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Original Research

The Effectiveness of Combining Empirical Decomposition Mode and Machine Learning Tools on Bitcoin Volatility Prediction

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ABSTRACT

This study explores whether combining Empirical Mode Decomposition (EMD) with machine learning models Artificial Neural Networks (ANN), Recurrent Neural Networks (RNN), and Long Short-Term Memory (LSTM)—can improve the accuracy of Bitcoin price volatility (VBTC) predictions. Utilizing daily Bitcoin price data from September 2011 to December 2024, the research, conducted using R software, compares the performance of hybrid models (EMD-ANN, EMD-RNN, EMD-LSTM) against standalone machine learning models and traditional time series methods like ARIMA. The results demonstrate that hybrid models significantly outperform their non-hybrid counterparts, with the EMD-RNN model achieving the highest accuracy, reducing Mean Absolute Error (MAE) by 95.76% and Root Mean Squared Error (RMSE) by 96.35%. The decomposition of VBTC into Intrinsic Mode Functions (IMFs) revealed distinct short-term and long-term volatility components, providing deeper insights into market behavior. The findings highlight the superiority of integrating EMD with machine learning for volatility forecasting, offering enhanced predictive accuracy and robustness. This research underscores the potential of advanced analytical techniques in improving risk management and investment strategies in highly volatile cryptocurrency markets.

1 Introduction

In financial economics, volatility is commonly understood as the irregular variations in return movements within financial markets over specific periods, such as daily, monthly, or yearly intervals [3, 19, 24]. This measure serves as an indicator of widespread uncertainty in financial markets, playing a central role in areas such as sentiment analysis, derivative pricing, risk management, and portfolio

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optimization [45, 46]. As a fundamental trait of financial time series, accurately estimating and predicting volatility is crucial for financial and economic studies [11]. Building a reliable volatility forecasting model entails identifying patterns or interactions within the variable's values over time while maintaining robustness and general applicability [4]. Despite their advantages in modeling nonlinear processes, machine learning approaches have limitations and are far from flawless. They often face challenges in detecting long-term dependencies in data and are prone to overfitting, particularly with highly noisy or large datasets [54]. Consequently, there is a pressing need for advanced modeling methods capable of accounting for both the data's nonlinear characteristics and its long-term dependencies. Decomposition techniques address this by isolating latent components in series and incorporating the effects of changing market conditions and economic environments into forecasting [12, 48]. Combining (1) sophisticated decomposition methods with (2) machine learning algorithms establishes a hybrid modeling strategy that enables a comprehensive analysis of stock and cryptocurrency market dynamics while substantially enhancing prediction accuracy [6, 18, 29]. This study introduces a novel approach by integrating Empirical Mode Decomposition (EMD) with advanced machine learning models—Artificial Neural Networks (ANN), Recurrent Neural Networks (RNN), and Long Short-Term Memory (LSTM)—to enhance Bitcoin volatility forecasting. This hybrid methodology leverages the strengths of both decomposition techniques and deep learning models to improve predictive accuracy and uncover latent volatility components.Given the oscillatory and noisy characteristics of volatility series, decomposition tools can help uncover latent components and account for the impacts of fluctuations in market conditions and economic environments when forecasting these series. Consequently, integrating decomposition tools with machine learning methods in a hybrid modeling approach has the potential to significantly enhance prediction accuracy. In this study, however, we primarily focus on applying machine learning techniques-namely ANN, RNN, and LSTM-to forecast the volatility of the Bitcoin cryptocurrency. Additionally, to assess the advantages offered by decomposition tools, empirical mode decomposition (EMD) will be combined with machine learning techniques to predict Bitcoin cryptocurrency volatility. Thus, the research intends to evaluate and compare model performance within both hybrid and non-hybrid frameworks. Ultimately, optimal models for volatility prediction will be proposed based on the findings. The article proceeds by discussing the theoretical framework and reviewing related literature, followed by outlining the methodology, presenting the findings, and concluding with the implications, limitations, and recommendations derived from the study.

