Intelligent Multimedia Processing & Communication Systems Journal

IMPCSJ

J IMPCS (2025) 19: 57-66
DOI 10.71856/IMPCS.2025.1204703

Research Paper

LMI-based Sensor-less Robust Predictive Control of Induction Motors by Torque Disturbance

Mehdi Safaeian[1], Seyed Hamidreza Mousavi[2*], Arash Shabani[3]

1,3. Assistant Professor, Department of Electrical Engineering, Hi.C., Islamic Azad University, Hidaj, Iran
2. Assistant Professor, Department of Electrical Engineering, Z.C., Islamic Azad University, Zanjan, Iran. **Corresponding Author*

Article Info

Article history:
Received: 31 Dec 2024
Accepted: 9 Feb 2025
DOR:

Keywords:
Induction Motor,
Linear Matrix Inequalities,
Predictive control,
System, Uncertainty.

ABSTRACT

In this paper, a new robust predictive control method for induction motors (IMs) is discussed. Linear matrix inequalities (LMIs) are employed, and feedback and observer matrices are designed in the presence of disturbances. To improve accuracy, a nonlinear motor model with parameter uncertainties and perturbations is used. With this nonlinear model, rotor speed is treated as one of the state variables and can be directly controlled using the proposed method. Additionally, uncertainty is incorporated to compensate for modeling errors or parameter mismatches. Finally, the effect of load torque is considered as a disturbance input to the system. Simulations and experimental results are presented at the end.

Publisher: Islamic Azad University – Zanjan Branch

I. Introduction

Industrial applications of AC motors have significantly increased in recent decades due to the development of new control methods, such as Field-Oriented Control (FOC) and Direct Torque Control (DTC). These methods enable easy and efficient control of AC motors. FOC is based on the differential equations of the machine and, through the use of PI controllers, provides good steady-state behavior. However, the heavy computational load results in a weak dynamic response [1]. In contrast, DTC offers fast and dynamic performance. Nevertheless, issues such as high torque ripple and variable switching frequency prevent DTC from being considered a comprehensive method [2]. A new switching table for Permanent Magnet Synchronous Motors (PMSM) to reduce torque ripple is proposed in [3], but it is not applicable to Induction Motors (IMs).

Model Predictive Torque Control (MPTC) uses a system model to predict torque in the next step and select the optimal input vector accordingly. In this method, a cost function is formed to minimize torque and flux errors, and optimization is based on this function to improve the system's output characteristics [4]. The main issue with predictive control is its dependence on parameter variations in the system model [5]. Since the predictions made by PTC are based on the system model, any inaccuracies or errors in the model lead to incorrect predictions. Therefore, it is necessary to correct parameter variations in PTC to achieve optimal results.

Linear Matrix Inequalities (LMI) is a classical control method that converts nonlinear equations into linear matrix inequalities. Using this method, important parameters can be easily represented as positive or negative definite matrices. Additionally, input or output constraints can be easily applied to LMI, and system stability can be checked effortlessly [6].

To address system parameter variations, [7] introduces an adaptive resistance estimation method for Direct Torque Control (DTC), while [8] suggests a new tuning method based on winding temperature monitoring. The authors of [9] propose an online model reference adaptive estimation method that uses a neural network to improve uncertain parameters. [10] extends a tube-based model predictive control approach to manage nonlinear systems with unstructured uncertainties. [11] introduces a robust model predictive control for discrete systems with bounded disturbances. However, the results of this method depend on the linearity of the system and cannot be easily extended to nonlinear systems. A new robust output-feedback model predictive control method is proposed in [12], which presents a stability result for a class of square, open-loop stable systems with hard constraints and model uncertainty.

The application of linear matrix inequalities in control is discussed in [6] and [13]. Here, LMI is used for robust control for the first time, and an LMI-based observer is introduced in [14]. The polytopic model is employed to reduce the impact of uncertainties. [15] improves this method to reduce computational efforts and extend the number of feasible initial states. [16] and [17] present new offline methods for LMI-based control. Specifically, [16] focuses on designing offline LMI control, while [17] extends this method to design feedback matrices and observers.

In this paper, a new LMI-based robust method is proposed for predictive control of an induction motor (IM). The key ideas of this paper are as follows:

- Using the LMI method for robust control of IMs.
- Using a nonlinear model to achieve better control of speed as one of the state variables and designing cost functions based on it.
- Implementing robust predictive control in the presence of both uncertainty and disturbance.
- Incorporating the effect of load torque as an input disturbance and designing new feedback and observer matrices based on it.
- Ensuring stability despite disturbances and uncertainties for the closed-loop system.

The results demonstrate that this method can reduce the effects of uncertainties and disturbances on torque load and rotor speed. First, the nonlinear equations of the IM with torque disturbance are presented and linearized. Then, linear matrix inequalities are briefly explained, and a new feedback matrix and observer are designed based on them. The stability theorem is also proven. Additionally, an improved predictive control method is described. Finally, simulation and experimental results validate the effectiveness of this method, and the conclusion is provided at the end.

II. Modelling of IM

In this section, a description of the nonlinear model of the induction motor (IM) is presented, followed by a discussion of its linearization.

a) Nonlinear Model

Equations of IM in the vector mode are as follows:

$$\frac{d\vec{\iota_s}}{dt} = (a_1 - jb_1\omega_m)\vec{\Psi}_s + (j\omega_m - a_2)\vec{\iota_s} + b_1\vec{v}_s \quad (1)$$

$$\frac{d\omega_m}{dt} = \frac{1}{J}\left(\frac{3}{2}pIm\left(\vec{\Psi}_s \times \vec{\iota_s}\right) - T_l\right) \quad (2)$$

$$\left(a_1 = \frac{1}{\delta L_s L_r}\ a_2 = \frac{R_s}{\delta L_s} + \frac{R_r}{\delta L_r}\ \ b_1 = \frac{1}{\delta L_s}\ \ b_2 = \frac{1}{\delta L_r}\right)$$

Where $\vec{\Psi}_s$, $\vec{\iota_s}$ are the stator flux and current respectively, ω_m is the rotor speed, $R_s, \vec{v}_s$ denotes the stator resistor and voltage, R_r is the rotor resistor, L_s, L_r are the stator and rotor inductances respectively, p is the number of pair poles and T_l represents load torque. After separating the real from the imaginary parts, this equation takes the following form.

