Original Research

Making Index for the intercity transportation network based on the modeling and preparation of mathematical method

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Abstract
The necessary condition but not the enough one in order to enhance productivity and optimizing the operation of the transportation network lines is to give a proper index to any of these routes. The common lines of the distinct routes, therefore, would give more complexity and importance to the matter. So, through modeling and using the mathematical method based on the preparation of software we have tried to solve the problem. First, the routes of the journeys between all origins and destinations on the network concerning they sometimes had no specific and constant moving plan were identified. The number of journey demands between them were processed by this software in which for any route and number of journeys a quantitative figure between zero and one with the name of index was calculated. The more the index is closer to the number one, the more valuable the activity of the route will be. In this way the capacity of the routes must be regarded as independent variables. It is notable that by using this mathematical method it would be possible to predict the critical situations; that is, with an omission of one or more routes from the network we can calculate the traffic load imposed on the other routes as well as their respective index variations. The evaluations of the said indices are the basic tools for the safety and traffic management as well as the analysis of the system capacity and transportation planning.

Keywords- Transportation systems management; The safety transportation networks and traffic management; The system capacity and transportation planning

INTRODUCTION

Through the identification of the ultimate limit of productivity and operation of the whole network lines we can obtain important data upon which the analysis of capacity, the safety and traffic management are performed. The goals of the transportation network and its sustainability are some permanent issues which have a close relation with the productivity and operating of the whole network lines from one hand and the category of safety improvement and accident prevention from the other.
In this regard it is necessary to refer in particular to the issue of optimum distribution of equipment's, technology and most skillful personnel across the vital railway lines. It is obvious that the consequences of blocking two distinct connecting sections of lines are not the same from the view of losses incurred.
Many scholars have focused on transportation network resilience because of its importance in society's well-being and recovery efforts after disturbances. Existing studies have suggested various definitions, indicators, and methods for assessing the

resilience of transportation networks. This variation is due to differences in the nature, scale, and impact of disturbances. This systematic literature review presents resilience assessment methods for transportation networks, indicators, and disturbance categories [9,11,12].

In terms of algorithm design, researchers have proposed numerous heuristic or metaheuristic algorithms to solve feeder bus network optimization problems, such as simulated annealing algorithm [7], genetic algorithm [10], [8], hybrid enhanced artificial bee colony algorithm [6], and a complex two-phase heuristic algorithm [5]. Reference [5] proposed a two-phase heuristic algorithm to optimize the route network design and frequency setting problems. Reference [10] proposed a genetic algorithm with two types of crossover operators and four types of mutation operators. Reference [6] proposed a hybrid enhanced artificial bee colony algorithm to solve the same problem.

So here the issue of loss estimation and identifying the effect of one or two optional blocked sections on the other lines is put forward. In this case the traffic loads of the blocked sections are imposed on the others. This cycle is occurred repeatedly which inevitably causes the other vehicles to change their routes by force. So, the network management faces with a new condition as an outcome of the former network with some deficiencies inside. Here we need a software program to be able to find the optimum shortest route for the new condition for all journeys and introduce them to the applicants. It means that there are still origins, destinations and journey applicants in which only some connecting lines are missed. In such conditions only some applicants postpone their journey without cancelling. The starting time for their journey depends on the information provided by the traffic management. Off course, the numbers of journeys arranged and put into action prior to the line blockage, are the first traffic waves which must be properly managed and controlled.

These factors show the intensive dependence of the transportation network on the software of "finding the shortest route in the intermodal transportation network" which in its turn would add to its significance. But in this paper through application of the said software algorithm on identifying the connecting lines of the routes, first we calculate the number of traffic movements on each line in respect to the quantity of journey applicants between all origins and destinations.

Then based on these data we will give an index to the mathematic formula of the said line. It is obvious that only based on the identification of the critical network lines and using a proper and optimized plan it would be possible to have a sustainable transport system.

For all line-based transit systems like bus, metro and tram, the routes of the lines and the frequencies at which they are operated are determining for the operational performance of the system. However, as transit line planning happens early in the planning process, it is not straightforward to predict the effects of line planning decisions on relevant performance indicators. This challenge has in more than 40 years of research on transit line planning let to many different models[1, 2, 3, 4].

STATEMENT OF THE PROBLEM AND ITS MATHEMATICAL METHOD

The best managerial decisions on the way to improve the issues of traffic and safety as well as the economy and transportation planning is to give a proper index to the city streets. This important issue can be achieved based on some assumptions such as the number of applicants for inter-city and outer-city journeys, number of private, public and state-owned vehicles together with the city map. Without disturbing the generality of the matter, we assume that the city junctions and squares, metro stations, busses and taxies are the set of connecting points named as the origins and destinations. Based on this assumption, we name any direct connection between two optional points as a curved or connecting line.

According to the mathematical logic, if the network consists of n number of connecting points, then there would be $n(n-1)$ number of journey demands. The numbers of network connecting lines have no influence on the number of journey demands. As an example, for a network with three connecting points there are six conditions of journey demands as following:

1- Number of vehicles of the journey applicants from the first point (origin) to the second point (destination).
2- Number of vehicles of the journey applicants from the first point (origin) to the third point (destination).
3- Number of vehicles of the journey applicants from the second point (origin) to the first point (destination).
4- Number of vehicles of the journey applicants from the second point (origin) to the third point (destination).
5- Number of vehicles of the journey applicants from the third point (origin) to the first point (destination).
6- Number of vehicles of the journey applicants from the third point (origin) to the sixth point (destination).