2 Theoretical Fundamentals and Research Background

Currently, research on volatility prediction primarily focuses on two main approaches: stochastic methods and data-driven methods [2, 14, 26, 29]:

Stochastic methods, such as autoregressive conditional heteroskedasticity (ARCH) models and their enhanced variants, including generalized autoregressive conditional heteroskedasticity (GARCH) and exponential autoregressive conditional heteroskedasticity (EGARCH) models, have been extensively applied to forecast volatility [1, 49]. These econometric models are widely recognized for their solid statistical underpinnings and their ability to capture linear patterns in time series data [19, 21, 31]. They demonstrate strong performance in handling stable and linear datasets [44]. However, the primary drawback of econometric models lies in their reliance on linearity assumptions, rendering them ineffective at addressing the inherently nonlinear and dynamic nature of volatility [4, 20]. Put simply, traditional econometric models struggle to adequately handle the nonlinear and dynamic aspects of financial market volatility, which is frequently shaped by unexpected economic events [33]. To address the shortcomings of these traditional models, particularly in dealing with nonlinear and complex datasets, data-driven techniques such as machine learning-an integral branch of artificial intelligence-have emerged as powerful alternatives and are gaining increasing traction [42, 43].

Data-driven methods, which derive patterns and relationships directly from data, can be seen as an overarching extension of stochastic techniques. These methods encompass a broad range of approaches, including least absolute shrinkage and selection operator (LASSO), support vector regression (SVR), artificial neural networks (ANN), recurrent neural networks (RNN), long short-term memory (LSTM), gated recurrent units (GRU), bidirectional gated recurrent units (BiGRU), among others [7, 32, 38, 43]. Their ability to accurately detect nonlinear relationships within datasets, paired with a flexible structure that facilitates adaptive learning and adjustments, allows for more dynamic and adaptable modeling of various components in financial markets [17, 37]. These attributes enable them to deliver superior performance in forecasting market volatility [8, 22].Advanced techniques for time series decomposition include Wavelet Analysis (WA), Empirical Mode Decomposition (EMD), and the Hilbert-Huang Transform (HHT), each offering distinct features and applications in data analysis. Wavelet Analysis utilizes wavelets as mathematical bases to decompose signals within the time-frequency domain, proving highly effective in applications such as signal processing, data compression, and noise reduction, due to its ability to emphasize local signal features. One key advantage of Wavelet Analysis is its capacity to perform multiscale signal decomposition simultaneously, though the complexity of selecting the appropriate mother wavelet can be a challenge. Empirical Mode Decomposition, on the other hand, is an entirely empirical method that decomposes signals into Intrinsic Mode Functions (IMFs) without relying on predefined mathematical bases. This approach excels in analyzing non-stationary and nonlinear data but is notably susceptible to noise. The Hilbert-Huang Transform combines EMD with the Hilbert transform, deriving localized time-frequency representations by applying the Hilbert transform to IMFs. While highly proficient in analyzing intricate data and uncovering hidden structures in signals, the Hilbert-Huang Transform may experience reduced performance in noisy conditions. In summary, Wavelet Analysis is ideal for multiscale analysis, Empirical Mode Decomposition for studying non-stationary data, and the Hilbert-Huang Transform for precise analysis and identifying concealed patterns within data [8, 9, 15, 16, 34].

Lin et al. (2012) introduced a hybrid forecasting model that combines Empirical Mode Decomposition (EMD) with Least Squares Support Vector Regression (LSSVR) to address the nonlinear and non-stationary characteristics of financial time series such as exchange rates. In this approach, EMD is used to break down exchange rate dynamics into multiple Intrinsic Mode Functions (IMFs) and a residual component. Subsequently, LSSVR is employed to separately forecast these IMFs and the residual value, and the aggregated predictions result in the final estimated exchange rate. Their findings demonstrate that the EMD-LSSVR hybrid model achieves superior performance compared to EMD-ARIMA (Autoregressive Integrated Moving Average) models, as well as standalone LSSVR and ARIMA models without time series decomposition [27]. Zhang et al. (2017) highlighted that time series data often comprises various elements, including trends, seasonality, and jumps, each governed by distinct coefficients in the data-generating process. Utilizing a unified time series model for aggregated data can be both resource-intensive and less accurate. By calculating component-specific coefficients through multiresolution wavelet analysis, they showed that forecasting accuracy improves for aggregated data, as this method alleviates the constraint of applying identical coefficients across all data components [53].

