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Vibration of circular sandwich plates with FG face sheets on the Pasternak elastic foundation

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Abstract

In this paper, the free vibration behaviour of the circular sandwich plates with two functionally graded face sheets on the Pasternak elastic foudation is investigated in a clamped boundary condition based on a high order sandwich plate theory. By considering the inplane stresses of the core the theory is modified and the obtained equation is more accurate. The material properties of the functionally graded face sheets and the homogeneous core are assumed to be temperature- dependent. The functionally graded materials vary continuosly through the thickness according to a power - law distribution. The governing equations of the motion are derived by using Hamilton's principle and a Glaerkin method is used to solve the equations. To verify the results of the present method, they are compared with the finite element results which obtained by Abaqus software and for special cases with the results in some literatures which a good agreement is found between them.

***Keywords*:** Vibration, Circular, Sandwich plate, FGM, Pasternak elastic foundation.

Introduction

Due to the high flexural stiffness to weight ratio, sandwich structures have a wide application in the modern industries such aerospace, transportation, naval and construction structures. Sandwiches include two thin and stiff faces that cover a thick and lightweight core which usually is flexible. The separation of face sheets by a soft core increases the bending rigidity of the plate at an expenses of small weight [1]. Application of classical composite material in high temperature environments cause to the failure, delamination and thermal stress concentration. Japanese researchers proposed functionally graded materials (FGMs) to overcome these problem. FGMs are microscopic inhomogeneous materials which gradually graded from a metal surface to a ceramic one [2]. Investigation on these materials have been increased by material researchers. Mollarazi et al. studied the vibration behaviour of FGM cylinder. The materials were functionally graded in the radial direction from a silicon carbide to stainless steel [3]. Dai et al. studied the Thermoelastic responses of FG hollow cylinder by using a power law rule to model the material properties variation [4]. Sofiyev investigated the vibration and stability of clamped conical shells. Power law rule and exponential rule were used to model the FGM variation [5]. Kim et al. analyzed the nonlinear behaviour of the FGM plates and shells. By using a sigmoid function, the material properties were modelled [6]. Dai et al. studied the low velocity impact effects on the nonlinear responses of the FG circular plates [7].
Plates on the elastic foundations have been widely adopted by many researchers to model the interaction between elastic media and plates for various engineering plate problems. Tahouneh and Yas investigated the 3-D free vibration of thick FG annular sector plate on a Pasternak foundation [8]. Singh and Harsha analyzed the nonlinear dynamic of FG sandwich plate on Pasternak foundation under thermal environment [9]. Gao et el. Studied the stability of composite orthotropic plate on elastic foundation under thermal environment [10]. Keleshteri et al. studied nonlinear bending of FG-CNTRC annular plates with variable thickness on elastic foundation [11].
There are different approaches to investigate the mechanical behaviour of plates such as shear deformation plate theory, 3D elastic theory, energy and finite element method [12]. By applying a finite element approach, Prakash and Ganapathi studied the mechanical behaviour of FG circular plates [13]. Civalek by using FSDT, investigated the static responses of thick composite plates [14]. Cohen studied the buckling of the laminated plate based on a transverse shear deformation theory [15]. In these theories the core height is constant, but in fact the thickness of the sandwich plates are variable. So, the core should be considered as a flexible layer that compressed transversely. In the classical theories, the localized effects in the core can't be calculated, so to consider these effects, Frostig et al. presented a high order theory [16]. Malekzadeh et al. studied the dynamic responses of the composite sandwich panels based on an improved high order sandwich plate theory [17]. Mantari et al. utilised a HSDT to find the frequencies of functionally graded plates located on elastic foundation [18]. Rahmani et al. studied the nonlinear buckling of different types of porous FG sandwich beams with temperature-dependent material

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based on a high order theory of the sandwich beam. [19]. With a high order theory Salami et al. inspected the bending in rather thick faces sandwich beams with a soft core which satisfied the stress compatibility condition at interface [20]. Frostig et al. investigated the nonlinear wrinkling of a functionally graded core sandwich panel by employing a modified high order theory [21].

A review in literature shows there are limited articles that consider the dependency of the material to the temperature for both faces and core in studying of the mechanical behaviour. Shahrjerdi et al. analyzed the vibration characteristics of temperature-dependent solar FG plates by applying the second-order shear deformation theory [22]. Frostig and Thomsen numerically investigated the vibration of sandwich plates consisted the core that its material was temperature dependent [23]. Pandey and Pradyumna by utilising the layer-wise theory explored the frequency responses of the FG sandwich plates made of the temperature dependent materials [24].

An important kind of sandwich structures that used in high temperature surroundings is the FG circular sandwich plate. Many researchers have explored the vibration behaviour of the circular sandwich plates. Sherif discussed the frequencies characteristics of the clamped circular sandwich plates by applying the FSDT. The core was viscoelastic and shear stress and rotary inertia were considered [25]. Chan II Park derived the frequency equations of the uniform thickness circular plate with clamped boundary condition [26]. By exerting a 3D elasticity procedure Nie and Zhong investigated the frequencies characteristic of the FG circular plates in various boundary conditions [27]. Ebrahimi et al. studied the vibration characteristics of FG circular plate which merged with two piezoelectric layers in different boundary condition [28]. Lal and Rani investigated the free vibrations of circular sandwich plates in different boundary conditions by utilising the FSDT [29].

As a result of review in the accessible literatures, it's found that there is no studying on the vibration of circular sandwich plates on the Pasternak elastic foundation by using a modified high order sandwich plate theory and considering the temperature dependent material for both faces and core. In this study, by applying a high order theory which modified by considering the flexibility of the core in the thickness direction and in-plane stresses of the core, vibration behaviour of circular sandwich plates are investigated in the uniform temperature distributions. Sandwiches consist of two FG faces which cover a homogeneous core. FG material properties are temperature and location dependent which graded in according to power law rule. The homogeneous materials are temperature dependent, too. Unlike the most papers, high order stresses and thermal stress resultants, in plane stresses and thermal stresses of the core and face sheets are considered at the same time. Boundary condition is clamped and equations are derived based on the Hamilton's energy principle. To obtain the frequencies, a Galerkin method is applied.

Formulation

In order to investigate the vibration behaviour of clamped functionally graded circular sandwich plates and obtain the governing equations of the motion, Hamilton's energy principle is applied which consists of the variation of the kinetic and strain energy. The main equation is as follow [30]:

The variation of kinetic and the strain energy are δK and δU, respectively; t is the time coordinate that varies between the times t_1 and t_2; δ is the variation operator. The variation of the kinetic energy is calculated as follows:

$$\int_{t1}^{t2}\left(-\delta K+\delta U\right)dt=0 \tag{1}$$

$$\begin{aligned}\int_{t1}^{t2}\delta K dt &= -\int_{t1}^{t2}\{\int_0^a\int_0^{2\pi}\int_{-\frac{h_t}{2}}^{\frac{h_t}{2}}\rho_t\left(z_t,T_t\right)(\ddot{u}_t\,\delta u_t+\ddot{\upsilon}_t\,\delta\upsilon_t+\ddot{w}_t\,\delta w_t)rdrd\theta dz_t\\ &+\int_0^a\int_0^{2\pi}\int_{-\frac{h_b}{2}}^{\frac{h_b}{2}}\rho_b\left(z_b,T_b\right)(\ddot{u}_b\,\delta u_b+\ddot{\upsilon}_b\,\delta\upsilon_b+\ddot{w}_b\,\delta w_b)rdrd\theta dz_b\\ &+\int_0^a\int_0^{2\pi}\int_{-\frac{h_c}{2}}^{\frac{h_c}{2}}\rho_c\left(T_c\right)(\ddot{u}_c\,\delta u_c+\ddot{\upsilon}_c\,\delta\upsilon_c+\ddot{w}_c\,\delta w_c)rdrd\theta dz_c\}dt\end{aligned} \tag{2}$$

where (··) indicates the second derivative with respect to time; The density is "ρ" which in the functionally graded layers is the function of the displacement and the temperature, and in the homogeneous layer is just a function of the temperature; The top and bottom face sheets and the core, are indicated with "t", "b" and "c", respectively.