Every journey consists of several points and lines which are determined through the algorithm of "optimized route identification" shown by W_{ij}^{Opt} in which the $i = 1,2,\ldots,n$ & $j = 1,2,\ldots,n$. In any network there are some common connecting points and lines. So, we must pay a close look at these common sections (said above points and lines). Therefore, the issue of ideal distribution of equipment and power in the transportation networks depends on the analysis of the distinct routes with the common parts. In this way we would calculate the number of moving vehicles on each line (line traffic load) through a mathematical formula. This important issue can be achieved by considering the number of vehicles of the journey applicants between all origins and destinations. So, we assume that the matrix of the journey applicants between all origins and destinations exist which is shown as following:

$$H = [h_{ij}] \tag{1}$$

In which h_{ij} is the number of vehicles of journey applicants from origin i to destination j. Also, we assume l_{rs} as a street or highway which directly connects the points r and s to each other. The direction of movement is from r to s in which $s \in \{1,2,....,n\}$ & $r \in \{1,2,....,n\}$.

All the distinct lines which have a common member of l_{rs} are the member of the set A_{rs} too

$$A_{rs} = \left\{ W_{ij}^{Opt} \middle| l_{rs} \in W_{ij}^{Opt} \right\} \tag{2}$$

Now we show the number of the moving vehicles from l_{rs} with $x_{l_{rs}}$ which is calculated by the following formula.

$$x_{l_{rs}} = \sum_{W_{ij}^{Opt} \in A_{rs}} h_{ij} \tag{3}$$

After calculating the number of movements for each line, we show the index of street or highway l_{rs} with K_{rs} and calculate it by the following formula:

$$K_{rs} = 1 - \frac{1}{x_{l_{rs}}} \tag{4}$$

Obviously, whatever the $x_{l_{rs}}$ is bigger, the $\frac{1}{x_{l_{rs}}}$ becomes smaller and in contrast the $1 - \frac{1}{x_{l_{rs}}}$ gets bigger which is between zero and one. The closer the index to one the more valuable it will be.

ALGORITHM OF IDENTIFYING THE OPTIMUM ROUTE LINES BETWEEN ORIGINS AND DESTINATIONS

Based on the real condition the structure of the network consists of n number of connecting points in the form of directional graph $G = (X, E)$ in which $X = \{i | i = 1,2,3,\ldots,n\}$ is the set of points and $E = \left\{ (l_{rs}) \middle| r \in \{1,2,\ldots,n\} \,\&\, s \in \{1,2,\ldots,n\} \right\}$ set of network lines.

Step one: we set up and show the matrix of the network connections with $C = [c_{ij}]$. The numbers one and zero are the members of this square matrix $n \times n$.

In which $c_{ij} = 1$ if and only if the connection point i is directly connected to the connecting point j. Otherwise, $c_{ij} = 0$.

Step two: we simultaneously introduce two matrices of $A(k)$ and $M(k)$ respectively named as journey expenses and journey lines identification matrix. The drivers would drive their vehicles based on these two matrices. Each member of the matrix $A(k)$ is calculated an assumed on the basis of one variable or a combination of several distinct and main variables such as time, length and fuel cost of the journey. It is recommended to only calculate the time concerning the connection of new metro stations to the transportation network. The second is the matrix whose members identify the optimum route lines for any journey between the origins and destinations. The k a natural figure which shows the conditions of being in the origin, destination and before reaching the destination. We respectively give some values to the k. This process is stopped when $M(k) = M(k-1)$.

First we assume $k = 1$.In this condition we are at the beginning of the movement. If the connecting point x_i is directly connected to the x_j, then the value of the $a_{ij}(1)$ member from matrix $A(1)$ would be equal to t_{ij}. Otherwise, we consider the infant penalty for this member, that is, $a_{ij}(1) = \infty$. In fact, the members of the assumed $A(1)$ matrix exist. The $M(0)$ does not exist, so we must add one to the k.

But at first for the $k \geq 2$ we introduce two matrices of $A(k)$ and $M(k)$ by the following formulas:

$$A(k) = [a_{ij}(k) | a_{ij}(k) = Min\{a_{ij}(k-1) \,\&\, a_{is^*}(k-1) + a_{s^*j}(k-1)\}] \tag{5}$$

In which

$$a_{is^*}(k-1) + a_{s^*j}(k-1) = Min\{a_{is}(k-1) + a_{sj}(k-1) | i \neq s \,\&\, j \neq s\} \tag{6}$$

$$M(k) = \left[m_{ij}(k) \begin{cases} m_{ij}(k-1); & a_{ij}(k) = a_{ij}(k-1) \\ m_{s^*j}(k-1); & a_{ij}(k) = a_{is^*}(k-1) + a_{s^*j}(k-1) \end{cases} \right] \tag{7}$$

Step three: we put $k = 2$. Then we calculate two matrices of $A(2)$ and $M(2)$.

If $M(2) \neq M(1)$, then the processing of this algorithm has finished. Otherwise, we add one to the k and once again we calculate two matrices of $A(k)$ and $M(k)$.

Step four: For identifying the optimum route between each optional origin and destination of i and j we refer to m_{ij}. We assume $m_{ij} = q_1$ in which $q_1 \in N$. There are two conditions.

Condition one: $q_1 = i$, then the route from i to j is direct.

Condition two: $q_1 \neq i$, then we look at the member of m_{iq_1}. We assume $m_{iq_1} = q_2$ then again two conditions occur.

Condition one: $q_2 = i$, then the route from i to j consists of two branches. The first branch from i to q_1 and the second one from q_1 to $q_2 = i$.

Condition two: $q_2 \neq i$, then we look at the member of m_{iq2}. We assume $m_{iq_2} = q_3$ then again two conditions occur and so on.

