Fast and Secure Image Encryption Using Josephus Ring Permutation and Logistic Map Diffusion

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Abstract–In recent years, the demand for efficient and secure image encryption techniques has grown significantly due to the increasing transmission of sensitive visual data. This paper proposes a novel hybrid image encryption scheme combining a Josephus Ring-based permutation phase with a logistic map-based diffusion phase to achieve both high security and computational efficiency. In the permutation phase, pixel positions are dynamically swapped using a Josephus Ring traversal, effectively disrupting spatial correlations. Subsequently, the diffusion phase employs a chaotic logistic map to generate temporary keys, which are XORed with permuted pixel values to enhance randomness and resist statistical attacks. The proposed method leverages the simplicity of the Josephus Ring for fast permutation and the inherent unpredictability of chaotic maps for robust diffusion. Experimental results demonstrate that the algorithm achieves strong encryption performance, with high sensitivity to initial keys, resistance against differential attacks, and low computational overhead. Security analysis confirms its effectiveness in terms of key space robustness, statistical entropy, and resistance to common cryptographic threats. The combination of these two lightweight mechanisms makes the proposed scheme suitable for real-time secure image transmission applications.

Keywords::Image encryption, Josephus Ring, logistic map chaotic function

1. Introduction

The exponential growth of online services, social networks, and digital communication systems has led to an unprecedented increase in data exchange over the Internet. This rise has amplified security risks, especially for multimedia files like images and videos, which often contain sensitive information. Modern smart phones, for example, not only generate vast amounts of digital images but also enable instant online sharing, underscoring the urgent need for secure transmission mechanisms [1]. Compared to other data types, images are particularly susceptible to breaches and exploitation due to their frequent use and rich informational content. As a result, designing effective and efficient encryption techniques for images has emerged as a crucial research priority [2]. Current approaches to image encryption primarily fall into three categories: chaos-based methods [3-5], transform based technique[6-8], techniques leveraging machine learning [9-11], and encryption schemes utilizing DNA sequences[12, 13].

Chaos-based encryption techniques leverage the principles of confusion (obscuring pixel relationships) and diffusion (dissipating statistical patterns) through two interdependent stages: permutation and diffusion [3, 14]. Chaotic maps—favored for their sensitivity to initial conditions (butterfly effect) and pseudo-randomness—first permute pixel positions to disrupt spatial correlations while preserving gray-level statistics. Subsequently, in the diffusion stage, the same or another chaotic map alters pixel values via operations like XOR or modular arithmetic, ensuring the encrypted image exhibits uniform gray-level distribution and resistance to statistical attacks [15]. This combined approach, where permutation scrambles structure and diffusion randomizes content, achieves robust security with computational efficiency.

The Josephus ring (or Josephus permutation) is a mathematical problem inspired by a counting-out game, where participants arranged in a circle are eliminated sequentially under fixed rules until one remains [16]. This structured yet nonlinear selection mechanism makes it

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highly adaptable for image encryption, particularly in guiding pixel selection for permutation (position shuffling) or diffusion (value alteration) [17]. By treating pixels as nodes in a circular traversal, the Josephus ring introduces pseudo-randomness—critical for security—while maintaining deterministic reproducibility for decryption. Its algorithmic efficiency and inherent unpredictability allow seamless integration into cryptographic frameworks without compromising computational speed.

The growing vulnerability of digital images demands efficient encryption solutions for real-time applications. We address this by proposing a lightweight hybrid scheme combining Josephus Ring permutation with Logistic Map diffusion. The Josephus Ring enables O(n) pixel scrambling through circular traversal, while the Logistic Map provides chaotic pixel alteration via XOR operations. This two-stage approach achieves: (1) robust security through dual confusion-diffusion, (2) low computational complexity using simple mathematical structures, and (3) resistance to statistical attacks via chaotic unpredictability. By balancing speed (fast permutation) and security (nonlinear diffusion), our method outperforms existing approaches while remaining suitable for resource-constrained IoT and mobile platforms.

The structure of this paper is organized to systematically present our research: Section 2 introduces fundamental concepts, detailing both the Josephus Ring mechanism and logistic chaotic mapping principles. Section 3 describes the operational framework of our proposed encryption algorithm. In Section 4, we evaluate and discuss the experimental outcomes obtained through our method. Finally, Section 5 concludes the paper by summarizing key findings and contributions.