To model the properties of the FGMs which usually include ceramic and metal and vary gradually in the thickness direction, a power law rule is applied.

$$P_j\left(z_j,T\right)=g\left(z_j\right)P_{ce}^j\left(T\right)+\left[1-g\left(z_j\right)\right]P_m^j\left(T\right),\quad j=(t,b) \tag{3}$$

$$g\left(z_t\right)=\left(\frac{\frac{h_t}{2}-z_t}{h_t}\right)^N;\, g\left(z_b\right)=\left(\frac{\frac{h_b}{2}+z_b}{h_b}\right)^N \tag{4}$$

Where N is the power law index; P is the material properties such as young module, Poisson's ratio, density. Since these sandwich structures are applied in high temperature conditions, it is necessary to consider that the FGMs and homogeneous materials are temperature dependent. This dependency is expressed as a nonlinear function of temperature as follows [31].

$$P = C_0\left(C_{-1}T^{-1} + 1 + C_1 T + C_2 T^2 + C_3 T^3\right) \tag{5}$$

Where "C"s are unique coefficients of temperature for each material; and $T=T_0+\Delta T$, which T_0 is the room temperature.

Inspired by Kirchhoff's assumptions, a classical theory of plates in polar coordinate, is employed to model the displacement fields of the face-sheets as [32]:

$$u_j(r,\theta,z,t) = u_{0j}(r,\theta,t) - z_j \frac{\partial w(r,\theta,t)}{\partial r} \tag{6}$$

$$\upsilon_j(r,\theta,z,t) = \upsilon_{0j}(r,\theta,t) - \frac{z_j}{r}\frac{\partial w(r,\theta,t)}{\partial r}, (j=t,b) \tag{7}$$

$$w_j(r,\theta,z,t) = w_{0j}(r,\theta,t) \tag{8}$$

where "0" denotes values with correspondence to the central plane of the layers. "u" and "υ" are the in-plane deformations in the "r" and "θ" directions and "w" is the transverse deflections of the faces. Also, the kinematic relations of the core are considered as polynomial pattern with the unknown coefficients, u_k and ν_k (k= 0,1,2,3), for the in-plane and w_l (l = 0,1,2) for vertical displacement components which obtained by the variational principle [31]:

$$u_j(r,\theta,z,t) = u_{0j}(r,\theta,t) - z_j \frac{\partial w(r,\theta,t)}{\partial r} \tag{9}$$

$$\upsilon_j(r,\theta,z,t) = \upsilon_{0j}(r,\theta,t) - \frac{z_j}{r}\frac{\partial w(r,\theta,t)}{\partial r}, (j=t,b) \tag{10}$$

$$w_j(r,\theta,z,t) = w_{0j}(r,\theta,t) \tag{11}$$

In this theory, the compatibility conditions assume that the faces are sticked to the core completely and the interface displacements between the core and the face sheets can be obtained as follows:

$$u_c(z_c=-h_c/2) = u_t(z_t=h_t/2) \quad \upsilon_c(z_c=-h_c/2) = \upsilon_t(z_t=h_t/2) \quad w_c(z_c=-h_c/2) = w_t \tag{12}$$

$$u_b(z_b=-h_b/2) = u_c(z_c=h_c/2) \quad \upsilon_b(z_b=-h_b/2) = \upsilon_c(z_c=h_c/2) \quad w_b = w_c(z_c=h_c/2) \tag{13}$$

The variation of the total strain energy includes all mechanical and thermal stresses and linear and nonlinear strains of the layers of the sandwich plates that make the mechanical and thermal energy [20]. In addition, the compatibility conditions at the interfaces of the core and the face-sheets are the constraints and attended in the Hamilton's principle in terms of six Lagrange multipliers. By considering the effects in-plane stresses of the core in this formulation, is as follows:

$$\begin{aligned}
\delta U_p = &\int_{v_t}(\sigma^t_{rr}\delta\in^t_{rr} + \sigma^{tT}_{rr}\delta d^t_{rr} + \sigma^t_{\theta\theta}\delta\in^t_{\theta\theta} + \sigma^{tT}_{\theta\theta}\delta d^t_{\theta\theta} + \tau^t_{r\theta}\delta\gamma^t_{r\theta})dv + \\
&\int_{v_b}(\sigma^b_{rr}\delta\in^b_{rr} + \sigma^{bT}_{rr}\delta d^b_{rr} + \sigma^b_{\theta\theta}\delta\in^b_{\theta\theta} + \sigma^{bT}_{\theta\theta}\delta d^b_{\theta\theta} + \tau^b_{r\theta}\delta\gamma^b_{r\theta})dv + \\
&\int_{v_{core}}(\sigma^c_{rr}\delta\in^c_{rr} + \sigma^c_{\theta\theta}\delta\in^c_{\theta\theta} + \sigma^c_{zz}\delta\in^c_{zz} + \tau^c_{r\theta}\delta\gamma^c_{r\theta} + \tau^c_{rz}\delta\gamma^c_{rz} + \tau^c_{\theta z}\delta\gamma^c_{\theta z})dv \\
&+\delta\int_0^a\int_0^{2\pi}[\lambda_{rt}\left(u_t\left(z_t=\frac{h_t}{2}\right) - u_c\left(z_c-\frac{h_c}{2}\right)\right) + \lambda_{\theta t}\left(v_t\left(z_t=\frac{h_t}{2}\right) - v_c\left(z_c=-\frac{h_c}{2}\right)\right) \\
&+\lambda_{zt}\left(w_t\left(z_t=\frac{h_t}{2}\right) - w_c\left(z_c=-\frac{h_c}{2}\right)\right) + \lambda_{rb}(u_c\left(z_c=\frac{h_c}{2}\right) - u_b\left(z_b=-\frac{h_b}{2}\right) + \lambda_{\theta b} \\
&\left(v_c\left(z_c=\frac{h_c}{2}\right) - v_b\left(z_b=-\frac{h_b}{2}\right)\right) + \lambda_{zb}\left(w_c\left(z_c=\frac{h_c}{2}\right) - w_b\right)]rdrd\theta
\end{aligned} \tag{14}$$

"σ_{rr}" , "$\sigma_{\theta\theta}$" and "$\tau_{r\theta}$" display the normal and shear stresses; "ε_{rr}" , "$\varepsilon_{\theta\theta}$" and "$\gamma_{r\theta}$" are the linear normal and shear strains of the layers; "σ_{rr}^T" and "$\sigma_{\theta\theta}^T$" express the thermal stresses and "d_{rr}" and "$d_{\theta\theta}$" are the non-linear strains in the faces; "σ_{zz}^c" and "ε_{zz}^c" present the lateral normal stress and strain in the core; "τ_{rz}^c", "$\tau_{\theta z}^c$", "γ_{rz}^c" and "$\gamma_{\theta z}^c$" declare the shear stresses and shear strains in the thickness direction of the core; "λ_r","λ_θ" and "λ_z" are the Lagrange multipliers at the face sheet-core interfaces.