We assume after n number of irritation the $m_{iq_n} = i$. So, the optimum route between the connecting points I and j consist of n branches as following:

$$W_{ij}^{Opt} : \{(i \to q_1), (q_1 \to q_2), (q_2 \to q_3), \to .... \to (q_n \to j)\} \tag{8}$$

PROBLEM MODELING

The origins and destinations of the transportation network in the 14 connecting points have been gathered in the figure 1. These points have been connected to each other through 40 numbers of curve-shaped lines. (20 two directional curve-shaped lines). The process of gathering the origins and destinations has only been carried out for the ease of function. In fact, it means that there are some distinct transportation networks inside the very nature of each single points of the main network adhering to these calculations and discussions. The figures on the curved lines demonstrate the time period between two connected points. Without disturbing the generality of the matter, it is assumed that the time for going and return is equal.

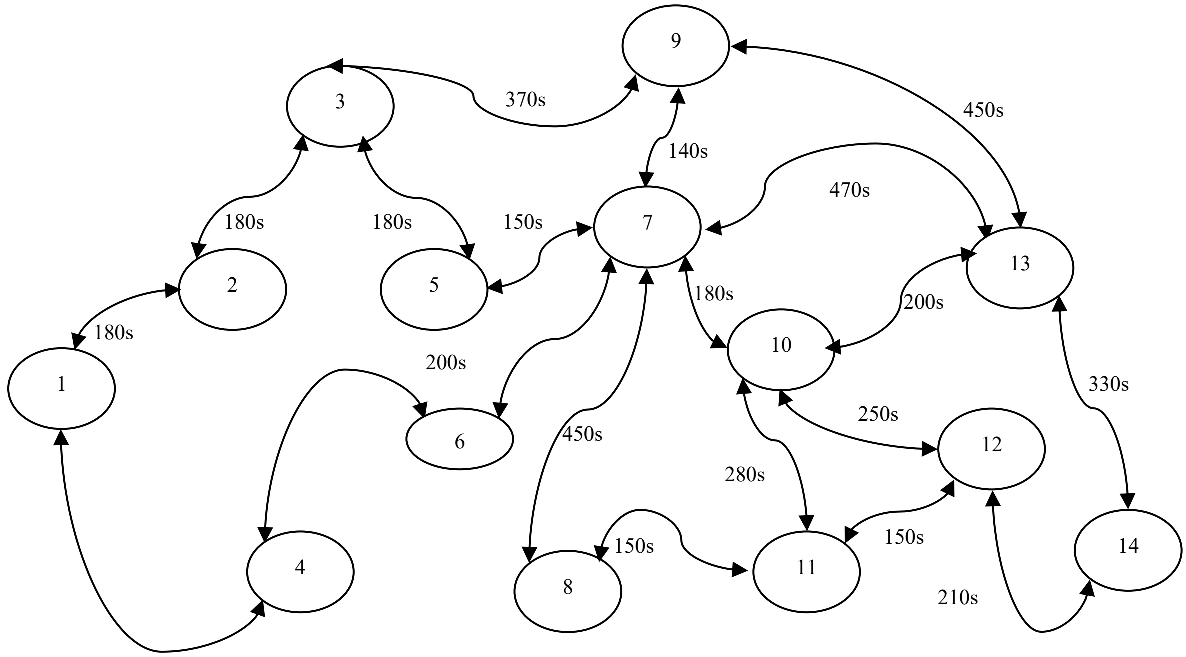

FIGURE 1
A MODEL OF THE 14 CONNECTING POINTS HAVE BEEN GATHERED

As an example, $t_{14} = t_{41} = 250s$ .As it was formerly discussed there are $14 \times 13 = 182$ number of journey demands between its all origins and destinations. By the contribution of statistics its outcomes are assumed by the matrix H .

$$H = \begin{bmatrix} 0 & 11 & 5 & 25 & 1 & 7 & 1 & 22 & 9 & 5 & 9 & 17 & 10 & 11 \\ 5 & 0 & 1 & 5 & 3 & 6 & 21 & 17 & 5 & 12 & 9 & 14 & 10 & 12 \\ 20 & 3 & 0 & 13 & 1 & 11 & 1 & 11 & 1 & 5 & 7 & 1 & 3 & 7 \\ 3 & 1 & 8 & 0 & 8 & 2 & 9 & 3 & 5 & 10 & 13 & 12 & 11 & 13 \\ 9 & 2 & 5 & 1 & 0 & 9 & 8 & 3 & 8 & 11 & 45 & 64 & 13 & 43 \\ 12 & 7 & 8 & 8 & 9 & 0 & 9 & 1 & 1 & 12 & 18 & 11 & 9 & 22 \\ 9 & 13 & 7 & 5 & 8 & 8 & 0 & 5 & 7 & 11 & 7 & 3 & 8 & 5 \\ 2 & 1 & 7 & 11 & 9 & 1 & 1 & 0 & 3 & 11 & 7 & 7 & 17 & 8 \\ 9 & 15 & 1 & 14 & 1 & 1 & 1 & 9 & 0 & 11 & 7 & 1 & 5 & 11 \\ 24 & 15 & 2 & 4 & 27 & 5 & 3 & 73 & 3 & 0 & 7 & 19 & 7 & 12 \\ 3 & 3 & 7 & 3 & 20 & 3 & 1 & 9 & 8 & 11 & 0 & 9 & 7 & 5 \\ 12 & 7 & 1 & 5 & 1 & 1 & 7 & 3 & 3 & 11 & 11 & 0 & 1 & 7 \\ 8 & 9 & 3 & 7 & 8 & 3 & 2 & 11 & 5 & 11 & 7 & 9 & 0 & 18 \\ 5 & 11 & 9 & 5 & 4 & 7 & 1 & 10 & 1 & 11 & 7 & 10 & 15 & 0 \end{bmatrix}$$

