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Quadrants Dynamic Fuzzy Histogram Equalization for Color Images with Brightness Preservation

Nooshin Allahbakhshi[1], Yashar Salami[2]*, Mohammad Bagher Karimi[3], Kioumars Abdi[4], Yaser Pourshadlou[5]

1. Department of Computer and Information Technology Engineering, Khoy Branch, Islamic Azad University, Khoy, Iran
2. Faculty of Computer and Information Technologies, Cappadocia University, Nevsehir, Turkey
3. Department of Computer Engineering, Tabriz branch, Islamic Azad University, Tabriz, Iran
4. Kapadokya Üniversitesi, Kapadokya Meslek Yüksekokulu, Bilgisayar Programcılığı, Nevşehir, Türkiye
5. Department of Electrical and Electronics Engineering, Tabriz branch, Islamic Azad University, Tabriz, Iran

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**Corresponding Author's Email Address:*
Yashar.salami@gmail.com

Abstract

Contrast enhancement is essential in image processing and contributes to image enhancement. Histogram equalization is perhaps the most common way operators enhance the contrast of digital images. Easy and handy, this method often has too much contrast enhancement, making the output images' visual quality look unnatural. Moreover, it usually cannot also preserve the mean of the image substantially. This paper presents a color image equalization technique that takes a better guess to conserve the brightness. In other words, it is a method based on some image histogram modification using fuzzy and a clipping process for equalization rate applied to the original image. Initially, the histogram is split into two parts hinged on the mean gray level. Then, it is divided into four sections by calculating an average of the two sub histograms. The dynamic equalization is defined for a new range, and the sub-histogram equalization is independent. The simulation results prove that this new method can significantly improve the spatial characteristics of color images and keep a high brightness level.

1. Introduction

In today's digital era, the rise of the Internet of Things [1], [2] and blockchain technology [3][4] has transformed data-driven applications [5][6], highlighting the need for secure and reliable information exchange[4], [7]. This reliance on trustworthy multimedia and image data makes digital image processing increasingly vital, a field that has historically advanced in response to real-world demands.

In the 1960s, NASA's Ranger 7 spacecraft transmitted unclear television images of the Moon's surface to Earth, marking the initial steps toward global human-space communication [8]. With the increasing demand for extracting image details and identifying suitable landing sites for the Apollo missions, the field of digital image processing emerged, leading to the rapid advancement and adoption of this technology [9]. However, the images obtained from these early transmissions were often affected by noise and distortions, including blurring and image fading, significantly diminishing their clarity and quality[10]. As a result, removing noise and addressing visual defects such as improper lighting parameters and poor color composition have become critical concerns in image processing. All techniques and methods employed to enhance image quality and reduce visual imperfections fall under the domain of image processing [11].

Since its inception in 1964, image processing has witnessed remarkable growth and has extended beyond space research to various fields.[12]. It is now integral to various applications such as medicine, speech recognition, handwriting recognition, archaeology, astronomy, biology, nuclear medicine, and industries including aerospace, packaging, automotive, pharmaceuticals, medical diagnostics[13], [14] , and meteorology[15]. One standard method for improving image quality is image enhancement, particularly contrast enhancement. Image enhancement involves adjusting the intensity values of an input image so that the output image appears visually enhanced. The primary objective of image enhancement is to make the information in the image more interpretable for human viewers or to optimize it as an input for automated image processing systems.[16].

Histogram equalization (HE) is a widely recognized method for contrast enhancement that redistributes an image's intensity values.[17], [18]. The fundamental concept of HE is to map the input image intensity values to new intensities using a cumulative distribution function (CDF)[19]. This process effectively broadens and flattens the image's histogram, improving overall contrast. Initially, histogram equalization transforms the original image’s histogram into a uniform distribution based on the average grayscale levels. Consequently, the average brightness of the output image is centered around the mean brightness of the input image. This adjustment is particularly significant for images with low or high brightness, as it enhances contrast. Subsequently, the second phase of HE performs contrast enhancement based on the overall content of the image.[20].

