b_6^6 - a_5b_7^6 \\
& - a_5b_8^6 - 6a_6^3b_5^2b_6^2 + 6a_6^2b_5^2b_6^3 - 6a_7^3b_5^2b_7^2 + 6a_7^2b_5^2b_7^3 - 6a_8^3b_5^2b_8^2 + 6a_8^2b_5^2b_8^3
\end{aligned}
$$

$$
\begin{aligned}
y_{10} = {} & a_1^6b_5 - 3a_1^4b_1^2b_5 + 6a_1^3a_5^2b_1^2 + 6a_1^3b_1^2b_5^2 - 6a_1^2a_5^2b_1^3 + 3a_1^2b_1^4b_5 - 6a_1^2b_1^3b_5^2 + a_2^6b_5 - 3a_2^4b_2^2b_5 \\
& + 6a_2^3a_5^2b_2^2 + 6a_2^3b_2^2b_5^2 - 6a_2^2a_5^2b_2^3 + 3a_2^2b_2^4b_5 - 6a_2^2b_2^3b_5^2 + a_3^6b_5 - 3a_3^4b_3^2b_5 + 6a_3^3a_5^2b_3^2 \\
& + 6a_3^3b_3^2b_5^2 - 6a_3^2a_5^2b_3^3 + 3a_3^2b_3^4b_5 - 6a_3^2b_3^3b_5^2 + a_4^6b_5 - 3a_4^4b_4^2b_5 + 6a_4^3a_5^2b_4^2 + 6a_4^3b_4^2b_5^2 \\
& - 6a_4^2a_5^2b_4^3 + 3a_4^2b_4^4b_5 - 6a_4^2b_4^3b_5^2 + a_5^6b_5 + 6a_5^5b_5^2 - 9a_5^4b_5^3 + 6a_5^3b_5^4 + 6a_5^2a_6^3b_6^2 - 6a_5^2a_6^2b_6^3 \\
& + 6a_5^2a_7^3b_7^2 - 6a_5^2a_7^2b_7^3 + 6a_5^2a_8^3b_8^2 - 6a_5^2a_8^2b_8^3 - 3a_5^2b_5^5 + a_6^6b_5 - 3a_6^4b_5b_6^2 + 6a_6^3b_5^2b_6^2 \\
& - 6a_6^2b_5^2b_6^3 + 3a_6^2b_5b_6^4 + a_7^6b_5 - 3a_7^4b_5b_7^2 + 6a_7^3b_5^2b_7^2 - 6a_7^2b_5^2b_7^3 + 3a_7^2b_5b_7^4 + a_8^6b_5 - 3a_8^4b_5b_8^2 \\
& + 6a_8^3b_5^2b_8^2 - 6a_8^2b_5^2b_8^3 + 3a_8^2b_5b_8^4 - b_1^6b_5 - b_2^6b_5 - b_3^6b_5 - b_4^6b_5 - b_5^7 - b_5b_6^6 - b_5b_7^6 - b_5b_8^6
\end{aligned}
$$

$$
\begin{aligned}
y_{11} = {} & a_1^6a_6 - 3a_1^4a_6b_1^2 + 6a_1^3a_6^2b_1^2 - 6a_1^3b_1^2b_6^2 - 6a_1^2a_6^2b_1^3 + 3a_1^2a_6b_1^4 + 6a_1^2b_1^3b_6^2 + a_2^6a_6 - 3a_2^4a_6b_2^2 \\
& + 6a_2^3a_6^2b_2^2 - 6a_2^3b_2^2b_6^2 - 6a_2^2a_6^2b_2^3 + 3a_2^2a_6b_2^4 + 6a_2^2b_2^3b_6^2 + a_3^6a_6 - 3a_3^4a_6b_3^2 + 6a_3^3a_6^2b_3^2 \\
& - 6a_3^3b_3^2b_6^2 - 6a_3^2a_6^2b_3^3 + 3a_3^2a_6b_3^4 + 6a_3^2b_3^3b_6^2 + a_4^6a_6 - 3a_4^4a_6b_4^2 + 6a_4^3a_6^2b_4^2 - 6a_4^3b_4^2b_6^2 \\
& - 6a_4^2a_6^2b_4^3 + 3a_4^2a_6b_4^4 + 6a_4^2b_4^3b_6^2 + a_5^6a_6 - 3a_5^4a_6b_5^2 + 6a_5^3a_6^2b_5^2 - 6a_5^3b_5^2b_6^2 - 6a_5^2a_6^2b_5^3 \\
& + 3a_5^2a_6b_5^4 + 6a_5^2b_5^3b_6^2 + a_6^7 + 3a_6^5b_6^2 - 6a_6^4b_6^3 - 3a_6^3b_6^4 + 6a_6^2a_7^3b_7^2 - 6a_6^2a_7^2b_7^3 + 6a_6^2a_8^3b_8^2 \\
& - 6a_6^2a_8^2b_8^3 + 6a_6^2b_6^5 + a_6a_7^6 - 3a_6a_7^4b_7^2 + 3a_6a_7^2b_7^4 + a_6a_8^6 - 3a_6a_8^4b_8^2 + 3a_6a_8^2b_8^4 - a_6b_1^6 - a_6b_2^6 - a_6b_3^6 \\
& - a_6b_4^6 - a_6b_5^6 - a_6b_6^6 - a_6b_7^6 - a_6b_8^6 - 6a_7^3b_6^2b_7^2 + 6a_7^2b_6^2b_7^3 - 6a_8^3b_6^2b_8^2 + 6a_8^2b_6^2b_8^3
\end{aligned}
$$

$$
\begin{aligned}
y_{12} = {} & a_1^6b_6 - 3a_1^4b_1^2b_6 + 6a_1^3a_6^2b_1^2 + 6a_1^3b_1^2b_6^2 - 6a_1^2a_6^2b_1^3 + 3a_1^2b_1^4b_6 - 6a_1^2b_1^3b_6^2 + a_2^6b_6 - 3a_2^4b_2^2b_6 \\
& + 6a_2^3a_6^2b_2^2 + 6a_2^3b_2^2b_6^2 - 6a_2^2a_6^2b_2^3 + 3a_2^2b_2^4b_6 - 6a_2^2b_2^3b_6^2 + a_3^6b_6 - 3a_3^4b_3^2b_6 + 6a_3^3a_6^2b_3^2 \\
& + 6a_3^3b_3^2b_6^2 - 6a_3^2a_6^2b_3^3 + 3a_3^2b_3^4b_6 - 6a_3^2b_3^3b_6^2 + a_4^6b_6 - 3a_4^4b_4^2b_6 + 6a_4^3a_6^2b_4^2 + 6a_4^3b_4^2b_6^2 \\
& - 6a_4^2a_6^2b_4^3 + 3a_4^2b_4^4b_6 - 6a_4^2b_4^3b_6^2 + a_5^6b_6 - 3a_5^4b_5^2b_6 + 6a_5^3a_6^2b_5^2 + 6a_5^3b_5^2b_6^2 - 6a_5^2a_6^2b_5^3 \\
& + 3a_5^2b_5^4b_6 - 6a_5^2b_5^3b_6^2 + a_6^6b_6 + 6a_6^5b_6^2 - 9a_6^4b_6^3 + 6a_6^3b_6^4 + 6a_6^2a_7^3b_7^2 - 6a_6^2a_7^2b_7^3 + 6a_6^2a_8^3b_8^2 \\
& - 6a_6^2a_8^2b_8^3 - 3a_6^2b_6^5 + a_7^6b_6 - 3a_7^4b_6b_7^2 + 6a_7^3b_6^2b_7^2 - 6a_7^2b_6^2b_7^3 + 3a_7^2b_6b_7^4 + a_8^6b_6 - 3a_8^4b_6b_8^2 \\
& + 6a_8^3b_6^2b_8^2 - 6a_8^2b_6^2b_8^3 + 3a_8^2b_6b_8^4 - b_1^6b_6 - b_2^6b_6 - b_3^6b_6 - b_4^6b_6 - b_5^6b_6 - b_6^7 - b_6b_7^6 - b_6b_8^6
\end{aligned}
$$

$$
\begin{aligned}
y_{13} = {} & a_1^6a_7 - 3a_1^4a_7b_1^2 + 6a_1^3a_7^2b_1^2 - 6a_1^3b_1^2b_7^2 - 6a_1^2a_7^2b_1^3 + 3a_1^2a_7b_1^4 + 6a_1^2b_1^3b_7^2 + a_2^6a_7 - 3a_2^4a_7b_2^2 \\
& + 6a_2^3a_7^2b_2^2 - 6a_2^3b_2^2b_7^2 - 6a_2^2a_7^2b_2^3 + 3a_2^2a_7b_2^4 + 6a_2^2b_2^3b_7^2 + a_3^6a_7 - 3a_3^4a_7b_3^2 + 6a_3^3a_7^2b_3^2 \\
& - 6a_3^3b_3^2b_7^2 - 6a_3^2a_7^2b_3^3 + 3a_3^2a_7b_3^4 + 6a_3^2b_3^3b_7^2 + a_4^6a_7 - 3a_4^4a_7b_4^2 + 6a_4^3a_7^2b_4^2 - 6a_4^3b_4^2b_7^2 \\
& - 6a_4^2a_7^2b_4^3 + 3a_4^2a_7b_4^4 + 6a_4^2b_4^3b_7^2 + a_5^6a_7 - 3a_5^4a_7b_5^2 + 6a_5^3a_7^2b_5^2 - 6a_5^3b_5^2b_7^2 - 6a_5^2a_7^2b_5^3 \\
& + 3a_5^2a_7b_5^4 + 6a_5^2b_5^3b_7^2 + a_6^6a_7 - 3a_6^4a_7b_6^2 + 6a_6^3a_7^2b_6^2 - 6a_6^3b_6^2b_7^2 - 6a_6^2a_7^2b_6^3 + 3a_6^2a_7b_6^4 \\
& + 6a_6^2b_6^3b_7^2 + a_7^7 + 3a_7^5b_7^2 - 6a_7^4b_7^3 - 3a_7^3b_7^4 + 6a_7^2a_8^3b_8^2 - 6a_7^2a_8^2b_8^3 + 6a_7^2b_7^5 + a_7a_8^6 - 3a_7a_8^4b_8^2 \\
& + 3a_7a_8^2b_8^4 - a_7b_1^6 - a_7b_2^6 - a_7b_3^6 - a_7b_4^6 - a_7b_5^6 - a_7b_6^6 - a_7b_7^6 - a_7b_8^6 - 6a_8^3b_7^2b_8^2 + 6a_8^2b_7^2b_8^3
\end{aligned}
$$

$$
\begin{aligned}
y_{14} = {} & a_1^6 b_7 - 3a_1^4 b_1^2 b_7 + 6a_1^3 a_7^2 b_1^2 + 6a_1^3 b_1^2 b_7^2 - 6a_1^2 a_7^2 b_1^3 + 3a_1^2 b_1^4 b_7 - 6a_1^2 b_1^3 b_7^2 + a_2^6 b_7 - 3a_2^4 b_2^2 b_7 \\
& + 6a_2^3 a_7^2 b_2^2 + 6a_2^3 b_2^2 b_7^2 - 6a_2^2 a_7^2 b_2^3 + 3a_2^2 b_2^4 b_7 - 6a_2^2 b_2^3 b_7^2 + a_3^6 b_7 - 3a_3^4 b_3^2 b_7 + 6a_3^3 a_7^2 b_3^2 \\
& + 6a_3^3 b_3^2 b_7^2 - 6a_3^2 a_7^2 b_3^3 + 3a_3^2 b_3^4 b_7 - 6a_3^2 b_3^3 b_7^2 + a_4^6 b_7 - 3a_4^4 b_4^2 b_7 + 6a_4^3 a_7^2 b_4^2 + 6a_4^3 b_4^2 b_7^2 \\
& - 6a_4^2 a_7^2 b_4^3 + 3a_4^2 b_4^4 b_7 - 6a_4^2 b_4^3 b_7^2 + a_5^6 b_7 - 3a_5^4 b_5^2 b_7 + 6a_5^3 a_7^2 b_5^2 + 6a_5^3 b_5^2 b_7^2 - 6a_5^2 a_7^2 b_5^3 \\
& + 3a_5^2 b_5^4 b_7 - 6a_5^2 b_5^3 b_7^2 + a_6^6 b_7 - 3a_6^4 b_6^2 b_7 + 6a_6^3 a_7^2 b_6^2 + 6a_6^3 b_6^2 b_7^2 - 6a_6^2 a_7^2 b_6^3 + 3a_6^2 b_6^4 b_7 \\
& - 6a_6^2 b_6^3 b_7^2 + a_7^6 b_7 + 6a_7^5 b_7^2 - 9a_7^4 b_7^3 + 6a_7^3 b_7^4 + 6a_7^2 a_8^3 b_8^2 - 6a_7^2 a_8^2 b_8^3 - 3a_7^2 b_7^5 + a_8^6 b_7 - 3a_8^4 b_7 b_8^2 \\
& + 6a_8^3 b_7^2 b_8^2 - 6a_8^2 b_7^2 b_8^3 + 3a_8^2 b_7 b_8^4 - b_1^6 b_7 - b_2^6 b_7 - b_3^6 b_7 - b_4^6 b_7 - b_5^6 b_7 - b_6^6 b_7 - b_7^7 - b_7 b_8^6
\end{aligned}
$$

$$
\begin{aligned}
y_{15} = {} & a_1^6 a_8 - 3a_1^4 a_8 b_1^2 + 6a_1^3 a_8^2 b_1^2 - 6a_1^3 b_1^2 b_8^2 - 6a_1^2 a_8^2 b_1^3 + 3a_1^2 a_8 b_1^4 + 6a_1^2 b_1^3 b_8^2 + a_2^6 a_8 - 3a_2^4 a_8 b_2^2 \\
& + 6a_2^3 a_8^2 b_2^2 - 6a_2^3 b_2^2 b_8^2 - 6a_2^2 a_8^2 b_2^3 + 3a_2^2 a_8 b_2^4 + 6a_2^2 b_2^3 b_8^2 + a_3^6 a_8 - 3a_3^4 a_8 b_3^2 + 6a_3^3 a_8^2 b_3^2 \\
& - 6a_3^3 b_3^2 b_8^2 - 6a_3^2 a_8^2 b_3^3 + 3a_3^2 a_8 b_3^4 + 6a_3^2 b_3^3 b_8^2 + a_4^6 a_8 - 3a_4^4 a_8 b_4^2 + 6a_4^3 a_8^2 b_4^2 - 6a_4^3 b_4^2 b_8^2 \\
& - 6a_4^2 a_8^2 b_4^3 + 3a_4^2 a_8 b_4^4 + 6a_4^2 b_4^3 b_8^2 + a_5^6 a_8 - 3a_5^4 a_8 b_5^2 + 6a_5^3 a_8^2 b_5^2 - 6a_5^3 b_5^2 b_8^2 - 6a_5^2 a_8^2 b_5^3 \\
& + 3a_5^2 a_8 b_5^4 + 6a_5^2 b_5^3 b_8^2 + a_6^6 a_8 - 3a_6^4 a_8 b_6^2 + 6a_6^3 a_8^2 b_6^2 - 6a_6^3 b_6^2 b_8^2 - 6a_6^2 a_8^2 b_6^3 + 3a_6^2 a_8 b_6^4 \\
& + 6a_6^2 b_6^3 b_8^2 + a_7^6 a_8 - 3a_7^4 a_8 b_7^2 + 6a_7^3 a_8^2 b_7^2 - 6a_7^3 b_7^2 b_8^2 - 6a_7^2 a_8^2 b_7^3 + 3a_7^2 a_8 b_7^4 + 6a_7^2 b_7^3 b_8^2 + a_8^7 \\
& + 3a_8^5 b_8^2 - 6a_8^4 b_8^3 - 3a_8^3 b_8^4 + 6a_8^2 b_8^5 - a_8 b_1^6 - a_8 b_2^6 - a_8 b_3^6 - a_8 b_4^6 - a_8 b_5^6 - a_8 b_6^6 - a_8 b_7^6 - a_8 b_8^6
\end{aligned}
$$