The members of this matrix are assumptive which are considered on the basis of thousand-fold bunches. As an example, the member h_{12} is a symbol of eleven thousands of movements from the origin one to the destination two. The demand for the values of the movements is performed within the unit of time.

$$A(4)=\begin{bmatrix} 0 & 180 & 360 & 250 & 540 & 470 & 670 & 1120 & 730 & 850 & 1130 & 1100 & 1050 & 1210 \\ 180 & 0 & 180 & 430 & 360 & 650 & 510 & 960 & 550 & 690 & 970 & 940 & 980 & 1120 \\ 360 & 180 & 0 & 610 & 180 & 530 & 330 & 780 & 370 & 510 & 790 & 760 & 710 & 1940 \\ 200 & 430 & 610 & 0 & 570 & 220 & 420 & 870 & 560 & 600 & 880 & 800 & 890 & 1220 \\ 540 & 360 & 180 & 570 & 0 & 350 & 150 & 600 & 290 & 330 & 610 & 580 & 530 & 790 \\ 470 & 650 & 530 & 220 & 350 & 0 & 200 & 650 & 340 & 380 & 660 & 630 & 580 & 840 \\ 670 & 510 & 330 & 420 & 150 & 200 & 0 & 450 & 140 & 180 & 460 & 430 & 380 & 540 \\ 1120 & 960 & 780 & 870 & 600 & 650 & 450 & 0 & 590 & 430 & 150 & 300 & 630 & 510 \\ 730 & 550 & 370 & 560 & 290 & 340 & 140 & 590 & 0 & 320 & 600 & 570 & 450 & 780 \\ 850 & 690 & 510 & 600 & 330 & 380 & 180 & 430 & 320 & 0 & 280 & 250 & 200 & 460 \\ 1130 & 970 & 790 & 880 & 610 & 660 & 460 & 150 & 600 & 280 & 0 & 150 & 480 & 360 \\ 1100 & 940 & 760 & 850 & 580 & 630 & 430 & 300 & 570 & 250 & 150 & 0 & 450 & 210 \\ 1050 & 980 & 710 & 800 & 530 & 580 & 380 & 630 & 450 & 200 & 480 & 450 & 0 & 330 \\ 1210 & 1120 & 940 & 1220 & 790 & 840 & 540 & 510 & 780 & 460 & 360 & 210 & 330 & 0 \end{bmatrix}$$

$$M(4)=\begin{bmatrix} 0 & 1 & 2 & 1 & 3 & 4 & 6 & 7 & 3 & 7 & 10 & 10 & 10 & 12 \\ 2 & 0 & 2 & 1 & 3 & 4 & 5 & 7 & 3 & 7 & 10 & 10 & 10 & 12 \\ 2 & 3 & 0 & 1 & 3 & 7 & 5 & 7 & 3 & 7 & 10 & 10 & 10 & 12 \\ 4 & 1 & 2 & 0 & 7 & 4 & 6 & 7 & 7 & 7 & 10 & 10 & 10 & 12 \\ 2 & 3 & 5 & 6 & 0 & 7 & 5 & 7 & 7 & 7 & 10 & 10 & 10 & 12 \\ 4 & 1 & 5 & 6 & 7 & 0 & 6 & 7 & 7 & 7 & 10 & 10 & 10 & 12 \\ 4 & 3 & 5 & 6 & 7 & 7 & 0 & 7 & 7 & 7 & 10 & 10 & 10 & 12 \\ 4 & 3 & 5 & 6 & 7 & 7 & 8 & 0 & 7 & 11 & 8 & 11 & 10 & 12 \\ 2 & 3 & 9 & 6 & 7 & 7 & 9 & 7 & 0 & 7 & 10 & 10 & 9 & 13 \\ 4 & 3 & 5 & 6 & 7 & 7 & 10 & 11 & 7 & 0 & 10 & 10 & 10 & 12 \\ 4 & 3 & 5 & 6 & 7 & 7 & 10 & 11 & 7 & 11 & 11 & 11 & 10 & 12 \\ 4 & 3 & 5 & 6 & 7 & 7 & 10 & 11 & 7 & 12 & 0 & 0 & 10 & 12 \\ 4 & 3 & 5 & 6 & 7 & 7 & 10 & 11 & 13 & 13 & 10 & 10 & 0 & 13 \\ 4 & 3 & 5 & 6 & 7 & 7 & 10 & 11 & 13 & 12 & 14 & 14 & 14 & 0 \end{bmatrix}$$

For example, we identify the best route from the second origin to the eighth destination. According to the algorithm we refer to the member of m_{28} from the matrix $M(4)$ and it is seen that $m_{28} = 7$.