Several HE methods have been proposed to preserve image brightness while improving contrast. These methods are typically classified into partitioned histogram equalization (PHE) and dynamic partitioned histogram equalization (DPHE) [21], [22]. Both approaches rely on statistical information to divide the original histogram into multiple sub-histograms. The primary distinction between them is that DPHE assigns a new dynamic range instead of utilizing the original range. PHE-based methods include brightness-preserving bi-histogram equalization (BBHE)[23], [24] and multi-peak histogram equalization with brightness preservation (MPHEBP)[25]. In contrast, DPHE methods are fewer in number, with examples such as dynamic histogram equalization (DHE) and brightness-preserving dynamic histogram equalization (BPDHE) [26]. Furthermore, to enhance images captured in low-light conditions, quarter dynamic histogram equalization (QDHE) has been introduced [27]. Another method was presented.

In recent years, additional methods have been developed, including color image enhancement based on gamma encoding and histogram equalization [28], and low-contrast enhancement for color images using intuitionistic fuzzy sets with adaptive histogram equalization [29]. In another paper, they present a method that uses a variational approach including an energy function to determine local transformations in the luminance (L) and chroma (C) channels of the CIE LCH color space[30] . Another paper introduces exposure-based recursive histogram equalization techniques along with an energy curve instead of the conventional histogram[31] . Another paper presented is fuzzy logic-based histogram segmentation based on maximum and minimum peaks, which combines it with an entropy-controlled coefficient correction system[32].

The proposed technique offers a fresh method for these problems, characterized by fuzzy histograms with an idealized membership function.

The strategy successfully counters issues of grey or color imprecisions; it not only eradicates the random variations but also prevents loss of intensity levels without extra smoothing. Fuzzy statistics indeed deliver a significant gain in performance, as is demonstrated through experimental results, too. Merging quarter histograms and fuzzy statistics improves the capability of the proposed method to solve incompletely developed narrow sub-histogram problems, as low-light images will be better reconstructed.

1.1. Contribution

Image processing and contrast enhancement are important and are number one in image enhancement. Histogram Equalization is perhaps the most popular way for operators to brighten contrast on digital images. Easy and handy Sadly, this method also has the problem of output images themselves looking artificial due to the notorious contrast enhancement. Also, it is hard to keep the mean of the image at a significant level. This paper introduces a color equalization method of images, which gives a better idea of maintaining lightness. It is a Fuzzy and Blending method of equalization rate applied to the original image that is reduced to level histogram modification tandem. In the beginning, the histogram is split into two parts according to the mean gray level, then divided into four parts, which uses an average of two sub-histograms. Dynamic equalization is described independently for a new range and sub-histogram equalization. Simulation results indicate that this novel method effectively affects much of spatial color images in terms of improvement, keeping high brightness.

1.2. Paper organization

The paper's organization is as follows: In the second section, the phases of the proposed method are described in detail, outlining each step systematically. The third section covers the simulation environment and discusses the obtained results, providing an in-depth analysis of the proposed approach's performance and effectiveness. Finally, the concluding section summarizes the key findings and reflects on the implications of the results.

2. Proposed Scheme

The proposed method begins by transforming the image from the RGB color space to the HSV color space, allowing for better image contrast and intensity manipulation. This transformation is crucial because the HSV color space separates the intensity (value) component from the color information, making enhancing the contrast of low-light and low-contrast images easier. Next, the input histogram is converted into a fuzzy histogram to smooth out the intensity values and prepare the data for more precise processing. The fuzzy histogram introduces a level of uncertainty, helping to preserve image details during the enhancement process. A clipping process is then applied to tackle saturation issues, which often arise in contrast enhancement. The mean intensity value of the image is used as a threshold, and intensity values exceeding this threshold are clipped, preventing over-bright areas and ensuring that the enhancement remains natural. The mean intensity of the image is used to divide the fuzzy histogram into four parts, and then it is in the mean range. This separated way allows for adjusting the contrast of each sub-histogram in its dynamic range, so you slowly increase saturation for all intensity levels but not so much the brightness. After equalization, the image was converted from HSV color space to RGB. This step aims to enhance contrast further but keep the image's original color. The proposed method increases contrast in such images, improving visual clarity and detail preservation. Next, a detailed description of each step in this process is given in the following parts.