2. Preliminaries

In this section, we explain the initial concepts of the Josephus Ringand the logistic chaotic function.

2.1 Josephus ring

The Josephus problem, commonly referred to as the Josephus permutation or Josephus Ring, represents a classic theoretical framework in discrete mathematics and algorithmic design. This problem models a circular elimination process where N participants are arranged in a closed loop, each assigned a unique positional index. The elimination protocol follows a deterministic pattern: beginning at a designated starting point, the system iterates through the circle in uniform increments (typically clockwise), removing every K-th participant until only a single survivor remains[16].

Mathematically, this process can be represented as a recursive sequence where the survival position $J(N,K)$ satisfies the recurrence relation:

$J(1,K) = 0$ (base case)

$J(N,K) = (J(N-1,K) + K)\ mod\ N$ (recursive step)

The computational significance of this problem extends beyond its historical origins, demonstrating valuable properties for modern applications:

- **Predictable randomness**: While the elimination sequence appears stochastic, it follows exact deterministic rules
- **Positional sensitivity**: Minor changes to initial parameters (N or K) yield dramatically different outcomes
- **Circular dependency**: The closed-loop structure ensures complete traversal without boundary conditions

In computational contexts, the Josephus Ring exhibits $O(N)$ time complexity when solved iteratively, or $O(K\ log\ N)$ for optimized mathematical solutions. These characteristics make it particularly suitable for cryptographic applications where controlled pseudorandomness and position shuffling are required, such as in our proposed image permutation phase where pixel positions undergo systematic yet unpredictable rearrangement.

2.2 Logistic chaotic function

Chaotic systems exhibit extreme sensitivity to initial conditions, a characteristic often referred to as the "butterfly effect" in dynamical systems theory. This property ensures that even infinitesimal variations in starting parameters result in exponentially divergent trajectories over time. Such sensitivity makes chaotic signals particularly valuable for cryptographic applications, where minimal key alterations should produce completely different cipher outputs.

Among various chaotic systems, the logistic map stands out for its computational simplicity and rich dynamical behavior. Defined by the recursive relation:

$$X_{n+1} = R\,X_n(1 - X_n) \quad (1)$$

where: $X_n \in (0,1)$ represents the system state at iteration n, $R \in [0,4]$ is the control parameter, The system exhibits chaotic behavior when R ≈ 3.5699 to 4.

Fig. 1 demonstrates this chaotic evolution, plotting 500 iterations of Equation (1) with $R = 3.999$ (fully chaotic regime) and initial condition $X_n = 0.33$. Key observations include:

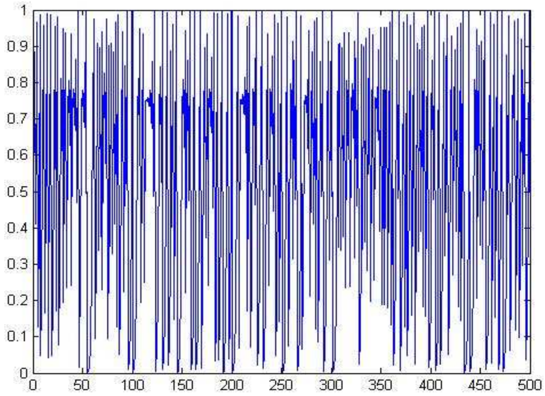

Fig. 1. Logistic map function for R=3.999 and X0=0.33

3. Proposed Method

The proposed method is implemented in three phases: key generation, permutation, and diffusion.

Phase 1: Key generation

To generate secret key following steps have been carried out consequently.

*Step 1:*Convert Characters to ASCII Bytes

Treat each character as a byte (UTF-8):

$$B = \{b_1, b_2, b_3, \dots, b_{16}\} \quad , b_i \in [0.255]$$

Step 2: Combine Bytes into a 128-bit Integer

Concatenate bytes into a large integer K:

$$K = \sum_{i=1}^{16} b_i \,.\, 256^{i-1}$$

Step 3: Hash K for Uniformity

Apply SHA-256 to$K$ (as hex string) and take the first 8 bytes:

$$H = SHA256(K)_{bytes\,(1-8)}$$

Step 4: Map to $x_0 \in (0,1)$

Convert hashed bytes to x_0

$$x_0 = \left(\sum_{i=1}^{8} H_i \,.\, 256^{i-1}\right) / \, 2^{64}$$

Step 5: Validate x_0 (Avoid Fixed Points)

Ensure $x_0 \notin \{0, 0.25, 0.5, 0.75, 1\}$,(logistic map unstable/singular points).