Thus, we must consider m_{27}. Since $m_{28} = 7$ then we refer to the m_{25}. As $m_{25} = 3$ we will consider m_{23}. It is seen that $m_{23} = 2$ so the algorithm process terminates.
In this way the best route from the second origin to the eighth destination is identified which consists of the following connecting lines:

$$W_{28}^{Opt} : (2,3) \to (3,5) \to (5,7) \to (7,8)$$

Moreover, the value of the member a_{28} from the matrix $A(4)$ is equal to nine hundred and sixty seconds which shows the least period of time for traveling from the second origin to the eight destinations. The identification of the optimum route between all origins and destinations as well as their traveling time is shown in Table I.
Since it was assumed that the going and returning times on the curved lines are the same it is seen that the travel direction of W_{82}^{Opt} is exactly opposite to that of W_{28}^{Opt} and their traveling time is also equal. So, the results are Symmetric which can be avoided. In this way based on the said algorithm, the identification of the route between each pair of two optional points - so called as origin and destination- can be achieved.
According to the Figure 1 of the network curved line the sets of A_{rs} are as following:

$A_{12}, A_{21}, A_{23}, A_{32}, A_{35}, A_{53}, A_{46}, A_{64}, A_{57}, A_{75}, A_{67}, A_{76}, A_{78}, A_{87}, A_{79}, A_{97}, A_{710}, A_{107}, A_{713}, A_{137}, A_{811}, A_{118}, A_{913}, A_{139}, A_{1011}, A_{1110}, A_{1012}, A_{1210}, A_{1013}, A_{1310}, A_{1112}, A_{1211}, A_{1214}, A_{1412}$ From Table I any single member of these sets is obtained. As an example:

$$A_{12} = \{W_{12}^{Opt}, W_{13}^{Opt}, W_{15}^{Opt}, W_{19}^{Opt}, W_{42}^{Opt}, W_{43}^{Opt}, W_{62}^{Opt}\}.$$

Now we calculate $x_{l_{12}}$ according to the formula (3) which is equal to:

$$x_{l_{12}} = \sum_{W_{ij}^{Opt} \in A_{12}} h_{ij} = h_{12} + h_{13} + h_{15} + h_{19} + h_{42} + h_{43} + h_{62} = 42$$

Based on the above result obtained by the formula (4) the index of the street or highway l_{12} is calculated. We show this index with K_{12} which is equal to:

$$K_{12} = 1 - \frac{1}{x_{l_{12}}} = 1 - \frac{1}{42} = 0.9762 \text{ but } K_{21} = 1 - \frac{1}{x_{l_{21}}} = 1 - \frac{1}{67} = 0.9851.$$

According to this method the index of all lines of the Figure 1 are calculated whose results are shown in the Table II sorted from the highest weight in four digits decimal.