2.1. Calculation of the Fuzzy Histogram

After converting the image to the HSV color space, the input histogram is transformed into a fuzzy histogram, as Equation (1) describes. Here, fuzzy Histogram(i) represents the frequency of gray levels around intensity i. The function I(x, y) denotes the gray level intensity at pixel coordinates (x, y), and its corresponding fuzzy value is computed based on the fuzzy histogram.

$$\text{fuzzyHistogram}(i), i \in \{0,1,2,\ldots,L-1\} \quad (1)$$

Where fuzzyHistogram(i) is actually the number of repetitions of gray levels around i. I(x,y) represents the gray value and represents its fuzzy value. a membership function must first be defined using Equation (2) to achieve this transformation. In the proposed method, a triangular membership function is employed. The fuzzy triangular membership function is defined a s follows:

$$\mu_{\tilde{I}(x,y)} = \max\left(0.1 - \frac{I(x,y) - i}{4}\right) \quad (2)$$

$\mu_{\tilde{I}(x,y)}$ is the fuzzy triangular membership function . This equation accumulates the fuzzy values corresponding to each gray level, thus creating the fuzzy histogram, which is subsequently used for further processing in the algorithm. The parameters a and c in the equation are adjustable elements of the membership function, allowing for control over the shape and sensitivity of the fuzzy set. The gray level intensity i is an essential component, as it is used to compute the fuzzy value for each gray level in the histogram.

The reasons for using the fuzzy triangular membership function include: gradual modeling of changes in brightness, increasing accuracy in processing brightness levels, reducing the effect of noise and improving image smoothness, and preserving details in dark or bright areas.

Equation (3) calculates the fuzzy histogram, incorporating these parameters to accurately reflect the distribution of gray levels in the image in a fuzzy framework. This approach enhances the histogram's ability to capture subtle differences in intensity, making it more effective in contrast enhancement and image segmentation.

$$\text{fuzzyHistogram}(i) = \text{fuzzyHistogram}(i) + \sum_x \sum_y \mu_{\tilde{I}(x,y)i} \quad (3)$$

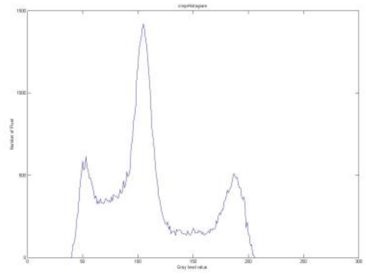


Figure 1. Crisp histogram of the input image

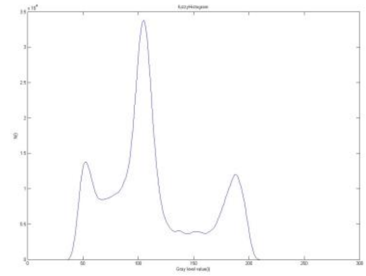


Figure 2. Fuzzy histogram obtained from the image

Figure1 shows the crisp histogram of the input image,which is transformed into Figure 2 by applying the fuzzy process that can be performed using equations (2) and (3). The fuzzy statistic is able to apply more precision to the gray value than the classic crisp histogram, thus producing a smooth histogram.

2.2. Application of Clipping Process Based on a Threshold Value

One key reason for utilizing the clipping process is to regulate the equalization rate, thereby avoiding unnatural image processing and excessive enhancement that could distort the original image. To simplify and reduce computational complexity, the mean intensity value of the image can be used to determine the clipping point, which is the approach adopted in this study. In the clipping process, the mean intensity of the input image is first calculated, and this value is taken as the clipping point or threshold (denoted as TCT_CTC). This threshold is then applied to the fuzzy histogram obtained from the image. Specifically, all values in the fuzzy histogram that exceed this threshold are replaced by the threshold value itself, while values below the threshold remain unchanged. This method ensures that the image's intensities are controlled, preventing overly bright or saturated regions, which ultimately enhances the image's overall quality and clarity.