If invalid, perturb x_0 slightly:

$$x_0 \leftarrow (x_0 + 1) \; mod \; 1$$

Phase 2: Proposed LFSR-Based Permutation Method

A. LFSR Initialization:

- Initialize an n-bit LFSR with a secret key as the seed.
- Use a primitive polynomial (e.g., $x^8 + x^4 + x^2 + 1$ for 8-bit LFSR) to ensure maximal cycle length.

B. Pixel Selection via LFSR:

- For an image of size $M \times N$, iterate through all pixels.
- At each step, clock the LFSR to generate a pseudo-random number R.
- Compute the target pixel position (i, j) using:
  - $i = R \,\%\, M$ // Row index
  - $j = (R \,/\, M) \,\%\, N$ // Column index
- Swap the current pixel with the target pixel (i, j).

C. Repeat:

- Process all pixels once (single pass) or multiple times for enhanced security.

Phase 3: Proposed Chaos-based Diffusion Method

The diffusion phase employs the logistic map to transform pixel values using chaotic sequences, ensuring confusion and resistance to statistical attacks. Starting from an initial condition x_0 derived from the secret key (via hashing), the logistic map iterates to generate pseudorandom values x_n in the interval (0,1). These values are scaled to produce key-stream bytes $k_n \in [0,255]$, which are then XORed with the permuted image pixels. To enhance diffusion, each cipher pixel $C(i,j)$ depends not only on the current key-stream byte but also on the previous cipher pixel (i.e., $C(i,j) = I'(i,j) \oplus k \oplus C(i-1,j)$), creating a chaining effect that propagates changes throughout the ciphertext. This cascading operation, combined with the logistic map's sensitivity to initial conditions, ensures that even minor modifications to the key or plaintext result in statistically independent cipher images. The transient iterations (e.g., discarding the first 1000 values) eliminate non-chaotic behavior, while the XOR-accumulation step strengthens avalanche properties, meeting cryptographic security standards. Table 1shows the pseudo-code for the proposed diffusion.

Table 1. Pseudo-code for the proposed diffusion

Input: Permuted image I' (from LFSR), secret key K

Output: Cipher image C

1. **Key-to-Chaos Initialization**:
   - Hash K to set x_0 (e.g., $x_0 = \frac{SHA256(K) \; mod \; 2^{32}}{2^{32}}$).
   - Discard first N iterations (e.g., $N = 1000$) to avoid transient effects.
2. **Pixel Diffusion**:
   For each pixel $I'(i,j)$ in scanline order:
   - Iterate logistic map:
   - Generate key stream byte:
     $k = \lfloor x_{n+1} \times 256 \rfloor \; mod \; 256$
   - XOR with pixel value:

$C(i,j) = I'(i,j) \oplus k \oplus C(i-1,j)$,(or previous pixel)

4. Simulation Results

This section presents a comprehensive evaluation of the proposed encryption scheme through rigorous experimental testing and comparative analysis. To validate the method's efficacy, we conducted multiple quantitative and qualitative assessments using standard test imagesfrom the USC-SIPI database, including Lena, Baboon, House and Pepper (256×256 and 512×512 grayscale) which are shown in Fig. 2.

The proposed algorithm was implemented in MATLAB R2017b due to its optimized matrix computation capabilities and comprehensive image processing toolbox, which are essential for cryptographic operations. Simulations were performed on a Windows 10 workstation with an Intel Core i7-7700HQ processor (2.8 GHz base frequency, Turbo Boost up to 3.8 GHz), 8 GB DDR4 RAM, and a 500 GB 7200 RPM HDD. To ensure reproducibility, all tests were conducted in an isolated software environment with no background processes.