$$
\begin{aligned}
y_{16} = {} & a_1^6 b_8 - 3a_1^4 b_1^2 b_8 + 6a_1^3 a_8^2 b_1^2 + 6a_1^3 b_1^2 b_8^2 - 6a_1^2 a_8^2 b_1^3 + 3a_1^2 b_1^4 b_8 - 6a_1^2 b_1^3 b_8^2 + a_2^6 b_8 - 3a_2^4 b_2^2 b_8 \\
& + 6a_2^3 a_8^2 b_2^2 + 6a_2^3 b_2^2 b_8^2 - 6a_2^2 a_8^2 b_2^3 + 3a_2^2 b_2^4 b_8 - 6a_2^2 b_2^3 b_8^2 + a_3^6 b_8 - 3a_3^4 b_3^2 b_8 + 6a_3^3 a_8^2 b_3^2 \\
& + 6a_3^3 b_3^2 b_8^2 - 6a_3^2 a_8^2 b_3^3 + 3a_3^2 b_3^4 b_8 - 6a_3^2 b_3^3 b_8^2 + a_4^6 b_8 - 3a_4^4 b_4^2 b_8 + 6a_4^3 a_8^2 b_4^2 + 6a_4^3 b_4^2 b_8^2 \\
& - 6a_4^2 a_8^2 b_4^3 + 3a_4^2 b_4^4 b_8 - 6a_4^2 b_4^3 b_8^2 + a_5^6 b_8 - 3a_5^4 b_5^2 b_8 + 6a_5^3 a_8^2 b_5^2 + 6a_5^3 b_5^2 b_8^2 - 6a_5^2 a_8^2 b_5^3 \\
& + 3a_5^2 b_5^4 b_8 - 6a_5^2 b_5^3 b_8^2 + a_6^6 b_8 - 3a_6^4 b_6^2 b_8 + 6a_6^3 a_8^2 b_6^2 + 6a_6^3 b_6^2 b_8^2 - 6a_6^2 a_8^2 b_6^3 + 3a_6^2 b_6^4 b_8 \\
& - 6a_6^2 b_6^3 b_8^2 + a_7^6 b_8 - 3a_7^4 b_7^2 b_8 + 6a_7^3 a_8^2 b_7^2 + 6a_7^3 b_7^2 b_8^2 - 6a_7^2 a_8^2 b_7^3 + 3a_7^2 b_7^4 b_8 - 6a_7^2 b_7^3 b_8^2 + a_8^6 b_8 \\
& + 6a_8^5 b_8^2 - 9a_8^4 b_8^3 + 6a_8^3 b_8^4 - 3a_8^2 b_8^5 - b_1^6 b_8 - b_2^6 b_8 - b_3^6 b_8 - b_4^6 b_8 - b_5^6 b_8 - b_6^6 b_8 - b_7^6 b_8 - b_8^7
\end{aligned}
$$

is a solution in $\mathbb{K}$ of $x_1^3 + x_2^3 + \cdots + x_{15}^3 + x_{16}^3 = y_1^3 + y_2^3 + \cdots + y_{15}^3 + y_{16}^3$.

3. Solutions of a quartic Diophantine equation

Regarding the equation $x_1^4 + x_2^4 + \cdots + x_{2n-1}^4 + x_{2n}^4 = y_1^4 + y_2^4 + \cdots + y_{2n-1}^4 + y_{2n}^4$, we shall establish the following

Theorem 3.1 Let n be an arbitrary integer $\geqslant 2$. Given elements $a_1, \ldots, a_n, b_1, \ldots, b_n$ of $\mathbb{K}$ such that $a_i > b_i > 0$ for $i = 1, \ldots, n$, let us define, for each $k = 1, \ldots, n$,

$$
\begin{aligned}
x_{2k-1} &= a_k w + c_{2k-1}, \\
x_{2k} &= b_k w - c_{2k}, \\
y_{2k-1} &= a_k w + c_{2k}, \\
y_{2k} &= b_k w + c_{2k-1},
\end{aligned}
\tag{4}
$$

where $c_{2k-1} = \mu(a_k^3 + b_k^3)$, $c_{2k} = \mu(a_k^3 - b_k^3)$,

$$
\begin{aligned}
w = {} & (b_1 - a_1)(a_1^3 + b_1^3)^3 + (b_1 + a_1)(a_1^3 - b_1^3)^3 + \cdots \\
& + (b_n - a_n)(a_n^3 + b_n^3)^3 + (b_n + a_n)(a_n^3 - b_n^3)^3
\end{aligned}
$$

and

$$\mu = 6\big(a_1^3\, b_1^3\, (a_1^2 - b_1^2) + \cdots + a_n^3\, b_n^3\, (a_n^2 - b_n^2)\big). \tag{5}$$

Then $x_1, \ldots, x_{2n}, y_1, \ldots, y_{2n}$, defined in (4), is a solution in $\mathbb{K}$ of the Diophantine equation

$$x_1^4 + x_2^4 + \cdots + x_{2n-1}^4 + x_{2n}^4 = y_1^4 + y_2^4 + \cdots + y_{2n-1}^4 + y_{2n}^4. \tag{6}$$

Proof. We shall argue as in [3], where the cases $n = 2$ and $n = 3$ have been considered when $\mathbb{K} = \mathbb{Q}$. Indeed, for $w \in \mathbb{K}^*$, let us write

$$\begin{aligned} &x_1 = a_1 w + c_1, \quad x_2 = b_1 w - c_2, \quad x_3 = a_2 w + c_3, \quad x_4 = b_2 w - c_4, \\ &\quad \ldots, \quad x_{2n-1} = a_n w + c_{2n-1}, \qquad x_{2n} = b_n w - c_{2n}, \\ &y_1 = a_1 w + c_2, \quad y_2 = b_1 w + c_1, \quad y_3 = a_2 w + c_4, \quad y_4 = b_2 w + c_3, \\ &\quad \ldots, \quad y_{2n-1} = a_n w + c_{2n}, \quad y_{2n} = b_n w + c_{2n-1}, \end{aligned}$$

where $c_1, \ldots, c_{2n}$ are elements of $\mathbb{K}$ (clearly, if $w = 0$, then $x_1 = c_1, x_2 = -c_2, \ldots, x_{2n-1} = c_{2n-1}, x_{2n} = -c_{2n}, y_1 = c_2, y_2 = c_1, \ldots, y_{2n-1} = c_{2n}, y_{2n} = c_{2n-1}$ would be a solution of the equation for all $c_1, c_2, \ldots, c_{2n-1}, c_{2n} \in \mathbb{K}$).

Substituting the above-mentioned elements $x_1, \ldots, x_{2n}$, $y_1, \ldots, y_{2n}$ of $\mathbb{K}$ in the equation under consideration, we obtain a quartic equation in w in which the first and last coefficients are 0. The coefficient of w^3 is

$$4\Big(a_1^3(c_1 - c_2) - b_1^3(c_1 + c_2) + \cdots + a_n^3(c_{2n-1} - c_{2n}) - b_n^3(c_{2n-1} + c_{2n})\Big).$$

Thus, by taking

$$\begin{aligned} c_1 &= a_1^3 + b_1^3, \\ c_2 &= a_1^3 - b_1^3, \\ &\vdots \\ c_{2n-1} &= a_n^3 + b_n^3, \\ c_{2n} &= a_n^3 - b_n^3, \end{aligned}$$

we get

$$\begin{aligned} c_1 + c_2 &= 2a_1^3, \\ c_1 - c_2 &= 2b_1^3, \\ &\vdots \\ c_{2n-1} + c_{2n} &= 2a_n^3, \\ c_{2n-1} - c_{2n} &= 2b_n^3, \end{aligned}$$

and the coefficient of w^3 will be 0. Therefore the equation for w will simplify to

$$\begin{aligned} &6(a_1^2(c_1^2 - c_2^2) + b_1^2(c_2^2 - c_1^2) + \cdots + a_n^2(c_{2n-1}^2 - c_{2n}^2) + b_n^2(c_{2n}^2 - c_{2n-1}^2))\, w^2 \\ &= 4((b_1 - a_1)c_1^3 + (b_1 + a_1)c_2^3 + \cdots + (b_n - a_n)c_{2n-1}^3 + (b_n + a_n)c_{2n}^3)w. \end{aligned}$$

Moreover since, for each $k = 1, \dots, n$,

$$c_{2k-1}^2 - c_{2k}^2 = (c_{2k-1} - c_{2k})(c_{2k-1} + c_{2k}) = 4a_k^3 b_k^3,$$

we arrive at

$$\begin{aligned} &6(a_1^3 b_1^3(a_1^2 - b_1^2) + \cdots + a_n^3 b_n^3(a_n^2 - b_n^2))\, w^2 \\ &= \mu w^2 = ((b_1 - a_1)(a_1^3 + b_1^3)^3 + (b_1 + a_1)(a_1^3 - b_1^3)^3 + \cdots \\ &\cdots + (b_n - a_n)(a_n^3 + b_n^3)^3 + (b_n + a_n)(a_n^3 - b_n^3)^3)w \end{aligned}$$

(by hypothesis μ, defined in (5), is a positive element of $\mathbb{K}$). Finally, multiplying w and each c_i $(i = 1, \dots, 2n)$ by the element $\mu \in \mathbb{K}$, we obtain elements of $\mathbb{K}$ that we will still denote by w and c_i $(i = 1, \dots, 2n)$ which, when substituted in (4), furnish the solution $x_1, \dots, x_{2n}, y_1, \dots, y_{2n}$ in $\mathbb{K}$ of the Diophantine equation (6). This completes the proof. ■

Remark 2 *Under the conditions of Theorem 3.1, $x_1, \dots, x_{2n}, y_1, \dots, y_{2n}$ belong to $\mathbb{L}$ if $a_1, \dots, a_n, b_1, \dots, b_n$ belong to $\mathbb{L}$.*

In view of Theorem 3.1, one can use `SymPy` to guarantee the validity of the following (see code at the appendix)

***Example* 3.2** If $a_i > b_i > 0$ $(i = 1, \dots, 8)$ are elements of $\mathbb{K}$,

$$\begin{aligned} x_1 = {} & 2a_1^{10}b_1 - 6a_1^6b_1^5 + 6a_1^5b_1^6 + 6a_1^4b_1^7 + 6a_1^3a_2^5b_2^3 - 6a_1^3a_2^3b_2^5 + 6a_1^3a_3^5b_3^3 - 6a_1^3a_3^3b_3^5 + 6a_1^3a_4^5b_4^3 \\ & - 6a_1^3a_4^3b_4^5 + 6a_1^3a_5^5b_5^3 - 6a_1^3a_5^3b_5^5 + 6a_1^3a_6^5b_6^3 - 6a_1^3a_6^3b_6^5 + 6a_1^3a_7^5b_7^3 - 6a_1^3a_7^3b_7^5 \\ & + 6a_1^3a_8^5b_8^3 - 6a_1^3a_8^3b_8^5 - 6a_1^3b_1^8 - 2a_1^2b_1^9 + 2a_1a_2^9b_2 - 6a_1a_2^7b_2^3 + 6a_1a_2^3b_2^7 - 2a_1a_2b_2^9 + 2a_1a_3^9b_3 \\ & - 6a_1a_3^7b_3^3 + 6a_1a_3^3b_3^7 - 2a_1a_3b_3^9 + 2a_1a_4^9b_4 - 6a_1a_4^7b_4^3 + 6a_1a_4^3b_4^7 - 2a_1a_4b_4^9 + 2a_1a_5^9b_5 - 6a_1a_5^7b_5^3 \\ & + 6a_1a_5^3b_5^7 - 2a_1a_5b_5^9 + 2a_1a_6^9b_6 - 6a_1a_6^7b_6^3 + 6a_1a_6^3b_6^7 - 2a_1a_6b_6^9 + 2a_1a_7^9b_7 - 6a_1a_7^7b_7^3 + 6a_1a_7^3b_7^7 \\ & - 2a_1a_7b_7^9 + 2a_1a_8^9b_8 - 6a_1a_8^7b_8^3 + 6a_1a_8^3b_8^7 - 2a_1a_8b_8^9 + 6a_2^5b_1^3b_2^3 - 6a_2^3b_1^3b_2^5 + 6a_3^5b_1^3b_3^3 \\ & - 6a_3^3b_1^3b_3^5 + 6a_4^5b_1^3b_4^3 - 6a_4^3b_1^3b_4^5 + 6a_5^5b_1^3b_5^3 - 6a_5^3b_1^3b_5^5 + 6a_6^5b_1^3b_6^3 - 6a_6^3b_1^3b_6^5 \\ & + 6a_7^5b_1^3b_7^3 - 6a_7^3b_1^3b_7^5 + 6a_8^5b_1^3b_8^3 - 6a_8^3b_1^3b_8^5 \end{aligned}$$