$$\frac{d\Psi_{sd}}{dt} = -R_s i_{sd} + v_{sd}, \tag{3}$$

$$\frac{d\Psi_{sq}}{dt} = -R_s i_{sq} + v_{sq} \tag{4}$$

$$\frac{di_{sd}}{dt} = a_1\Psi_{sd} + b_1\omega_m\Psi_{sq} - a_2 i_{sd} - \omega_m i_{sq} + b_1 v_{sd} \tag{5}$$

$$\frac{di_{sq}}{dt} = -b_1\omega_m\Psi_{sd} + a_1\Psi_{sq} + \omega_m i_{sd} - a_2 i_{sq} + b_1 v_{sq} \tag{6}$$

$$\frac{d\omega_m}{dt} = \frac{1}{J}\left(\frac{3}{2}p\left(\Psi_{sd} i_{sq} - \Psi_{sq} i_{sd}\right) - T_l\right) \tag{7}$$

And finally, this model can be rewritten as state equations:

$$\begin{cases} \dot{x}_1 = -R_s x_3 + v_{sd} \\ \dot{x}_2 = -R_s x_4 + v_{sq} \\ \dot{x}_3 = a_1 x_1 - (b_1 x_2 + x_4)x_5 - a_2 x_3 + b_1 v_{sd} \\ \dot{x}_4 = (-b_1 x_1 + x_3)x_5 + a_1 x_2 - a_2 x_4 + b_1 v_{sq} \\ \dot{x}_5 = \frac{1}{J}\left(\frac{3}{2}p(x_1 x_4 - x_2 x_3) - T_l\right) \end{cases} \tag{8}$$

where $x_1 = \Psi_{sd}$ $x_2 = \Psi_{sq}$ $x_3 = i_{sd}$ $x_4 = i_{sq}$ $x_5 = \omega_m$.

b) Linearized Model

For linearization, we must first obtain the equilibrium point by $\dot{X} = 0$ [16]:

$$\begin{cases} -R_s\bar{x}_3 + \bar{v}_{sd} = 0 \\ -R_s\bar{x}_4 + \bar{v}_{sq} = 0 \\ a_1\bar{x}_1 - (b_1\bar{x}_2 + \bar{x}_4)\bar{x}_5 - a_2\bar{x}_3 + b_1\bar{v}_{sd} = 0 \\ (-b_1\bar{x}_1 + \bar{x}_3)\bar{x}_5 + a_1\bar{x}_2 - a_2\bar{x}_4 + b_1\bar{v}_{sq} = 0 \\ \frac{1}{J}\left(\frac{3}{2}p(\bar{x}_1 x_4 - \bar{x}_2\bar{x}_3) - \bar{T}_l\right) = 0 \end{cases} \tag{9}$$

Values $\bar{x}_1\ \bar{x}_2 \dots \bar{x}_5$ are equilibrium points obtained from the values $\bar{v}_{sd}\ \bar{v}_{sq}\ \bar{T}_l$. For convenience, this relationship can be expressed in a matrix form:

$$K(\bar{x}_5)\bar{X} = G\bar{u} \tag{3}$$

$$\begin{bmatrix} 0 & 0 & -R_s & 0 \\ 0 & 0 & 0 & -R_s \\ a_1 & b_1\bar{x}_5 & -a_2 & -\bar{x}_5 \\ -b_1\bar{x}_5 & a_1 & \bar{x}_5 & -a_2 \end{bmatrix}\begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \\ \bar{x}_3 \\ \bar{x}_4 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \\ -b_1 & 0 \\ 0 & -b_1 \end{bmatrix}\begin{bmatrix} \bar{v}_{sd} \\ \bar{v}_{sq} \end{bmatrix} \tag{4}$$

$$\bar{X} = K^{-1}(\bar{x}_5)G\bar{u} \tag{5}$$

$$\bar{T}_l = \frac{3}{2}p(\bar{x}_1 x_4 - \bar{x}_2\bar{x}_3) \tag{6}$$

The equilibrium points chosen are $\bar{v}_{sd}\ \bar{v}_{sq}\ \bar{x}_5$. These values are chosen so that the system is asymptotically stable. Using Taylor explanation of the nonlinear equations and considering the linear parts, the main system around the equilibrium point can be stated as follows.

$$\dot{\check{X}} = A\check{x}(t) + B\check{u}(t) + B_w\check{w}(t) \tag{7}$$

$$A = \left[\frac{\partial f_i}{\partial x_j}\right]_{\substack{1\le i\le 5 \\ 1\le j\le 5}} \tag{8}$$

$$A = \begin{bmatrix} 0 & 0 & -R_s & 0 & 0 \\ 0 & 0 & 0 & -R_s & 0 \\ a_1 & -b_1\bar{x}_5 & -a_2 & -\bar{x}_5 & -(b_1\bar{x}_2 + \bar{x}_4) \\ b_1\bar{x}_5 & a_1 & \bar{x}_5 & -a_2 & -b_1\bar{x}_1 + \bar{x}_3 \\ \mu\bar{x}_4 & -\mu\bar{x}_3 & -\mu\bar{x}_2 & \mu\bar{x}_1 & 0 \end{bmatrix}, \tag{9}$$

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ b_1 & 0 \\ 0 & b_1 \\ 0 & 0 \end{bmatrix} \quad B_w = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -\frac{1}{J} \end{bmatrix}$$

Where $\mu = \frac{3p}{2J}$ $\check{u}(t) = u(t) - \bar{u}$ $\check{x}(t) = x(t) - \bar{x}$ $\check{w}(t) = w(t) - \bar{w}$, $u(t) = [v_{sd}\quad v_{sq}]^T$ $w(t) = T_l$ $X = [\Psi_{sd}\quad \Psi_{sq}\quad i_{sd}\quad i_{sq}\quad \omega_m]^T$.

Remark 1: Owing to space constraints, the discretizing procedure is omitted in this paper. Therefore, these state matrices are used as an IM model after discretization.

