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Comparative Study of Geometric and Numerical Cognitions of Structures in the Architectural Design Process

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ABSTRACT: Structural design extends beyond merely creating forms to withstand forces; it incorporates various technical and non-technical elements within architectural design. The true value of structural knowledge emerges during the design process when these elements are thoroughly understood. Achieving this understanding necessitates the provision of diverse types of structural knowledge capable of informing design and analysis. Additionally, by exploring various structural design methodologies, two principal types of cognition — "geometric" and "numerical" — can significantly help designers navigate the complex requirements of structural design. This research aims to investigate the different forms of structured knowledge relevant to architectural design, ultimately addressing the central question of how geometric and numerical comprehension of structures contributes to the architectural design process. The research employs a descriptive-analytical method with a comparative study approach. It begins by defining and detailing the characteristics of geometric and numerical cognition within the design framework, subsequently elucidating their interrelationship in architectural design. Findings suggest that geometric and numerical cognitions of structures, articulated through distinct languages, correspond to various roles in designing mechanical and spatial aspects, as well as different levels of structural comprehension at various design stages. Geometric cognition, due to its linguistic proximity to the design step, has the capability to examine and apply structural knowledge from the initial stages of form design to its actual construction phase. On the other hand, numerical cognition primarily focuses on the detailed examination of the mechanical and load-bearing aspects of the structure in its analytical step.

Keywords: *Design process, Structural knowledge, Geometric cognition, Numerical cognition, Comparative studies.*

INTRODUCTION

Various studies have examined structural knowledge in architectural research, covering a wide range of topics: historical research (Addis, 2007), the collaboration between architects and engineers (Yu et al., 2022; Luyten, 2012; Larsen, 2003), and various methods of structural design (Lewis, 2005; Lai et al., 2024). Furthermore, the approach of most studies focuses on the acquisition and understanding of structural concepts for architects (Millais, 2005). The primary goal of these writings is to bring architects closer to contemporary structural knowledge and re-establish a connection with it. However, the passage of time has revealed a lack of practical success in these methods within architectural design (Mainstone, 2001).

The main issue in most of these studies is the insufficient and inadequate attention to how structural knowledge is present in the architectural design process (Mahmoudi, 2012). In other words, the issue of structural knowledge in architecture extends beyond the method of learning to the transfer of structural knowledge content within the design process. Consequently, the structure faces challenges in its position and method of transfer within architectural works.

Understanding structural knowledge is an essential necessity throughout the process of shaping architectural form. The integration of innovative, economical, and feasible structures within the building design process has always been a critical topic in architecture. Structural knowledge encompasses various aspects, some of which are necessary for the design and construction of buildings, while others are utilized for evaluating and optimizing the structure itself. Therefore, structures within the realm of architecture, which is an inseparable part of the architectural space, can only be recognized through

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examining the diverse aspects of structural knowledge. A significant issue is the translation of this knowledge into architectural design and how it manifests in the design process. This challenge arises from an inadequate understanding of structural knowledge and its influence during different stages of the design process. In other words, which type and extent of structural knowledge can be beneficial to architects amidst the multitude of design requirements?

In general, structural knowledge includes two primary types of understanding: geometric and numerical, within the design process (Haghir et al., 2021; Mainstone, 2001; Mahmoudi, 2012).

Until the mid-18th century, the use of drawings and geometric relationships in the design and construction of structures was common. Humanity, after advancing in foundational perceptions, succeeded in designing and understanding structures through geometric methods. This approach utilized drawings and spatial and geometric relationships in the design and construction of structures. For example, in Western architecture, geometry, as a practical art, enhanced the level of design knowledge throughout the Middle Ages. Developed geometry facilitated the review of proposed designs and employed profound practical applications in the construction process, enabling the precise fabrication of building components. Such advancements could inspire builders to consider constructing larger and taller buildings (Addis, 2003). Similarly, in Iranian architecture during this period (Mohammadi et al., 2019; Rezazadeh & Etessam, 2020), when architecture was still based on trial and experience, the stability of structures, their dimensions and proportions, and the relationship between architecture and structural elements were based on geometric understanding, which gradually expanded. In fact, geometry served as a guide for architects in ensuring proportions and fundamental harmony among structural components, regulating dimensions, and understanding and visualizing the spatial forces acting within the load-bearing structure (Omranipour, 2005).

After this period, particularly in the early 19th century, numerical and computational methods dominated structural understanding and design. This approach was grounded in theoretical physics and largely departed from traditional design fundamentals. During this time, structural theories entered the field of education and academia due to the influence of polytechnic schools and advancements in the construction industry and new materials (including iron). Proficiency in precise calculations and adequate levels of safety using new materials became possible. These skills led to a complete separation of the architectural profession from engineering, which had previously been determined solely by occupational structure (Saint, 2007). To date, numerical understanding has been utilized in various ways in structural design. Analytical programs are capable of determining stress, deflection, and dynamic behavior for very complex structures using advanced methods. However, these tools are less frequently applied in the conceptual design of structures (Mueller, 2014).

The research objectives aim to provide a comprehensive understanding of how geometric and numerical cognitions interact in the architectural design process, ultimately contributing to the field of architecture. Accordingly, this research aims to elucidate the role of two types of understanding—geometric and numerical—in relation to the steps of structural design and analysis within the architectural design process. Therefore, the following questions form the basis of this study:

- What aspects of structural design do geometric and numerical cognitions address?
- What is the role of geometric and numerical cognitions of structures within the design process?

Theorical Framework

Structural Aspects in Architectural Design

Structural considerations play a crucial role in architectural design, significantly influencing a building's performance, environmental impact, and overall design outcomes. The integration of structural engineering principles in the early design stages can lead to more efficient and sustainable buildings (Felicioni et al., 2023).

To understand the place and logic of structural knowledge in the architectural design process, all its details must be examined. This necessitates theorizing to establish solid foundations across various fields of knowledge, allowing for deeper insights. This endeavor requires an examination of the qualities of structures as part of the architectural expression, which can influence or be influenced by the architectural space. Therefore, in this research, building structures are considered at least as a component of architectural composition.

The term "structural aspects" refers to the fundamental elements of a structure that help in understanding its roles and basis within architectural design. Lawson refers to these as "constraints" (Lawson, 2008). Architects and designers identify various aspects of structural participation in the architectural design process; however, most of them share common characteristics.