2.3. Fuzzy Histogram Segmentation

The steps involved in the segmentation process based on the mean of the fuzzy histogram of the image are crucial for enhancing image analysis. Here is an expanded explanation of the process:

2.3.1. Segmentation of the Fuzzy Histogram: The process begins by dividing the fuzzy histogram into two sub-histograms based on the mean value of the entire histogram. This segmentation is achieved by separating the pixel intensity values into two categories: those above the mean and those below the mean. These two sub-histograms correspond to higher and lower intensity values relative to the mean, ensuring that the image's darker and brighter areas are distinctly addressed.

2.3.2. Calculation of Sub-Histogram Means: After the initial division, the mean value of each sub-histogram is calculated. These new mean values become the key separation points between the sub-histograms. By doing so, the algorithm identifies significant intensity boundaries within high and low-intensity regions, preparing the data for further segmentation.

2.3.2. Division into Four Sub-Histograms: The fuzzy histogram is divided into four distinct sub-histograms using the previously calculated mean values as separation points. Each sub-histogram represents a different image intensity range, with two sub-histograms dedicated to the lower intensities and two to the higher intensities. This finer segmentation allows for a more precise analysis of the intensity distribution across the image.

This segmentation process divides the image into distinct regions based on intensity levels and enhances the overall image analysis by providing more detailed information about how intensity values are distributed. As a result, it significantly improves the quality and clarity of the image, making it easier to analyze subtle details, contrasts, and textures. The additional information gained from this segmentation plays a key role in various image processing tasks, including contrast enhancement, edge detection, and texture analysis.

2.4. Assignment of New Gray Level Ranges

Dynamic equalization processes each sub-histogram to achieve a balanced and precise equalization. By dynamically assigning a gray level range based on the span of gray levels and the total number of pixels within each sub-histogram, this method ensures that each sub-histogram receives an appropriately tailored range for optimal equalization. This approach significantly improves the equalization of each sub-histogram, minimizing the risk of losing image details or encountering intensity saturation effects, which are common in conventional equalization techniques.

The dynamic range assignment, designed to enhance different image sections adaptively, is mathematically formulated using Equations (4) and (5). These equations dictate how the gray level ranges are distributed, ensuring that the intensity transitions between different parts of the image remain smooth while preventing over-compression or excessive expansion of the gray levels. The result is an image with better-preserved details, contrast, and overall quality.

$$\mathrm{span_i} = \mathrm{high_i} - \mathrm{low_i} \tag{4}$$

$$\mathrm{range_i} = (\mathrm{L} - 1) \times \mathrm{span_i} / \sum_{k=1}^{4} \mathrm{span_k} \tag{5}$$

The parameters $high_i$ and low_i are the maximum and minimum intensity values under histogram i are respectively. The dynamic range used by sub-histogram i in the input image is denoted by $\mathrm{span_i}$ While the dynamic range applied in the output image is denoted by $\mathrm{range_i}$ For the i-the sub-histogram, the new dynamic range is assigned to the interval $[\mathrm{i_{start}}\ \mathrm{i_{end}}]$ Which is determined by Equations (6) and (7).

$$\mathrm{i_{start}} = (\mathrm{i} - 1)_{\mathrm{end}} + 1 \tag{6}$$

$$i_{end} = i_{start} + range_i \quad (7)$$

The first value i_{start} is the smallest intensity value of the new dynamic range.

2.5. Dynamic Equalization of Each Sub-histogram

Dynamic equalization for each sub-histogram facilitates the individualized equalization of every sub-histogram, preventing the issues of under or over-equalization in different regions of the image. This is achieved by allocating unique, non-overlapping gray level ranges to each sub-histogram. This ensures that gray levels from distinct sub-histograms are not mapped to the same gray level in the final output image. The method guarantees that the entire gray level spectrum is utilized efficiently, maintaining the contrast and detail across all image regions.