$$\begin{aligned} x_2 = {} & 2a_1^9b_1^2 - 6a_1^8b_1^3 - 6a_1^7b_1^4 + 6a_1^6b_1^5 + 6a_1^5b_1^6 - 6a_1^3a_2^5b_2^3 + 6a_1^3a_2^3b_2^5 - 6a_1^3a_3^5b_3^3 + 6a_1^3a_3^3b_3^5 \\ & - 6a_1^3a_4^5b_4^3 + 6a_1^3a_4^3b_4^5 - 6a_1^3a_5^5b_5^3 + 6a_1^3a_5^3b_5^5 - 6a_1^3a_6^5b_6^3 + 6a_1^3a_6^3b_6^5 - 6a_1^3a_7^5b_7^3 \\ & + 6a_1^3a_7^3b_7^5 - 6a_1^3a_8^5b_8^3 + 6a_1^3a_8^3b_8^5 - 2a_1b_1^{10} + 2a_2^9b_1b_2 - 6a_2^7b_1b_2^3 + 6a_2^5b_1^3b_2^3 - 6a_2^3b_1^3b_2^5 \\ & + 6a_2^3b_1b_2^7 - 2a_2b_1b_2^9 + 2a_3^9b_1b_3 - 6a_3^7b_1b_3^3 + 6a_3^5b_1^3b_3^3 - 6a_3^3b_1^3b_3^5 + 6a_3^3b_1b_3^7 - 2a_3b_1b_3^9 + 2a_4^9b_1b_4 \\ & - 6a_4^7b_1b_4^3 + 6a_4^5b_1^3b_4^3 - 6a_4^3b_1^3b_4^5 + 6a_4^3b_1b_4^7 - 2a_4b_1b_4^9 + 2a_5^9b_1b_5 - 6a_5^7b_1b_5^3 + 6a_5^5b_1^3b_5^3 \\ & - 6a_5^3b_1^3b_5^5 + 6a_5^3b_1b_5^7 - 2a_5b_1b_5^9 + 2a_6^9b_1b_6 - 6a_6^7b_1b_6^3 + 6a_6^5b_1^3b_6^3 - 6a_6^3b_1^3b_6^5 + 6a_6^3b_1b_6^7 - 2a_6b_1b_6^9 \\ & + 2a_7^9b_1b_7 - 6a_7^7b_1b_7^3 + 6a_7^5b_1^3b_7^3 - 6a_7^3b_1^3b_7^5 + 6a_7^3b_1b_7^7 - 2a_7b_1b_7^9 + 2a_8^9b_1b_8 - 6a_8^7b_1b_8^3 + 6a_8^5b_1^3b_8^3 \\ & - 6a_8^3b_1^3b_8^5 + 6a_8^3b_1b_8^7 - 2a_8b_1b_8^9 \end{aligned}$$

$$
\begin{aligned}
x_3 = {} & 2a_1^9a_2b_1 - 6a_1^7a_2b_1^3 + 6a_1^5a_2^3b_1^3 + 6a_1^5b_1^3b_2^3 - 6a_1^3a_2^3b_1^5 + 6a_1^3a_2b_1^7 - 6a_1^3b_1^5b_2^3 - 2a_1a_2b_1^9 + 2a_2^{10}b_2 \\
& - 6a_2^6b_2^5 + 6a_2^5b_2^6 + 6a_2^4b_2^7 + 6a_2^3a_3^5b_3^3 - 6a_2^3a_3^3b_3^5 + 6a_2^3a_4^5b_4^3 - 6a_2^3a_4^3b_4^5 + 6a_2^3a_5^5b_5^3 \\
& - 6a_2^3a_5^3b_5^5 + 6a_2^3a_6^5b_6^3 - 6a_2^3a_6^3b_6^5 + 6a_2^3a_7^5b_7^3 - 6a_2^3a_7^3b_7^5 + 6a_2^3a_8^5b_8^3 - 6a_2^3a_8^3b_8^5 - 6a_2^3b_2^8 \\
& - 2a_2^2b_2^9 + 2a_2a_3^9b_3 - 6a_2a_3^7b_3^3 + 6a_2a_3^3b_3^7 - 2a_2a_3b_3^9 + 2a_2a_4^9b_4 - 6a_2a_4^7b_4^3 + 6a_2a_4^3b_4^7 - 2a_2a_4b_4^9 \\
& + 2a_2a_5^9b_5 - 6a_2a_5^7b_5^3 + 6a_2a_5^3b_5^7 - 2a_2a_5b_5^9 + 2a_2a_6^9b_6 - 6a_2a_6^7b_6^3 + 6a_2a_6^3b_6^7 - 2a_2a_6b_6^9 + 2a_2a_7^9b_7 \\
& - 6a_2a_7^7b_7^3 + 6a_2a_7^3b_7^7 - 2a_2a_7b_7^9 + 2a_2a_8^9b_8 - 6a_2a_8^7b_8^3 + 6a_2a_8^3b_8^7 - 2a_2a_8b_8^9 + 6a_3^5b_2^3b_3^3 - 6a_3^3b_2^3b_3^5 \\
& + 6a_4^5b_2^3b_4^3 - 6a_4^3b_2^3b_4^5 + 6a_5^5b_2^3b_5^3 - 6a_5^3b_2^3b_5^5 + 6a_6^5b_2^3b_6^3 - 6a_6^3b_2^3b_6^5 + 6a_7^5b_2^3b_7^3 \\
& - 6a_7^3b_2^3b_7^5 + 6a_8^5b_2^3b_8^3 - 6a_8^3b_2^3b_8^5
\end{aligned}
$$

$$
\begin{aligned}
x_4 = {} & 2a_1^9b_1b_2 - 6a_1^7b_1^3b_2 - 6a_1^5a_2^3b_1^3 + 6a_1^5b_1^3b_2^3 + 6a_1^3a_2^3b_1^5 + 6a_1^3b_1^7b_2 - 6a_1^3b_1^5b_2^3 - 2a_1b_1^9b_2 + 2a_2^9b_2^2 \\
& - 6a_2^8b_2^3 - 6a_2^7b_2^4 + 6a_2^6b_2^5 + 6a_2^5b_2^6 - 6a_2^3a_3^5b_3^3 + 6a_2^3a_3^3b_3^5 - 6a_2^3a_4^5b_4^3 + 6a_2^3a_4^3b_4^5 - 6a_2^3a_5^5b_5^3 \\
& + 6a_2^3a_5^3b_5^5 - 6a_2^3a_6^5b_6^3 + 6a_2^3a_6^3b_6^5 - 6a_2^3a_7^5b_7^3 + 6a_2^3a_7^3b_7^5 - 6a_2^3a_8^5b_8^3 + 6a_2^3a_8^3b_8^5 - 2a_2b_2^{10} \\
& + 2a_3^9b_2b_3 - 6a_3^7b_2b_3^3 + 6a_3^5b_2^3b_3^3 - 6a_3^3b_2^3b_3^5 + 6a_3^3b_2b_3^7 - 2a_3b_2b_3^9 + 2a_4^9b_2b_4 - 6a_4^7b_2b_4^3 + 6a_4^5b_2^3b_4^3 \\
& - 6a_4^3b_2^3b_4^5 + 6a_4^3b_2b_4^7 - 2a_4b_2b_4^9 + 2a_5^9b_2b_5 - 6a_5^7b_2b_5^3 + 6a_5^5b_2^3b_5^3 - 6a_5^3b_2^3b_5^5 + 6a_5^3b_2b_5^7 - 2a_5b_2b_5^9 \\
& + 2a_6^9b_2b_6 - 6a_6^7b_2b_6^3 + 6a_6^5b_2^3b_6^3 - 6a_6^3b_2^3b_6^5 + 6a_6^3b_2b_6^7 - 2a_6b_2b_6^9 + 2a_7^9b_2b_7 - 6a_7^7b_2b_7^3 + 6a_7^5b_2^3b_7^3 \\
& - 6a_7^3b_2^3b_7^5 + 6a_7^3b_2b_7^7 - 2a_7b_2b_7^9 + 2a_8^9b_2b_8 - 6a_8^7b_2b_8^3 + 6a_8^5b_2^3b_8^3 - 6a_8^3b_2^3b_8^5 + 6a_8^3b_2b_8^7 - 2a_8b_2b_8^9
\end{aligned}
$$

$$
\begin{aligned}
x_5 = {} & 2a_1^9a_3b_1 - 6a_1^7a_3b_1^3 + 6a_1^5a_3^3b_1^3 + 6a_1^5b_1^3b_3^3 - 6a_1^3a_3^3b_1^5 + 6a_1^3a_3b_1^7 - 6a_1^3b_1^5b_3^3 - 2a_1a_3b_1^9 \\
& + 2a_2^9a_3b_2 - 6a_2^7a_3b_2^3 + 6a_2^5a_3^3b_2^3 + 6a_2^5b_2^3b_3^3 - 6a_2^3a_3^3b_2^5 + 6a_2^3a_3b_2^7 - 6a_2^3b_2^5b_3^3 - 2a_2a_3b_2^9 + 2a_3^{10}b_3 \\
& - 6a_3^6b_3^5 + 6a_3^5b_3^6 + 6a_3^4b_3^7 + 6a_3^3a_4^5b_4^3 - 6a_3^3a_4^3b_4^5 + 6a_3^3a_5^5b_5^3 - 6a_3^3a_5^3b_5^5 + 6a_3^3a_6^5b_6^3 \\
& - 6a_3^3a_6^3b_6^5 + 6a_3^3a_7^5b_7^3 - 6a_3^3a_7^3b_7^5 + 6a_3^3a_8^5b_8^3 - 6a_3^3a_8^3b_8^5 - 6a_3^3b_3^8 - 2a_3^2b_3^9 + 2a_3a_4^9b_4 \\
& - 6a_3a_4^7b_4^3 + 6a_3a_4^3b_4^7 - 2a_3a_4b_4^9 + 2a_3a_5^9b_5 - 6a_3a_5^7b_5^3 + 6a_3a_5^3b_5^7 - 2a_3a_5b_5^9 + 2a_3a_6^9b_6 - 6a_3a_6^7b_6^3 \\
& + 6a_3a_6^3b_6^7 - 2a_3a_6b_6^9 + 2a_3a_7^9b_7 - 6a_3a_7^7b_7^3 + 6a_3a_7^3b_7^7 - 2a_3a_7b_7^9 + 2a_3a_8^9b_8 - 6a_3a_8^7b_8^3 + 6a_3a_8^3b_8^7 \\
& - 2a_3a_8b_8^9 + 6a_4^5b_3^3b_4^3 - 6a_4^3b_3^3b_4^5 + 6a_5^5b_3^3b_5^3 - 6a_5^3b_3^3b_5^5 + 6a_6^5b_3^3b_6^3 - 6a_6^3b_3^3b_6^5 + 6a_7^5b_3^3b_7^3 \\
& - 6a_7^3b_3^3b_7^5 + 6a_8^5b_3^3b_8^3 - 6a_8^3b_3^3b_8^5
\end{aligned}
$$

$$
\begin{aligned}
x_6 = {} & 2a_1^9b_1b_3 - 6a_1^7b_1^3b_3 - 6a_1^5a_3^3b_1^3 + 6a_1^5b_1^3b_3^3 + 6a_1^3a_3^3b_1^5 + 6a_1^3b_1^7b_3 - 6a_1^3b_1^5b_3^3 - 2a_1b_1^9b_3 \\
& + 2a_2^9b_2b_3 - 6a_2^7b_2^3b_3 - 6a_2^5a_3^3b_2^3 + 6a_2^5b_2^3b_3^3 + 6a_2^3a_3^3b_2^5 + 6a_2^3b_2^7b_3 - 6a_2^3b_2^5b_3^3 - 2a_2b_2^9b_3 \\
& + 2a_3^9b_3^2 - 6a_3^8b_3^3 - 6a_3^7b_3^4 + 6a_3^6b_3^5 + 6a_3^5b_3^6 - 6a_3^3a_4^5b_4^3 + 6a_3^3a_4^3b_4^5 - 6a_3^3a_5^5b_5^3 + 6a_3^3a_5^3b_5^5 \\
& - 6a_3^3a_6^5b_6^3 + 6a_3^3a_6^3b_6^5 - 6a_3^3a_7^5b_7^3 + 6a_3^3a_7^3b_7^5 - 6a_3^3a_8^5b_8^3 + 6a_3^3a_8^3b_8^5 - 2a_3b_3^{10} + 2a_4^9b_3b_4 \\
& - 6a_4^7b_3b_4^3 + 6a_4^5b_3^3b_4^3 - 6a_4^3b_3^3b_4^5 + 6a_4^3b_3b_4^7 - 2a_4b_3b_4^9 + 2a_5^9b_3b_5 - 6a_5^7b_3b_5^3 + 6a_5^5b_3^3b_5^3 \\
& - 6a_5^3b_3^3b_5^5 + 6a_5^3b_3b_5^7 - 2a_5b_3b_5^9 + 2a_6^9b_3b_6 - 6a_6^7b_3b_6^3 + 6a_6^5b_3^3b_6^3 - 6a_6^3b_3^3b_6^5 + 6a_6^3b_3b_6^7 - 2a_6b_3b_6^9 \\
& + 2a_7^9b_3b_7 - 6a_7^7b_3b_7^3 + 6a_7^5b_3^3b_7^3 - 6a_7^3b_3^3b_7^5 + 6a_7^3b_3b_7^7 - 2a_7b_3b_7^9 + 2a_8^9b_3b_8 - 6a_8^7b_3b_8^3 + 6a_8^5b_3^3b_8^3 \\
& - 6a_8^3b_3^3b_8^5 + 6a_8^3b_3b_8^7 - 2a_8b_3b_8^9
\end{aligned}
$$

$$
\begin{aligned}
x_7 = {} & 2a_1^9a_4b_1 - 6a_1^7a_4b_1^3 + 6a_1^5a_4^3b_1^3 + 6a_1^5b_1^3b_4^3 - 6a_1^3a_4^3b_1^5 + 6a_1^3a_4b_1^7 - 6a_1^3b_1^5b_4^3 - 2a_1a_4b_1^9 \\
& + 2a_2^9a_4b_2 - 6a_2^7a_4b_2^3 + 6a_2^5a_4^3b_2^3 + 6a_2^5b_2^3b_4^3 - 6a_2^3a_4^3b_2^5 + 6a_2^3a_4b_2^7 - 6a_2^3b_2^5b_4^3 - 2a_2a_4b_2^9 \\
& + 2a_3^9a_4b_3 - 6a_3^7a_4b_3^3 + 6a_3^5a_4^3b_3^3 + 6a_3^5b_3^3b_4^3 - 6a_3^3a_4^3b_3^5 + 6a_3^3a_4b_3^7 - 6a_3^3b_3^5b_4^3 - 2a_3a_4b_3^9 + 2a_4^{10}b_4 \\
& - 6a_4^6b_4^5 + 6a_4^5b_4^6 + 6a_4^4b_4^7 + 6a_4^3a_5^5b_5^3 - 6a_4^3a_5^3b_5^5 + 6a_4^3a_6^5b_6^3 - 6a_4^3a_6^3b_6^5 + 6a_4^3a_7^5b_7^3 \\
& - 6a_4^3a_7^3b_7^5 + 6a_4^3a_8^5b_8^3 - 6a_4^3a_8^3b_8^5 - 6a_4^3b_4^8 - 2a_4^2b_4^9 + 2a_4a_5^9b_5 - 6a_4a_5^7b_5^3 + 6a_4a_5^3b_5^7 - 2a_4a_5b_5^9 \\
& + 2a_4a_6^9b_6 - 6a_4a_6^7b_6^3 + 6a_4a_6^3b_6^7 - 2a_4a_6b_6^9 + 2a_4a_7^9b_7 - 6a_4a_7^7b_7^3 + 6a_4a_7^3b_7^7 - 2a_4a_7b_7^9 + 2a_4a_8^9b_8 \\
& - 6a_4a_8^7b_8^3 + 6a_4a_8^3b_8^7 - 2a_4a_8b_8^9 + 6a_5^5b_4^3b_5^3 - 6a_5^3b_4^3b_5^5 + 6a_6^5b_4^3b_6^3 - 6a_6^3b_4^3b_6^5 + 6a_7^5b_4^3b_7^3 \\
& - 6a_7^3b_4^3b_7^5 + 6a_8^5b_4^3b_8^3 - 6a_8^3b_4^3b_8^5
\end{aligned}
$$