Schlaich argues that the art of building is indivisible, and thus one should never focus solely on its technical aspects but also consider aesthetic and functional dimensions (Schlaich, 2006). Billington's views align with Schlaich's, as he identifies three key characteristics of outstanding structural designs: efficiency, economy, and elegance, which expand the concept of structural art (Billington, 1985).

In another approach, three main factors are considered for designing a new structure or assessing an existing one (Lin et al., 2016):

- **Performance:** The structure must possess specific functional characteristics to meet user needs.
- **Structural Integrity:** The structure must be constructible, structurally reliable, and economically feasible.
- **Aesthetic Experience:** The structure should symbolize social and aesthetic values for users and harmonize with the surrounding environment.

Sandaker, in a more comprehensive and precise definition, views the role of structure beyond mechanical factors: "In design, there is a close relationship between structure, architectural space, and visual expression; therefore, merely serving a load-bearing function is insufficient. If the goal is to understand structural elements, a comprehensive examination is necessary, including space and context." He introduces three factors for understanding structures in architecture:

- The scientific aspect, which includes logical concepts and structural efficiency such as stability, balance, stiffness, and strength.
- The technological aspect, encompassing production, methods, and

construction processes.

- The spatial aspect of the structure, which includes aesthetic experience and architectural functionality (Sandaker, 2010).

Understanding each of these three aspects of structural design requires different modes of thinking. It is important to contemplate the load-bearing behavior of the structure, the construction process, and the type of users involved. Since the loading conditions and structural interaction scenarios can make identifying new structures challenging, they must be assessed based on scientific understanding (Reid, 2010). Understanding the scientific aspect of a structure can be achieved through sustainability thinking (durability). Given the increasing risks of unexpected instabilities, new structures must be designed to mitigate potential consequences of any instability. The functional thinking of the structure can also encompass a range of spatial criteria for the building, including usability, contextuality, and morphology (Sandaker, 2010). Ultimately, as the design agenda expands, the feasibility and practicality of the project are assessed. Feasibility represents thoughtful consideration and study of physical constraints in the construction technology phase (Amit, 2007). In Table 1, the various aspects of structural design are presented based on the approaches of some experts. Additionally, in Fig. 1, the model of the aspects of structural design considered in this research is presented.

Design and Analysis Theories of Structures

Referring to specialized structural and architectural resources, two distinct approaches can be identified in defining the concepts of "structural design" and "structural analysis," which have content differences:

- **First Approach:** In some specialized structural resources, based on definitions in structural engineering (Pedron, 2006; Khadka, 2015), structural analysis is conducted first through loading and then structural design follows. Here, analysis means determining the internal forces of the members. In other words, structural analysis is defined by calculating the applied loads and the behavior of structural members under certain loads. Then, based on the existing forces and the calculated loads, the design phase is carried out by finding the dimensions, shape, and size of the members. In other words, structural design is defined as the process of finding the properties of safety,

Table 1: Various aspects of structural design in architecture from the perspectives of some experts (In the continuation of this research, the three aspects of scientific, technological, and spatial structure will be emphasized.)

Structural Design Aspects			Source
Scientific	Technological	Spatial	(Sandaker ,2010)
Efficiency	Economy	Elegance	(Billington, 1985)
Force concepts		Spatial organization	(Schodek & Bechthold, 2013)
Integrity		Performance and aesthetic experience	(Lin & Huang, 2016)
Mechanical		Spatial	(Schlaich, 2006)
Efficiency		Functional and visual efficiency	(Mueller, 2013)

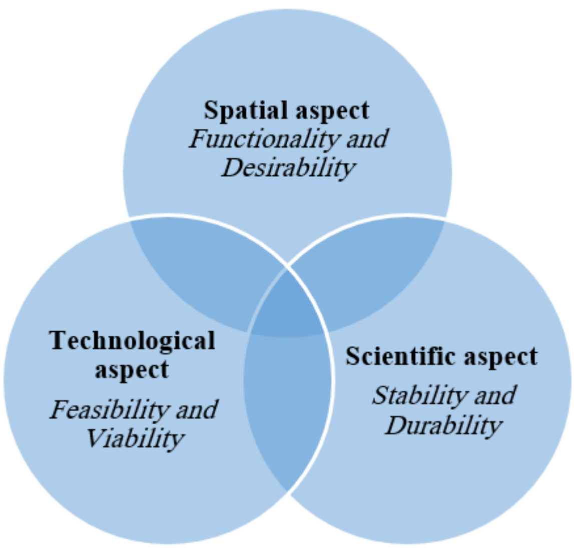

Fig. 1: Types of aspects and design thinking in structural architecture

strength, and economy of the structure. Here, structural design is also based on numerical analyses and focuses on mechanical issues, with its language being mathematical (Voyatzaki & Spiridonidis, 2009).

- **Second Approach:** In some specialized structural resources that are mainly presented for architects and structural designers (Macdonald, 2019; Luyten, 2012), the formation of a structure in the architectural design process is divided into two main steps: the structural design step and the structural analysis step. In the structural design step, the overall form and arrangement of the structure are planned. Therefore, this design step primarily refers to the "conceptual design of the structure," which is mainly carried out by the architect; then, in the "structural analysis" step, the structural calculations are performed, and the precise dimensions of the various structural elements are determined, with the primary goal being optimization. This step is mainly defined in the domain of structural engineer responsibilities.

Thus, for most engineers, structural design only refers to the calculations of the dimensions of sections or connections. On the other hand, architects focus on geometric issues in designing and visualizing the structure. The precise tools allow the structural engineer to input the geometry of the structure (proposed by the architect) into software to simulate loads, determine the dimensions of physical arrangements, and optimize them (Voyatzaki & Spiridonidis, 2009).

In this research, the concepts of "structural design and structural analysis" refer to the second approach.

MATERIALS AND METHODS

This research employs a descriptive-analytical method with a comparative approach in mind. The study aims to describe and analyze the role of geometric and numerical cognitions in the design process by conducting a comparative analysis based on specific examples and data reasoning. Through the presentation of appropriate findings, the study seeks to provide beneficial results that enhance the application of these cognitions, particularly in scientific research. To achieve the objectives of this research, various technical and non-technical aspects of structures are considered as the main components in the description and analysis of geometric and numerical cognitions of structures.