III. Cost function design

Several parameters can be used to define a cost function. Owing to the importance of control issues, stator current an d flux, and rotor speed were selected. Hence, cost function i s defined as follows:

$$J(k) = \frac{\left|\Psi_s^* - \hat{\Psi}_s(k+1)\right|}{\Psi_s^{*2}} + \frac{\left|i_s^* - \hat{\imath}_s(k+1)\right|}{i_s^{*2}} + \frac{\left|\omega_m^* - \hat{\omega}_m(k+1)\right|}{\omega_m^{*\,2}} \tag{10}$$

where Ψ_s^*, i_s^* and ω_m^* are nominal values of the stator flu x and current and rotor speed. Besides, $\widehat{\Psi}_s(k+1)\ \hat{I}_s(k+1)$ $and\ \widehat{\omega}_m(k+1)$ are the predicted values of these parameter s at the next time-sampling. Thus, cost function is expresse d in standard quadratic form:

$$J_\infty(k)=\sum_{i=0}^{\infty}\big(x^T(k+i|k)Q_1x(k+i|k) + u^T(k+i|k)Ru(k+i|k)\big) \tag{18}$$

Where:

$x(k)=\begin{bmatrix}\widetilde{\Psi}_{sd} & \widetilde{\Psi}_{sq} & \tilde{I}_{sd} & \tilde{I}_{sq} & \widetilde{\omega}_m\end{bmatrix}^T$ $u(k)=\begin{bmatrix}v_{sd} & v_{sq}\end{bmatrix}^T$,

$$Q_1=\begin{bmatrix} q_{11} & 0 & 0 & 0 & 0\\ 0 & q_{22} & 0 & 0 & 0\\ 0 & 0 & q_{33} & 0 & 0\\ 0 & 0 & 0 & q_{44} & 0\\ 0 & 0 & 0 & 0 & q_{55}\end{bmatrix}>0$$

$$R=\begin{bmatrix} r_{11} & 0\\ 0 & r_{11}\end{bmatrix}>0$$

$$\left(q_{11}=\frac{1}{{\Psi_{sd}^*}^2}\ q_{22}=\frac{1}{{\Psi_{sq}^*}^2}\ q_{33}=\frac{1}{{I_{sd}^*}^2}\ q_{44}=\frac{1}{{I_{sq}^*}^2}\ q_{55}=\frac{1}{{\omega_m^*}^2}\ r_{11}=r_{22}=1\right)$$

Where $\widetilde{\Psi}_{sd}=\Psi_{sd}^*-\widehat{\Psi}_{sd}(k+1)$ $\widetilde{\Psi}_{sq}=\Psi_{sq}^*-\widehat{\Psi}_{sq}(k+1)$, $\tilde{I}_{sd}=I_{sd}^*-\hat{I}_{sd}(k+1)$ $\tilde{I}_{sq}=I_{sq}^*-\hat{I}_{sq}(k+1)$, $\widetilde{\omega}_m=\omega_m^*-\widehat{\omega}_m(k+1)$. It is assumed that the nominal current, flu x and speed are constant although it is simple to rewrite the equation to follow a special curve.

IV. Linear Matrix inequalities

This section gives a brief introduction to LMIs and their a pplication to optimization problems [6]. A linear matrix ineq uality is a matrix equation as follows:

$$F(x)=F_0+\sum_{i=0}^{l}x_iF_i>0 \tag{19}$$

where $x_1, x_2, \dots, x_l$ are decision variables and F_i represen ts real and symmetric matrices. Moreover, $F(x)>0$ means F(x) is positive definite. Usually, optimization problem lead s to minimizing a cost function due to $F(x)>0$. In fact, co nvex nonlinear inequalities can be converted into an LMI w ith Schur lemma, or nonlinear equations can be transformed into linear matrix inequalities with this compliment.

Lemma 1: Schur Complement [5]and[6]: For a given sym metric matrix $M=\begin{bmatrix}A & B\\ B^T & D\end{bmatrix}$, where $A=A^T$, $B=C^T$ and $D=D^T$, the condition $M<0$ is equivalent to $D<0$, $A-BD^{-1}B^T<0$.

LMI is often used to solve problems that variables are de cision-making matrix. In these cases, the problem is not clea rly understood in terms of LMI, and only unknown matrixes are expressed as variables for consideration.

The LMI can be used to perform robust analysis in the pr esence of uncertainty. In this case, the problem of controllin g flux and torque in the IM changes to minimize the cost fu nction J. Among the various options, the cost of the form is considered.

$$minimize\ \ J_\infty(k)$$

$$J_\infty(k)=\sum_{i=0}^{\infty}\big(x^T(k+i|k)Q_1x(k+i|k) + u^T(k+i|k)Ru(k+i|k)\big) \tag{11}$$

where Q_1, R are weighing matrixes introduced in the pre vious section ($Q_1>0, R>0$). To complete the discussion, a model for IMs with polytopic uncertainties are explained, a nd then the design of the feedback matrix and the observer a re discussed.

a) Motor LTV model with disturbance

There are several ways to display the uncertainties in a sy stem. One of these involves displaying a system with polyty pic uncertainties as a linear time-varying (LTV) system. Thi s model is defined as follows:

$$x(k+1)=A(k)\,x(k)+B(k)\,u(k)$$
$$y(k)=C\,x(k), \tag{12}$$

$$[A(k)\ \ B(k)]\in\Omega$$
$$\Omega=Co\,\{[A_1\ \ B_1]\,[A_2\ \ B_2]\,\dots\,[A_L\ \ B_L]\}$$

Co represents a convex hull, meaning that there are $\lambda_1\ \lambda_2\ \dots\ \lambda_L$, which is why their sum is equal to 1 and:

$$[A\ \ B]=\sum_{i=1}^{L}\lambda_i[A_i\ \ B_i] \tag{13}$$

Based on uncertainties intended for rotor and stator resist ance, this model can be used as a model of IM:

$$x(k+1)=A(k)x(k)+B(k)u(k)+B_ww(k)$$
$$y(k)=C\,x(k), \tag{14}$$

$$\begin{pmatrix}R_{s_{min}}\le R_s\le R_{s_{max}}\\ R_{r_{min}}\le R_r\le R_{r_{max}}\end{pmatrix}$$

Remark 2: There are several methods in the literature to determine disturbances more precisely [16],[17].The effect o f load torque is often not measurable; therefore, it is added a s a disturbance variable to the system in this paper.

b) Design feedback matrix

According to the system model and the disturbance inten ded, feedback matrix $u(k)=Fx(k)$ is obtained from the fo llowing theorem.