$$
\begin{aligned}
x_8 = {} & 2a_1^9b_1b_4 - 6a_1^7b_1^3b_4 - 6a_1^5a_4^3b_1^3 + 6a_1^5b_1^3b_4^3 + 6a_1^3a_4^3b_1^5 + 6a_1^3b_1^7b_4 - 6a_1^3b_1^5b_4^3 - 2a_1b_1^9b_4 \\
& + 2a_2^9b_2b_4 - 6a_2^7b_2^3b_4 - 6a_2^5a_4^3b_2^3 + 6a_2^5b_2^3b_4^3 + 6a_2^3a_4^3b_2^5 + 6a_2^3b_2^7b_4 - 6a_2^3b_2^5b_4^3 - 2a_2b_2^9b_4 \\
& + 2a_3^9b_3b_4 - 6a_3^7b_3^3b_4 - 6a_3^5a_4^3b_3^3 + 6a_3^5b_3^3b_4^3 + 6a_3^3a_4^3b_3^5 + 6a_3^3b_3^7b_4 - 6a_3^3b_3^5b_4^3 - 2a_3b_3^9b_4 \\
& + 2a_4^9b_4^2 - 6a_4^8b_4^3 - 6a_4^7b_4^4 + 6a_4^6b_4^5 + 6a_4^5b_4^6 - 6a_4^3a_5^5b_5^3 + 6a_4^3a_5^3b_5^5 - 6a_4^3a_6^5b_6^3 + 6a_4^3a_6^3b_6^5 \\
& - 6a_4^3a_7^5b_7^3 + 6a_4^3a_7^3b_7^5 - 6a_4^3a_8^5b_8^3 + 6a_4^3a_8^3b_8^5 - 2a_4b_4^{10} + 2a_5^9b_4b_5 - 6a_5^7b_4b_5^3 + 6a_5^5b_4^3b_5^3 \\
& - 6a_5^3b_4^3b_5^5 + 6a_5^3b_4b_5^7 - 2a_5b_4b_5^9 + 2a_6^9b_4b_6 - 6a_6^7b_4b_6^3 + 6a_6^5b_4^3b_6^3 - 6a_6^3b_4^3b_6^5 + 6a_6^3b_4b_6^7 - 2a_6b_4b_6^9 \\
& + 2a_7^9b_4b_7 - 6a_7^7b_4b_7^3 + 6a_7^5b_4^3b_7^3 - 6a_7^3b_4^3b_7^5 + 6a_7^3b_4b_7^7 - 2a_7b_4b_7^9 + 2a_8^9b_4b_8 - 6a_8^7b_4b_8^3 + 6a_8^5b_4^3b_8^3 \\
& - 6a_8^3b_4^3b_8^5 + 6a_8^3b_4b_8^7 - 2a_8b_4b_8^9
\end{aligned}
$$

$$
\begin{aligned}
x_9 = {} & 2a_1^9a_5b_1 - 6a_1^7a_5b_1^3 + 6a_1^5a_5^3b_1^3 + 6a_1^5b_1^3b_5^3 - 6a_1^3a_5^3b_1^5 + 6a_1^3a_5b_1^7 - 6a_1^3b_1^5b_5^3 - 2a_1a_5b_1^9 \\
& + 2a_2^9a_5b_2 - 6a_2^7a_5b_2^3 + 6a_2^5a_5^3b_2^3 + 6a_2^5b_2^3b_5^3 - 6a_2^3a_5^3b_2^5 + 6a_2^3a_5b_2^7 - 6a_2^3b_2^5b_5^3 - 2a_2a_5b_2^9 \\
& + 2a_3^9a_5b_3 - 6a_3^7a_5b_3^3 + 6a_3^5a_5^3b_3^3 + 6a_3^5b_3^3b_5^3 - 6a_3^3a_5^3b_3^5 + 6a_3^3a_5b_3^7 - 6a_3^3b_3^5b_5^3 - 2a_3a_5b_3^9 \\
& + 2a_4^9a_5b_4 - 6a_4^7a_5b_4^3 + 6a_4^5a_5^3b_4^3 + 6a_4^5b_4^3b_5^3 - 6a_4^3a_5^3b_4^5 + 6a_4^3a_5b_4^7 - 6a_4^3b_4^5b_5^3 - 2a_4a_5b_4^9 + 2a_5^{10}b_5 \\
& - 6a_5^6b_5^5 + 6a_5^5b_5^6 + 6a_5^4b_5^7 + 6a_5^3a_6^5b_6^3 - 6a_5^3a_6^3b_6^5 + 6a_5^3a_7^5b_7^3 - 6a_5^3a_7^3b_7^5 + 6a_5^3a_8^5b_8^3 \\
& - 6a_5^3a_8^3b_8^5 - 6a_5^3b_5^8 - 2a_5^2b_5^9 + 2a_5a_6^9b_6 - 6a_5a_6^7b_6^3 + 6a_5a_6^3b_6^7 - 2a_5a_6b_6^9 + 2a_5a_7^9b_7 - 6a_5a_7^7b_7^3 \\
& + 6a_5a_7^3b_7^7 - 2a_5a_7b_7^9 + 2a_5a_8^9b_8 - 6a_5a_8^7b_8^3 + 6a_5a_8^3b_8^7 - 2a_5a_8b_8^9 + 6a_6^5b_5^3b_6^3 - 6a_6^3b_5^3b_6^5 + 6a_7^5b_5^3b_7^3 \\
& - 6a_7^3b_5^3b_7^5 + 6a_8^5b_5^3b_8^3 - 6a_8^3b_5^3b_8^5
\end{aligned}
$$

$$
\begin{aligned}
x_{10} = {} & 2a_1^9b_1b_5 - 6a_1^7b_1^3b_5 - 6a_1^5a_5^3b_1^3 + 6a_1^5b_1^3b_5^3 + 6a_1^3a_5^3b_1^5 + 6a_1^3b_1^7b_5 - 6a_1^3b_1^5b_5^3 - 2a_1b_1^9b_5 \\
& + 2a_2^9b_2b_5 - 6a_2^7b_2^3b_5 - 6a_2^5a_5^3b_2^3 + 6a_2^5b_2^3b_5^3 + 6a_2^3a_5^3b_2^5 + 6a_2^3b_2^7b_5 - 6a_2^3b_2^5b_5^3 - 2a_2b_2^9b_5 \\
& + 2a_3^9b_3b_5 - 6a_3^7b_3^3b_5 - 6a_3^5a_5^3b_3^3 + 6a_3^5b_3^3b_5^3 + 6a_3^3a_5^3b_3^5 + 6a_3^3b_3^7b_5 - 6a_3^3b_3^5b_5^3 - 2a_3b_3^9b_5 \\
& + 2a_4^9b_4b_5 - 6a_4^7b_4^3b_5 - 6a_4^5a_5^3b_4^3 + 6a_4^5b_4^3b_5^3 + 6a_4^3a_5^3b_4^5 + 6a_4^3b_4^7b_5 - 6a_4^3b_4^5b_5^3 - 2a_4b_4^9b_5 \\
& + 2a_5^9b_5^2 - 6a_5^8b_5^3 - 6a_5^7b_5^4 + 6a_5^6b_5^5 + 6a_5^5b_5^6 - 6a_5^3a_6^5b_6^3 + 6a_5^3a_6^3b_6^5 - 6a_5^3a_7^5b_7^3 + 6a_5^3a_7^3b_7^5 \\
& - 6a_5^3a_8^5b_8^3 + 6a_5^3a_8^3b_8^5 - 2a_5b_5^{10} + 2a_6^9b_5b_6 - 6a_6^7b_5b_6^3 + 6a_6^5b_5^3b_6^3 - 6a_6^3b_5^3b_6^5 + 6a_6^3b_5b_6^7 - 2a_6b_5b_6^9 \\
& + 2a_7^9b_5b_7 - 6a_7^7b_5b_7^3 + 6a_7^5b_5^3b_7^3 - 6a_7^3b_5^3b_7^5 + 6a_7^3b_5b_7^7 - 2a_7b_5b_7^9 + 2a_8^9b_5b_8 - 6a_8^7b_5b_8^3 + 6a_8^5b_5^3b_8^3 \\
& - 6a_8^3b_5^3b_8^5 + 6a_8^3b_5b_8^7 - 2a_8b_5b_8^9
\end{aligned}
$$

$$\begin{aligned}
x_{11} = {} & 2a_1^9a_6b_1 - 6a_1^7a_6b_1^3 + 6a_1^5a_6^3b_1^3 + 6a_1^5b_1^3b_6^3 - 6a_1^3a_6^3b_1^5 + 6a_1^3a_6b_1^7 - 6a_1^3b_1^5b_6^3 - 2a_1a_6b_1^9 \\
& + 2a_2^9a_6b_2 - 6a_2^7a_6b_2^3 + 6a_2^5a_6^3b_2^3 + 6a_2^5b_2^3b_6^3 - 6a_2^3a_6^3b_2^5 + 6a_2^3a_6b_2^7 - 6a_2^3b_2^5b_6^3 - 2a_2a_6b_2^9 \\
& + 2a_3^9a_6b_3 - 6a_3^7a_6b_3^3 + 6a_3^5a_6^3b_3^3 + 6a_3^5b_3^3b_6^3 - 6a_3^3a_6^3b_3^5 + 6a_3^3a_6b_3^7 - 6a_3^3b_3^5b_6^3 - 2a_3a_6b_3^9 \\
& + 2a_4^9a_6b_4 - 6a_4^7a_6b_4^3 + 6a_4^5a_6^3b_4^3 + 6a_4^5b_4^3b_6^3 - 6a_4^3a_6^3b_4^5 + 6a_4^3a_6b_4^7 - 6a_4^3b_4^5b_6^3 - 2a_4a_6b_4^9 \\
& + 2a_5^9a_6b_5 - 6a_5^7a_6b_5^3 + 6a_5^5a_6^3b_5^3 + 6a_5^5b_5^3b_6^3 - 6a_5^3a_6^3b_5^5 + 6a_5^3a_6b_5^7 - 6a_5^3b_5^5b_6^3 - 2a_5a_6b_5^9 + 2a_6^{10}b_6 \\
& - 6a_6^6b_6^5 + 6a_6^5b_6^6 + 6a_6^4b_6^7 + 6a_6^3a_7^5b_7^3 - 6a_6^3a_7^3b_7^5 + 6a_6^3a_8^5b_8^3 - 6a_6^3a_8^3b_8^5 - 6a_6^3b_6^8 - 2a_6^2b_6^9 \\
& + 2a_6a_7^9b_7 - 6a_6a_7^7b_7^3 + 6a_6a_7^3b_7^7 - 2a_6a_7b_7^9 + 2a_6a_8^9b_8 - 6a_6a_8^7b_8^3 + 6a_6a_8^3b_8^7 - 2a_6a_8b_8^9 + 6a_7^5b_6^3b_7^3 \\
& - 6a_7^3b_6^3b_7^5 + 6a_8^5b_6^3b_8^3 - 6a_8^3b_6^3b_8^5
\end{aligned}$$

$$\begin{aligned}
x_{12} = {} & 2a_1^9b_1b_6 - 6a_1^7b_1^3b_6 - 6a_1^5a_6^3b_1^3 + 6a_1^5b_1^3b_6^3 + 6a_1^3a_6^3b_1^5 + 6a_1^3b_1^7b_6 - 6a_1^3b_1^5b_6^3 - 2a_1b_1^9b_6 \\
& + 2a_2^9b_2b_6 - 6a_2^7b_2^3b_6 - 6a_2^5a_6^3b_2^3 + 6a_2^5b_2^3b_6^3 + 6a_2^3a_6^3b_2^5 + 6a_2^3b_2^7b_6 - 6a_2^3b_2^5b_6^3 - 2a_2b_2^9b_6 \\
& + 2a_3^9b_3b_6 - 6a_3^7b_3^3b_6 - 6a_3^5a_6^3b_3^3 + 6a_3^5b_3^3b_6^3 + 6a_3^3a_6^3b_3^5 + 6a_3^3b_3^7b_6 - 6a_3^3b_3^5b_6^3 - 2a_3b_3^9b_6 \\
& + 2a_4^9b_4b_6 - 6a_4^7b_4^3b_6 - 6a_4^5a_6^3b_4^3 + 6a_4^5b_4^3b_6^3 + 6a_4^3a_6^3b_4^5 + 6a_4^3b_4^7b_6 - 6a_4^3b_4^5b_6^3 - 2a_4b_4^9b_6 \\
& + 2a_5^9b_5b_6 - 6a_5^7b_5^3b_6 - 6a_5^5a_6^3b_5^3 + 6a_5^5b_5^3b_6^3 + 6a_5^3a_6^3b_5^5 + 6a_5^3b_5^7b_6 - 6a_5^3b_5^5b_6^3 - 2a_5b_5^9b_6 \\
& + 2a_6^9b_6^2 - 6a_6^8b_6^3 - 6a_6^7b_6^4 + 6a_6^6b_6^5 + 6a_6^5b_6^6 - 6a_6^3a_7^5b_7^3 + 6a_6^3a_7^3b_7^5 - 6a_6^3a_8^5b_8^3 + 6a_6^3a_8^3b_8^5 \\
& - 2a_6b_6^{10} + 2a_7^9b_6b_7 - 6a_7^7b_6b_7^3 + 6a_7^5b_6^3b_7^3 - 6a_7^3b_6^3b_7^5 + 6a_7^3b_6b_7^7 - 2a_7b_6b_7^9 + 2a_8^9b_6b_8 - 6a_8^7b_6b_8^3 \\
& + 6a_8^5b_6^3b_8^3 - 6a_8^3b_6^3b_8^5 + 6a_8^3b_6b_8^7 - 2a_8b_6b_8^9
\end{aligned}$$