For sub-histogram iii, which operates within the range $[i_{start} \quad i_{end}]$, the transfer function responsible for equalizing the output histogram is mathematically expressed by Equation (8). This equation governs how pixel intensities are redistributed within the designated range, ensuring optimal enhancement without introducing artifacts or excessive brightness changes. The adaptive nature of this approach contributes to more accurate image enhancement, particularly in cases with varied lighting conditions or uneven intensity distributions across different parts of the image.

$$y(x) = (i_{start} - i_{end}) \times cdf(X_k) + i_{start}\, cdf(X_k) \quad (8)$$

$cdf(X_k)$ is the cumulative distribution function in that sub histogram. This formula is the same as the HE formula, but instead of maximum and minimum intensity, i_{start} and i_{end} are used in the dynamic range of the output.

3 Simulation

3.1. Simulation Environment

A proposed method was implemented and checked on a Fujitsu laptop (4 GB RAM, 128 GB disk, Intel Core i5). The simulations were performed using MATLAB, a very well-known platform commonly used for image processing and algorithm development.

The critical aspect of this approach was accomplished using extensive libraries (built-in functions) in the MATLAB environment and specialized toolboxes to leverage fuzzy histogram equalization and intricate following calculations, ensuring that the simulations establish correct and efficient results.

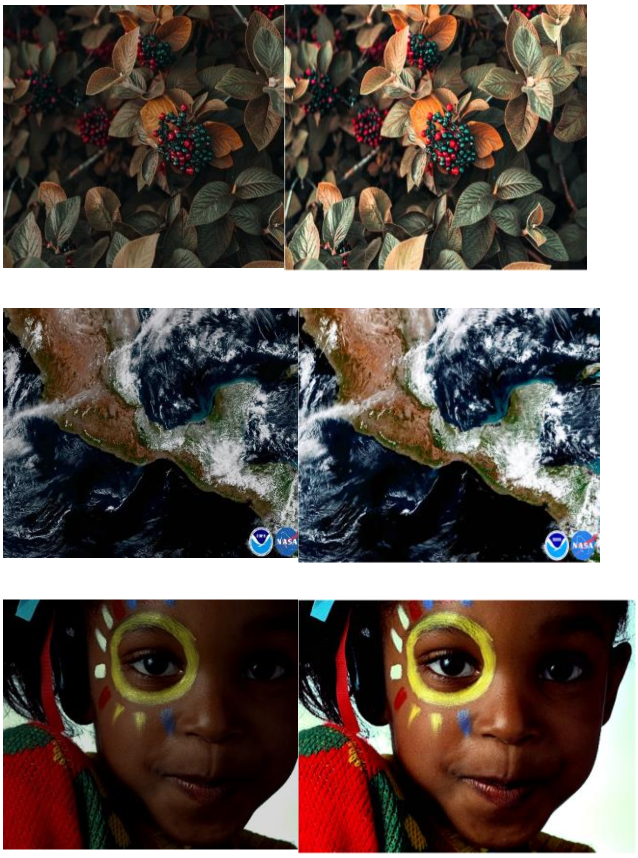
For testing and evaluating the method, we used the test suite of the publicly available Kodak Lossless True Color Image, available from https://r0k.us/graphics/kodak/. This dataset is typically used in image processing research, as the images are highly quality and provide a reliable basis for evaluating contrast enhancement algorithms' performance. The dataset also includes several images that allow for an in-depth assessment of how well the method performs, particularly in low-contrast and low-light situations. This simulation environment, along with the Kodak dataset, assures a statistically sound testing ground to judge the efficacy of contrast enhancement methodology in enhancing contrast without compromising image quality for clarity and details.