RESULTS AND DISCUSSIONS

Design Approach in Geometric Cognition of Structures

For a long time, geometry has always been a fundamental knowledge in the architectural design process; however, it has rarely been used as a research domain. The emergence of free forms in contemporary architecture has completely changed this situation, challenging the geometry of architectural designs much more than before. Nowadays, architects utilize digital technology from the automotive and aerospace industries for architectural design and construction. This leads to numerous problems, as architectural programs, including aesthetics, statics, and construction technologies, differ significantly from many other industries (Pottmann, 2013).

Major aspects emphasized in some definitions of design include being purposeful, focused on creating form, structure, and meaningful order in objects, and paying attention to aesthetic dimensions while also addressing functional, economic, and social aspects. Nadimi, in expanding the concept of engineering, considers the term "geometry" as closely associated with the act of "design": "Familiar interpretations and attributes such as the fitting of members with the environment, moderation and neatness, harmony of components, order of distances, and proportions in the final composition bring geometry closer to the concept of quality and beauty" (Nadimi, 2012).

Therefore, architecture is the organization of space, and geometry is one of the sciences that has long been used to organize forms and the physicality of space, as well as to arrange the movement of structural forces in buildings. With its dual capacity—quality and quantity—it can serve as a unifying factor for concepts such as architecture and structure. Efficient structures often have geometric solutions (Breish et al., 2024; Tam et al., 2015). Thus, geometry is considered a visual thought. Visual thinking is particularly related to the design stage, in which the architect has the most significant impact. Generally, envisioning and recording spatial organization is recognized as an activity that distinguishes architects from other individuals involved in building creation (reflecting visual thinking). Charles Eames, as an architect, furniture designer, filmmaker, and exhibition designer, responds to the question, "What is your definition of design?" by saying, "It's a way of organizing elements in the best possible manner to achieve a specific goal." This refers to the relationship between design and future events, predicting them using any suitable method, including drawing, modeling, and digital simulation; however, in architecture, a visual thought must exist beforehand (Brawne, 2003).

Geometry plays a significant role in the design of structures broadly (Ibid). The use of advanced geometric knowledge has laid the groundwork for understanding, designing, and constructing some of the most significant achievements in the history of structures. Japanese wooden structures, complex advancements in medieval and Gothic architecture, are just examples of this. The developments of the 20th century in structural construction technology, including conical concrete shells and geodesic domes, demonstrate an extraordinary complexity in spatial and structural conceptualization. New material technologies, combined with new architectural and structural concepts, contain a wide range of forms. Polyhedra and various hyperbolic spatial organizations have created a diverse architectural geometric vocabulary (Medaković et al., 2024).

Moreover, advancements in architectural and structural technologies refer to a wide range of thin surfaces, tensile membranes, beam-cable structures, and more. For the application of these advanced structures, Valvani believes there is a need for greater understanding in the field of morphology, including: anticlastic surfaces, diamond-shaped structures, polyhedra, spiral geometries, and many hyperbolic spatial surfaces. In fact, there exists a structure of geometries that have different configurations compared to Euclidean geometry. In other words, it focuses on various non-numeric aspects of structural morphology. For instance, understanding parabolic geometry, often sinusoidal, is suitable for designing and visualizing tensile membranes. Concepts in topology can be very significant in studying specific types of structures, such as deformable or expandable structures. Recent developments in fractal geometry can also provide a useful tool for visualizing tree-like structures and a method for analyzing self-similar systems. Velasquez refers to geometry as a tool for visual organization and proportions in design: "The main idea is to examine the internal components and simplify them so that they can facilitate the construction of the product. This requires a balance between beauty and function, aiming to refine the shape of the product to achieve an attractive and beautiful product" (Velásquez, 2013). When designing a product, the main challenge is the correct placement of components alongside one another. Geometry is essentially a way to logically explain many design decisions in both functional and aesthetic domains.

The necessity of geometric knowledge for structural designers has been recognized by many researchers over time, even from the early years (Bursill, 2002). Real or mental concepts of space, visualization of many shapes, initial understanding, modeling, and other skills can only be developed through geometric understanding. The lack of geometric knowledge, among other factors, reduces the applicability of structural knowledge for designers and engineers because many concepts include geometric principles (Kurrer, 2008). Researchers believe (Liapi, 2002) that understanding the geometry of structures is of great importance because: 1) it enhances the ability to mentally comprehend space; 2) it connects mathematics to the real world and the design space; 3) it helps in understanding abstract ideas through the interpretation of geometric models; 4) it facilitates the simplification and easy recognition of structures; 5) it provides order and structure to the design process; and 6) it enhances the power of imagination, creativity, spatial perception, and complex thinking.

Moreover, geometry plays an important role in the design and construction of structural elements both locally and globally (Liapi, 2002). Geometry, for example, affects how loads are distributed within a structure, such that different forms lead to varying internal forces (Ellis et al., 2003). Kent and Noss emphasize the significance of geometry not only for spatial perception and the construction process but also for understanding and awareness of structural behavior. Additionally, a structural sense in the design and construction of structures is very beneficial; part of this capability is achieved through geometric cognition (Kent & Noss, 2002). Therefore, some experts believe (Van Niekerk, 2010) that catastrophic structural failures have resulted from a misunderstanding of the geometry of structures.

While the production of various forms was once limited by traditional methods, modern technologies have led to a true revolution in geometry. Nowadays, we are faced with significant changes, and the tools at our disposal seem virtually limitless. However, the increase in possibilities does not necessarily correlate with a deeper understanding of geometry (Pottmann et al., 2007). Beghini views geometric cognition as a design method in architecture and structures aimed at achieving both aesthetic appeal and structural sustainability (Beghini et al., 2014). Scale and construction technologies present new challenges for engineering and design, which can be effectively mitigated through a comprehensive understanding of geometry (Pottmann et al., 2007). Therefore, since the design, analysis, and construction of structures primarily depend on the definition of their form, the geometric foundation of a structure serves as an important intermediary between its understanding and design. Thus, structure and geometry are inextricably linked. In other words, optimal structural design can be manifested in geometry (Romeo, 2010).