$$\begin{aligned}
x_{13} = {} & 2a_1^9a_7b_1 - 6a_1^7a_7b_1^3 + 6a_1^5a_7^3b_1^3 + 6a_1^5b_1^3b_7^3 - 6a_1^3a_7^3b_1^5 + 6a_1^3a_7b_1^7 - 6a_1^3b_1^5b_7^3 - 2a_1a_7b_1^9 \\
& + 2a_2^9a_7b_2 - 6a_2^7a_7b_2^3 + 6a_2^5a_7^3b_2^3 + 6a_2^5b_2^3b_7^3 - 6a_2^3a_7^3b_2^5 + 6a_2^3a_7b_2^7 - 6a_2^3b_2^5b_7^3 - 2a_2a_7b_2^9 \\
& + 2a_3^9a_7b_3 - 6a_3^7a_7b_3^3 + 6a_3^5a_7^3b_3^3 + 6a_3^5b_3^3b_7^3 - 6a_3^3a_7^3b_3^5 + 6a_3^3a_7b_3^7 - 6a_3^3b_3^5b_7^3 - 2a_3a_7b_3^9 \\
& + 2a_4^9a_7b_4 - 6a_4^7a_7b_4^3 + 6a_4^5a_7^3b_4^3 + 6a_4^5b_4^3b_7^3 - 6a_4^3a_7^3b_4^5 + 6a_4^3a_7b_4^7 - 6a_4^3b_4^5b_7^3 - 2a_4a_7b_4^9 \\
& + 2a_5^9a_7b_5 - 6a_5^7a_7b_5^3 + 6a_5^5a_7^3b_5^3 + 6a_5^5b_5^3b_7^3 - 6a_5^3a_7^3b_5^5 + 6a_5^3a_7b_5^7 - 6a_5^3b_5^5b_7^3 - 2a_5a_7b_5^9 \\
& + 2a_6^9a_7b_6 - 6a_6^7a_7b_6^3 + 6a_6^5a_7^3b_6^3 + 6a_6^5b_6^3b_7^3 - 6a_6^3a_7^3b_6^5 + 6a_6^3a_7b_6^7 - 6a_6^3b_6^5b_7^3 - 2a_6a_7b_6^9 + 2a_7^{10}b_7 \\
& - 6a_7^6b_7^5 + 6a_7^5b_7^6 + 6a_7^4b_7^7 + 6a_7^3a_8^5b_8^3 - 6a_7^3a_8^3b_8^5 - 6a_7^3b_7^8 - 2a_7^2b_7^9 + 2a_7a_8^9b_8 - 6a_7a_8^7b_8^3 \\
& + 6a_7a_8^3b_8^7 - 2a_7a_8b_8^9 + 6a_8^5b_7^3b_8^3 - 6a_8^3b_7^3b_8^5
\end{aligned}$$

$$\begin{aligned}
x_{14} = {} & 2a_1^9b_1b_7 - 6a_1^7b_1^3b_7 - 6a_1^5a_7^3b_1^3 + 6a_1^5b_1^3b_7^3 + 6a_1^3a_7^3b_1^5 + 6a_1^3b_1^7b_7 - 6a_1^3b_1^5b_7^3 - 2a_1b_1^9b_7 \\
& + 2a_2^9b_2b_7 - 6a_2^7b_2^3b_7 - 6a_2^5a_7^3b_2^3 + 6a_2^5b_2^3b_7^3 + 6a_2^3a_7^3b_2^5 + 6a_2^3b_2^7b_7 - 6a_2^3b_2^5b_7^3 - 2a_2b_2^9b_7 \\
& + 2a_3^9b_3b_7 - 6a_3^7b_3^3b_7 - 6a_3^5a_7^3b_3^3 + 6a_3^5b_3^3b_7^3 + 6a_3^3a_7^3b_3^5 + 6a_3^3b_3^7b_7 - 6a_3^3b_3^5b_7^3 - 2a_3b_3^9b_7 \\
& + 2a_4^9b_4b_7 - 6a_4^7b_4^3b_7 - 6a_4^5a_7^3b_4^3 + 6a_4^5b_4^3b_7^3 + 6a_4^3a_7^3b_4^5 + 6a_4^3b_4^7b_7 - 6a_4^3b_4^5b_7^3 - 2a_4b_4^9b_7 \\
& + 2a_5^9b_5b_7 - 6a_5^7b_5^3b_7 - 6a_5^5a_7^3b_5^3 + 6a_5^5b_5^3b_7^3 + 6a_5^3a_7^3b_5^5 + 6a_5^3b_5^7b_7 - 6a_5^3b_5^5b_7^3 - 2a_5b_5^9b_7 \\
& + 2a_6^9b_6b_7 - 6a_6^7b_6^3b_7 - 6a_6^5a_7^3b_6^3 + 6a_6^5b_6^3b_7^3 + 6a_6^3a_7^3b_6^5 + 6a_6^3b_6^7b_7 - 6a_6^3b_6^5b_7^3 - 2a_6b_6^9b_7 \\
& + 2a_7^9b_7^2 - 6a_7^8b_7^3 - 6a_7^7b_7^4 + 6a_7^6b_7^5 + 6a_7^5b_7^6 - 6a_7^3a_8^5b_8^3 + 6a_7^3a_8^3b_8^5 - 2a_7b_7^{10} + 2a_8^9b_7b_8 - 6a_8^7b_7b_8^3 \\
& + 6a_8^5b_7^3b_8^3 - 6a_8^3b_7^3b_8^5 + 6a_8^3b_7b_8^7 - 2a_8b_7b_8^9
\end{aligned}$$

$$
\begin{aligned}
x_{15} = {} & 2a_1^9a_8b_1 - 6a_1^7a_8b_1^3 + 6a_1^5a_8^3b_1^3 + 6a_1^5b_1^3b_8^3 - 6a_1^3a_8^3b_1^5 + 6a_1^3a_8b_1^7 - 6a_1^3b_1^5b_8^3 - 2a_1a_8b_1^9 \\
& + 2a_2^9a_8b_2 - 6a_2^7a_8b_2^3 + 6a_2^5a_8^3b_2^3 + 6a_2^5b_2^3b_8^3 - 6a_2^3a_8^3b_2^5 + 6a_2^3a_8b_2^7 - 6a_2^3b_2^5b_8^3 - 2a_2a_8b_2^9 \\
& + 2a_3^9a_8b_3 - 6a_3^7a_8b_3^3 + 6a_3^5a_8^3b_3^3 + 6a_3^5b_3^3b_8^3 - 6a_3^3a_8^3b_3^5 + 6a_3^3a_8b_3^7 - 6a_3^3b_3^5b_8^3 - 2a_3a_8b_3^9 \\
& + 2a_4^9a_8b_4 - 6a_4^7a_8b_4^3 + 6a_4^5a_8^3b_4^3 + 6a_4^5b_4^3b_8^3 - 6a_4^3a_8^3b_4^5 + 6a_4^3a_8b_4^7 - 6a_4^3b_4^5b_8^3 - 2a_4a_8b_4^9 \\
& + 2a_5^9a_8b_5 - 6a_5^7a_8b_5^3 + 6a_5^5a_8^3b_5^3 + 6a_5^5b_5^3b_8^3 - 6a_5^3a_8^3b_5^5 + 6a_5^3a_8b_5^7 - 6a_5^3b_5^5b_8^3 - 2a_5a_8b_5^9 \\
& + 2a_6^9a_8b_6 - 6a_6^7a_8b_6^3 + 6a_6^5a_8^3b_6^3 + 6a_6^5b_6^3b_8^3 - 6a_6^3a_8^3b_6^5 + 6a_6^3a_8b_6^7 - 6a_6^3b_6^5b_8^3 - 2a_6a_8b_6^9 \\
& + 2a_7^9a_8b_7 - 6a_7^7a_8b_7^3 + 6a_7^5a_8^3b_7^3 + 6a_7^5b_7^3b_8^3 - 6a_7^3a_8^3b_7^5 + 6a_7^3a_8b_7^7 - 6a_7^3b_7^5b_8^3 - 2a_7a_8b_7^9 + 2a_8^{10}b_8 \\
& - 6a_8^6b_8^5 + 6a_8^5b_8^6 + 6a_8^4b_8^7 - 6a_8^3b_8^8 - 2a_8^2b_8^9
\end{aligned}
$$

$$
\begin{aligned}
x_{16} = {} & 2a_1^9b_1b_8 - 6a_1^7b_1^3b_8 - 6a_1^5a_8^3b_1^3 + 6a_1^5b_1^3b_8^3 + 6a_1^3a_8^3b_1^5 + 6a_1^3b_1^7b_8 - 6a_1^3b_1^5b_8^3 - 2a_1b_1^9b_8 \\
& + 2a_2^9b_2b_8 - 6a_2^7b_2^3b_8 - 6a_2^5a_8^3b_2^3 + 6a_2^5b_2^3b_8^3 + 6a_2^3a_8^3b_2^5 + 6a_2^3b_2^7b_8 - 6a_2^3b_2^5b_8^3 - 2a_2b_2^9b_8 \\
& + 2a_3^9b_3b_8 - 6a_3^7b_3^3b_8 - 6a_3^5a_8^3b_3^3 + 6a_3^5b_3^3b_8^3 + 6a_3^3a_8^3b_3^5 + 6a_3^3b_3^7b_8 - 6a_3^3b_3^5b_8^3 - 2a_3b_3^9b_8 \\
& + 2a_4^9b_4b_8 - 6a_4^7b_4^3b_8 - 6a_4^5a_8^3b_4^3 + 6a_4^5b_4^3b_8^3 + 6a_4^3a_8^3b_4^5 + 6a_4^3b_4^7b_8 - 6a_4^3b_4^5b_8^3 - 2a_4b_4^9b_8 \\
& + 2a_5^9b_5b_8 - 6a_5^7b_5^3b_8 - 6a_5^5a_8^3b_5^3 + 6a_5^5b_5^3b_8^3 + 6a_5^3a_8^3b_5^5 + 6a_5^3b_5^7b_8 - 6a_5^3b_5^5b_8^3 - 2a_5b_5^9b_8 \\
& + 2a_6^9b_6b_8 - 6a_6^7b_6^3b_8 - 6a_6^5a_8^3b_6^3 + 6a_6^5b_6^3b_8^3 + 6a_6^3a_8^3b_6^5 + 6a_6^3b_6^7b_8 - 6a_6^3b_6^5b_8^3 - 2a_6b_6^9b_8 \\
& + 2a_7^9b_7b_8 - 6a_7^7b_7^3b_8 - 6a_7^5a_8^3b_7^3 + 6a_7^5b_7^3b_8^3 + 6a_7^3a_8^3b_7^5 + 6a_7^3b_7^7b_8 - 6a_7^3b_7^5b_8^3 - 2a_7b_7^9b_8 \\
& + 2a_8^9b_8^2 - 6a_8^8b_8^3 - 6a_8^7b_8^4 + 6a_8^6b_8^5 + 6a_8^5b_8^6 - 2a_8b_8^{10}
\end{aligned}
$$

$$
\begin{aligned}
y_1 = {} & 2a_1^{10}b_1 - 6a_1^6b_1^5 - 6a_1^5b_1^6 + 6a_1^4b_1^7 + 6a_1^3a_2^5b_2^3 - 6a_1^3a_2^3b_2^5 + 6a_1^3a_3^5b_3^3 - 6a_1^3a_3^3b_3^5 + 6a_1^3a_4^5b_4^3 \\
& - 6a_1^3a_4^3b_4^5 + 6a_1^3a_5^5b_5^3 - 6a_1^3a_5^3b_5^5 + 6a_1^3a_6^5b_6^3 - 6a_1^3a_6^3b_6^5 + 6a_1^3a_7^5b_7^3 - 6a_1^3a_7^3b_7^5 \\
& + 6a_1^3a_8^5b_8^3 - 6a_1^3a_8^3b_8^5 + 6a_1^3b_1^8 - 2a_1^2b_1^9 + 2a_1a_2^9b_2 - 6a_1a_2^7b_2^3 + 6a_1a_2^3b_2^7 - 2a_1a_2b_2^9 + 2a_1a_3^9b_3 \\
& - 6a_1a_3^7b_3^3 + 6a_1a_3^3b_3^7 - 2a_1a_3b_3^9 + 2a_1a_4^9b_4 - 6a_1a_4^7b_4^3 + 6a_1a_4^3b_4^7 - 2a_1a_4b_4^9 + 2a_1a_5^9b_5 - 6a_1a_5^7b_5^3 \\
& + 6a_1a_5^3b_5^7 - 2a_1a_5b_5^9 + 2a_1a_6^9b_6 - 6a_1a_6^7b_6^3 + 6a_1a_6^3b_6^7 - 2a_1a_6b_6^9 + 2a_1a_7^9b_7 - 6a_1a_7^7b_7^3 + 6a_1a_7^3b_7^7 \\
& - 2a_1a_7b_7^9 + 2a_1a_8^9b_8 - 6a_1a_8^7b_8^3 + 6a_1a_8^3b_8^7 - 2a_1a_8b_8^9 - 6a_2^5b_1^3b_2^3 + 6a_2^3b_1^3b_2^5 - 6a_3^5b_1^3b_3^3 \\
& + 6a_3^3b_1^3b_3^5 - 6a_4^5b_1^3b_4^3 + 6a_4^3b_1^3b_4^5 - 6a_5^5b_1^3b_5^3 + 6a_5^3b_1^3b_5^5 - 6a_6^5b_1^3b_6^3 + 6a_6^3b_1^3b_6^5 \\
& - 6a_7^5b_1^3b_7^3 + 6a_7^3b_1^3b_7^5 - 6a_8^5b_1^3b_8^3 + 6a_8^3b_1^3b_8^5
\end{aligned}
$$

$$
\begin{aligned}
y_2 = {} & 2a_1^9b_1^2 + 6a_1^8b_1^3 - 6a_1^7b_1^4 - 6a_1^6b_1^5 + 6a_1^5b_1^6 + 6a_1^3a_2^5b_2^3 - 6a_1^3a_2^3b_2^5 + 6a_1^3a_3^5b_3^3 - 6a_1^3a_3^3b_3^5 \\
& + 6a_1^3a_4^5b_4^3 - 6a_1^3a_4^3b_4^5 + 6a_1^3a_5^5b_5^3 - 6a_1^3a_5^3b_5^5 + 6a_1^3a_6^5b_6^3 - 6a_1^3a_6^3b_6^5 + 6a_1^3a_7^5b_7^3 \\
& - 6a_1^3a_7^3b_7^5 + 6a_1^3a_8^5b_8^3 - 6a_1^3a_8^3b_8^5 - 2a_1b_1^{10} + 2a_2^9b_1b_2 - 6a_2^7b_1b_2^3 + 6a_2^5b_1^3b_2^3 - 6a_2^3b_1^3b_2^5 \\
& + 6a_2^3b_1b_2^7 - 2a_2b_1b_2^9 + 2a_3^9b_1b_3 - 6a_3^7b_1b_3^3 + 6a_3^5b_1^3b_3^3 - 6a_3^3b_1^3b_3^5 + 6a_3^3b_1b_3^7 - 2a_3b_1b_3^9 + 2a_4^9b_1b_4 \\
& - 6a_4^7b_1b_4^3 + 6a_4^5b_1^3b_4^3 - 6a_4^3b_1^3b_4^5 + 6a_4^3b_1b_4^7 - 2a_4b_1b_4^9 + 2a_5^9b_1b_5 - 6a_5^7b_1b_5^3 + 6a_5^5b_1^3b_5^3 \\
& - 6a_5^3b_1^3b_5^5 + 6a_5^3b_1b_5^7 - 2a_5b_1b_5^9 + 2a_6^9b_1b_6 - 6a_6^7b_1b_6^3 + 6a_6^5b_1^3b_6^3 - 6a_6^3b_1^3b_6^5 + 6a_6^3b_1b_6^7 - 2a_6b_1b_6^9 \\
& + 2a_7^9b_1b_7 - 6a_7^7b_1b_7^3 + 6a_7^5b_1^3b_7^3 - 6a_7^3b_1^3b_7^5 + 6a_7^3b_1b_7^7 - 2a_7b_1b_7^9 + 2a_8^9b_1b_8 - 6a_8^7b_1b_8^3 + 6a_8^5b_1^3b_8^3 \\
& - 6a_8^3b_1^3b_8^5 + 6a_8^3b_1b_8^7 - 2a_8b_1b_8^9
\end{aligned}
$$