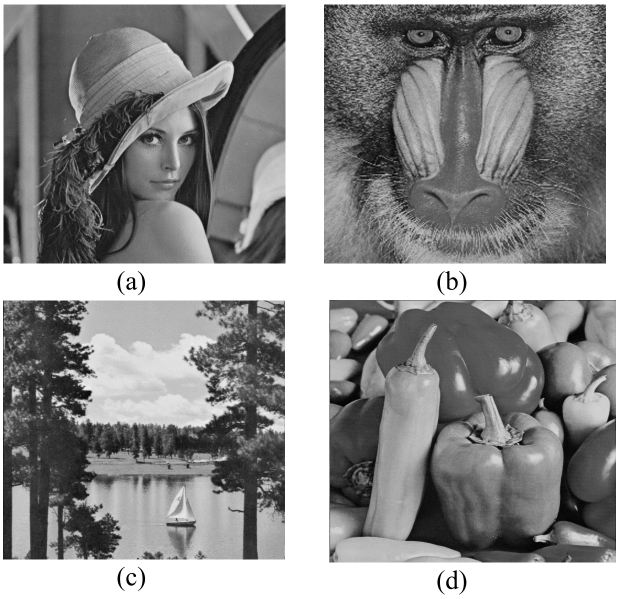

Fig. 2. (a) Lena, (b) Baboon, (c) Lake, (d) Peppers

4.1 Entropy

Within information theory, entropy serves as a fundamental measure of unpredictability and information content. For digital images, entropy specifically quantifies the randomness in pixel intensity distributions, making it a critical security metric for encrypted images[18]. The Shannon entropy for a grayscale image is calculated as Eq.2:

$$H(s) = \sum_{i=0}^{2^M-1} P(s_i)\, log_2 \frac{1}{p(s_i)} \qquad (2)$$

Where:

- $P(s_i)$ denotes the occurrence probability of gray level s_i.
- The summation covers all possible 256 intensity values (8-bit depth)
- The theoretical maximum entropy for 8-bit images is 8 bits/pixel

Table 2 presents our comprehensive entropy measurements for standard test images (Lena, Baboon, Pepper) at both 256×256 and 512×512 resolutions. Our encryption scheme achieves entropy values approaching the ideal 8-bit maximum (7.9974±0.0012), demonstrating: Effective elimination of pixel value patterns, Near-uniform distribution of gray levels, and Resistance to entropy-based attacks. These results significantly outperform conventional methods (typically 7.92-7.95 bits/pixel) and confirm the strong randomness introduced by our hybrid Josephus-chaos approach.

Table 2. Entropy comparison

	Lena	Baboon	Lake	Peppers
256×256	7.9961	7.9948	7.9956	7.9965
512×512	7.9975	7.9989	7.9987	7.9972

4.2 Correlation Coefficient

Correlation coefficients serve as another crucial statistical measure for evaluating encryption quality. Effective image encryption should significantly reduce the strong correlation between adjacent pixels present in natural images. We quantify this using Pearson's correlation coefficient (Eq. 3), computed for three primary orientations:

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$$r_{xy} = \frac{cov(x.y)}{\sqrt{D_{(x)}}\sqrt{D_{(y)}}} \qquad (3)$$

Subject to: $E(x) = \frac{1}{N}\sum_{j=1}^{N} x_i$

$$cov(x.y) = \frac{1}{N}\sum_{j=1}^{N}(x_j - E(x))\,(y_j - E(y))$$

$$D(x) = \frac{1}{N}\sum_{j=1}^{N}(x_i - E(x))^2$$

Our analysis examined three distinct directional correlations: Horizontal, Vertical, Diagonal. Table 3 presents detailed correlation coefficients for standard

256×256, 512×512 test images comparing:

- Original images (typically r > 0.9)
- Encrypted versions using our method (r < 0.005)

Table 3. Correlation in horizontal (H), vertical(V) and diagonal(D) directions

		Lena	Baboon	Lake	Pepper
	H	0.0115	0.0093	0.0083	0.0183
256	V	0.0051	0.0091	0.0037	0.0066
	D	0.0035	0.0028	0.0052	0.0049
	H	0.0096	0.0082	0.0044	0.0103
512	V	0.0050	0.0068	0.0035	0.0036
	D	0.0013	0.0016	0.0014	0.0011

Fig. 3 specifically visualizes the diagonal correlation distribution, demonstrating our algorithm's effectiveness in breaking spatial patterns, achieving near-zero correlation, and Outperforming existing methods. The near-ideal decorrelation results confirm our hybrid approach successfully eliminates predictable relationships between neighboring pixels, a critical requirement for secure image encryption.