Aspects of Geometric Cognition of Structures

Although the use of geometric understanding is not a solution to the problem in itself, it can open pathways for the optimal application of structural knowledge in the design process (Ugail, 2011). In fact, geometry is a language that can serve as a tool in designing aesthetic and abstract ideas as well as in optimization and sustainability (Beghini et al., 2014). Knowledge of geometry facilitates a more accurate description of proposed designs in the construction industry and allows for its practical application in the construction process, enabling the precise fabrication of building components. Such advancements can empower builders to construct larger and taller buildings (Addis, 2003).

The main approach in the field of geometric cognition can be attributed to Unwin's theory (Unwin, 2014). Unwin has analyzed geometry at various levels in the design of architectural structures based on studies of traditional to modern buildings. He examines the various factors of understanding and the processes that create or modify architectural structures. According to Unwin's approach, the geometry of a building structure can arise from the features of "spatial or social performance" and "the construction process" alongside the inherent and "existing" characteristics of geometry, which he considers factors of control and acceptance in structural production.

A. Theoretical Geometry: The first factor in the emergence of building geometry is its inherent characteristics and existing geometric rules. This factor in geometric understanding of structures is defined based on existing scientific theories. In an era of rapid technological change, architects must be proficient in structural science and possess an intuitive understanding of its behavior, enabling them to theorize and design beyond conventional structural forms (Luyten, 2012). The theoretical geometry of a structure defines the relationship between form and the stability and durability of the structure. The relationship between concepts of efficiency and geometry is a familiar subject in the field of structural morphology (Turrin, 2011). Geometry, as the fundamental science of forms, aids in the process of composition and design in architecture (Leopold, 2006). Fuller refers to geometry as the "science of structures." For example, in his research, he developed a systematic method for dividing a sphere. His structural thinking, starting from Platonic solids, culminated in geodesic networks and ultimately geodesic domes (Beghini et al., 2014). His practical geometry was used for various purposes, from tents to domes. Fuller was interested in a design space that encompassed all known mathematics from Euclidean to non-Euclidean geometry (Simitch et al., 2014). In another example, Nervi conveyed the spatial expression of structures through geometric language and hanging models. The essence of geometry, while logical and systematic in mathematics, also allows for optional and free combinations of structures, serving as an inherent tool for the designer to combine architectural pieces (Romeo, 2010). Sandaker believes that in the scientific study of structures, understanding "overall geometry" holds a special position. In this context, in addition to addressing the overall geometry of the structural system based on structural concepts, it is examined what geometry the shape of the structure closely resembles (Sandaker, 2010). For instance, an arch structure may be classified as parabolic, chain-like, or semi-circular. Therefore, the overall geometry of any structure suggests the behavior of the structural system, which must include one of the active, semi-active, or inactive groups.

In a wide range of ways that affect the mechanical behavior of structures, nearly all of them can have geometric outcomes on form, which is referred to as the grammar of structures. Moreover, the management of forces through geometry influences form and facilitates the integration of the structure with architectural language. Diest believed in "the resistance of form through geometry" and the use of geometry to shape structures and provide efficient and conceptual architecture (Pedreschi, 2014). His structures emphasized the lightness and efficiency of materials, achieved through a combination of engineering analysis, experience, and attention to the capabilities of geometry. Often, design masterpieces utilize strength in geometric form to minimize material and construction costs while maximizing structural capability and aesthetics. Therefore, one of the factors in the mechanical understanding of structures is "the relationship between the geometry of the structure and forces" (Mueller et al., 2013), which pertains to the understanding of structural science.

B. Spatial Geometry: The second geometric factor arises from the spatial performance of architecture, known as spatial geometry. In the realm of structural understanding, this concept examines the architectural performance of structures in building design. In fact, functional geometry recognizes the means by which designers can adapt and establish structures based on architectural performance. The main geometric design processes include an efficient description of geometric shape (theoretical geometry) and the integration of shape with its functional analysis (Ugail, 2011). For this purpose, geometric thinking that can generate diverse designs and relate to their functional capability in the early stages of the design process is desirable. In architectural structures, Engel considers the fundamental role of the structure to be the production of form and space (Engel, 2009). There is a connection between the overall organization of the structural system and the organization of architectural space, which defines the geometry of the building (Mora et al., 2004). The relationship between structural geometry and architectural form and space can be represented in various ways. The relationship between architectural form and structural geometry is not always straightforward. One can choose a structural strategy and allow it to define the desired space, or compel the structure to respond to the architectural space. Therefore, various geometric relationships can be defined between structure and space: dominant structural order, dominant spatial order, or a harmonious relationship between the two, in which spatial and structural orders are one and the same. There is also a fourth state where spatial organization is separated from the structure so that they can coordinate with each other, each conforming to its own logic without the constraints imposed by the other.

For example, in one instance, the geometry and configuration of structural components are highly integrated or coordinated with the architectural form, primarily utilizing basic geometric shapes. In architectural history, valuable buildings can be found that were constructed with a tradition of structural acceptance. In the northern dome of the Jameh Mosque of Isfahan, the spatial and mechanical aspects are well combined in a perfectly balanced form (Ghannad, 2003). As shown in Fig. 2, the structure of this dome is shaped using geometric order at various levels. At the first level, the geometry transforms into the form of the main structure, which is a square space. Each side of this square space is defined by three main arches, with the middle arch having a wider span compared to the other two, visually dominating the space. This visual space provides geometric characteristics such as symmetry, structural balance, and harmony. At the second level, the geometry of the northern dome is based on precise mathematical proportions. The square base is divided into nine square units, such that each of the arches of the square space is aligned with this division. The central section of the structure serves as a transitional area resulting from a series of geometric transformations starting from a square base, then forming an octagon, and ultimately intersecting with the circular base of the dome to create a hexadecagon. The corner arches frame four pendentives. These pendentives are base structures that transform the square base into a circular dome. The result of this arrangement is a unique structure that creates surfaces and rotational forms within a geometric space. Structurally, the construction process of a dome involves mechanical principles, geometric transformations, and mathematical analysis. At the third level, geometry, as a symbol, conveys meanings that go beyond their physical applications. Accordingly, geometry creates a distance between concept and perception, transforming an intangible idea into a tangible form. Architecture has become a venue for the visualization of sacred art, and thus, under the influence of Pythagorean spatial concepts, geometry is used to create physical form.