$$
\begin{aligned}
y_3 = {} & 2a_1^9 a_2 b_1 - 6a_1^7 a_2 b_1^3 + 6a_1^5 a_2^3 b_1^3 - 6a_1^5 b_1^3 b_2^3 - 6a_1^3 a_2^3 b_1^5 + 6a_1^3 a_2 b_1^7 + 6a_1^3 b_1^5 b_2^3 - 2a_1 a_2 b_1^9 + 2a_2^{10} b_2 \\
& - 6a_2^6 b_2^5 - 6a_2^5 b_2^6 + 6a_2^4 b_2^7 + 6a_2^3 a_3^5 b_3^3 - 6a_2^3 a_3^3 b_3^5 + 6a_2^3 a_4^5 b_4^3 - 6a_2^3 a_4^3 b_4^5 + 6a_2^3 a_5^5 b_5^3 \\
& - 6a_2^3 a_5^3 b_5^5 + 6a_2^3 a_6^5 b_6^3 - 6a_2^3 a_6^3 b_6^5 + 6a_2^3 a_7^5 b_7^3 - 6a_2^3 a_7^3 b_7^5 + 6a_2^3 a_8^5 b_8^3 - 6a_2^3 a_8^3 b_8^5 + 6a_2^3 b_2^8 \\
& - 2a_2^2 b_2^9 + 2a_2 a_3^9 b_3 - 6a_2 a_3^7 b_3^3 + 6a_2 a_3^3 b_3^7 - 2a_2 a_3 b_3^9 + 2a_2 a_4^9 b_4 - 6a_2 a_4^7 b_4^3 + 6a_2 a_4^3 b_4^7 - 2a_2 a_4 b_4^9 \\
& + 2a_2 a_5^9 b_5 - 6a_2 a_5^7 b_5^3 + 6a_2 a_5^3 b_5^7 - 2a_2 a_5 b_5^9 + 2a_2 a_6^9 b_6 - 6a_2 a_6^7 b_6^3 + 6a_2 a_6^3 b_6^7 - 2a_2 a_6 b_6^9 + 2a_2 a_7^9 b_7 \\
& - 6a_2 a_7^7 b_7^3 + 6a_2 a_7^3 b_7^7 - 2a_2 a_7 b_7^9 + 2a_2 a_8^9 b_8 - 6a_2 a_8^7 b_8^3 + 6a_2 a_8^3 b_8^7 - 2a_2 a_8 b_8^9 - 6a_3^5 b_2^3 b_3^3 + 6a_3^3 b_2^3 b_3^5 \\
& - 6a_4^5 b_2^3 b_4^3 + 6a_4^3 b_2^3 b_4^5 - 6a_5^5 b_2^3 b_5^3 + 6a_5^3 b_2^3 b_5^5 - 6a_6^5 b_2^3 b_6^3 + 6a_6^3 b_2^3 b_6^5 - 6a_7^5 b_2^3 b_7^3 \\
& + 6a_7^3 b_2^3 b_7^5 - 6a_8^5 b_2^3 b_8^3 + 6a_8^3 b_2^3 b_8^5
\end{aligned}
$$

$$
\begin{aligned}
y_4 = {} & 2a_1^9 b_1 b_2 - 6a_1^7 b_1^3 b_2 + 6a_1^5 a_2^3 b_1^3 + 6a_1^5 b_1^3 b_2^3 - 6a_1^3 a_2^3 b_1^5 + 6a_1^3 b_1^7 b_2 - 6a_1^3 b_1^5 b_2^3 - 2a_1 b_1^9 b_2 + 2a_2^9 b_2^2 \\
& + 6a_2^8 b_2^3 - 6a_2^7 b_2^4 - 6a_2^6 b_2^5 + 6a_2^5 b_2^6 + 6a_2^3 a_3^5 b_3^3 - 6a_2^3 a_3^3 b_3^5 + 6a_2^3 a_4^5 b_4^3 - 6a_2^3 a_4^3 b_4^5 + 6a_2^3 a_5^5 b_5^3 \\
& - 6a_2^3 a_5^3 b_5^5 + 6a_2^3 a_6^5 b_6^3 - 6a_2^3 a_6^3 b_6^5 + 6a_2^3 a_7^5 b_7^3 - 6a_2^3 a_7^3 b_7^5 + 6a_2^3 a_8^5 b_8^3 - 6a_2^3 a_8^3 b_8^5 - 2a_2 b_2^{10} \\
& + 2a_3^9 b_2 b_3 - 6a_3^7 b_2 b_3^3 + 6a_3^5 b_2^3 b_3^3 - 6a_3^3 b_2^3 b_3^5 + 6a_3^3 b_2 b_3^7 - 2a_3 b_2 b_3^9 + 2a_4^9 b_2 b_4 - 6a_4^7 b_2 b_4^3 + 6a_4^5 b_2^3 b_4^3 \\
& - 6a_4^3 b_2^3 b_4^5 + 6a_4^3 b_2 b_4^7 - 2a_4 b_2 b_4^9 + 2a_5^9 b_2 b_5 - 6a_5^7 b_2 b_5^3 + 6a_5^5 b_2^3 b_5^3 - 6a_5^3 b_2^3 b_5^5 + 6a_5^3 b_2 b_5^7 - 2a_5 b_2 b_5^9 \\
& + 2a_6^9 b_2 b_6 - 6a_6^7 b_2 b_6^3 + 6a_6^5 b_2^3 b_6^3 - 6a_6^3 b_2^3 b_6^5 + 6a_6^3 b_2 b_6^7 - 2a_6 b_2 b_6^9 + 2a_7^9 b_2 b_7 - 6a_7^7 b_2 b_7^3 + 6a_7^5 b_2^3 b_7^3 \\
& - 6a_7^3 b_2^3 b_7^5 + 6a_7^3 b_2 b_7^7 - 2a_7 b_2 b_7^9 + 2a_8^9 b_2 b_8 - 6a_8^7 b_2 b_8^3 + 6a_8^5 b_2^3 b_8^3 - 6a_8^3 b_2^3 b_8^5 + 6a_8^3 b_2 b_8^7 - 2a_8 b_2 b_8^9
\end{aligned}
$$

$$
\begin{aligned}
y_5 = {} & 2a_1^9 a_3 b_1 - 6a_1^7 a_3 b_1^3 + 6a_1^5 a_3^3 b_1^3 - 6a_1^5 b_1^3 b_3^3 - 6a_1^3 a_3^3 b_1^5 + 6a_1^3 a_3 b_1^7 + 6a_1^3 b_1^5 b_3^3 - 2a_1 a_3 b_1^9 \\
& + 2a_2^9 a_3 b_2 - 6a_2^7 a_3 b_2^3 + 6a_2^5 a_3^3 b_2^3 - 6a_2^5 b_2^3 b_3^3 - 6a_2^3 a_3^3 b_2^5 + 6a_2^3 a_3 b_2^7 + 6a_2^3 b_2^5 b_3^3 - 2a_2 a_3 b_2^9 + 2a_3^{10} b_3 \\
& - 6a_3^6 b_3^5 - 6a_3^5 b_3^6 + 6a_3^4 b_3^7 + 6a_3^3 a_4^5 b_4^3 - 6a_3^3 a_4^3 b_4^5 + 6a_3^3 a_5^5 b_5^3 - 6a_3^3 a_5^3 b_5^5 + 6a_3^3 a_6^5 b_6^3 \\
& - 6a_3^3 a_6^3 b_6^5 + 6a_3^3 a_7^5 b_7^3 - 6a_3^3 a_7^3 b_7^5 + 6a_3^3 a_8^5 b_8^3 - 6a_3^3 a_8^3 b_8^5 + 6a_3^3 b_3^8 - 2a_3^2 b_3^9 + 2a_3 a_4^9 b_4 \\
& - 6a_3 a_4^7 b_4^3 + 6a_3 a_4^3 b_4^7 - 2a_3 a_4 b_4^9 + 2a_3 a_5^9 b_5 - 6a_3 a_5^7 b_5^3 + 6a_3 a_5^3 b_5^7 - 2a_3 a_5 b_5^9 + 2a_3 a_6^9 b_6 - 6a_3 a_6^7 b_6^3 \\
& + 6a_3 a_6^3 b_6^7 - 2a_3 a_6 b_6^9 + 2a_3 a_7^9 b_7 - 6a_3 a_7^7 b_7^3 + 6a_3 a_7^3 b_7^7 - 2a_3 a_7 b_7^9 + 2a_3 a_8^9 b_8 - 6a_3 a_8^7 b_8^3 + 6a_3 a_8^3 b_8^7 \\
& - 2a_3 a_8 b_8^9 - 6a_4^5 b_3^3 b_4^3 + 6a_4^3 b_3^3 b_4^5 - 6a_5^5 b_3^3 b_5^3 + 6a_5^3 b_3^3 b_5^5 - 6a_6^5 b_3^3 b_6^3 + 6a_6^3 b_3^3 b_6^5 - 6a_7^5 b_3^3 b_7^3 \\
& + 6a_7^3 b_3^3 b_7^5 - 6a_8^5 b_3^3 b_8^3 + 6a_8^3 b_3^3 b_8^5
\end{aligned}
$$

$$
\begin{aligned}
y_6 = {} & 2a_1^9 b_1 b_3 - 6a_1^7 b_1^3 b_3 + 6a_1^5 a_3^3 b_1^3 + 6a_1^5 b_1^3 b_3^3 - 6a_1^3 a_3^3 b_1^5 + 6a_1^3 b_1^7 b_3 - 6a_1^3 b_1^5 b_3^3 - 2a_1 b_1^9 b_3 \\
& + 2a_2^9 b_2 b_3 - 6a_2^7 b_2^3 b_3 + 6a_2^5 a_3^3 b_2^3 + 6a_2^5 b_2^3 b_3^3 - 6a_2^3 a_3^3 b_2^5 + 6a_2^3 b_2^7 b_3 - 6a_2^3 b_2^5 b_3^3 - 2a_2 b_2^9 b_3 \\
& + 2a_3^9 b_3^2 + 6a_3^8 b_3^3 - 6a_3^7 b_3^4 - 6a_3^6 b_3^5 + 6a_3^5 b_3^6 + 6a_3^3 a_4^5 b_4^3 - 6a_3^3 a_4^3 b_4^5 + 6a_3^3 a_5^5 b_5^3 - 6a_3^3 a_5^3 b_5^5 \\
& + 6a_3^3 a_6^5 b_6^3 - 6a_3^3 a_6^3 b_6^5 + 6a_3^3 a_7^5 b_7^3 - 6a_3^3 a_7^3 b_7^5 + 6a_3^3 a_8^5 b_8^3 - 6a_3^3 a_8^3 b_8^5 - 2a_3 b_3^{10} + 2a_4^9 b_3 b_4 \\
& - 6a_4^7 b_3 b_4^3 + 6a_4^5 b_3^3 b_4^3 - 6a_4^3 b_3^3 b_4^5 + 6a_4^3 b_3 b_4^7 - 2a_4 b_3 b_4^9 + 2a_5^9 b_3 b_5 - 6a_5^7 b_3 b_5^3 + 6a_5^5 b_3^3 b_5^3 \\
& - 6a_5^3 b_3^3 b_5^5 + 6a_5^3 b_3 b_5^7 - 2a_5 b_3 b_5^9 + 2a_6^9 b_3 b_6 - 6a_6^7 b_3 b_6^3 + 6a_6^5 b_3^3 b_6^3 - 6a_6^3 b_3^3 b_6^5 + 6a_6^3 b_3 b_6^7 - 2a_6 b_3 b_6^9 \\
& + 2a_7^9 b_3 b_7 - 6a_7^7 b_3 b_7^3 + 6a_7^5 b_3^3 b_7^3 - 6a_7^3 b_3^3 b_7^5 + 6a_7^3 b_3 b_7^7 - 2a_7 b_3 b_7^9 + 2a_8^9 b_3 b_8 - 6a_8^7 b_3 b_8^3 + 6a_8^5 b_3^3 b_8^3 \\
& - 6a_8^3 b_3^3 b_8^5 + 6a_8^3 b_3 b_8^7 - 2a_8 b_3 b_8^9
\end{aligned}
$$

$$
\begin{aligned}
y_7 = {} & 2a_1^9a_4b_1 - 6a_1^7a_4b_1^3 + 6a_1^5a_4^3b_1^3 - 6a_1^5b_1^3b_4^3 - 6a_1^3a_4^3b_1^5 + 6a_1^3a_4b_1^7 + 6a_1^3b_1^5b_4^3 - 2a_1a_4b_1^9 \\
& + 2a_2^9a_4b_2 - 6a_2^7a_4b_2^3 + 6a_2^5a_4^3b_2^3 - 6a_2^5b_2^3b_4^3 - 6a_2^3a_4^3b_2^5 + 6a_2^3a_4b_2^7 + 6a_2^3b_2^5b_4^3 - 2a_2a_4b_2^9 \\
& + 2a_3^9a_4b_3 - 6a_3^7a_4b_3^3 + 6a_3^5a_4^3b_3^3 - 6a_3^5b_3^3b_4^3 - 6a_3^3a_4^3b_3^5 + 6a_3^3a_4b_3^7 + 6a_3^3b_3^5b_4^3 - 2a_3a_4b_3^9 + 2a_4^{10}b_4 \\
& - 6a_4^6b_4^5 - 6a_4^5b_4^6 + 6a_4^4b_4^7 + 6a_4^3a_5^5b_5^3 - 6a_4^3a_5^3b_5^5 + 6a_4^3a_6^5b_6^3 - 6a_4^3a_6^3b_6^5 + 6a_4^3a_7^5b_7^3 \\
& - 6a_4^3a_7^3b_7^5 + 6a_4^3a_8^5b_8^3 - 6a_4^3a_8^3b_8^5 + 6a_4^3b_4^8 - 2a_4^2b_4^9 + 2a_4a_5^9b_5 - 6a_4a_5^7b_5^3 + 6a_4a_5^3b_5^7 - 2a_4a_5b_5^9 \\
& + 2a_4a_6^9b_6 - 6a_4a_6^7b_6^3 + 6a_4a_6^3b_6^7 - 2a_4a_6b_6^9 + 2a_4a_7^9b_7 - 6a_4a_7^7b_7^3 + 6a_4a_7^3b_7^7 - 2a_4a_7b_7^9 + 2a_4a_8^9b_8 \\
& - 6a_4a_8^7b_8^3 + 6a_4a_8^3b_8^7 - 2a_4a_8b_8^9 - 6a_5^5b_4^3b_5^3 + 6a_5^3b_4^3b_5^5 - 6a_6^5b_4^3b_6^3 + 6a_6^3b_4^3b_6^5 - 6a_7^5b_4^3b_7^3 \\
& + 6a_7^3b_4^3b_7^5 - 6a_8^5b_4^3b_8^3 + 6a_8^3b_4^3b_8^5
\end{aligned}
$$