Theorem 1: If $x(k|k)$ is the measured value of state variables x at the sampling time k and we assume uncertainties Ω to be polytopic, then the minimization problem based on the robust performance intended by the cost function yield the following feedback matrix:

$$F = YQ(k)^{-1} \tag{15}$$

where γ, Q, Y are obtained from LMIs below:

$$\begin{bmatrix} 1 & x(k|k)^T \\ x(k|k) & Q(k) \end{bmatrix} \geq 0 \;\; Q(k) > 0 \tag{16}$$

$$\begin{bmatrix} Q & QC^T & QQ_1^{1/2} & Y^T R^{1/2} & QA_j^T + Y \\ CQ & \gamma^2 \mathrm{I} & 0 & 0 & 0 \\ Q_1^{1/2}Q & 0 & \gamma \mathrm{I} & 0 & 0 \\ R^{1/2}Y & 0 & 0 & \gamma \mathrm{I} & 0 \\ A_jQ + B_jY & 0 & 0 & 0 & Q - B_w\gamma \end{bmatrix} \geq 0 \;\; j = 1 \ldots 4 \tag{17}$$

Proof: To obtain an upper limit for the cost function in sampling time k the following Lyapunov function is defined:

$$V(x) = x^T P(k)x, P(k) > 0 \tag{18}$$

We suppose that $V(x)$ satisfies this equation:

$$V\big(x(k+i+1|k)\big) - V\big(x(k+i|k)\big) \leq -[x(k+i|k)^T Q_1 x(k+i|k) + u(k+i|k)^T R u(k+i|k)] \tag{28}$$

Adding () from $i = 0$ to $i = \infty$ and considering $x(\infty|k) = 0$ leads to:

$$\max J_\infty(k) \leq V(x(k|k)) \leq \gamma \tag{29}$$

This equation determines an upper bound for $J_\infty(k)$. We rewrite the $V(x(k|k)) \leq \gamma$ as below:

$$x(k)^T P(k)x(k) \leq \gamma \tag{19}$$

With substitution $P = \gamma Q(k)^{-1}$, the equation will be as follows:

$$1 - x(k)^T Q(k)^{-1} x(k) \geq 0 \tag{20}$$

Finally, the Schur compliment, (20) leads to.

$$\begin{bmatrix} 1 & x(k|k)^T \\ x(k|k) & Q(k) \end{bmatrix} \geq 0 \;\; Q(k) > 0$$

To prove the second part, we substitute $u(k+i|k) = Fx(k+i|k)$ in () and simplify first side:

$$\begin{aligned} V\big(x(k+i+1|k)\big) - V\big(x(K+i|k)\big) &= \big(x(k+i|k)^T(A+BF)^T + w(k+i|k)^T {B_w}^T\big)P\big((A+BF)x(k+i|k) + B_w w(k+i|k)\big) \\ &\quad - x(k+i|k)^T P(k)x(k+i|k) \end{aligned} \tag{21}$$

The second side equals to:

$$\begin{aligned} [x(k+i|k)^T Q_1 x(k+i|k) &+ u(k+i|k)^T R u(k+i|k)] \\ &= x(k)^T Q_1 x(k) + \big(Fx(k)\big)^T R\big(Fx(k)\big) \\ &= x(k+i|k)^T (Q_1 + F^T RF)x(k+i|k) \end{aligned} \tag{22}$$

Substituting (21) and (22) in () leads to:

$$\begin{aligned} \big(x(k+i|k)^T(A+BF)^T &+ w(k+i|k)^T {B_w}^T\big)P\big((A+BF)x(k+i|k) + B_w w(k+i|k)\big) \\ &- x(k+i|k)^T P(k)x(k+i|k) \leq -x(k+i|k)^T (Q_1 + F^T RF)x(k+i|k) \end{aligned} \tag{23}$$

which equals to:

$$V\begin{bmatrix} x(k+i|k)^T \\ w(k+i|k)^T \end{bmatrix} \begin{bmatrix} \begin{pmatrix} (A+BF)^T P(A+BF) - P \\ +Q_1 + F^T RF \end{pmatrix} & (\\ {B_w}^T P(A+BF) & \end{bmatrix} \leq 0 \tag{24}$$

From H_∞ condition or $\frac{\|y\|_2}{\|w\|_2} \leq \gamma$ we get:

$$y(k)^T y(k) \leq \gamma\, w(k)^T w(k) \tag{25}$$

Or:

$$x(k)^T \frac{C^T C}{\gamma \mathrm{I}} x(k) - w(k)^T w(k) \leq 0 \tag{26}$$

Adding (26) to (24) leads to:

$$\begin{bmatrix} x(k+i|k)^T \\ w(k+i|k)^T \end{bmatrix} \begin{bmatrix} \begin{pmatrix} (A+BF)^T P(A+BF) - P \\ +Q_1 + F^T RF + \dfrac{C^T C}{\gamma \mathrm{I}} \end{pmatrix} & (A \\ {B_w}^T P(A+BF) & I \end{bmatrix} \leq 0 \tag{38}$$

This equation will be true only:

$$\begin{bmatrix} \begin{pmatrix} (A+BF)^T P(A+BF) - P \\ +Q_1 + F^T RF + \dfrac{C^T C}{\gamma \mathrm{I}} \end{pmatrix} & (A+BF)^T P B_w \\ {B_w}^T P(A+BF) & {B_w}^T P B_w - 1 \end{bmatrix} \leq 0 \tag{39}$$

With substitution $P = \gamma Q(k)^{-1}$ and $F = YQ(k)^{-1}$ in (), we have:

$$\begin{bmatrix} \begin{pmatrix} \gamma Q(k)^{-1} - (A + BYQ(k)^{-1})^T \gamma Q(k)^{-1}(A + BY \\ -Q_1 - (YQ(k)^{-1})^T RYQ(k)^{-1} - \dfrac{C^T C}{\gamma \mathrm{I}} \end{pmatrix} \\ -{B_w}^T \gamma Q(k)^{-1}(A + BYQ(k)^{-1}) \end{bmatrix} \geq 0 \tag{27}$$

Now, after multiplying $Q(k)$ from both sides and after some simplification we have:

$$\begin{bmatrix} \begin{pmatrix} Q - Q\frac{Q_1}{\gamma I}Q - Y^T\frac{R}{\gamma I}Y \\ -(QA^T + Y^TB^T)Q^{-1}(AQ+BY) \\ -Q\frac{C^TC}{\gamma^2 I}Q \end{pmatrix} & -(QA^T + Y \\ -QB_w{}^TQ^{-1}(AQ+BY) & \frac{QQ}{\gamma I} - Q \end{bmatrix} \geq 0 \quad (28)$$

With the Schur compliment the equation changes to:

$$Q - QQ_1{}^{1/2}(\gamma I)^{-1}Q_1{}^{1/2}Q - Y^TR^{1/2}(\gamma I)^{-1}R^{1/2}Y - QC^T(\gamma^2 I)^{-1}CQ - (QA^T + Y^TB^T)\left(Q^{-1} + Q^{-1}B_w\left(\frac{1}{\gamma I} - B_w{}^TQ^{-1}B_w\right)^{-1}B_w{}^TQ^{-1}\right)(AQ + BY) \geq 0 \quad (29)$$