C. Practical Geometry: The third geometric factor is construction or practical geometry, which influences the methods of production and execution. Practical skill encompasses a range of construction, assembly of materials, and techniques that are physical, quantifiable, and objective. Structures form the basis of construction (Leopold, 2006). Ove Arup believes that "design is nothing more than demonstrating the logical method of construction, which includes all drawings, specifications, explanations, and detailed instructions regarding what should be built and how it should be built" (Addis, 2003). Geometry is a key and determining factor in the method of constructing a form; in other words, the geometry of many architectural structures arises from their method of construction; therefore, structural geometry is influential in the construction process (Chai et al., 2023). Candela and Isler emphasized the importance of the construction process in design. Both were interested in simplifying forms through geometric methods to reduce and eliminate complex calculations (Adriaenssens et al., 2014). For example, Candela utilized a method of "geometric modification" to reduce construction costs and facilitate execution. He used straight wooden boards as molds to shape surfaces with double-curvature parabolic-hyperbolic forms (Lee et al., 2009). The diversity of construction can be observed at many stages, including the assembly of components, the interaction and relationship of each element with the overall structure, the properties of materials, and construction methods. The shape of components, their formation, and their connections to other parts reflect the methods of production (Morales et al., 2023), which vary according to the diverse structural forms. For example, a structural element may consist of solid, truss, or I-shaped sections.

Therefore, technological characteristics primarily influence the local geometry. Additionally, the specific properties of structural materials

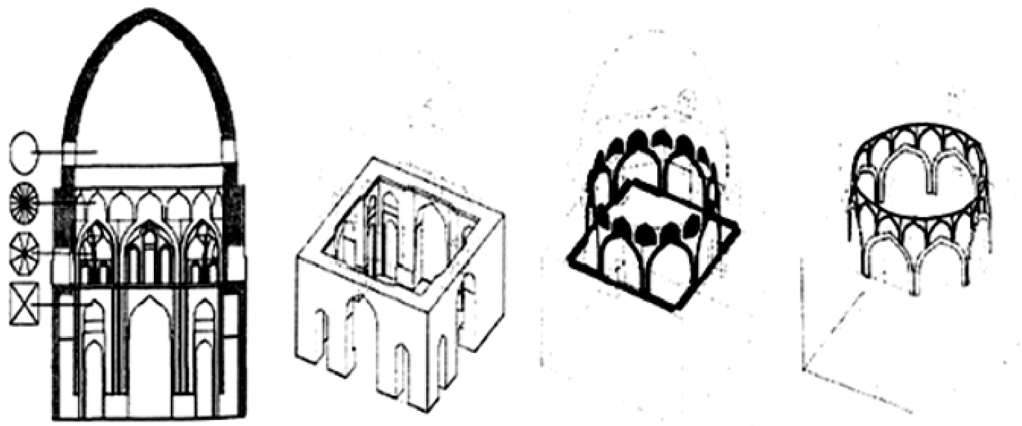

Fig. 2: Basic geometric surfaces and the transitional area of the northern dome of the Jameh Mosque of Isfahan (Source: Ghannad, 2003).

address the details within the local geometry. As shown in Fig. 3, two structures with similar overall geometries may contain different local geometries. Furthermore, Pottmann argues that considering the main aspects of construction often requires a redesign phase after defining the primary geometry. For example, Frank Gehry was one of the first to use expandable curved surfaces in the production of free forms. These surfaces consist of a series of straight lines, each having a constant tangent surface along them. This is a desirable feature for economical and easy construction (Pottmann, 2013).

Therefore, in the design of structures, knowledge of geometry can be examined from three perspectives: the theoretical aspect, which provides reasons and resolves doubts scientifically; in the realm of structures, the scientific aspect pertains to the laws of statics, equilibrium, and material properties (Sandaker, 2008). Practical geometry is also necessary for assessing the limitations that construction technology should not exceed (Addis, 2003). At the stage of spatial geometry, the designer needs to have precise knowledge of techniques and strategies to actualize structural knowledge within the architectural space. In other words, this level enables the emergence of ideas within the context and structure of architecture. The framework of the correlation of geometric cognition on aspects of structural design is Shown in Fig. 4.

Analytical Approach in Numerical Cognition of Structures

Engineering knowledge currently offers several methods to examine structural behavior. These methods are primarily based on numerical and algebraic techniques, with some examples being designed and analyzed using advanced software. Numerical language in structural design has been utilised in various forms. Generally, specialized structural tools include two types: form analysis tools and form-finding tools. While the first type focuses on calculating internal stresses in order to measure or evaluate structural components, the second type assesses the structure according to a set of mechanical constraints and boundary conditions (Fivet et al., 2014), with both approaches emphasizing structural optimization.

Throughout the history of construction, designers and engineers have employed various optimization methods, including isostatic lines, manual calculations, and form-finding techniques to achieve some desirable structural shapes (Consolini et al., 2010). One of the most recent methods, widely used by Frei Otto, involved using physical models (soap bubbles) to create structures with the least possible surface area (Adriaenssens et al., 2014). Additionally, manual calculations were employed to find optimal shapes for conventional structures such as arches, columns, and cantilevers.

Structural optimization is a numerical method (Fan et al., 2021) aimed

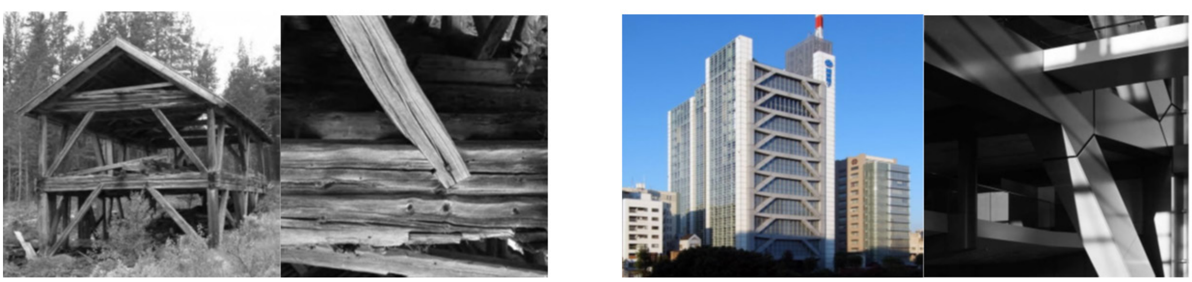

Fig. 3: Left: An old wooden building in Norway; Right: The Tokyo Skytree, a metal structure designed by Norman Foster (Sandaker, 2010); both wooden and steel materials possess the ability to withstand tensile and compressive stresses as well as shear and bending stresses. Despite the similarity in the overall geometry of the structures, their local geometries differ due to variations in material types and construction technologies.