$$
\begin{aligned}
y_8 = {} & 2a_1^9b_1b_4 - 6a_1^7b_1^3b_4 + 6a_1^5a_4^3b_1^3 + 6a_1^5b_1^3b_4^3 - 6a_1^3a_4^3b_1^5 + 6a_1^3b_1^7b_4 - 6a_1^3b_1^5b_4^3 - 2a_1b_1^9b_4 \\
& + 2a_2^9b_2b_4 - 6a_2^7b_2^3b_4 + 6a_2^5a_4^3b_2^3 + 6a_2^5b_2^3b_4^3 - 6a_2^3a_4^3b_2^5 + 6a_2^3b_2^7b_4 - 6a_2^3b_2^5b_4^3 - 2a_2b_2^9b_4 \\
& + 2a_3^9b_3b_4 - 6a_3^7b_3^3b_4 + 6a_3^5a_4^3b_3^3 + 6a_3^5b_3^3b_4^3 - 6a_3^3a_4^3b_3^5 + 6a_3^3b_3^7b_4 - 6a_3^3b_3^5b_4^3 - 2a_3b_3^9b_4 \\
& + 2a_4^9b_4^2 + 6a_4^8b_4^3 - 6a_4^7b_4^4 - 6a_4^6b_4^5 + 6a_4^5b_4^6 + 6a_4^3a_5^5b_5^3 - 6a_4^3a_5^3b_5^5 + 6a_4^3a_6^5b_6^3 - 6a_4^3a_6^3b_6^5 \\
& + 6a_4^3a_7^5b_7^3 - 6a_4^3a_7^3b_7^5 + 6a_4^3a_8^5b_8^3 - 6a_4^3a_8^3b_8^5 - 2a_4b_4^{10} + 2a_5^9b_4b_5 - 6a_5^7b_4b_5^3 + 6a_5^5b_4^3b_5^3 \\
& - 6a_5^3b_4^3b_5^5 + 6a_5^3b_4b_5^7 - 2a_5b_4b_5^9 + 2a_6^9b_4b_6 - 6a_6^7b_4b_6^3 + 6a_6^5b_4^3b_6^3 - 6a_6^3b_4^3b_6^5 + 6a_6^3b_4b_6^7 - 2a_6b_4b_6^9 \\
& + 2a_7^9b_4b_7 - 6a_7^7b_4b_7^3 + 6a_7^5b_4^3b_7^3 - 6a_7^3b_4^3b_7^5 + 6a_7^3b_4b_7^7 - 2a_7b_4b_7^9 + 2a_8^9b_4b_8 - 6a_8^7b_4b_8^3 + 6a_8^5b_4^3b_8^3 \\
& - 6a_8^3b_4^3b_8^5 + 6a_8^3b_4b_8^7 - 2a_8b_4b_8^9
\end{aligned}
$$

$$
\begin{aligned}
y_9 = {} & 2a_1^9a_5b_1 - 6a_1^7a_5b_1^3 + 6a_1^5a_5^3b_1^3 - 6a_1^5b_1^3b_5^3 - 6a_1^3a_5^3b_1^5 + 6a_1^3a_5b_1^7 + 6a_1^3b_1^5b_5^3 - 2a_1a_5b_1^9 \\
& + 2a_2^9a_5b_2 - 6a_2^7a_5b_2^3 + 6a_2^5a_5^3b_2^3 - 6a_2^5b_2^3b_5^3 - 6a_2^3a_5^3b_2^5 + 6a_2^3a_5b_2^7 + 6a_2^3b_2^5b_5^3 - 2a_2a_5b_2^9 \\
& + 2a_3^9a_5b_3 - 6a_3^7a_5b_3^3 + 6a_3^5a_5^3b_3^3 - 6a_3^5b_3^3b_5^3 - 6a_3^3a_5^3b_3^5 + 6a_3^3a_5b_3^7 + 6a_3^3b_3^5b_5^3 - 2a_3a_5b_3^9 \\
& + 2a_4^9a_5b_4 - 6a_4^7a_5b_4^3 + 6a_4^5a_5^3b_4^3 - 6a_4^5b_4^3b_5^3 - 6a_4^3a_5^3b_4^5 + 6a_4^3a_5b_4^7 + 6a_4^3b_4^5b_5^3 - 2a_4a_5b_4^9 + 2a_5^{10}b_5 \\
& - 6a_5^6b_5^5 - 6a_5^5b_5^6 + 6a_5^4b_5^7 + 6a_5^3a_6^5b_6^3 - 6a_5^3a_6^3b_6^5 + 6a_5^3a_7^5b_7^3 - 6a_5^3a_7^3b_7^5 + 6a_5^3a_8^5b_8^3 \\
& - 6a_5^3a_8^3b_8^5 + 6a_5^3b_5^8 - 2a_5^2b_5^9 + 2a_5a_6^9b_6 - 6a_5a_6^7b_6^3 + 6a_5a_6^3b_6^7 - 2a_5a_6b_6^9 + 2a_5a_7^9b_7 - 6a_5a_7^7b_7^3 \\
& + 6a_5a_7^3b_7^7 - 2a_5a_7b_7^9 + 2a_5a_8^9b_8 - 6a_5a_8^7b_8^3 + 6a_5a_8^3b_8^7 - 2a_5a_8b_8^9 - 6a_6^5b_5^3b_6^3 + 6a_6^3b_5^3b_6^5 - 6a_7^5b_5^3b_7^3 \\
& + 6a_7^3b_5^3b_7^5 - 6a_8^5b_5^3b_8^3 + 6a_8^3b_5^3b_8^5
\end{aligned}
$$

$$
\begin{aligned}
y_{10} = {} & 2a_1^9b_1b_5 - 6a_1^7b_1^3b_5 + 6a_1^5a_5^3b_1^3 + 6a_1^5b_1^3b_5^3 - 6a_1^3a_5^3b_1^5 + 6a_1^3b_1^7b_5 - 6a_1^3b_1^5b_5^3 - 2a_1b_1^9b_5 \\
& + 2a_2^9b_2b_5 - 6a_2^7b_2^3b_5 + 6a_2^5a_5^3b_2^3 + 6a_2^5b_2^3b_5^3 - 6a_2^3a_5^3b_2^5 + 6a_2^3b_2^7b_5 - 6a_2^3b_2^5b_5^3 - 2a_2b_2^9b_5 \\
& + 2a_3^9b_3b_5 - 6a_3^7b_3^3b_5 + 6a_3^5a_5^3b_3^3 + 6a_3^5b_3^3b_5^3 - 6a_3^3a_5^3b_3^5 + 6a_3^3b_3^7b_5 - 6a_3^3b_3^5b_5^3 - 2a_3b_3^9b_5 \\
& + 2a_4^9b_4b_5 - 6a_4^7b_4^3b_5 + 6a_4^5a_5^3b_4^3 + 6a_4^5b_4^3b_5^3 - 6a_4^3a_5^3b_4^5 + 6a_4^3b_4^7b_5 - 6a_4^3b_4^5b_5^3 - 2a_4b_4^9b_5 \\
& + 2a_5^9b_5^2 + 6a_5^8b_5^3 - 6a_5^7b_5^4 - 6a_5^6b_5^5 + 6a_5^5b_5^6 + 6a_5^3a_6^5b_6^3 - 6a_5^3a_6^3b_6^5 + 6a_5^3a_7^5b_7^3 - 6a_5^3a_7^3b_7^5 \\
& + 6a_5^3a_8^5b_8^3 - 6a_5^3a_8^3b_8^5 - 2a_5b_5^{10} + 2a_6^9b_5b_6 - 6a_6^7b_5b_6^3 + 6a_6^5b_5^3b_6^3 - 6a_6^3b_5^3b_6^5 + 6a_6^3b_5b_6^7 - 2a_6b_5b_6^9 \\
& + 2a_7^9b_5b_7 - 6a_7^7b_5b_7^3 + 6a_7^5b_5^3b_7^3 - 6a_7^3b_5^3b_7^5 + 6a_7^3b_5b_7^7 - 2a_7b_5b_7^9 + 2a_8^9b_5b_8 - 6a_8^7b_5b_8^3 + 6a_8^5b_5^3b_8^3 \\
& - 6a_8^3b_5^3b_8^5 + 6a_8^3b_5b_8^7 - 2a_8b_5b_8^9
\end{aligned}
$$

$$
\begin{aligned}
y_{11} &= 2a_1^9a_6b_1 - 6a_1^7a_6b_1^3 + 6a_1^5a_6^3b_1^3 - 6a_1^5b_1^3b_6^3 - 6a_1^3a_6^3b_1^5 + 6a_1^3a_6b_1^7 + 6a_1^3b_1^5b_6^3 - 2a_1a_6b_1^9 \\
&+ 2a_2^9a_6b_2 - 6a_2^7a_6b_2^3 + 6a_2^5a_6^3b_2^3 - 6a_2^5b_2^3b_6^3 - 6a_2^3a_6^3b_2^5 + 6a_2^3a_6b_2^7 + 6a_2^3b_2^5b_6^3 - 2a_2a_6b_2^9 \\
&+ 2a_3^9a_6b_3 - 6a_3^7a_6b_3^3 + 6a_3^5a_6^3b_3^3 - 6a_3^5b_3^3b_6^3 - 6a_3^3a_6^3b_3^5 + 6a_3^3a_6b_3^7 + 6a_3^3b_3^5b_6^3 - 2a_3a_6b_3^9 \\
&+ 2a_4^9a_6b_4 - 6a_4^7a_6b_4^3 + 6a_4^5a_6^3b_4^3 - 6a_4^5b_4^3b_6^3 - 6a_4^3a_6^3b_4^5 + 6a_4^3a_6b_4^7 + 6a_4^3b_4^5b_6^3 - 2a_4a_6b_4^9 \\
&+ 2a_5^9a_6b_5 - 6a_5^7a_6b_5^3 + 6a_5^5a_6^3b_5^3 - 6a_5^5b_5^3b_6^3 - 6a_5^3a_6^3b_5^5 + 6a_5^3a_6b_5^7 + 6a_5^3b_5^5b_6^3 - 2a_5a_6b_5^9 + 2a_6^{10}b_6 \\
&- 6a_6^6b_6^5 - 6a_6^5b_6^6 + 6a_6^4b_6^7 + 6a_6^3a_7^5b_7^3 - 6a_6^3a_7^3b_7^5 + 6a_6^3a_8^5b_8^3 - 6a_6^3a_8^3b_8^5 + 6a_6^3b_6^8 - 2a_6^2b_6^9 \\
&+ 2a_6a_7^9b_7 - 6a_6a_7^7b_7^3 + 6a_6a_7^3b_7^7 - 2a_6a_7b_7^9 + 2a_6a_8^9b_8 - 6a_6a_8^7b_8^3 + 6a_6a_8^3b_8^7 - 2a_6a_8b_8^9 - 6a_7^5b_6^3b_7^3 \\
&+ 6a_7^3b_6^3b_7^5 - 6a_8^5b_6^3b_8^3 + 6a_8^3b_6^3b_8^5
\end{aligned}
$$

$$
\begin{aligned}
y_{12} &= 2a_1^9b_1b_6 - 6a_1^7b_1^3b_6 + 6a_1^5a_6^3b_1^3 + 6a_1^5b_1^3b_6^3 - 6a_1^3a_6^3b_1^5 + 6a_1^3b_1^7b_6 - 6a_1^3b_1^5b_6^3 - 2a_1b_1^9b_6 \\
&+ 2a_2^9b_2b_6 - 6a_2^7b_2^3b_6 + 6a_2^5a_6^3b_2^3 + 6a_2^5b_2^3b_6^3 - 6a_2^3a_6^3b_2^5 + 6a_2^3b_2^7b_6 - 6a_2^3b_2^5b_6^3 - 2a_2b_2^9b_6 \\
&+ 2a_3^9b_3b_6 - 6a_3^7b_3^3b_6 + 6a_3^5a_6^3b_3^3 + 6a_3^5b_3^3b_6^3 - 6a_3^3a_6^3b_3^5 + 6a_3^3b_3^7b_6 - 6a_3^3b_3^5b_6^3 - 2a_3b_3^9b_6 \\
&+ 2a_4^9b_4b_6 - 6a_4^7b_4^3b_6 + 6a_4^5a_6^3b_4^3 + 6a_4^5b_4^3b_6^3 - 6a_4^3a_6^3b_4^5 + 6a_4^3b_4^7b_6 - 6a_4^3b_4^5b_6^3 - 2a_4b_4^9b_6 \\
&+ 2a_5^9b_5b_6 - 6a_5^7b_5^3b_6 + 6a_5^5a_6^3b_5^3 + 6a_5^5b_5^3b_6^3 - 6a_5^3a_6^3b_5^5 + 6a_5^3b_5^7b_6 - 6a_5^3b_5^5b_6^3 - 2a_5b_5^9b_6 \\
&+ 2a_6^9b_6^2 + 6a_6^8b_6^3 - 6a_6^7b_6^4 - 6a_6^6b_6^5 + 6a_6^5b_6^6 + 6a_6^3a_7^5b_7^3 - 6a_6^3a_7^3b_7^5 + 6a_6^3a_8^5b_8^3 - 6a_6^3a_8^3b_8^5 \\
&- 2a_6b_6^{10} + 2a_7^9b_6b_7 - 6a_7^7b_6b_7^3 + 6a_7^5b_6^3b_7^3 - 6a_7^3b_6^3b_7^5 + 6a_7^3b_6b_7^7 - 2a_7b_6b_7^9 + 2a_8^9b_6b_8 - 6a_8^7b_6b_8^3 \\
&+ 6a_8^5b_6^3b_8^3 - 6a_8^3b_6^3b_8^5 + 6a_8^3b_6b_8^7 - 2a_8b_6b_8^9
\end{aligned}
$$