TABLE I
OPTIMUM ROUTES BETWEEN ALL ORIGINS AND DESTINATIONS

$W_{12}^{Opt}:\xrightarrow{1,2}180$	$W_{21}^{Opt}:\xrightarrow{2,1}180$	$W_{31}^{Opt}:\xrightarrow{3,2,1}360$	$W_{41}^{Opt}:\xrightarrow{4,1}250$	$W_{51}^{Opt}:\xrightarrow{5,3,2,1}540$
$W_{13}^{Opt}:\xrightarrow{1,2,3}360$	$W_{23}^{Opt}:\xrightarrow{2,3}180$	$W_{32}^{Opt}:\xrightarrow{3,2}180$	$W_{42}^{Opt}:\xrightarrow{4,1,2}430$	$W_{52}^{Opt}:\xrightarrow{5,3,2}360$
$W_{14}^{Opt}:\xrightarrow{1,4}250$	$W_{24}^{Opt}:\xrightarrow{2,1,4}430$	$W_{34}^{Opt}:\xrightarrow{3,2,1,4}610$	$W_{43}^{Opt}:\xrightarrow{4,1,2,3}610$	$W_{53}^{Opt}:\xrightarrow{5,3}180$
$W_{15}^{Opt}:\xrightarrow{1,2,3,5}540$	$W_{25}^{Opt}:\xrightarrow{2,3,5}360$	$W_{35}^{Opt}:\xrightarrow{3,5}180$	$W_{45}^{Opt}:\xrightarrow{4,6,7,5}570$	$W_{54}^{Opt}:\xrightarrow{5,7,6,4}570$
$W_{16}^{Opt}:\xrightarrow{1,4,6}470$	$W_{26}^{Opt}:\xrightarrow{2,1,4,6}650$	$W_{36}^{Opt}:\xrightarrow{3,5,7,6}530$	$W_{46}^{Opt}:\xrightarrow{4,6}220$	$W_{56}^{Opt}:\xrightarrow{5,7,6}350$
$W_{17}^{Opt}:\xrightarrow{1,4,6,7}670$	$W_{27}^{Opt}:\xrightarrow{2,3,5,7}510$	$W_{37}^{Opt}:\xrightarrow{3,5,7}330$	$W_{47}^{Opt}:\xrightarrow{4,6,7}420$	$W_{57}^{Opt}:\xrightarrow{5,7}150$
$W_{18}^{Opt}:\xrightarrow{1,4,6,7,8}1120$	$W_{28}^{Opt}:\xrightarrow{2,3,5,7,8}960$	$W_{38}^{Opt}:\xrightarrow{3,5,7,8}780$	$W_{48}^{Opt}:\xrightarrow{4,6,7,8}870$	$W_{58}^{Opt}:\xrightarrow{5,7,8}600$
$W_{19}^{Opt}:\xrightarrow{1,2,3,9}730$	$W_{29}^{Opt}:\xrightarrow{2,3,9}550$	$W_{39}^{Opt}:\xrightarrow{3,9}370$	$W_{49}^{Opt}:\xrightarrow{4,6,7,9}560$	$W_{59}^{Opt}:\xrightarrow{5,7,9}290$
$W_{110}^{Opt}:\xrightarrow{1,4,6,7,10}850$	$W_{210}^{Opt}:\xrightarrow{2,3,5,7,10}690$	$W_{310}^{Opt}:\xrightarrow{3,5,7,10}510$	$W_{410}^{Opt}:\xrightarrow{4,6,7,10}600$	$W_{510}^{Opt}:\xrightarrow{5,7,10}330$
$W_{111}^{Opt}:\xrightarrow{1,4,6,7,10,11}1130$	$W_{211}^{Opt}:\xrightarrow{2,3,5,7,10,11}970$	$W_{311}^{Opt}:\xrightarrow{3,5,7,10,11}790$	$W_{411}^{Opt}:\xrightarrow{4,6,7,10,11}880$	$W_{511}^{Opt}:\xrightarrow{5,7,10,11}610$
$W_{112}^{Opt}:\xrightarrow{1,4,6,7,10,12}1100$	$W_{212}^{Opt}:\xrightarrow{2,3,5,7,10,12}940$	$W_{312}^{Opt}:\xrightarrow{3,5,7,10,12}760$	$W_{412}^{Opt}:\xrightarrow{4,6,7,10,12}850$	$W_{512}^{Opt}:\xrightarrow{5,7,10,12}580$
$W_{113}^{Opt}:\xrightarrow{1,4,6,7,10,13}1050$	$W_{213}^{Opt}:\xrightarrow{2,3,5,7,10,13}980$	$W_{313}^{Opt}:\xrightarrow{3,5,7,10,13}710$	$W_{413}^{Opt}:\xrightarrow{4,6,7,10,13}800$	$W_{513}^{Opt}:\xrightarrow{5,7,10,13}530$
$W_{114}^{Opt}:\xrightarrow{1,4,6,7,10,12,14}1210$	$W_{214}^{Opt}:\xrightarrow{2,3,5,7,10,12,14}1120$	$W_{314}^{Opt}:\xrightarrow{3,5,7,10,12,14}940$	$W_{414}^{Opt}:\xrightarrow{4,6,7,10,12,14}1220$	$W_{514}^{Opt}:\xrightarrow{5,7,10,12,14}950$
$W_{61}^{Opt}:\xrightarrow{6,4,1}470$	$W_{71}^{Opt}:\xrightarrow{7,6,4,1}670$	$W_{81}^{Opt}:\xrightarrow{8,7,6,4,1}1120$	$W_{91}^{Opt}:\xrightarrow{9,3,2,1}730$	$W_{101}^{Opt}:\xrightarrow{10,7,6,4,1}850$
$W_{62}^{Opt}:\xrightarrow{6,4,1,2}650$	$W_{72}^{Opt}:\xrightarrow{7,5,3,2}510$	$W_{82}^{Opt}:\xrightarrow{8,7,5,3,2}960$	$W_{92}^{Opt}:\xrightarrow{9,3,2}550$	$W_{102}^{Opt}:\xrightarrow{10,7,5,3,2}690$
$W_{63}^{Opt}:\xrightarrow{6,7,5,3}530$	$W_{73}^{Opt}:\xrightarrow{7,5,3}330$	$W_{83}^{Opt}:\xrightarrow{8,7,5,3}780$	$W_{93}^{Opt}:\xrightarrow{9,3}370$	$W_{103}^{Opt}:\xrightarrow{10,7,5,3}510$
$W_{64}^{Opt}:\xrightarrow{6,4}220$	$W_{74}^{Opt}:\xrightarrow{7,6,4}420$	$W_{84}^{Opt}:\xrightarrow{8,7,6,4}870$	$W_{94}^{Opt}:\xrightarrow{9,7,6,4}560$	$W_{104}^{Opt}:\xrightarrow{10,7,6,4}600$
$W_{65}^{Opt}:\xrightarrow{6,7,5}350$	$W_{75}^{Opt}:\xrightarrow{7,5}150$	$W_{85}^{Opt}:\xrightarrow{8,7,5}600$	$W_{95}^{Opt}:\xrightarrow{9,7,5}290$	$W_{105}^{Opt}:\xrightarrow{10,7,5}330$