Using the Sherman-Morrison matrix inverse equation, we get:

$$Q - QQ_1{}^{1/2}(\gamma I)^{-1}Q_1{}^{1/2}Q - Y^TR^{1/2}(\gamma I)^{-1}R^{1/2}Y - QC^T(\gamma^2 I)^{-1}CQ - (QA^T + Y^TB^T)\left(Q - B_w\gamma B_w{}^T\right)^{-1}(AQ + BY) \geq 0 \quad (30)$$

Introducing this matrix:

$$F_{11} = Q, \quad F_{12} = \begin{bmatrix} QC^T & QQ_1{}^{1/2} & Y^TR^{1/2} & QA^T + Y^TB^T \end{bmatrix}$$

$$F_{21} = \begin{bmatrix} CQ \\ Q_1{}^{1/2}Q \\ R^{1/2}Y \\ AQ+BY \end{bmatrix} \quad F_{22} = \begin{bmatrix} \gamma^2 I & 0 & 0 & 0 \\ 0 & \gamma I & 0 & 0 \\ 0 & 0 & \gamma I & 0 \\ 0 & 0 & 0 & Q - B_w\gamma B_w{}^T \end{bmatrix}$$

Finally, using the Schur compliment, we have:

$$\begin{bmatrix} Q & QC^T & QQ_1{}^{1/2} & Y^TR^{1/2} & QA_j{}^T + Y \\ CQ & \gamma^2 I & 0 & 0 & 0 \\ Q_1{}^{1/2}Q & 0 & \gamma I & 0 & 0 \\ R^{1/2}Y & 0 & 0 & \gamma I & 0 \\ A_jQ + B_jY & 0 & 0 & 0 & Q - B_w\gamma \end{bmatrix} \geq 0$$

□

V. Observer design and stability theorem

Owing to the difficulties and costs of measuring the motor's flux and speed, it is better to use an observer to estimate their values. Observer design can be made as follows [16] and [17]. The motor model with the observer is as follows:

$$\hat{x}(k+1) = A(k)\hat{x}(k) + B(k)u(k) + B_ww(k) + L_p\big(y(k) - C\hat{x}(k)\big) = \big(A(k) - L_pC\big)\hat{x}(k) + L_pCx(k) + B(k)u(k) + B_ww(k)$$

$$e(k+1) = x(k+1) - \hat{x}(k+1) = \big(A_0 - L_pC\big)\big(x(k) - \hat{x}(k)\big) + f(x(k)u(k)) \quad (31)$$

Where $f\big(x(k), u(k), w(k)\big) = (A(k) - A_0)x(k) + (B(k) - B_0)u(k) + B_ww(k)$. A_0 , B_0 are the values of the state matrix at steady state. The only nominal error is checked at the designing stage, but these statements are not considered. Besides, the relationship between the observer and the state feedback is investigated in a separate theorem. The convergence speed of the dynamic error $e(k+1)$ can be reduced by the rate of $0 < \rho < 1$ so that there is $P > 0$ to satisfy the following relationship:

$$\rho^2 e^T(k)Pe(k) \geq e^T(k+1)Pe(k+1) \quad (32)$$

Simplifying and substituting $L_p = Q^{-1}Y$ leads to the LMI below:

$$\begin{bmatrix} Q\rho^2 & QA_0 - YC \\ A_0{}^TQ - C^TY^T & Q \end{bmatrix} \geq 0 \quad (33)$$

Proof: Substitute $e(k+1) = \big(A_0 - L_pC\big)\big(x(k) - \hat{x}(k)\big)$ in (32) :

$$e^T(k)\left(\rho^2 P - \big(A_0 - L_pC\big)^T P\big(A_0 - L_pC\big)\right)e(k) \geq 0, \quad (34)$$

This equation will be true only:

$$\left(\rho^2 P - \big(A_0 - L_pC\big)^T P\big(A_0 - L_pC\big)\right) \geq 0 \quad (48)$$

Replacing $L_p = Q^{-1}Y$ and $P = Q^{-1}$ leads to:

$$Q\rho^2 - (A_0{}^TQ - C^TY^T)Q^{-1}(QA_0 - YC) \geq 0 \quad (49)$$

With the Schur compliment, the above equation changes to the following:

$$\begin{bmatrix} Q\rho^2 & QA_0 - YC \\ A_0{}^TQ - C^TY^T & Q \end{bmatrix} \geq 0$$

□

In order to investigate the stability of the closed loop system in this part, we assume that feedback matrix $F(k)$ is performed offline for the sake of simplicity and does not relate to $\hat{x}(k)$. Re-rewriting state equations results in:

$$x(k+1) = A(k)x(k) + B(k)F(k)\hat{x}(k) + B_ww(k) \quad (35)$$

$$\hat{x}(k+1) = A_0\hat{x}(k) + B_0u(k) + L_p\big(y(k) - C\hat{x}(k)\big) = \big(A_0 + B_0F(k) - L_pC\big)\hat{x}(k) + L_pCx(k) \quad (36)$$

The augmented closed loop system will be as follows:

$$\chi(k+1) = A_{poly}(k)\chi(k) + B_w w(k) \tag{37}$$

$$\left(A_{poly}(k) = \begin{bmatrix} A(k) & B(k)F(k) \\ L_p C & A_0 + B_0 F(k) - L_p C \end{bmatrix}\right)$$

Theorem 2: The above closed loop system is stable if th ere is $Q > 0$ for all Ω in such a way that:

$$\begin{bmatrix} Q & Q{C_y}^T & Q{A^T}_{poly} \\ C_y Q & \gamma I & 0 \\ A_{poly}Q & 0 & Q - B_w {B_w}^T \end{bmatrix} \geq 0 \tag{38}$$

Where:

$$A_{poly}(k) = \begin{bmatrix} A(k) & B(k)F(k) \\ L_p C & A_0 + B_0 F(k) - L_p C \end{bmatrix}, C_y = \begin{bmatrix} C \\ 0_{n\times n} \end{bmatrix} \tag{39}$$

.