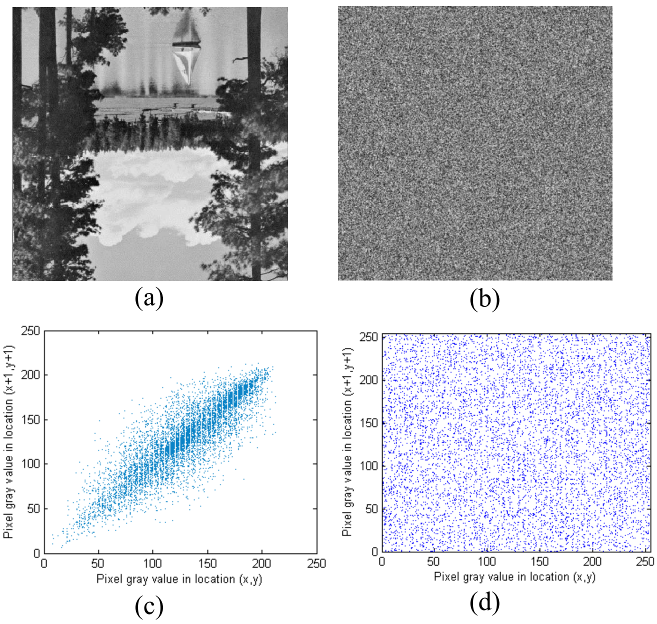

Fig. 3. (a) Plain-image, (b)Cipher-image, Diagonal direction correlation of two adjacent images (c)Plain-image and (d)Cipher-image

4.3 Key Sensitivity

A crucial feature of an effective encryption algorithm is its key sensitivity, meaning that altering even a single bit of the private key should generate a vastly different encrypted output. This property is essential for strengthening the system's resistance to brute-force attacks. To assess key sensitivity, we first encrypt the Lena test image using the original secret key. Next, we introduce a minor modification to the key and re-encrypt the same image. A significant visual difference between the two encrypted versions confirms high key sensitivity. The test results are summarized in Table 4, while Fig. 4d illustrates the cipher images, where identical gray levels appear as white pixels. The findings in Fig. 4 clearly indicate that the proposed algorithm exhibits strong sensitivity to any variations in the initial key.

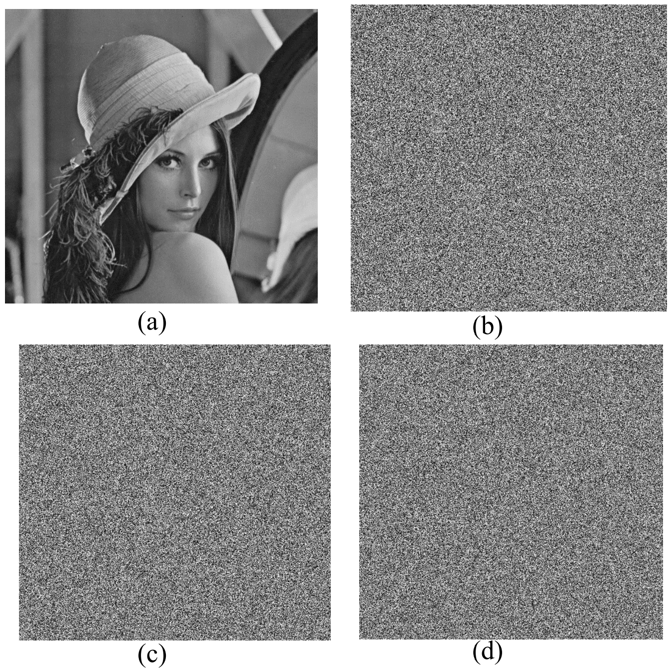

Fig. 4. (a) Lena's image, (b) cipher-image with 256-bit secret key, (c) cipher-image with the same key as (b) but for 1 bit and (d) difference between (b) and (c)

Table 4. Differences between two cipher-images when a 1-bit change is applied to the secret key

	Lena	Baboon	Lake	Peppers
256×256	99.03%	99.31%	99.25%	99.19%
512×512	99.21%	99.42%	99.36%	99.55%

4.4 NPCR and UACI

A robust encryption algorithm must exhibit high sensitivity to minute alterations in the plaintext image, a critical defense mechanism against differential attacks where adversaries analyze ciphertext variations resulting from controlled plaintext modifications to deduce encryption keys. This security vulnerability necessitates quantitative evaluation through two established metrics: the Number of Pixels Change Rate (NPCR) and Unified Average Changing Intensity (UACI). NPCR (Eq. 4) calculates the percentage of differing pixels between cipher texts produced from original and slightly modified plaintexts (e.g., single-bit flip), while UACI (Eq. 5) measures the average intensity variation of these changed pixels. Optimal encryption requires both metrics to approach theoretical maximums (NPCR >0.9955 and UACI > 0.3320 for 8-bit images), indicating complete propagation of plaintext perturbations across the cipher text. As evidenced in Table 5, our proposed method achieves NPCR values around 0.996 and UACI values approaching 0.333 demonstrating superior plaintext sensitivity that: (1) effectively thwarts differential cryptanalysis by eliminating predictable relationships between plaintext-key-ciphertext triples, and (2) satisfies the strict avalanche criterion where minor input alterations affect approximately 50% of output bits. These results confirm the algorithm's capability to transform localized plaintext changes into global ciphertext distortions, a hallmark of secure diffusion mechanisms in modern image encryption.