The potential energy of establishment of plate on the Pasternak elastic foundation can be calculated as follows:

$$U_F = \frac{1}{2}\int_0^{\varnothing}\int_0^R (k_w w_0^2 + k_s\left(\frac{\partial w_0}{\partial r}\right)^2 + k_s\left(\frac{1}{r}\frac{\partial w_0}{\partial r}\right)^2).rdrd\theta \tag{15}$$

Also, the variation of this energy δU_F is as follows:

$$\delta U_F = (2k_w w_0^c - 2k_s\frac{\partial w_0^c}{\partial r} - 2k_s r\frac{\partial^2 w_0^c}{\partial r^2} - 2k_s\frac{\partial^2 w_0^c}{\partial\theta^2})\delta w_0^c \tag{16}$$

Where k_w is the Winkler elastic coefficient of Pasternak foundation; k_s is the shear elastic coefficient of Pasternak foundation. Total potential energy of the plate is sum of the variation of the strain energy of plate and variation of the potential energy of the elastic foundation.

$$\delta U = \delta U_p + \delta U_f \tag{17}$$

Considering small deflection, the linear and nonlinear strain components for the faces can be declared as follows [33]:

$$\in^j_{rr}(r,\theta,z_j,t) = u_{0j,r}(r,\theta,t) - z_j w_{j,rr}(r,\theta,t) \tag{18}$$

$$\begin{aligned}
&\in_{\theta\theta}^{j}\left(r,\theta,z_j,t\right)=\frac{1}{r^2}\{ru_{0j}\left(r,\theta,t\right)-rv_{0j,\theta}\left(r,\theta,t\right)-z_j w_{j,\theta\theta}\left(r,\theta,t\right)-rz_j w_{j,r}\left(r,\theta,t\right)\}\\
&\in_{r\theta}^{j}\left(r,\theta,z_j,t\right)=\frac{1}{r^2}\{ru_{0j,\theta}\left(r,\theta,t\right)+r^2v_{0j,r}\left(r,\theta,t\right)-r\upsilon_{0j}\left(r,\theta,t\right)+2z_j w_{j,\theta}\left(r,\theta,t\right)-2z_j w_{j,r\theta}\left(r,\theta,t\right)\}\\
&d_{rr}^{j}\left(r,\theta,z_j,t\right)=\frac{1}{2}\left(w_j,r\right)^2\\
&d_{\theta\theta}^{j}\left(r,\theta,z_j,t\right)=\frac{1}{2}\left(\frac{w_{j,\theta}}{r}\right)^2
\end{aligned}\tag{19}$$

The"$()_{,i}$" expresses derivation with respect to i. The strain of the core can be defined as [34]:

$$\begin{aligned}
&\in_{rr}^{c}=u_{0,r}^{c}+z_c u_{1,r}^{c}+z_c^2u_{2,r}^{c}+z_c^3u_{3,r}^{c}\\
&\in_{\theta\theta}^{c}=\frac{1}{r}\left[v_{0,\theta}^{c}+z_c v_{1,\theta}^{c}+z_c^2v_{2,\theta}^{c}+z_c^3v_{3,\theta}^{c}\right]+\frac{1}{r}\left[u_0^c+z_c u_1^c+z_c^2u_2^c+z_c^3u_3^c\right]\\
&\in_{zz}^{c}=\left[w_1^c+2w_2^c z_c\right]\\
&\in_{r\theta}^{c}=\frac{1}{r}\left[u_{0,\theta}^{c}+z_c u_{1,\theta}^{c}+z_c^2u_{2,\theta}^{c}+z_c^3u_{3,\theta}^{c}\right]+\left[v_{0,\theta}^{c}+z_c v_{1,\theta}^{c}+z_c^2v_{2,\theta}^{c}+z_c^3v_{3,\theta}^{c}\right]-\frac{1}{r}\left[v_0^c+z_c v_1^c+z_c^2v_2^c+z_c^3v_3^c\right]\\
&\in_{rz}^{c}=\left[u_1^c+2u_2^c z_c+3u_3^c z_c^2\right]+\left[w_{0,r}^{c}+z_c w_{1r}^{c}+z_c^2w_{2,r}^{c}\right]\\
&\in_{\theta z}^{c}=\left[v_1^c+2v_2^c z_c+3v_3^c z_c^2\right]+\frac{1}{r}\left[w_{0,r}^{c}+z_c w_{1r}^{c}+z_c^2w_{2,r}^{c}\right]
\end{aligned}\tag{20}$$

In this model by substituting the expressions of the Eq. (2) and Eq. (14) according to the kinematic relations of the layers and using the interfaces relations, and after some algebraic operations the twenty three equations of motion are obtained, which included twenty three unknowns: six displacement unknowns for both face sheets in Eqs. (21)-(25), eleven displacement unknowns for the core in Eqs. (26)-(36), and six Lagrange multipliers in Eqs. (37)-(42).

$$-I_{\circ t}\ddot{u}_{\circ t}r+I_{1t}r\ddot{w}_{t,r}-N_{rr}^{t}-rN_{rr}^{t},r+N_{\theta\theta}^{t}-N_{r\theta,\theta}^{t}+r\lambda_{rt}=0\tag{21}$$

$$-I_{\circ t}\ddot{u}_{\circ t}r+I_{1t}r\ddot{w}_{t,r}-N_{\theta\theta,\theta}^{t}-rN_{r\theta}^{t},r-2N_{\theta}^{t}+r\lambda_{\theta t}=0\tag{22}$$

$$\begin{aligned}
&-I_{1t}\ddot{u}_{\circ t}-I_{1t}r\ddot{u}_{\circ t,r}+I_{2t}r\ddot{w}_{t,rr}-I_{1t}\ddot{v}_{\circ t,o}+\frac{I_{zt}}{r}\ddot{w}_{t,ro}-I_{\circ t}r\ddot{w}_{t}-zM_{rr,r}^{t}-rM_{rr,rr}^{t}\\
&-N_{rr}^{tT}w_{t,r}-rN_{rr,r}^{tT}w_{t,r}-rN_{rr}^{tT}w_{t,rr}+2M_{\theta\theta,r}^{t}-\frac{2}{rM_{\theta\theta,rr}^{t}}-\frac{N_{oo}^{tT}}{r}wt,\theta\theta^{-}N_{r\theta,r\theta}^{t}\\
&-M_{r\theta,r\theta}^{t}-\frac{4}{r}M\,_{r\theta,\theta}^{t}+\frac{ht}{2}\lambda_{rt}+\frac{rht}{2}\frac{5\lambda_{rt}}{5r}+\frac{ht}{z}\frac{5\lambda\theta t}{5o}+r\lambda_{tt}=0
\end{aligned}\tag{23}$$