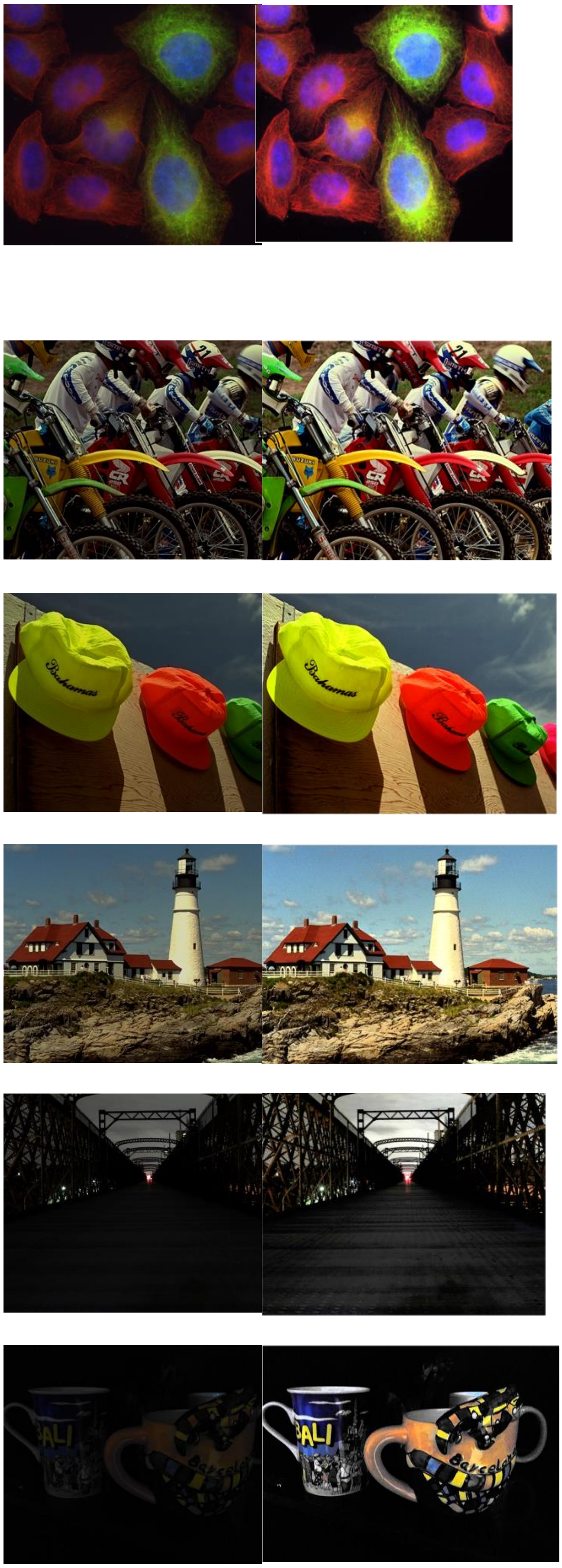
All of these components involve more than enough discernment to demonstrate that the method can deal with many imaging problems in different conditions.

3.2. Simulation result

The simulation results prove that the proposed method successfully resolved the deficiencies of the earlier techniques, showing significant increases in different areas compared to the earlier techniques as presented. Also, one benefit offered by this technique is the most effective reduction in saturation level while avoiding noise amplification, which is crucial to preserving sharpness and nuance within an image.

One of the benefits this method offers is to reduce or minimize saturation effects without amplifying noise, which will be a big part of ensuring the model can maintain quality and detail fracture in an image. On the contrary, most traditional strategies are strongly or improperly equalized, leading to losing some information in distinct image regions. Whilst this is not a problem solved with my method, the final image remains fine details and gains an enormous contrast throughout, and on the screenshot, it is a better visual. Of course, this previous approach is distinct because it takes this innovative attempt to create an even more flexible/dynamic equalization process. This guarantee is that the brightness and contrast of an image would be found in the better middle state without making it overexposed or underexposed. As the method performs well even in varying lighting environments, this added advantage makes it practical for many tasks. Figure 1 gives results of the original images and improved versions by using the proposed method to show that the proposed enhancement gives a greater detail fidelity and contrast improvement than an initially ordinary eye.