Proof: To prove stability a Lyapunov function has to be f ound in such a way that if $P > 0$ then for $V(\chi) = \chi^T P \chi$ we have $\dot{V}(\chi) < 0$ or:

$$V(\chi(k+1)) - V(\chi(k)) < 0 \tag{40}$$

Substitute (37) in (40):

$$\left(A_{poly}(k)\chi(k) + B_w w(k)\right)^T P \left(A_{poly}(k)\chi(k) + B_w w(k)\right) - \chi^T P \chi < 0 \tag{41}$$

Or:

$$\begin{pmatrix} \chi^T(k) \\ w^T(k) \end{pmatrix} \begin{pmatrix} {A^T}_{poly} P A_{poly} - P & {A^T}_{poly} P B_w \\ {B^T}_w P A_{poly} & {B^T}_w P B_w \end{pmatrix} \begin{pmatrix} \chi(k \\ w(k \end{pmatrix} < 0 \tag{42}$$

From H_∞ condition or $\frac{\|y\|_2}{\|w\|_2} \leq \gamma$ we have $y(k)^T y(k) \leq \gamma\, w(k)^T w(k)$ *or*:

$$\chi(k)^T \frac{{C_y}^T C_y}{\gamma I} \chi(k) - w(k)^T w(k) \leq 0 \tag{58}$$

Adding (42) and () leads to:

$$\begin{pmatrix} \chi^T(k) \\ w^T(k) \end{pmatrix} \begin{pmatrix} {A^T}_{poly} P A_{poly} - P + \frac{{C_y}^T C_y}{\gamma I} & {A^T}_{poly} P \\ {B^T}_w P A_{poly} & {B^T}_w P B_w \end{pmatrix} < 0 \tag{59}$$

This will be true only:

$$\begin{pmatrix} {A^T}_{poly} P A_{poly} - P + \frac{{C_y}^T C_y}{\gamma I} & {A^T}_{poly} P B_w \\ {B^T}_w P A_{poly} & {B^T}_w P B_w - 1 \end{pmatrix} < 0 \tag{43}$$

After replacing $L_p = Q^{-1}Y$ and $P = Q^{-1}$ and some simpli fication we have:

$$\begin{pmatrix} Q - Q{A^T}_{poly} Q^{-1} A_{poly} Q - Q\frac{{C_y}^T C_y}{\gamma I} Q & -Q{A^T}_p \\ -Q{B^T}_w Q^{-1} A_{poly} Q & QQ - Q \end{pmatrix} > 0 \tag{44}$$

Now we apply the Schur compliment:

$$Q - Q\frac{{C_y}^T C_y}{\gamma I} Q - \left(Q{A^T}_{poly}\right)(Q^{-1} + Q^{-1}B_w(1 - {B^T}_w Q^{-1} B_w)^{-1} {B^T}_w Q^{-1})\left(A_{poly}Q\right) > 0 \tag{45}$$

With the Sherman-Morrison matrix, the inverse equation l eads to:

$$Q - Q{C_y}^T(\gamma I)^{-1} C_y Q - \left(Q{A^T}_{poly}\right)\left(Q - B_w {B_w}^T\right)^{-1}\left(A_{poly}Q\right) > 0 \tag{46}$$

Now, select this matrix to simplify:

$$F_{11} = Q \qquad F_{12} = \begin{bmatrix} Q{C_y}^T & Q{A^T}_{poly} \end{bmatrix}$$

$$F_{21} = \begin{bmatrix} C_y Q \\ A_{poly} Q \end{bmatrix} \quad F_{22} = \begin{bmatrix} \gamma I & 0 \\ 0 & Q - B_w {B_w}^T \end{bmatrix}$$

Finally, using the Schur compliment we have:

$$\begin{bmatrix} Q & Q{C_y}^T & Q{A^T}_{poly} \\ C_y Q & \gamma I & 0 \\ A_{poly}Q & 0 & Q - B_w {B_w}^T \end{bmatrix} \geq 0$$

VI. Improved Predictive Control

Predictive control methods have undergone significant de velopments in recent years. The Predictive Torque Control (PTC) algorithm is specifically designed for AC motors to co ntrol electromagnetic torque. This method involves controlli ng electromagnetic torque by adjusting the phase angle betw een the stator and rotor fluxes, as well as the magnitude of t he stator flux. A cost function is used to optimize the system 's future behavior, and based on this function a selected volt age vector is applied at each sample time [5].

For an induction motor (IM), selecting the appropriate vo ltage vector allows for steering the torque and flux toward t heir reference values. This voltage vector adjusts the stator f lux amplitude and simultaneously alters the torque by modif ying the angle between the stator and rotor flux [12].

Despite its advantages, this method has some drawbacks, such as high computational requirements. The primary issue with predictive control methods is that the accuracy of the system model directly impacts torque and flux ripple. In methods like PTC or dead-beat control, inaccuracies in estimating system parameters can lead to forced oscillations [4]. This issue can be addressed using robust LMI-based control, as introduced in the previous section.

One of the main improvements of this paper is predicting voltage vector at the next sampling time. We use the nearest value of v_s based on the one obtained by the feedback matrix. Fig.1 shows how we select this voltage vector. In this case, v_2 is selected and e_k represents the voltage error ($v_0 = 0$).

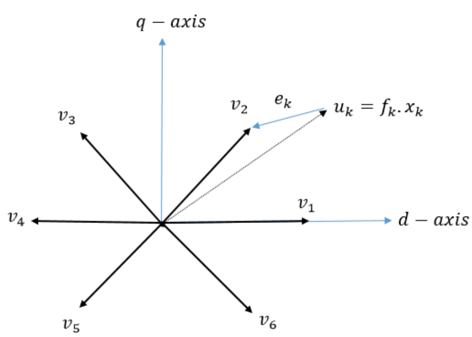

Fig.1. Voltage Vector selecting

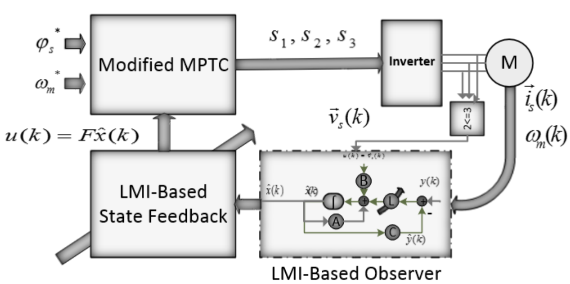

Fig.2 represents the block diagram of the control method. For performing a good prediction, the feedback matrix and observer were correct at every sample time while torque value in k+1 was predicted according to flux and current predictions.