$$-I_{\circ t}\ddot{u}_{\circ b}r+I_{1b}r\ddot{w}_{b,r}-N_{rr}^{b}-rN_{rr}^{b},r+N_{\theta\theta}^{b}-N_{r\theta,\theta}^{b}+r\lambda_{rb}=0\tag{24}$$

$$-I_{\circ t}\ddot{v}_{\circ b}r+I_{1b}r\ddot{w}_{b,r}-N_{\theta\theta,\theta}^{b}-rN_{r\theta}^{b},r+2N_{r\theta}^{b}-N_{r\theta,\theta}^{b}+r\lambda_{ob}=0\tag{25}$$

$$\begin{aligned}
&-I_{1b}\ddot{u}_{\circ b}-I_{1b}r\ddot{u}_{\circ b,r}+I_{zb}\ddot{w}_{b,r}+I_{zt}r\ddot{w}_{b,rr}+I_{1b}\ddot{v}_{\circ b,\theta}+\frac{Izb}{r\ddot{w}_{b,r\theta}}-I_{\circ b}r\ddot{w}_{b}-2M_{rr,r}^{b}\\
&-rM_{rr,rr}^{b}-N_{rr}^{bt}w_{b,r}-rN_{rr,r}^{bt}w_{b,r}-rN_{rr}^{bT}w_{b,rr}+2M_{\theta\theta,r}^{b}-\frac{z}{r}M_{\theta\theta,rr}^{b}\\
&-\frac{hb}{z}\lambda_{rb}\frac{-rhb}{z}\frac{5\lambda_{rb}}{5r}-\frac{hb}{z}\frac{5\lambda_{ob}}{5\theta}r\lambda_{zb}=0
\end{aligned}\tag{26}$$

$$-I_{\circ c}\ddot{u}_{\circ c}-I_{1c}r\ddot{u}_{1c}-I_{zc}r\ddot{u}_{zc}-I_{3c}r\ddot{u}_{3c}-R_{rr}^{c}-rR_{rr,r}^{c}+R_{o}^{c}-Q_{ro,o}^{c}-r\lambda_{rt}+r\lambda_{rb}=0\tag{27}$$

$$-I_{1c}\ddot{u}_{\circ c}-I_{zc}r\ddot{u}_{1c}-I_{3c}r\ddot{u}_{zc}-I_{3c}r\ddot{u}_{3c}-R_{rr}^{c}-rR_{rr,r}^{c}+R_{\theta}^{c}-Q_{r\theta,\theta}^{c}-r\lambda_{rt}+r\lambda_{rt}+r\lambda_{rt}+rr_{b}=0\tag{28}$$

$$\begin{aligned}
&-I_{1c}r\ddot{u}_{\circ c}-I_{zc}r\ddot{u}_{1c}-I_{3c}r\ddot{u}_{zc}-I_{4c}r\ddot{u}_{3c}-M_{r1}^{c}-rM_{r1,r}^{c}+M_{\theta 1}^{c}+rQ_{rc}-M_{Q1r\theta,\theta}^{c}+hc/zr\lambda_{r1}\\
&\qquad+hc/zr\lambda_{rb}=0
\end{aligned}\tag{29}$$

$$\begin{aligned}
&-I_{1c}\ddot{u}_{\circ c}-I_{zc}r\ddot{u}_{1c}-I_{4c}r\ddot{u}_{zc}-I_{5c}r\ddot{u}_{3c}-M_{r2}^{c}-rM_{rz,r}^{c}+M_{\theta z}^{c}+2rM_{Q1rc}-M_{Qzr\theta,\theta}^{c}-hc/4r\lambda_{y4}\\
&\qquad+hc/4r\lambda_{rb}=0
\end{aligned}\tag{30}$$

$$-M_{r3}^{c}-rM_{r3,r}^{c}+M_{\theta 3}^{c}+3rM_{Q2rc}-M_{Q3r\theta,\theta}^{c}+hc^3/8r\lambda_{rt}+hc^3/r\lambda_{rb}=0\tag{31}$$

$$-I_{\circ c}\ddot{v}_{\circ c}-I_{1c}r\ddot{v}_{1c}-I_{2c}r\ddot{v}_{zc}-I_{3c}r\ddot{v}_{3c}-R_{\theta.\theta}^{c}-2Q_{r\theta}^{c}+rQ_{r\theta.r}^{c}-r\lambda_{\theta t}+r\lambda_{\theta t}+r\lambda_{\theta b}=0\tag{32}$$

$$-I_{1c}r\ddot{v}_{\circ c}-I_{2c}r\ddot{v}_{1c}-I_{3c}r\ddot{v}_{zc}-I_{4c}r\ddot{v}_{3c}-M_{\theta 1.\theta}^{c}+Q_{\theta}c^{r}-rM_{Q1r\theta.r}^{c}+hc/zr\lambda\theta t+hc/zr\lambda\theta b=0\tag{33}$$

$$\begin{aligned}
&-I_{2c}r\ddot{v}_{\circ c}-I_{3c}r\ddot{v}_{1c}-I_{4c}r\ddot{v}_{zc}-I_{5c}r\ddot{v}_{3c}-M_{\theta 2.\theta}^{c}+2rM_{\mathrm{Q1\theta c}}-zM_{Q2r\theta}^{c}-rM_{Q2r\theta,r}^{c}+hc/4r\lambda\theta b\\
&\qquad+hc^2/r\lambda\theta b=0
\end{aligned}\tag{34}$$

$$\begin{aligned}
&-I_{3c}r\ddot{v}_{\circ c}-I_{4c}r\ddot{v}_{1c}-I_{5c}r\ddot{v}_{zc}-I_{6c}r\ddot{v}_{3c}-M_{\theta 3.\theta}^{c}+3rM_{Q2\theta c}-2M_{Q3r\theta}-rM_{Q3r\theta}^{c}+hc/zr\lambda\theta b\\
&\qquad+hc^3/zr\lambda\theta b=0
\end{aligned}\tag{35}$$

$$-I_{0c}r\ddot{v}_{\circ c}-I_{1c}r\ddot{v}_{1c}-I_{2c}r\ddot{v}_{zc}-Q_{\theta t.\theta}^{c}-Q_{rc}^{c}-rQ_{rc.r}^{c}-r\lambda tt-r\lambda tb=0\tag{36}$$

$$-I_{1c}r\ddot{\upsilon}_{\circ c}-I_{2c}r\ddot{\upsilon}_{1c}-I_{3c}r\ddot{\upsilon}_{2c}-rR_t^c-\frac{5M_{Q1\theta c.\theta}^c}{5\theta}-M_{Q1re}-r_{Q1rc.r}^c+hc/zr\lambda_{tt}-hc/zr\lambda tb=0 \tag{37}$$

$$u_{0t}-\frac{h_t}{2}\frac{\partial w_t}{\partial r}-u_{0c}+\frac{h_c}{2}u_{1c}-\frac{h_c^{\,2}}{4}u_{2c}+\frac{h_c^{\,3}}{8}u_{3c}=0 \tag{38}$$

$$\nu_{0t}-\frac{h_t}{2r}\frac{\partial w_t}{\partial\theta}-\nu_{0c}+\frac{h_c}{2}\nu_{1c}-\frac{h_c^{\,2}}{4}\nu_{2c}+\frac{h_c^{\,3}}{8}\nu_{3c}=0 \tag{39}$$