$W_{67}^{Opt}:\xrightarrow{6,7}200$	$W_{76}^{Opt}:\xrightarrow{7,6}200$	$W_{86}^{Opt}:\xrightarrow{8,7,6}650$	$W_{96}^{Opt}:\xrightarrow{9,7,6}340$	$W_{106}^{Opt}:\xrightarrow{10,7,6}380$
$W_{68}^{Opt}:\xrightarrow{6,7,8}650$	$W_{78}^{Opt}:\xrightarrow{7,8}150$	$W_{87}^{Opt}:\xrightarrow{8,7}450$	$W_{97}^{Opt}:\xrightarrow{9,7}140$	$W_{107}^{Opt}:\xrightarrow{10,7}180$
$W_{69}^{Opt}:\xrightarrow{6,7,9}340$	$W_{79}^{Opt}:\xrightarrow{7,9}140$	$W_{89}^{Opt}:\xrightarrow{8,7,9}590$	$W_{98}^{Opt}:\xrightarrow{9,7,8}590$	$W_{108}^{Opt}:\xrightarrow{10,11,8}430$
$W_{610}^{Opt}:\xrightarrow{6,7,10}380$	$W_{710}^{Opt}:\xrightarrow{7,10}180$	$W_{810}^{Opt}:\xrightarrow{8,11,10}430$	$W_{910}^{Opt}:\xrightarrow{9,7,10}320$	$W_{109}^{Opt}:\xrightarrow{10,9}320$
$W_{611}^{Opt}:\xrightarrow{6,7,10,11}660$	$W_{711}^{Opt}:\xrightarrow{7,10,11}460$	$W_{811}^{Opt}:\xrightarrow{8,11}150$	$W_{911}^{Opt}:\xrightarrow{9,7,10,11}600$	$W_{1011}^{Opt}:\xrightarrow{10,11}280$
$W_{612}^{Opt}:\xrightarrow{6,7,10,12}630$	$W_{712}^{Opt}:\xrightarrow{7,10,12}430$	$W_{812}^{Opt}:\xrightarrow{8,11,12}300$	$W_{912}^{Opt}:\xrightarrow{9,7,10,12}570$	$W_{1012}^{Opt}:\xrightarrow{10,12}250$
$W_{613}^{Opt}:\xrightarrow{6,7,10,13}580$	$W_{713}^{Opt}:\xrightarrow{7,10,13}380$	$W_{813}^{Opt}:\xrightarrow{8,11,10,13}630$	$W_{913}^{Opt}:\xrightarrow{9,.13}450$	$W_{1013}^{Opt}:\xrightarrow{10,13}200$
$W_{614}^{Opt}:\xrightarrow{6,7,10,12,14}840$	$W_{714}^{Opt}:\xrightarrow{7,10,12,14}540$	$W_{814}^{Opt}:\xrightarrow{8,11,12,14}510$	$W_{914}^{Opt}:\xrightarrow{9,7,10,12,14}780$	$W_{1014}^{Opt}:\xrightarrow{10,12,14}460$
$W_{111}^{Opt}:\xrightarrow{11,10,7,6,4,1}1130$	$W_{121}^{Opt}:\xrightarrow{12,10,7,6,4,1}1100$	$W_{131}^{Opt}:\xrightarrow{13,10,7,6,4,1}1050$	$W_{141}^{Opt}:\xrightarrow{14,12,10,7,6,4,1}1210$	
$W_{112}^{Opt}:\xrightarrow{11,10,7,5,1,2}970$	$W_{122}^{Opt}:\xrightarrow{12,10,7,5,3,2}940$	$W_{132}^{Opt}:\xrightarrow{13,10,7,5,3,2}980$	$W_{142}^{Opt}:\xrightarrow{14,12,10,7,5,3,2}1120$	
$W_{113}^{Opt}:\xrightarrow{11,10,7,5,3}790$	$W_{123}^{Opt}:\xrightarrow{12,10,7,5,3}760$	$W_{133}^{Opt}:\xrightarrow{13,10,7,5,3}710$	$W_{143}^{Opt}:\xrightarrow{14,12,10,7,5,3}940$	
$W_{114}^{Opt}:\xrightarrow{11,10,7,6,4}880$	$W_{124}^{Opt}:\xrightarrow{12,10,7,6,4}800$	$W_{134}^{Opt}:\xrightarrow{13,10,7,6,4}890$	$W_{144}^{Opt}:\xrightarrow{14,12,10,7,6,4}1220$	
$W_{115}^{Opt}:\xrightarrow{11,10,7,5}610$	$W_{125}^{Opt}:\xrightarrow{12,10,7,5}580$	$W_{135}^{Opt}:\xrightarrow{13,10,7,5}530$	$W_{145}^{Opt}:\xrightarrow{14,12,10,7,5}950$	
$W_{116}^{Opt}:\xrightarrow{11,10,7,6}660$	$W_{126}^{Opt}:\xrightarrow{12,10,7,6}630$	$W_{136}^{Opt}:\xrightarrow{13,10,7,6}580$	$W_{146}^{Opt}:\xrightarrow{14,12,10,7,6}840$	
$W_{117}^{Opt}:\xrightarrow{11,10,7}460$	$W_{127}^{Opt}:\xrightarrow{12,10,7}430$	$W_{137}^{Opt}:\xrightarrow{13,10,7}380$	$W_{147}^{Opt}:\xrightarrow{14,12,10,7}540$	
$W_{118}^{Opt}:\xrightarrow{11,8}150$	$W_{128}^{Opt}:\xrightarrow{12,11,8}300$	$W_{138}^{Opt}:\xrightarrow{13,10,11,8}630$	$W_{148}^{Opt}:\xrightarrow{14,12,11,8}510$	
$W_{119}^{Opt}:\xrightarrow{11,10,7,9}600$	$W_{129}^{Opt}:\xrightarrow{12,10,7,9}570$	$W_{139}^{Opt}:\xrightarrow{13,9}450$	$W_{149}^{Opt}:\xrightarrow{1412,10,7,9}780$	
$W_{1110}^{Opt}:\xrightarrow{11,10}280$	$W_{1210}^{Opt}:\xrightarrow{12,10,}250$	$W_{1310}^{Opt}:\xrightarrow{13,10}200$	$W_{1410}^{Opt}:\xrightarrow{14,12,10}460$	
$W_{1112}^{Opt}:\xrightarrow{11,12}150$	$W_{1211}^{Opt}:\xrightarrow{12,11}150$	$W_{1311}^{Opt}:\xrightarrow{13,10,11}480$	$W_{1411}^{Opt}:\xrightarrow{14,12,11}360$	
$W_{1113}^{Opt}:\xrightarrow{11,10,.13}480$	$W_{1213}^{Opt}:\xrightarrow{.12,10,13}450$	$W_{1312}^{Opt}:\xrightarrow{13,10,12}450$	$W_{1412}^{Opt}:\xrightarrow{14,12}210$	
$W_{1114}^{Opt}:\xrightarrow{11,12,14}360$	$W_{1214}^{Opt}:\xrightarrow{12,14}210$	$W_{1314}^{Opt}:\xrightarrow{.13,14}330$	$W_{1413}^{Opt}:\xrightarrow{14,13}330$	