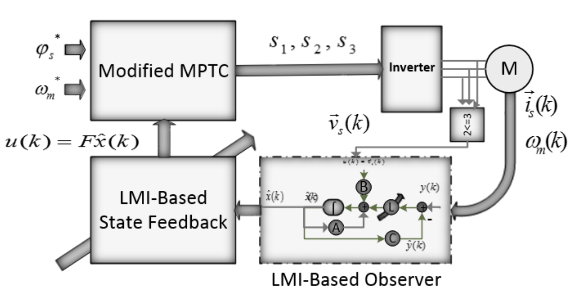

Fig.2. System Block Diagram

VII. Simulation Results

The proposed algorithm has been proved theoretically by Theorems 1 and 2, and experimentally by the simulation and hardware setup using the hardware-in-the-loop (HIS) method. In this method, a TI DSP f28335 board is used to evaluate the motor and inverter model, and outputs are sent to a D/A board using SCI protocol. Other parts are simulated in MATLAB. Fig.3 shows the block diagram and the real setup.

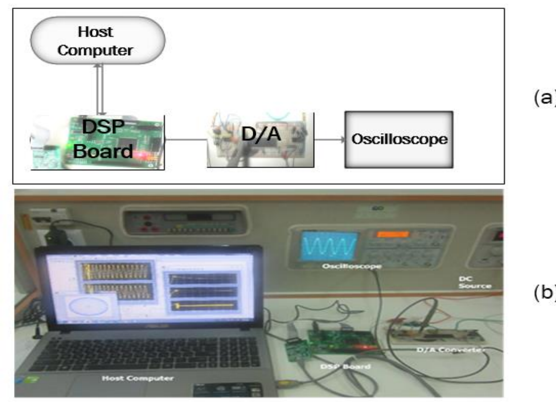

Fig.3. Experimental set-up (a) block diagram (b) real set-up

In this method, only a selected part of simulation is implemented in the DSP board and the results are sent back to the host PC. Therefore, all outputs could be shown in both the host PC and the oscilloscope. Table 1 shows the motor parameters.

TABLE I The IM parameters

Parameter Name	Value	Unit
R_r	0.500	Ω
R_s	0.180	Ω
L_r	0.056	H
L_s	0.0553	H
L_m	0.0538	H
ω_n	200	rpm
T_n	5	Nm
f_{sn}	50	Hz
I_n	5	A
V_n	220	V

To achieve optimum results, the new represented method is compared with [12] and root PTC. Besides, to examine the robustness of the method, 25% uncertainty is considered for stator and rotor resistance. Fig.4 shows the stator current, electric torque load, and the speed of the rotor and the stator flux without observer feedback at 200 rpm. Fig.5 shows the same test with robust observer feedback. It is obvious that feedback reduces torque oscillation. The total harmonic disto rtion (THD) of the stator flux and the current are reduced from 55.37 to 35.12 for the flux and from 38.46 to 26.68 for the current.

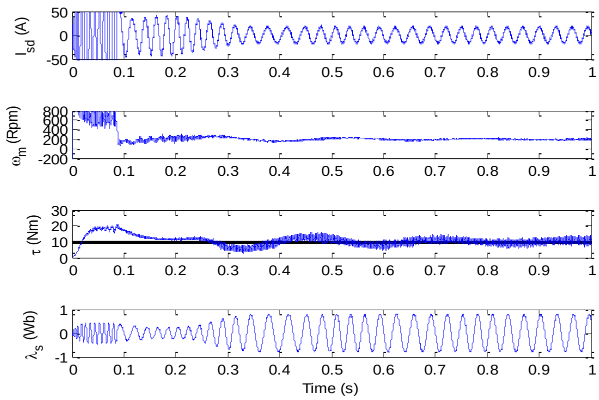

Fig.4. System Outputs without the observer (25% uncertainty, 200 rmp)

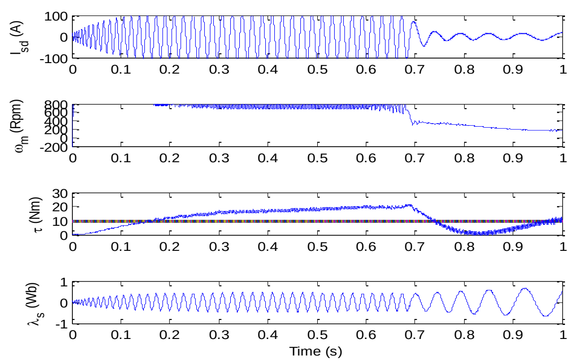

Fig.5. System Outputs with the robust observer (25% uncertainty, 200 rmp)

The low-speed performance of the algorithm has been studied in Fig.6. At 50 rpm, torque oscillation increases and speed predicted with more error. Fig.7 presents the effect of the robust feedback. The feedback has acceptable result at this speed. The THD of the stator flux changes from 67.89 to 35.36 while the THD of the stator current is reduced from 42.24 to 36.71.

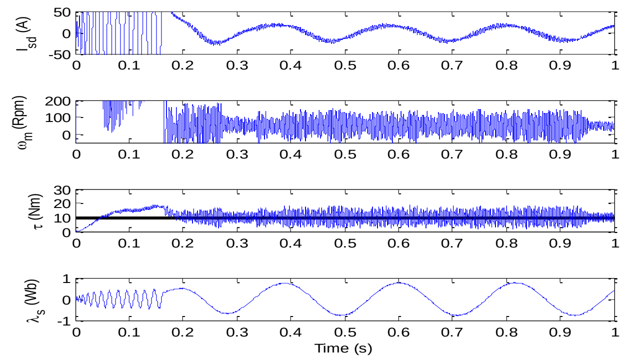

Fig.6. System Outputs without the observer (25% uncertainty, 50 rmp)

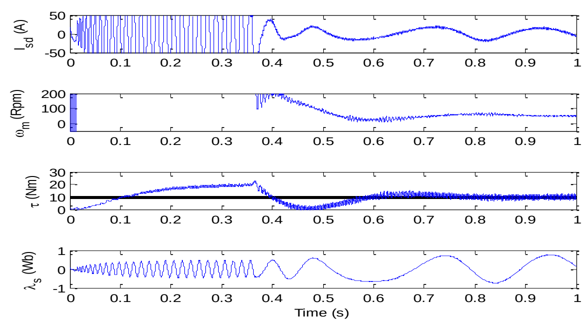

Fig.7. System Outputs with the observer (25% uncertainty, 50 rmp)

Fig.8 represents the exported output from MATLAB. In this figure, the motor and inverter model is simulated in the TI DSP board. Using the HIS method, all the output can be exported to D/A or returned to the host PC. Fig.8 shows IM outputs in the host PC. Fig.9 shows this test at a lower speed.