$$\mathrm{w}_{0t}-\mathrm{w}_{0c}+\frac{\mathrm{h}_c}{2}\mathrm{w}_{1c}-\frac{\mathrm{h}_c^{\,2}}{4}w_{2c}=0 \tag{40}$$

$$u_{0c}+\frac{h_c}{2}u_{1c}+\frac{h_c^{\,2}}{4}u_{2c}+\frac{h_c^{\,3}}{8}u_{3c}-u_{0b}-\frac{h_b}{2}\frac{\partial w_b}{\partial r}=0 \tag{41}$$

$$\nu_{0c}+\frac{h_c}{2}\nu_{1c}+\frac{h_c^{\,2}}{4}\nu_{2c}+\frac{h_c^{\,3}}{8}\nu_{3c}-\nu_{0b}-\frac{h_b}{2r}\frac{\partial w_b}{\partial\theta}=0 \tag{42}$$

$$w_{0c}+\frac{h_c}{2}w_{1c}+\frac{h_c^{\,2}}{4}w_{2c}-w_{0b}=0 \tag{43}$$

Stress resultants, moment resultants, thermal stress and moment resultants and inertia terms of the faces, and high order stress resultants of the core have been presented in references [35-36].

Finally, by substituting the high order stress resultants in the equations of the face sheets and the core in terms of the displacement components, the governing equations of motion are derived in terms of the twenty three unknowns. However, for a clamped circular sandwich plate, a Galerkin method solution could be established.

Clamped circular sandwich plate

In order to solve the equations of the free vibration of the clamped FG circular sandwich plate, a Galerkin method with twenty three trigonometric shape functions, which satisfy the boundary conditions, is established. The shape functions can be expressed as:

$$u_{0j}=\left[C_{uj}rsin\lambda r\right]e^{i\omega t}\quad ,j=(t,b) \tag{44}$$

$$w_{0j}=\left[C_{wj}\left(cos\lambda r+r\lambda sin\lambda r-\lambda a\right)\right]e^{i\omega t} \tag{45}$$

$$u_k=\left[C_{uk}rsin\lambda r\right]e^{i\omega t}\quad ,k=\left(0,1,2,3\right) \tag{46}$$

$$w_l=\left[C_{wl}\left(cos\lambda r+r\lambda sin\lambda r-\lambda a\right)\right]e^{i\omega t}\qquad l=\left(0,1,2\right) \tag{47}$$

$$\lambda_{rj}=\left[C_{\lambda_{rj}}rsin\lambda r\right]e^{i\omega t} \tag{48}$$

$$\upsilon_{0j}=\upsilon_k=\lambda_{\theta j}=0\qquad ,k=\left(0,1,2,3\right) \tag{49}$$

$$\lambda_{zj}=\left[C_{\lambda_{zj}}\left(cos\lambda r+r\lambda sin\lambda r-\lambda a\right)\right]e^{i\omega t} \tag{50}$$

where "C_{uj}‘C_{wj}‘C_{uk} ‘C_{wl}, $C_{\lambda rj}$ and $C_{\lambda zj}$" are fifteen unknown constants and $\lambda=\pi/2a((2n-1))$ that "n" is the wave number. Since the plate is axisymmetric, eight equations are eliminated and the number of the equations are reduced to fifteen. On the other hand, these fifteen equations are not independent and by a procedure the number of them are reduced. Lagrange constants can be isolated as the faces constants. It's seen that based on the compatibility conditions the unknown constants of the faces are dependent to the core constants. At last by some operations the number of the equations are reduced to seven in terms of the core unknown constants. The seven equations can be written in the 7*7 matrix form which include the mass, "M", and stiffness, "K", matrices in accordant to the Eq. (51) to obtain the constant Eigen values which equals to Eigen frequencies, ω_m , for every wave number, m.

$$\left(k_m-\omega_n^2M_m\right)C_m=0 \tag{51}$$

In Eq. (51), F_n is the Eigen vector which determines the seven unknown constants of the core. For simplicity, the fundamental frequency parameter defined that is non-dimensional as:

$$\bar{\omega}=\frac{\omega a^2}{h}\sqrt{\frac{\rho_0}{E_0}} \tag{52}$$

where "a" is the radius of circular sandwich plate; "h" is the total thickness of sandwich plate; ρ_0 is density equal to 1kg/m^3 and E_0 is the young module equal to 1 GPa.

Verification and Numerical results

To validate the approach of this work, the present results in a special case are compared with results of [13, 35] and FEM results of Abaqus software for a clamped isotropic circular plate with properties: E=380 (GPa), $\rho=3800\ (\frac{Kg}{m^3})$, $\upsilon=0.3$ and $\frac{h}{a}=0.2$ as shown in Table 1. Because, the theory of present analysis is different from the [13, 37], a discrepancy is found in the results. Also, discontinues model is used in Abaqus model that causes a little discrepancy with present analysis.

Table 1
Comparison of fundamental frequency parameters of present, [13, 37] and Abaqus results

m	Present result	[13]	[37]	ABAQUS
1	10.232	10.213	10.216	10.217
2	21.472	21.259	21.260	21.265

Now, another numerical problem will be discussed to more investigation the present approach. Consider a clamped FG circular sandwich plates as shown in Fig. 1.

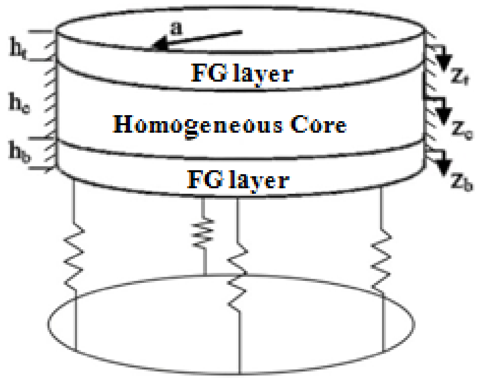


Fig 1. Schematic of FG circular sandwich plate on Pasternak foundation

The face sheets interior planes and the core are made of the stainless steel and the outer planes of the faces are made of silicon nitride. The properties of these materials are available in [30]. Variation of the material properties in each FG layer is correspond to the modified power-law function.

To validate the present method, numerical examples are simulated by Abaqus software, version 6.13. The continuum three dimensional and eight nodes hexagonal with the effect of thermal elements (C3D8T) are used to mesh the samples as shown in Fig. 2. In order to simulate the FG face-sheets and FG core in Abaqus, all FG layers are divided to 20 sub-layers and each sub layer has different properties according to the power law function. Also, the number of the elements in FEM are dependent to the convergence of the results. First 4000 elements are considered. By increasing the number of elements, it's seen that the variation of the results are high. But, after 12000 elements, there is a convergence between the results. So, it's found that 12000 elements are proper and increasing the elements more than 12000, just increases the time of the solving and doesn't have any important effect on the results. Also, explicit solution is used to solve the problem.

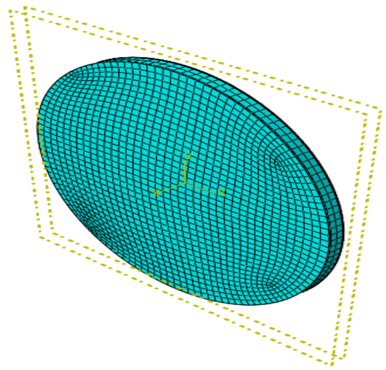


Fig 2. Finite element model for the clamped sandwich plate.