CONCLUSION

Undoubtedly, many of factors such as the population growth, the increase in number of vehicles and intercity and outer-city journeys as a result, improper development of spider web shaped of the network from inside and outside, lack of using the modern controlling technologies, not observing the rules and regulations by the drivers and pedestrians, the existence of some amateur or extra professional accident makers in this industry together with other elements would directly or indirectly add to the traffic load of the transportation network. Only through the modeling and preparation of the mathematical model it is possible to identify and analyze the transportation network. In fact, by identifying the capability of the transportation network it would be possible to solve some parts of the traffic problems or to control them or even to ensure sustainability of its goals.

Basically, the activity of indexing the transportation network lines is performed in order to identify the dead routes as well as to identify the whole capacity of the lines especially the critical ones with the aim of supporting them. From the Table II we observe that in the transportation network the Figure 1 is a fully dead line which has been hidden among all fully traffic loaded ones. By the contribution of this model the design of the new lines for decreasing the traffic load of the saturated lines can also be studied. One of the other advantages and paramount features of this method is an ability of its software to compare the change in the direction of movement of the network lines to correct and manage the operational activities. Although, the main and critical lines can easily be identified but the scientific and precise indexing to all lines to establish an intelligent traffic management would cause the degree of any fault and error to be led to zero.

To give an index to the network lines for developing a single and integrated management is a promising measure for rendering a comprehensive method a policy to introduce new transportation systems and technologies as well as to improve the national culture as a whole.

TABLE II
THE INDEX OF ALL LINES OF THE FIGURE 1

$K_{710} = 0.9979$	$K_{1213} = 0.9913$	$K_{97} = 0.9778$
$K_{57} = 0.9970$	$K_{23} = 0.9911$	$K_{12} = 0.9762$
$K_{1012} = 0.9966$	$K_{32} = 0.9909$	$K_{79} = 0.9722$
$K_{67} = 0.9959$	$K_{118} = k_{53} = 0.9906$	$K_{87} = 0.9714$
$K_{107} = 0.9956$	$K_{1013} = 0.9896$	$K_{1211} = 0.9643$
$K_{75} = 0.9949$	$K_{41} = 0.9894$	$K_{93} = 0.9600$
$K_{46} = 0.9942$	$K_{1412} = 0.9876$	$K_{1112} = 0.9513$
$K_{76} = 0.9940$	$K_{1310} = 0.9872$	$K_{1314} = 0.9444$
$K_{1214} = 0.9936$	$K_{78} = 0.9859$	$K_{1413} = k_{39} = 0.9333$
$K_{64} = K_{35} = 0.9931$	$K_{1110} = 0.9853$	$K_{913} = k_{139} = 0.8000$
$K_{14} = 0.9924$	$K_{21} = 0.9851$	
$K_{1011} = 0.9922$	$K_{811} = 0.9800$	
$K_{713} = k_{137} = 0$ (dead lines)		

REFERENCES

[1] An, K. and Lo, H. (2016). Two-phase stochastic program for transit network design under demand uncertainty. Transportation Research B, 84:157–181. *Journal of Urban Planning and Development . Volume 146, Issue 3*. https://doi.org/10.1061/(ASCE)UP.1943-5444.000060

[2] Badia, H., Estrada, M., and Robusté, F. (2014). Competitive transit network design in cities with radial street patterns. *Transportation Research Part B: Methodological*, 59:161–181. https://doi.org/10.1016/j.trb.2013.11.006

[3] Bel, G. and Holst, M. (2018). Evaluation of the impact of bus rapid transit on air pollution in mexico city. *Transport Policy*, 63, 209–220. https://doi.org/10.1016/j.tranpol.2018.01.001

[4] Beltran, B., Carrese, S., Cipriani, E., and Petrelli, M. (2009). Transit network design with allocation of green vehicles: A genetic algorithm approach. Transportation Research Part C: *Emerging Technologies*, 17(5), 475–483. https://doi.org/10.1016/j.trc.2009.04.008

[5] B. Barabino.(2009). Transit Bus Route Network Design: a model and its application in a real network. *WIT Transactions on The Built Environment*, 107, 369-382. https://doi.org/10.2495/UT090331

[6] Y. Jiang, W.Y. Szeto, and T.M. Ng.(2013). Transit Network Design: A Hybrid Enhanced Artificial Bee Colony Approach and a Case Study. *International Journal of Transportation Science and Technology*, 2, no. 3, 243-260. https://doi.org/10.1260/2046-0430.2.3.243

[7] Y. Kuo, C. Luo, and C. Wang, "Bus route design with limited travel time." Transport, 28, no. 4, 368-373. https://doi.org/10.3846/16484142.2013.865103

[8] S. Ngamchai, D.J. Lovell, and A.M. ASCE, (2005). Optimal Time Transfer in Bus Transit Route Network Design Using a Genetic Algorithm. *Proceedings of the Eastern Asia Society for Transportation Studies*, 5, pp, 477 - 489. https://www.researchgate.net/publication/228960835

[9] Sharma, SK, & George, SA. (2018). Modelling resilience of truckload transportation industry. *Benchmarking An International Journal*, 25, 2531–2545. https://doi.org/10.1108/BIJ-07- 2017-0188

[10] Twumasi-Boakye, R., and J. O. Sobanjo. 2017. Evaluation of transportation user costs based on simulated regional network models. *Journal of the Transportation Research Board*. 2612, 1. https://doi.org/10.3141/2612-14

[11] Twumasi-Boakye, R, & Sobanjo, JO. (2018). Resilience of Regional Transportation Networks Subjected to Hazard-Induced Bridge Damages. *Journal of Transportation Engineering, Part A: Systems,* 144, 10 .https://doi.org/10.1061/JTEPBS.0000186
[12] Wang, C, Ye, Z, & Wang, W. (2020). A Multi-Objective Optimization and Hybrid Heuristic Approach for Urban Bus Route Network Design. *IEEE Access*, 8, 12154–12167. https://doi.org/10.1109/ACCESS.2020.2966008.

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