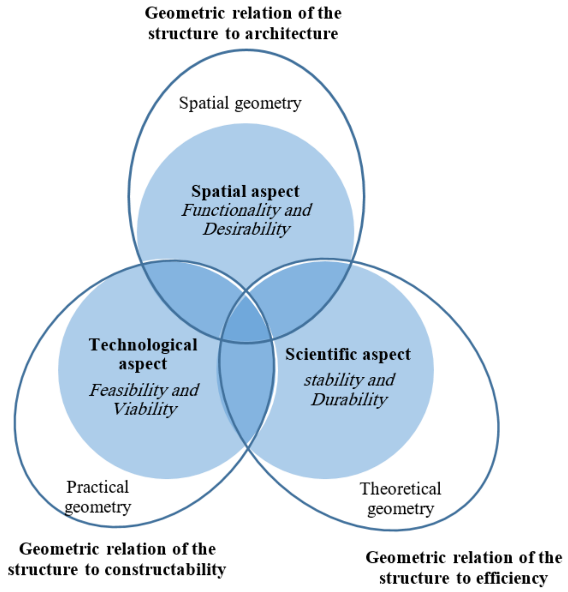

Fig. 4: Framework of the correlation of geometric cognition on aspects of structural design

at finding the best solution for the structural performance requirements, according to computational and mathematical constraints (Mueller et al., 2013). Therefore, in analysis using computer tools for structural optimization, the objective function is mathematically formulated, and parameters are defined numerically. In the literature, the fields of optimization for building structures are categorized into three types of optimisations: "shape, topology, and scale (dimensions)" (Kingman et al., 2014). Because many requirements and objectives, including visual impacts, spatial experience, and overall architectural values, are qualitative or even subjective, applying numerical calculations in the design of architectural (spatial) aspects can be difficult or impossible (Mueller et al., 2013).

Aspects of Numerical Cognition of Structures

An optimized structure is defined based on a set of requirements (constraints, including acceptable deformations) and objectives, which may include reducing materials and the weight of the structure for material savings, decreasing deformation and dynamic vibrations to ensure structural safety, and increasing the stiffness of the structure to enhance efficiency (Beghini et al., 2014). Optimization problems in various fields can be solved using two specific methods: the "global method and local method." Local methods lead to local optimization and, therefore, typically do not guarantee finding a globally optimal form. The basis of most local methods is approximate calculations. On the other hand, the most popular global methods are often based on mimicking observable phenomena in nature. These methods approach problem-solving through trial and error. Despite being designed to find overall optimization, there is no systematic guarantee of finding the optimal solution. Numerical calculations analyze the effectiveness of these methods in specific applications (Adriaenssens et al., 2014).

A crucial step in optimization is determining the overall shape ratio of the structure concerning the dimensions of various structural elements (Mueller et al., 2013). As shown in Fig. 5, in scale (or dimensional) optimization, variables are applied to the cross-sectional dimensions or their thickness (in this case, geometry and topology remain constant). For example, in structural design, constraints related to construction technology may be considered regarding the size of structural members. Other manufacturing constraints in optimization include casting, extrusion, and machining (Beghini et al., 2014). Optimization can also lead to increased geometric efficiency of the structure. In shape optimization, variables are applied to the joints and nodes of the structure, as well as the lines, curvatures, and surfaces forming the geometry of the structure (without changing topology). In practice, node coordinates are used as parameters for geometric modification. Topology optimization has developed as a result of rapid advancements in mathematical methods and the increasing computational power of modern computers. Topology optimization is defined by the composition (presence or absence) of structural elements and materials (Adriaenssens et al., 2014). Topology spatially distributes specific properties of the structure, including density, stress, and strain (Consolini et al., 2010). While traditional geometric problem-solving remains important in engineering (Torres-Peña et al., 2025), the integration of computational techniques like topology optimization (Li et al., 2023) allows for more sophisticated and optimized structural designs.

The outcome of this type of optimization can be a reduction in material consumption in a structure, making it a tool in specific projects for bridging architecture and structure to produce design options. Shape optimisation often includes optimising the dimensions of cross-sections as well. Additionally, topology optimization may also influence both

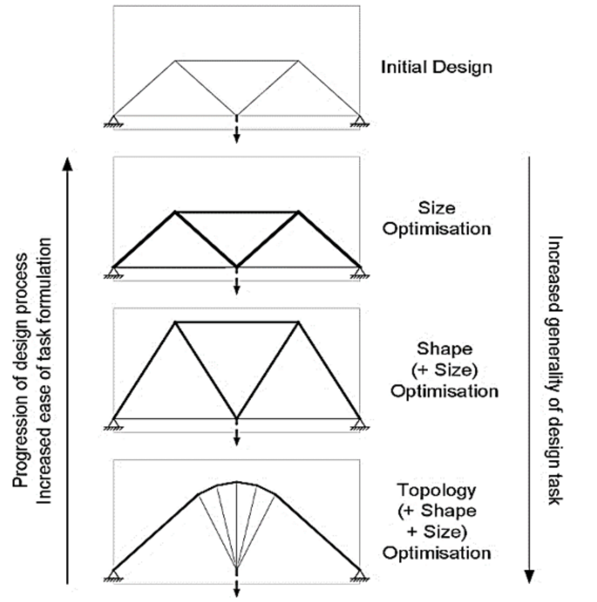

Fig. 5: Optimisation of a simple support truss structure with symmetrical concentrated load (Source: Baldock, 2007)

scale and shape optimization (Baldock, 2007). Therefore, topology optimization is the most general type among these three categories, examining information regarding the number, size, and shape of spans. Fig.6 shows the framework of the correlation of geometric

The Ratio of Geometric Cognition and Numerical Cognition of Structures in the Design Process