In Table 2 fundamental frequency parameters of this approach are compared with the FEM results by Abaqus software in the temperature of the room and for different power law indices in the case of 1-1-1, 2-1-2 and 1-8-1 sandwiches. It should be noted that in 1-8-1 sandwich, the core thickness is eight times of every face sheets thickness and the structure is symmetric. In these Tables, the discrepancies between the present results and FEM results are due to simulation method of FG layers in Abaqus software. There is a good agreement between the present study results and the FEM results obtained by Abaqus.

Table 2
Comparison of fundamental frequency parameters of the present method and Abaqus

Discrepancy(%)	Abaqus	Present method	N
	1-8-1		
2.15	0.151229	0.154492	0
3.49	0.148296	0.153475	0.2
7.75	0.140353	0.151232	1
	1-1-1		
4.81	0.180013	0.188672	0
7.9	0.1705121	0.183997	0.2

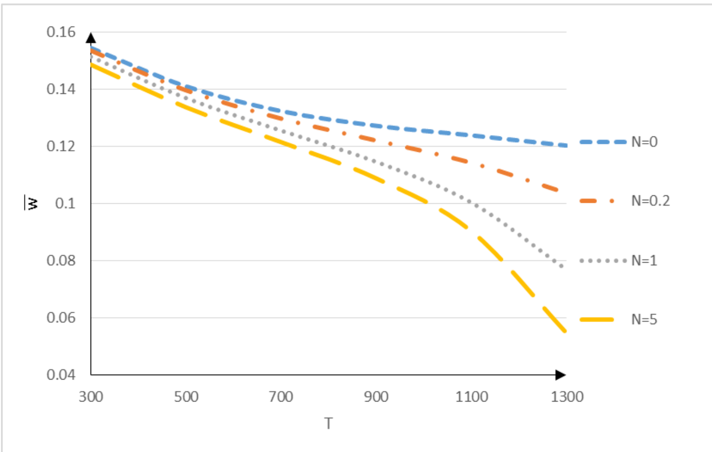
7.51	0.160833	0.172924	1
2-1-2			
6.91	0.192115	0.205392	0
5	0.190031	0.199543	0.2
7.19	0.173231	0.185699	1

In Table 3 fundamental frequency parameters of this approach are presented for different temperature and different Ksbar in the case of 1-1-1, 2-1-2 and 1-8-1 sandwiches.

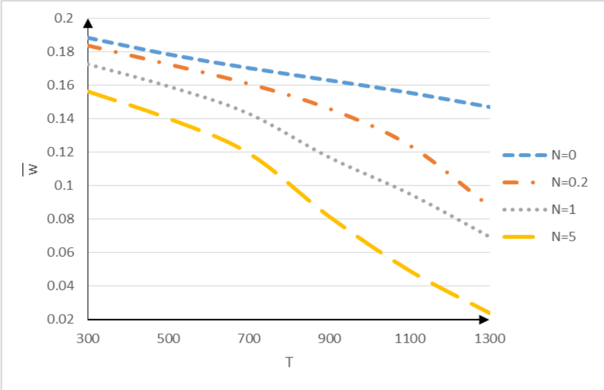
Table 3
Fundamental frequency parameters of the present method

1300 (k)	900 (k)	300 (k)	Ksbar
1-8-1			
0.120209	0.127166	0.154492	0
0.120244	0.1272	0.154531	10
0.120381	0.127335	0.154687	50
0120553	0.127502	0.154882	100
1-1-1			
0.14703	0.163126	0.188672	0
0.147031	0.163127	0.188673	10
0.1477032	0.163130	0.188674	50
0.147033	0.163132	0.188675	100
2-1-2			
0.162053	0.180315	0.205392	0
0.162054	0.180316	0.205393	10
0.162055	0.180317	0.205394	50
0.162056	0.180318	0.205395	100

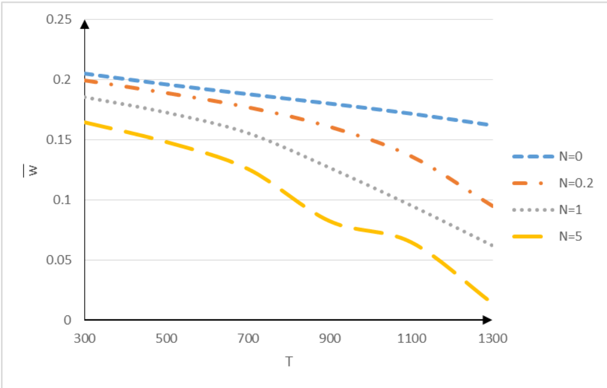
The frequency of the structures are dependent to the temperature variation. The effect of the uniform temperature distribution on the fundamental frequency parameter is depicted in Fig. 3 for three types of clamped circular FG sandwich plates in different power law indices. As shown in the figures, while the temperature is increased in a constant power law index, the fundamental frequency parameter decreases. According to Eq. (5), temperature rising reduces the strength of the material. With increasing the temperature, modulus of metal and ceramic decrease, but due to the microstructural reasons, decreasing the module of metal is more. So, increasing the temperature reduces the mechanical properties that is one of the most important reason in decreasing the frequency in high temperature. Also in a constant temperature, the fundamental frequency is decreased in the larger power law indices. Because, with increasing the power-law index the properties of the layers are tending to metal and the strength of the structure is decreased. It is obvious that the values of the fundamental frequency parameters in 2-1-2 sandwich are more than 1-1-1 one and the 1-8-1 sandwich is lower than 1-1-1 one.



1-8-1



1-1-1

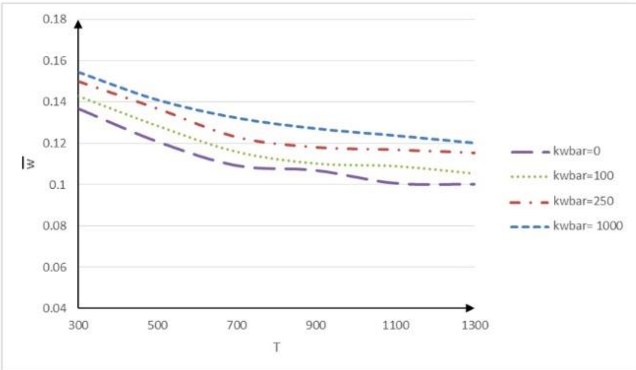


2-1-2

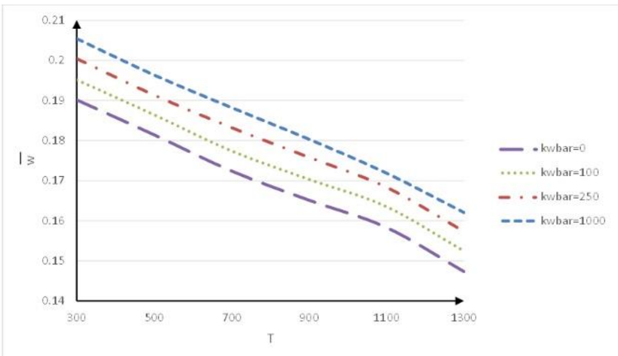
Fig 3. Frequencies changing with temperature in various power law indices for different circular sandwich plates

Frequencies changing with temperature in various Winkler elastic coefficient of Pasternak foundation for 1-8-1, 1-1-1 and 2-1-2 sandwich plates are shown in Fig 4.