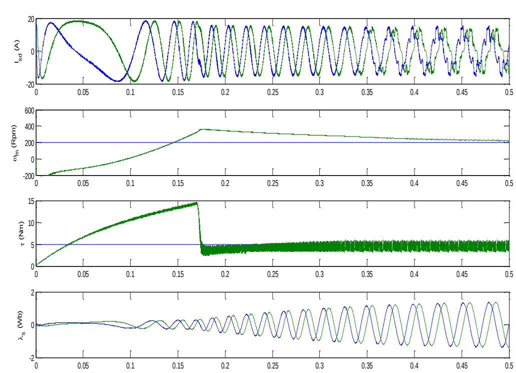

Fig.8. Process in the loop outputs exported to matlab (25% uncertainty, 200 rmp)

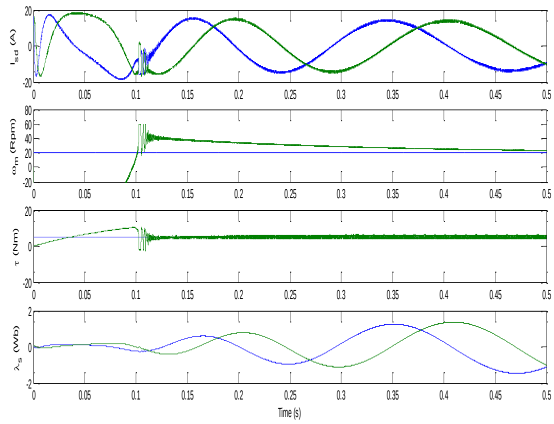

Fig.9. Process in the loop outputs exported to Matlab (25% uncertainty, 20 rmp)

VIII. Conclusion

In this paper, a new method for designing an observer and feedback matrix for robust predictive control of induction motors (IMs) is presented. In this approach, load torque is treated as an unknown disturbance. Additionally, a nonlinear IM model is utilized to enhance the accuracy of the modeling.

To validate this method, simulations and the HIS method are conducted and compared with root PTC methods. The results demonstrate that using the nonlinear motor model for designing the feedback matrix and observer improves the accuracy of the controller when compared to a linear model. Furthermore, this approach addresses one of the main challenges of predictive control methods: parameter uncertainty. In comparison to the other methods considered, the robust LMI-based observer reduces the Total Harmonic Distortion (THD) of both stator flux and current. Finally, the results indicate that this method effectively handles unknown load torque.

References

[1] V. Ambrozic, G. S. Buja, and R. Menis, "Band-constrained technique for direct torque control of induction motor," *IEEE Transactions on Industrial Electronics*, vol. 51, no. 4, pp. 776–784, 2004.

[2] A. G. M. A. Aziz, H. Rez, and A. A. Z. Diab, "Robust sensorless model-predictive torque flux control for high-performance induction motor drives," *Mathematics*, vol. 9, no. 4, p. 403, 2021. doi: 10.3390/math9040403.

[3] S. A. Davari, D. A. Khaburi, F. Wang, and R. M. Kennel, "Using full order and reduced order observers for robust sensorless predictive torque control of induction motors," *IEEE Transactions on Power Electronics*, vol. 27, no. 7, pp. 3424–3433, 2012.

[4] P. Falugi and D. Q. Mayne, "Getting robustness against unstructured uncertainty: A tube-based MPC approach," *IEEE Transactions on Automatic Control*, vol. 59, no. 5, pp. 1295–1301, 2014.

[5] L. E. Ghaoul, E. Feron, V. Balakrishnan, and S. P. Boyd, *Linear Matrix Inequalities in System and Control Theory*, vol. 15, SIAM, 1994.

[6] D.-J. Kim and B. Kim, "Linear matrix inequality-based robust model predictive speed control for a permanent magnetic synchronous motor with a disturbance observer," *Energies*, vol. 17, no. 4, p. 869, 2024. doi: 10.3390/en17040869.

[7] M. V. Kothare, V. Balakrishnan, and M. Morari, "Robust constrained model predictive control using linear matrix inequalities," *Automatica*, vol. 32, no. 10, pp. 1361–1379, 1996.

[8] B. S. Lee and R. Krishnan, "Adaptive stator resistance compensator for high performance direct torque controlled induction motor drives," *Proceedings of the IEEE Industry Applications Society Annual Meeting*, St. Louis, MO, USA, 1998.

[9] S. B. Lee, T. G. Habetler, R. G. Harley, and D. J. Gritter, "An evaluation of model-based stator resistance estimation for induction motor stator winding temperature monitoring," *Proceedings of the IEEE Industry Applications Conference*, 2002.

[10] C. Løvaas, M. M. Seron, and G. C. Goodwin, "Robust output-feedback MPC with integral action, " *IEEE Transactions on Automatic Control*, vol. 55, no. 7, pp. 1531–1543, 2010.

[11] D. Q. Mayne, M. M. Seron, and S. V. Raković, "Robust model predictive control of constrained linear systems with bounded disturbances," *Automatica*, vol. 41, no. 2, pp. 219–224, 2005.

[12] M. Nemec, D. Nedeljkovic, and V. Ambrozic, "Predictive torque control of induction machines using immediate flux control," *IEEE Transactions on Industrial Electronics*, vol. 54, no. 4, pp. 2009–2017, 2007.

[13] H. H. Nguyen and R. Findeisen, "LMI-based data-driven robust model predictive control," *arXiv preprint arXiv:2303.04777*, 2023. Available: https://arxiv.org/abs/2303.04777.

[14] N. D. Nguyen, N. N. Nam, C. Yoon, and Y. I. Lee, "Speed sensorless model predictive torque control of induction motors using a modified adaptive full-order observer," *IEEE Transactions on Industrial Electronics*, vol. 68, no. 4, pp. 3173–3183, 2021. doi: 10.1109/TIE.2020.2984567.

[15] S. E. Rezgui, S. Legrioui, A. Mehdi, and H. Benalla, "Robust IM control with MRAS-based speed and parameters estimation with ANN using exponential reaching law," *Proceedings of the International Conference on Systems and Control*, Metz, France, 2014.

[16] J. Rodríguez and P. Cortes, *Predictive Control of Power Converters and Electrical Drives*, Wiley-IEEE Press, 2012.

[17] A. Sivaprakasam and T. Manigandan, "An alternative scheme to reduce torque ripple and mechanical vibration in direct torque controlled permanent magnet synchronous motor," *Journal of Vibration and Control*, vol. 21, no. 5, pp. 855–871, 2015.