Structural design primarily involves non-numerical aspects (structural layout design) and numerical aspects (optimization) (Luyten, 2010). Herbert also describes the structural design process as a spectrum from form creation to numerical solutions (Herbert et al., 2013). Therefore, the structural design process can be divided into two main steps: first, there is an initial design stage where the overall form and arrangement of the structure are conceived, which is commonly referred to as "design"; then, in the second step, structural calculations and the determination of the dimensions of various construction elements and connections are carried out, which is referred to as "analysis"

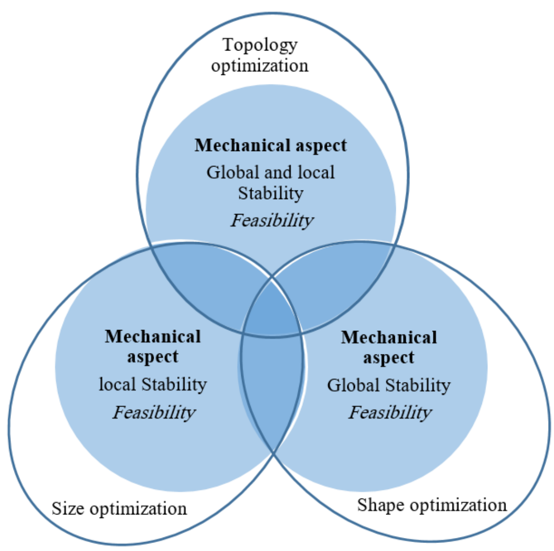

Fig. 6: Framework of the correlation of numerical cognition on aspects of structural design

(Mohammadi, 2012; Macdonald, 2019). It is essential to understand that analysis is not the same as design. Unfortunately, the term "design" is mistakenly applied in processes such as calculating steel beams or reinforcing rods in concrete columns. While design typically relates to producing a suitable overall geometry for the structure (Larsen et al., 2003).

Key aspects emphasized in definitions of design include purposefulness, a focus on creating form, structure, and meaningful order in objects, and attention to aesthetic dimensions while addressing functional, economic, and social aspects. Nadimi, in expanding the concept of engineering, considers the term geometry as synonymous with design: "Familiar interpretations and attributes such as the fitting of members to their environment, balance and neatness, harmony of components, order of spacing and proportions in the final composition, bring geometry closer to the concept of quality and beauty" (Nadimi, 2012).

On the other hand, all numerical methods have proven their importance over the years and have enabled designers to address the logical and precise behavior of structures. Numerical methods mainly focus on the mechanical aspects of structures; however, the need for extensive and complex structural information, the ambiguous status of architectural qualities in the design process, and the time-consuming and complicated stages—especially in specific structures—can be considered shortcomings of numerical methods. Although Larambeber had a deep interest in mathematical analysis, he realized that complex calculations were only useful for assessing the stability of structures (Lee et al., 2009). Furthermore, despite Gaudí having advanced studies in structural mechanics, he believed that numerical calculations were not essential for structural design. Instead, he favored creating models geometrically, which later became common among shell builders (Alpana, 2007). Luyten, as a structural engineer, also believes that in the early stages of the design process, there is no need for detailed precision and elaboration of many scientific concepts of structure (Luyten, 2012). A general understanding of structural proposals is often sufficient. Being overly precise slows down the design process, not merely due to the creation or transfer of various diagrams, but because of the overwhelming amount of information. An architect should understand the structure without the need to use quantitative concepts (especially in the conceptual and initial stages of the design process) (Ilkovic, 2014).

In contrast to geometric design, which is a divergent process proposing solutions to problems (the synthesis stage), selecting the optimal solution is a convergent process (the evaluation stage) (Luyten, 2012) and cannot be managed without numerical understanding and its limitations. With this perspective on design processes, for example, the creative process view, which divides into initial reception, preparation, incubation, illumination, and verification (Lawson, 2008), one can assert that the greatest seed of the geometric design of structures is planted in the first four stages, which are specific to structural design, while numerical analysis plays a role in the "verification" stage. Moreover, at the beginning of the design process, the geometry of the structure is defined. Initially, the overall geometry and then the local geometry are examined in terms of mechanical optimisation (Pedron, 2006). Therefore, the design stage is predominantly conducted through geometric understanding of the structure, while the analysis stage recognises the structure through numerical calculations. In the geometric design stage, the role of spatial aspects (i.e., the relationship between the structure's geometry and architectural space) acts as "internal constraints" (Lawson, 2008) in determining the overall form of the structure. Internal constraints provide greater freedom to the designer and constitute the basis of the design and a significant part of the design program. In contrast, "external constraints," such as optimising dimensions and connections against seismic forces or topology optimisation for material savings, are often applied during the analysis stage and have less flexibility in design.

In this regard, contemporary architecture and engineering extensively utilise various design and analytical tools, including computers, throughout the design process. Available tools include geometric design patterns, which serve as the form-giving aspect of the structure, and numerical analysis, which evaluates the structure's form. However, most available tools lack the necessary capabilities to utilize this potential: modeling tools consider geometry in the absence of numerical understanding, and structural analysis tools require predefined geometric forms (Mueller et al., 2013). The optimal combination of

Table 2: Comparison of geometric and numerical cognitions in the design of structural aspects

Cognitive aspects of structures	Geometric cognition			Numerical cognition		
	Scientific aspect	**Technological aspect**	**Spatial aspect**	**Mechanical aspect**		
	Theoretical geometry	Practical geometry	Spatial geometry	Shape optimization	Scale optimization	Topology optimization
Application in design	Structural efficiency	Constructability of the structure	Architectural quality of the structure	Determining outer lines, curvature, and surfaces	Precise determination of dimensions and thicknesses of structural elements (improved sections)	Determining the composition of structural elements and materials
Global and local limits	Global and Local geometry	Mainly local geometry	Mainly global geometry	Global optimization	Local optimization	Global and Local optimisation
Position in the design process	Form giving (Design stage)			Form evaluation (Analysis stage)		