$$
\begin{aligned}
y_{13} &= 2a_1^9a_7b_1 - 6a_1^7a_7b_1^3 + 6a_1^5a_7^3b_1^3 - 6a_1^5b_1^3b_7^3 - 6a_1^3a_7^3b_1^5 + 6a_1^3a_7b_1^7 + 6a_1^3b_1^5b_7^3 - 2a_1a_7b_1^9 \\
&+ 2a_2^9a_7b_2 - 6a_2^7a_7b_2^3 + 6a_2^5a_7^3b_2^3 - 6a_2^5b_2^3b_7^3 - 6a_2^3a_7^3b_2^5 + 6a_2^3a_7b_2^7 + 6a_2^3b_2^5b_7^3 - 2a_2a_7b_2^9 \\
&+ 2a_3^9a_7b_3 - 6a_3^7a_7b_3^3 + 6a_3^5a_7^3b_3^3 - 6a_3^5b_3^3b_7^3 - 6a_3^3a_7^3b_3^5 + 6a_3^3a_7b_3^7 + 6a_3^3b_3^5b_7^3 - 2a_3a_7b_3^9 \\
&+ 2a_4^9a_7b_4 - 6a_4^7a_7b_4^3 + 6a_4^5a_7^3b_4^3 - 6a_4^5b_4^3b_7^3 - 6a_4^3a_7^3b_4^5 + 6a_4^3a_7b_4^7 + 6a_4^3b_4^5b_7^3 - 2a_4a_7b_4^9 \\
&+ 2a_5^9a_7b_5 - 6a_5^7a_7b_5^3 + 6a_5^5a_7^3b_5^3 - 6a_5^5b_5^3b_7^3 - 6a_5^3a_7^3b_5^5 + 6a_5^3a_7b_5^7 + 6a_5^3b_5^5b_7^3 - 2a_5a_7b_5^9 \\
&+ 2a_6^9a_7b_6 - 6a_6^7a_7b_6^3 + 6a_6^5a_7^3b_6^3 - 6a_6^5b_6^3b_7^3 - 6a_6^3a_7^3b_6^5 + 6a_6^3a_7b_6^7 + 6a_6^3b_6^5b_7^3 - 2a_6a_7b_6^9 + 2a_7^{10}b_7 \\
&- 6a_7^6b_7^5 - 6a_7^5b_7^6 + 6a_7^4b_7^7 + 6a_7^3a_8^5b_8^3 - 6a_7^3a_8^3b_8^5 + 6a_7^3b_7^8 - 2a_7^2b_7^9 + 2a_7a_8^9b_8 - 6a_7a_8^7b_8^3 \\
&+ 6a_7a_8^3b_8^7 - 2a_7a_8b_8^9 - 6a_8^5b_7^3b_8^3 + 6a_8^3b_7^3b_8^5
\end{aligned}
$$

$$
\begin{aligned}
y_{14} &= 2a_1^9b_1b_7 - 6a_1^7b_1^3b_7 + 6a_1^5a_7^3b_1^3 + 6a_1^5b_1^3b_7^3 - 6a_1^3a_7^3b_1^5 + 6a_1^3b_1^7b_7 - 6a_1^3b_1^5b_7^3 - 2a_1b_1^9b_7 \\
&+ 2a_2^9b_2b_7 - 6a_2^7b_2^3b_7 + 6a_2^5a_7^3b_2^3 + 6a_2^5b_2^3b_7^3 - 6a_2^3a_7^3b_2^5 + 6a_2^3b_2^7b_7 - 6a_2^3b_2^5b_7^3 - 2a_2b_2^9b_7 \\
&+ 2a_3^9b_3b_7 - 6a_3^7b_3^3b_7 + 6a_3^5a_7^3b_3^3 + 6a_3^5b_3^3b_7^3 - 6a_3^3a_7^3b_3^5 + 6a_3^3b_3^7b_7 - 6a_3^3b_3^5b_7^3 - 2a_3b_3^9b_7 \\
&+ 2a_4^9b_4b_7 - 6a_4^7b_4^3b_7 + 6a_4^5a_7^3b_4^3 + 6a_4^5b_4^3b_7^3 - 6a_4^3a_7^3b_4^5 + 6a_4^3b_4^7b_7 - 6a_4^3b_4^5b_7^3 - 2a_4b_4^9b_7 \\
&+ 2a_5^9b_5b_7 - 6a_5^7b_5^3b_7 + 6a_5^5a_7^3b_5^3 + 6a_5^5b_5^3b_7^3 - 6a_5^3a_7^3b_5^5 + 6a_5^3b_5^7b_7 - 6a_5^3b_5^5b_7^3 - 2a_5b_5^9b_7 \\
&+ 2a_6^9b_6b_7 - 6a_6^7b_6^3b_7 + 6a_6^5a_7^3b_6^3 + 6a_6^5b_6^3b_7^3 - 6a_6^3a_7^3b_6^5 + 6a_6^3b_6^7b_7 - 6a_6^3b_6^5b_7^3 - 2a_6b_6^9b_7 \\
&+ 2a_7^9b_7^2 + 6a_7^8b_7^3 - 6a_7^7b_7^4 - 6a_7^6b_7^5 + 6a_7^5b_7^6 + 6a_7^3a_8^5b_8^3 - 6a_7^3a_8^3b_8^5 - 2a_7b_7^{10} + 2a_8^9b_7b_8 - 6a_8^7b_7b_8^3 \\
&+ 6a_8^5b_7^3b_8^3 - 6a_8^3b_7^3b_8^5 + 6a_8^3b_7b_8^7 - 2a_8b_7b_8^9
\end{aligned}
$$

$$
\begin{aligned}
y_{15} = {} & 2a_1^9a_8b_1 - 6a_1^7a_8b_1^3 + 6a_1^5a_8^3b_1^3 - 6a_1^5b_1^3b_8^3 - 6a_1^3a_8^3b_1^5 + 6a_1^3a_8b_1^7 + 6a_1^3b_1^5b_8^3 - 2a_1a_8b_1^9 \\
& + 2a_2^9a_8b_2 - 6a_2^7a_8b_2^3 + 6a_2^5a_8^3b_2^3 - 6a_2^5b_2^3b_8^3 - 6a_2^3a_8^3b_2^5 + 6a_2^3a_8b_2^7 + 6a_2^3b_2^5b_8^3 - 2a_2a_8b_2^9 \\
& + 2a_3^9a_8b_3 - 6a_3^7a_8b_3^3 + 6a_3^5a_8^3b_3^3 - 6a_3^5b_3^3b_8^3 - 6a_3^3a_8^3b_3^5 + 6a_3^3a_8b_3^7 + 6a_3^3b_3^5b_8^3 - 2a_3a_8b_3^9 \\
& + 2a_4^9a_8b_4 - 6a_4^7a_8b_4^3 + 6a_4^5a_8^3b_4^3 - 6a_4^5b_4^3b_8^3 - 6a_4^3a_8^3b_4^5 + 6a_4^3a_8b_4^7 + 6a_4^3b_4^5b_8^3 - 2a_4a_8b_4^9 \\
& + 2a_5^9a_8b_5 - 6a_5^7a_8b_5^3 + 6a_5^5a_8^3b_5^3 - 6a_5^5b_5^3b_8^3 - 6a_5^3a_8^3b_5^5 + 6a_5^3a_8b_5^7 + 6a_5^3b_5^5b_8^3 - 2a_5a_8b_5^9 \\
& + 2a_6^9a_8b_6 - 6a_6^7a_8b_6^3 + 6a_6^5a_8^3b_6^3 - 6a_6^5b_6^3b_8^3 - 6a_6^3a_8^3b_6^5 + 6a_6^3a_8b_6^7 + 6a_6^3b_6^5b_8^3 - 2a_6a_8b_6^9 \\
& + 2a_7^9a_8b_7 - 6a_7^7a_8b_7^3 + 6a_7^5a_8^3b_7^3 - 6a_7^5b_7^3b_8^3 - 6a_7^3a_8^3b_7^5 + 6a_7^3a_8b_7^7 + 6a_7^3b_7^5b_8^3 - 2a_7a_8b_7^9 + 2a_8^{10}b_8 \\
& - 6a_8^6b_8^5 - 6a_8^5b_8^6 + 6a_8^4b_8^7 + 6a_8^3b_8^8 - 2a_8^2b_8^9
\end{aligned}
$$

$$
\begin{aligned}
y_{16} = {} & 2a_1^9b_1b_8 - 6a_1^7b_1^3b_8 + 6a_1^5a_8^3b_1^3 + 6a_1^5b_1^3b_8^3 - 6a_1^3a_8^3b_1^5 + 6a_1^3b_1^7b_8 - 6a_1^3b_1^5b_8^3 - 2a_1b_1^9b_8 \\
& + 2a_2^9b_2b_8 - 6a_2^7b_2^3b_8 + 6a_2^5a_8^3b_2^3 + 6a_2^5b_2^3b_8^3 - 6a_2^3a_8^3b_2^5 + 6a_2^3b_2^7b_8 - 6a_2^3b_2^5b_8^3 - 2a_2b_2^9b_8 \\
& + 2a_3^9b_3b_8 - 6a_3^7b_3^3b_8 + 6a_3^5a_8^3b_3^3 + 6a_3^5b_3^3b_8^3 - 6a_3^3a_8^3b_3^5 + 6a_3^3b_3^7b_8 - 6a_3^3b_3^5b_8^3 - 2a_3b_3^9b_8 \\
& + 2a_4^9b_4b_8 - 6a_4^7b_4^3b_8 + 6a_4^5a_8^3b_4^3 + 6a_4^5b_4^3b_8^3 - 6a_4^3a_8^3b_4^5 + 6a_4^3b_4^7b_8 - 6a_4^3b_4^5b_8^3 - 2a_4b_4^9b_8 \\
& + 2a_5^9b_5b_8 - 6a_5^7b_5^3b_8 + 6a_5^5a_8^3b_5^3 + 6a_5^5b_5^3b_8^3 - 6a_5^3a_8^3b_5^5 + 6a_5^3b_5^7b_8 - 6a_5^3b_5^5b_8^3 - 2a_5b_5^9b_8 \\
& + 2a_6^9b_6b_8 - 6a_6^7b_6^3b_8 + 6a_6^5a_8^3b_6^3 + 6a_6^5b_6^3b_8^3 - 6a_6^3a_8^3b_6^5 + 6a_6^3b_6^7b_8 - 6a_6^3b_6^5b_8^3 - 2a_6b_6^9b_8 \\
& + 2a_7^9b_7b_8 - 6a_7^7b_7^3b_8 + 6a_7^5a_8^3b_7^3 + 6a_7^5b_7^3b_8^3 - 6a_7^3a_8^3b_7^5 + 6a_7^3b_7^7b_8 - 6a_7^3b_7^5b_8^3 - 2a_7b_7^9b_8 \\
& + 2a_8^9b_8^2 + 6a_8^8b_8^3 - 6a_8^7b_8^4 - 6a_8^6b_8^5 + 6a_8^5b_8^6 - 2a_8b_8^{10}
\end{aligned}
$$

is a solution in $\mathbb{K}$ of the equation

$$x_1^4 + x_2^4 + \cdots + x_{15}^4 + x_{16}^4 = y_1^4 + y_2^4 + \cdots + y_{15}^4 + y_{16}^4.$$

Appendix. Python Code

We can use the following code excerpts to generate solutions of the cubic and quartic Diophantine equations that we presented is this paper for any integer N.

We first import from `SymPy` library:

```
from sympy import symbols
from sympy import expand
from sympy import latex
```

The following function generates a solution of

$$x_1^3 + x_2^3 + \cdots + x_{2N-1}^3 + x_{2N}^3 = y_1^3 + y_2^3 + \cdots + y_{2N-1}^3 + y_{2N}^3$$

```
def solve_cubic(N):
    """ Return a solution of the cubic equation:
    x_1^3 + x_2^3 + ... + x_{2N}^3 = y_1^3 + y_2^3 + ... + y_{2N}^3 """
    # Create symbols a_i, b_i
    a = [symbols(f'a_{i}') for i in range(N + 1)]
    b = [symbols(f'b_{i}') for i in range(N + 1)]
    #  Create lists x and y to hold solution
    x, y = [None] * (2*N + 1), [None] * (2*N + 1)
    w = 0
    lbd = 0
    for k in range(1, N + 1):
        w += (a[k]**2 - b[k]**2)**3
        lbd += a[k]**2 * b[k]**2 * (a[k] - b[k])
    lbd *= 6
    for k in range(1, N + 1):
```

```
        c_even = lbd*(a[k]**2 - b[k]**2)
        c_odd = lbd*(a[k]**2 + b[k]**2)
        x[2*k - 1] = a[k]*w + c_odd
        x[2*k] =     b[k]*w - c_even
        y[2*k - 1] = a[k]*w + c_even
        y[2*k] =     b[k]*w + c_odd
    return x, y
```

The following function generates a solution of

$$x_1^4 + x_2^4 + \cdots + x_{2N-1}^4 + x_{2N}^4 = y_1^4 + y_2^4 + \cdots + y_{2N-1}^4 + y_{2N}^4$$

```
def solve_quartic(N):
    """ Return a solution of the quartic equation:
    x_1^4 + x_2^4 + ... + x_{2N}^4 = y_1^4 + y_2^4 + ... + y_{2N}^4 """
    # Create symbols a_i, b_i (created but not used a_0,b_0)
    a = [symbols(f'a_{i}') for i in range(N + 1)]
    b = [symbols(f'b_{i}') for i in range(N + 1)]
    # Create lists x, y to hold solution (x[0],y[0] are not used)
    x, y = [None] * (2*N + 1), [None] * (2*N + 1)
    w = 0
    mu = 0
    for k in range(1, N + 1):
        w += (b[k] - a[k])*(a[k]**3 + b[k]**3)**3
        w += (b[k] + a[k])*(a[k]**3 - b[k]**3)**3
        mu += a[k]**3 * b[k]**3 * (a[k]**2 - b[k]**2)
    mu *= 6
    for k in range(1, N + 1):
        c_even = mu * (a[k]**3 - b[k]**3)
        c_odd =  mu * (a[k]**3 + b[k]**3)
        x[2*k - 1] = a[k]*w + c_odd
        x[2*k] =     b[k]*w - c_even
        y[2*k - 1] = a[k]*w + c_even
        y[2*k] =     b[k]*w + c_odd
    return x, y
```

We can test a solution, with exponent $\mathtt{e} = 3$ or 4:

```
def check_cubic(x, y, e):
    """ Check if x,y is a solution of
    x_1^e + x_2^e + ... + x_{2N}^e = y_1^e + y_2^e + ... + y_{2N}^e """
    tot = 0
    for k in range(1, len(x)):
        tot += x[k]**e - y[k]**e
    print('Should be zero:', expand(tot))
```

To display a solution we can generate the LaTeX code with the following function.

```
def gen_latex(x, y):
    """ Return latex expression for each variable after
    simplifying the expression """
    xl, yl = [None] * len(x), [None] * len(x)
    # Given a polynomial, expand() will put it into a canonical form
    # of a sum of monomials.
    for k in range(1, len(x)):
        xl[k] = latex(expand(x[k]))
        yl[k] = latex(expand(y[k]))
    return xl, yl
```

With the following code we can generate and check the solution shown in Example 2.3.

```
x, y = solve_cubic(8)  # solve cubic equation
xl, yl = gen_latex(x, y)  # obtain latex expression
check_cubic(x, y)  # check this solution
```

This code takes, in a Intel i5-7200 CPU, about 2 seconds to generate the solution and 4 minutes and 24 seconds to check it.

With a similar code we can generate and check Example 3.3, with 2.5 seconds to generate it and 1 hour, 49 minutes and 8 seconds to check it.

References

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