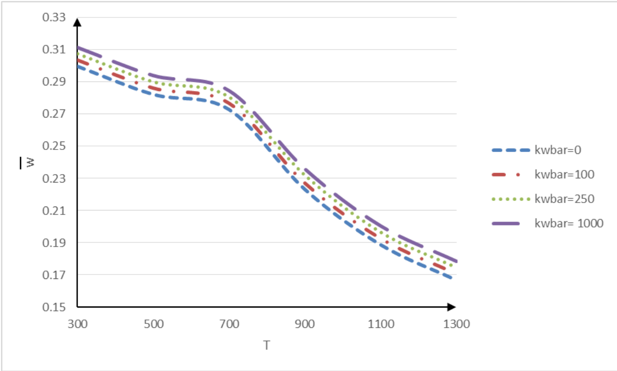
It is obvious that by increasing the Kwbar, the fundamental frequencies increase. The frequency parameter in the case of 2-1-2 is highest.



1-8-1



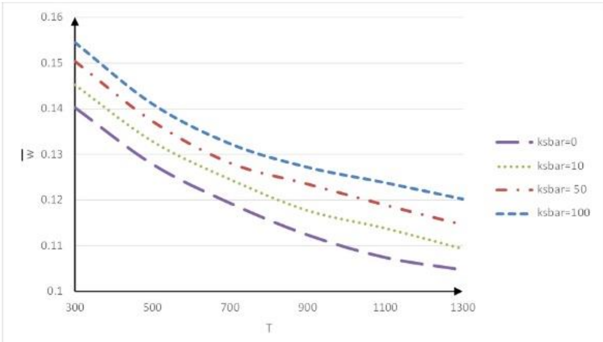
1-1-1



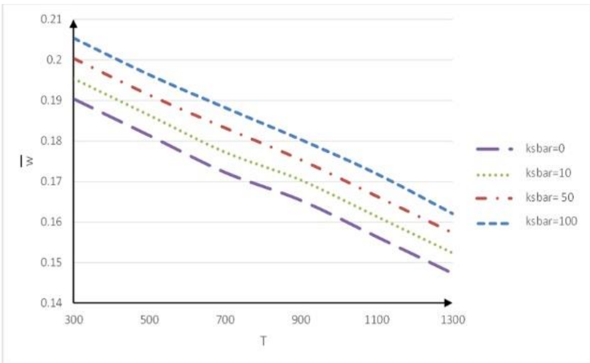
2-1-2

Fig 4. Frequencies changing with temperature in various Winkler elastic coefficient of Pasternak foundation for different circular sandwich plates

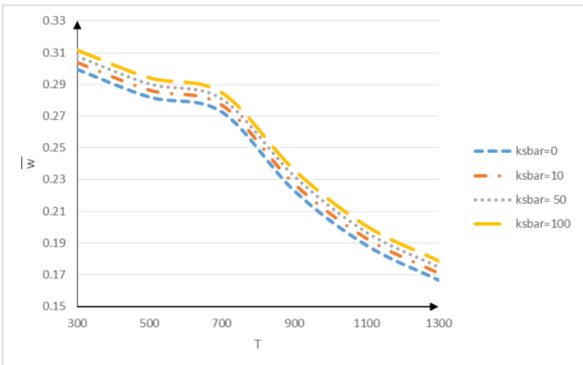
Frequency changing with temperature in various shear elastic coefficient of Pasternak foundation for 1-8-1, 1-1-1 and 2-1-2 sandwich plates are shown in Fig. 5. It is obvious by increasing the Ks, the fundamental frequencies increase. The frequency parameter in the case of 2-1-2 is highest.



1-8-1



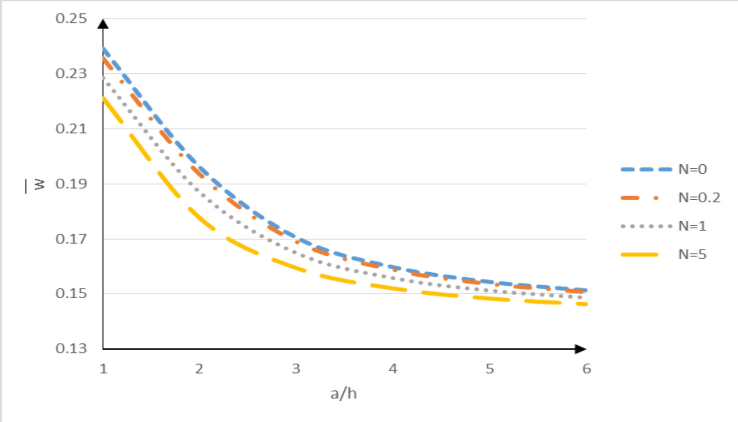
1-1-1



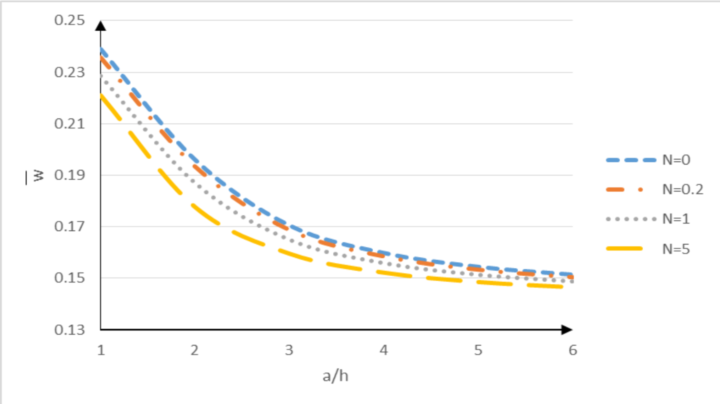
2-1-2

Fig 5. Frequencies changing with temperature in various shear elastic coefficient of Pasternak foundation for different circular sandwich plates

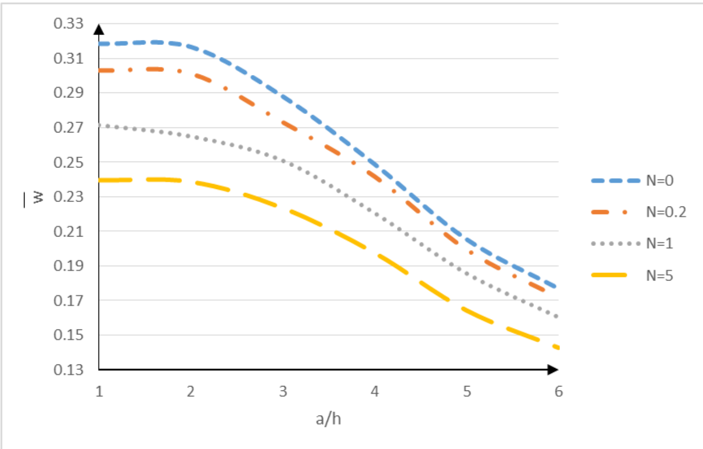
Variation of the fundamental frequency with radius to thickness ratio for different power law index for different types of sandwiches are shown in the Fig. 6. With increasing the ratio, the strength of the structure decreases, so by increasing this ratio the fundamental frequency decrease. The frequency parameter in the case of 2-1-2 is highest.