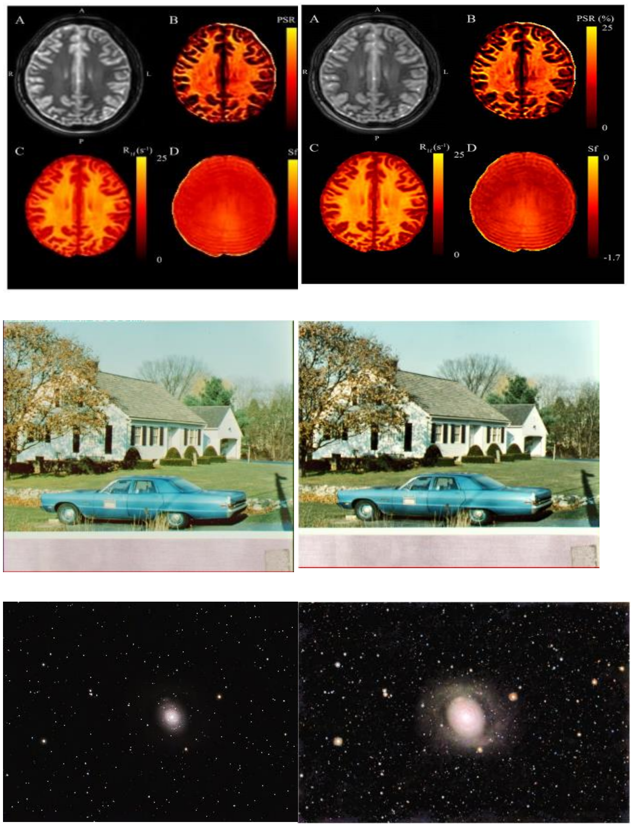


Fig 3. Original image and results of the proposed method.

4. Future Work

Future investigations may focus on merging advanced causal reasoning methods with biomarker detection techniques approaches that have been highlighted in recent studies on protein corona profiling and actual causality modeling as potential tools for application in image processing and contrast enhancement.

First, the methodology introduced in[33] which integrates mass spectrometry-based proteomics, machine learning, and causal analysis demonstrates the value of combining multi-modal data analysis with dynamic fuzzy histogram equalization. Within our framework, such integration could support adaptive tuning of histogram equalization parameters based on complex and heterogeneous datasets, thereby achieving improved contrast preservation in domain-specific imaging tasks, such as biomedical image interpretation.

Second, the causality-driven strategy described in[34] shows that pinpointing "actual causes" within variable interactions leads to system outputs that are more predictable and controllable. Applying this principle to our algorithm could help identify which image attributes or histogram regions act as the true causal drivers of perceived visual quality, thus enabling targeted enhancement methods that reduce the likelihood of unwanted artifacts.

Third, the formal reasoning approach for identifying root causes in engineered systems proposed in [35] could guide the design of automated diagnostic modules in our image enhancement pipeline. Such modules would be capable of detecting and isolating the primary sources of visual degradation such as uneven illumination or noise and activating selective enhancement processes tailored to each specific issue.

Overall, future research will aim to integrate our fuzzy dynamic histogram equalization method with causality-based analysis and cross-domain data fusion, in order to develop intelligent, context-aware enhancement systems. This path has the potential to extend the scope of our approach beyond general-purpose color image processing to specialized domains such as medical imaging, remote sensing, and industrial inspection, where maintaining high contrast alongside preservation of critical details is essential.

5. Conclusion

This paper proposes an original solution for enhancing color images using histogram equalization to preserve brightness. This solution overcomes some limitations of existing approaches by quenching the saturation, noise enhancement, and improper equalization, resulting in the preservation of image details. It is especially well-suited for low-light images. Fuzzy logic-based, dividing histograms into four sub-histograms results in better output and decreased computational overhead. Suggestions for future research are to use advanced fuzzy logic techniques, improve ways of maintaining brightness and preserving image details, and apply dynamic equalization methods via new boundary invariance.

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