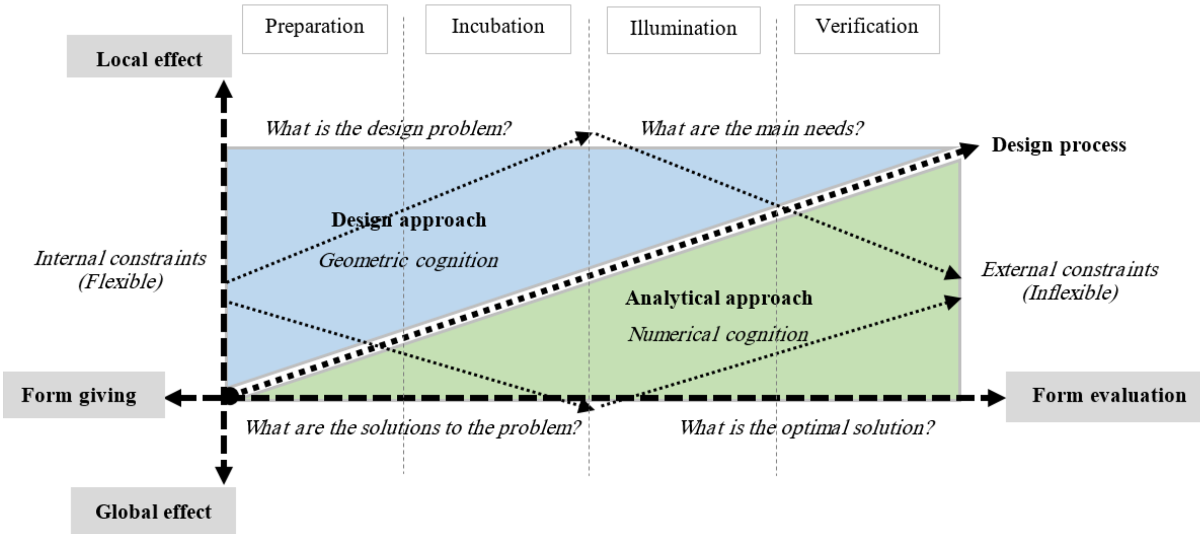

Fig.7: Diagram of the ratio between geometric and numerical cognitions of structures in the design process

these understandings offers a solution to overcome these weaknesses in structural design.

Table 2 and Fig. 7 show the comparison and also the ratio between geometric and numerical cognitions of the structure in the design process.

CONCLUSION

This research aimed to clarify the relationship between geometric cognition and numerical cognition of structural knowledge in the design process. A comparative investigation was employed in this regard. In the analytical perspective based on numerical understanding, which reflects the governing laws of force transmission, questions arise regarding engineering concepts including force, stress, strain, material resistance, and ultimately the mathematical formulas for problem-solving. In contrast, the questions from the design perspective based on geometric cognition focus on the overall form of the structure, the relationship between structural design and architectural design, and how to combine structural components. Valuable results can be extracted from this investigation in two domains:

Aspects of Geometric and Numerical Cognitions of Structures

By exploring the characteristics of existing structures and analysing the thoughts of experts, three important aspects were identified for aligning geometric and numerical cognitions of structural knowledge in the design process:

- **Scientific Aspect (Structural Logic):** This aspect examines the relationship between structures and concepts related to force. Those who have not reached this level of expertise in the design process do not fully possess the most explicit cognitive aspect of structures. Understanding this aspect requires both types of geometric cognition (i.e., theoretical geometry) and numerical cognition of structures, as well as proficiency in them. The broader a designer's technical knowledge of structures, the greater their capability in both geometric and numerical cognitions of the scientific aspect.

- **Technological Aspect (Constructability of Structures):** This aspect addresses the relationship between structures and the construction process. Sometimes, understanding the construction method can reveal the story behind the idea embodied in the structure's design form. In this realm, understanding the geometry of structure plays a crucial role. This aspect primarily affects the local geometry of the structure.

- **Spatial Aspect (Quality Enhancement of Structures):** By examining the relationship between structures and architecture, structures shift from the mechanical domain to the spatial domain. Enhancing the quality of a structure goes beyond mechanical understanding. In this domain, numerical (and quantitative) criteria take a back seat, while qualitative criteria take precedence. Optimising dimensions or topology does not determine the true value of the spatial aspect of a structure; therefore, geometric cognition of structures takes the forefront. The geometric cognition of the spatial aspect of a structure involves understanding how to integrate structural design with architectural design.

Levels of Geometric and Numerical Cognitions of Structures

By examining the relationship between geometric understanding and numerical understanding of structures in the context of architectural design, two fundamental levels of structural understanding can be identified:

- **Form-Giving Level:** While technical ability and familiarity with structural analysis are important, the significance of the design language for generating the structure's form cannot be overlooked before starting mathematical analysis. The contribution of geometric cognition to structural design is form-giving. At this level, the breadth of designers' geometric cognition is directly related to their understanding of the spatial and mechanical aspects of structures. In the process of form-giving, the form of the structure is primarily determined by the "internal constraints" of the spatial and mechanical aspects. Consequently, a crucial step in understanding structural form-giving is gaining a geometric understanding of structures and the relationships between the structure's geometry and architectural form.

- Form-Evaluation Level: The measurement of structure is directly related to the concept of optimisation, which is identified through numerical cognition and analytical language. This process is often influenced by "external constraints," particularly the mechanical aspects of the structure; therefore, it mainly involves numerical cognition of structures. It is clear that without the stage of structural analysis, the final design will not materialise, but its position can never be at the level of form-giving of the structure and the conceptual stages of the design process.

It was observed that, based on the type of relationship between structures and architecture, the relationship between geometric cognition and numerical cognition of structures varies. Structural design is crucial for architects and designers. This does not mean that the numerical optimisation stage is unimportant; rather, creating inappropriate details often leads to the failure of many structures. Both types of cognitions have their values in the design process, but part of the structural understanding has the capability and merit to be present in the step of "structural design," which involves everything from the initial stages of form generation to the actual construction phase. This highlights the important and valuable role of geometry, which has often been overlooked amid numerical difficulties and the expansion of design and analytical tools in the field of structural engineering.

AUTHOR CONTRIBUTIONS

M. Ahmadnejad Karimi designed the study, performed the literature review, collected initial data, and analysed and interpreted the data. M. Ahmadnejad Karimi categorised and analysed the research data. M. Mahmoudi Kamelabad and M. Azimi managed the research literature and continuously reviewed and revised the research results.

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CONFLICT OF INTERESt

The authors declare no potential conflict of interest regarding the publication of this work. In addition, the authors have witnessed ethical issues, including plagiarism, informed consent, misconduct, data fabrication or falsification, double publication and submission, and redundancy.

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