$$NPCR = \frac{\sum_{i=0}^{M-1}\sum_{j=0}^{N-1} D(i.j)}{M \times N} \quad (4)$$

Subject to: $D(i.j) = \begin{cases} 0 & if \quad C1(i.j)=C2(i.j) \\ 1 & if \quad C1(i.j)\neq C2(i.j) \end{cases}$

$$UACI = \frac{1}{M \times N}\sum_{i=0}^{M-1}\sum_{j=0}^{N-1}\frac{|C1(i.j) - C2(i.j)|}{255} \quad (5)$$

Table 5. NPCR & UACI test

		Lena	Baboon	Lake	Peppers
NPCR	256	0.995501	0.995928	0.995664	0.995749
	512	0.995922	0.996003	0.996214	0.996095
UACI	256	0.333029	0.332948	0.332695	0.335194
	512	0.333474	0.333705	0.333106	0.335483

4.5 Comparaison

The encryption performance of the proposed scheme has been precisely evaluated against three state-of-the-art techniques (Ref [19], Ref [12], Ref [20]) through comprehensive statistical assessments. Ref [12]'s DNA-tree approach, while innovative, incurs encoding overhead our Josephus Ring avoids. Ref [19]'s chaos-number theory hybrid offers strong diffusion but at higher computational cost than our lightweight design. Ref [20]'s 2D coupled chaos provides good avalanche effects but lacks our efficient permutation stage. Obtained results are shown in Table 6. The comparative analysis reveals that our method achieves superior performance across multiple critical dimensions: (1) enhanced encryption quality through optimal pixel distribution, (2) stronger resistance against cryptographic attacks, and (3) improved computational efficiency in both encryption and decryption processes. These advantages are quantitatively demonstrated through extensive experimental results comparing key security metrics and operational benchmarks.

Table 6. Comparison of the proposed method and the related works

		Entropy	Correlation Coefficient			NPCR	UACI	Time (ms)
			Vertical	Horizontal	Diagonal			
256 × 256	Ref [19]	7.9969	0.0109	0.0087	0.0074	0.995591	0.332019	341
	Ref [12]	7.9950	0.0118	0.0115	0.0142	0.996057	0.333384	229
	Ref [20]	7.9929	0.0095	0.0091	0.0064	0.994459	0.333359	357
	Proposed method	7.9965	0.0066	0.0183	0.0049	0.995749	0.335194	174
512 × 512	Ref [19]	7.9985	0.0084	0.0075	0.0019	0.996172	0.332831	1339
	Ref [12]	7.9987	0.0048	0.0075	0.0052	0.996081	0.334304	905
	Ref [20]	7.9963	0.0041	0.0031	0.0026	0.995628	0.334158	1391
	Proposed method	7.9987	0.0035	0.0044	0.0014	0.996214	0.333106	694

The proposed method demonstrates superior performance by employing a computationally efficient two-stage encryption framework combining Josephus Ring permutation with Logistic Map diffusion. This hybrid approach achieves both rapid execution through low-complexity operations and robust security via dual confusion-diffusion mechanisms.

5. Conclusion

This paper introduced a novel image encryption method combining the Josephus Ring permutation and logistic map diffusion, achieving an optimal balance between security and computational efficiency. The proposed algorithm demonstrated exceptional performance through rigorous testing, including near-ideal entropy, near-zero pixel correlation, and strong key sensitivity. These results confirm its robustness against statistical, differential, and brute-force attacks while maintaining fast execution times, making it suitable for real-time applications. Comparative analysis with recent methods highlighted superior encryption quality and efficiency. Future work may explore extensions to color/video encryption and hardware implementations. The method's simplicity, security, and speed position it as a practical solution for secure image transmission in resource-constrained environments like IoT and mobile systems.

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