1-8-1



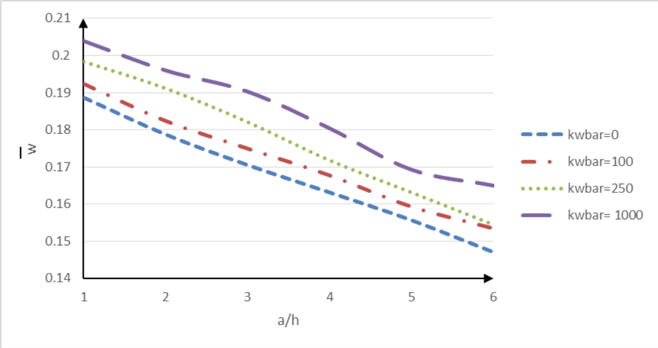
1-1-1



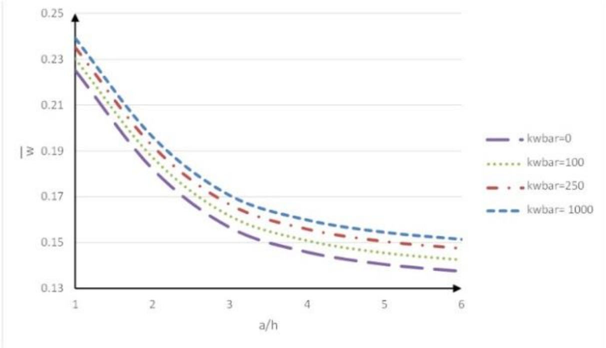
2-1-2

Fig 6. Variation of the fundamental frequency with radius to thickness ratio for different power law index for different circular sandwich plates

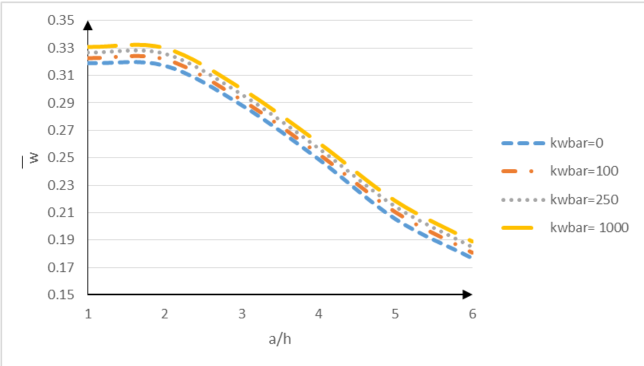
Variation of the fundamental frequency with radius to thickness ratio for different Winkler elastic coefficient for different types of sandwiches are shown in the Fig. 7. With increasing the ratio, the strength of the structure decreases, so by increasing this ratio the fundamental frequency decrease. With increasing the Winkler coefficient, the fundamental frequency increases.



1-8-1



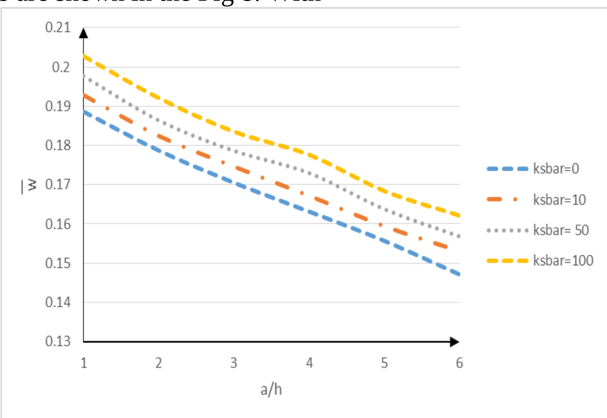
1-1-1



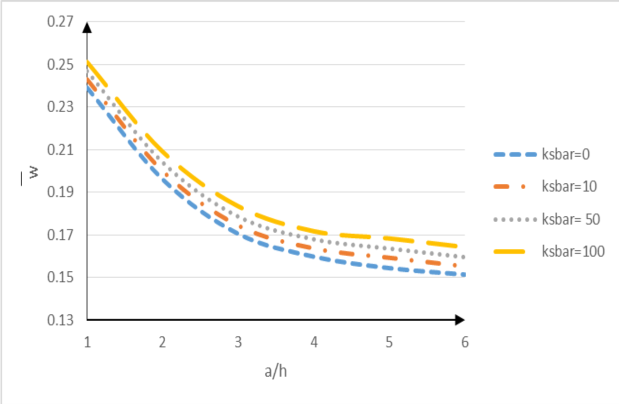
2-1-2

Fig 7. Variation of the fundamental frequency with radius to thickness ratio for different Winkler elastic coefficient for different circular sandwich plates

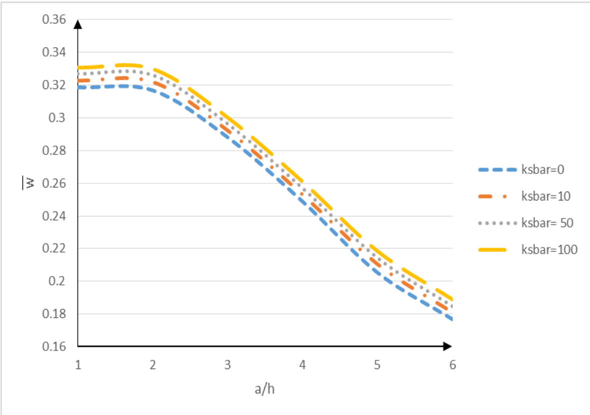
Variation of the fundamental frequency with radius to thickness ratio for different shear elastic coefficient for different types of sandwiches are shown in the Fig 8. With increasing the shear coefficient, the fundamental frequency increases.



1-8-1



1-1-1



2-1-2

Fig 8. Variation of the fundamental frequency with radius to thickness ratio for different shear elastic coefficient for different circular sandwich plates

Conclusion

Temperature dependent vibration behaviour of circular sandwich plates with FG face sheets on the Pasternak elastic foundation which is subjected to a uniform high temperature distribution were investigated based on the modified high order sandwich plate theory. Governing equations were derived based on the Hamilton's energy principle. Material properties of the FG layers were temperature and location dependent. Power law rule was employed to model the gradually variation of the properties in the FG layers. The homogeneous layer was temperature dependent, too. In plane and out of plane stresses of the core were considered at the same time. There are different methods to solve the equations which the Galerkin method is selected among them in a clamped boundary condition. In order to validate present approach, the numerical results which obtained by Abaqus software were compared to the results of this analytical approach and for a special case compared with some literature. Based on the results obtained by this approach and comparing with FEM results, there was a good agreement with them and the following conclusion can be drawn.

1. With increasing the temperature in a constant power law index, the fundamental frequency parameter decreases.
2. While power law index is increased in a constant temperature, the fundamental frequency parameter decreases.
3. By increasing the radius to thickness ratio, the fundamental frequency parameter decreases.
4. With increasing the winkler coefficient, the fundamental frequency increases.
5. With increasing the shear coefficient, the fundamental frequency increases.
6. In the 2-1-2 sandwich, the fundamental frequency is the highest and in the 1-8-1 one is the lowest.

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