Risse (2019) utilized a combination of discrete wavelet transform and support vector regression to forecast gold price dynamics, demonstrating that this approach outperforms other forecasting methods.

His findings highlighted that the influence of short-term and long-term trends varies over the evaluation period [36]. Lin et al. (2020) introduced a novel hybrid model for forecasting crude oil prices, accounting for factors such as long-term memory, asymmetry, heavy-tailed distributions, nonlinearity, and non-stationarity. Empirical evidence revealed that the WPD–EMD–ARMA–FIGARCH-M hybrid model performs particularly well during periods of significant events. Stability assessments confirmed that this model surpasses traditional forecasting techniques [28]. Zaj et al. (2022) aimed to forecast Bitcoin prices using the Grey model, the artificial neural network with backpropagation, and the integrated Grey neural network model. Their findings revealed that for daily Bitcoin price estimation, the artificial neural network with backpropagation exhibited the lowest absolute error (5.6%) compared to the Grey model and the integrated model. Additionally, for monthly Bitcoin price forecasting, the integrated model outperformed the other two, achieving the lowest absolute error (9%) [50]. Tavakoli et al. (2022) utilized the Kullback-Leibler, Lin-Wang, and triangular information metrics, as loss functions to evaluate predictive performance. Their findings indicate that the capsule network, when employing the triangular information criterion, effectively forecasts Bitcoin prices over medium- and long-term periods of 10, 90, and 180 days. As a result, its prediction accuracy reaches 94% for long-term forecastsand 64% for medium-term predictions [41].

Zhang et al. (2024) combined wavelet, ARIMA, and LSTM methodologies to develop a hybrid model that showed superior performance in forecasting the stock price index compared to other models. Their findings indicated that LSTM excels in handling noisy residual data, while ARIMA is more effective for less noisy signals. A trading strategy derived from these forecasts demonstrated robust returns under diverse market conditions, particularly in managing risk effectively. Additionally, they examined the effect of the COVID-19 pandemic on forecasting performance, concluding that the model effectively adapts to varying data structures, achieving improved accuracy [52]. Souropanis and Vivian (2023) applied wavelet analysis to forecast S&P500 index volatility, finding that technical indicators excel in predicting short-frequency components, complemented by macroeconomic variables for longer frequencies. They emphasized that incorporating methods addressing the frequency dimension significantly enhances prediction accuracy [39]. Zhao et al. (2023) forecasted crude oil inventory using a hybrid model combining wavelet, ARDL, and SVR, observing that inventory's buffering effect is more evident in the long term, while speculative effects, particularly on the supply side, accumulate in the short term and increase market risks. This approach demonstrated improved forecasting accuracy, achieving a 19% enhancement over institutional predictions [56]. Lastly, Dezhkam and Manzouri (2023) proposed the HHT-XGB model, integrating the Hilbert-Huang Transform (HHT) for feature engineering and the XGBoost method as a price trend classifier. This hybrid approach improved portfolio performance by 99.8% compared to models using unprocessed financial data. Backtesting results revealed that the HHT-XGB strategy consistently outperforms benchmark approaches, even in challenging market conditions [10]. Koosha et al. (2023) proposed an ensemble learning approach to improve the accuracy of predicting Bitcoin's peak and bottom prices by integrating multiple machine learning models. Their findings demonstrate that the ensemble learning model significantly outperforms individual machine learning models, offering superior accuracy and reliability in predicting Bitcoin price fluctuations [23].

Recognizing diesel's essential role in mitigating carbon emissions, Yan et al. (2024) developed a hybrid learning model based on time-frequency interval decomposition (TFIDE) to forecast diesel prices and evaluate the nonlinear effects of global low-carbon development trends on these prices. The model integrates three components: two-dimensional empirical mode decomposition (BEMD), interval multilayer perceptron (IMLP) networks, and threshold autoregressive interval (TARI) models. Initially, BEMD decomposes weekly diesel price intervals into multiple intricate intrinsic mode functions (IMFs) and a residual component. Subsequently, the IMLP model forecasts the IMFs, while the TARI model predicts the residual values using carbon reduction technology indicators and carbon emission considerations. Finally, the aggregated predictions yield the final interval forecasting outcomes for diesel prices. Experimental results reveal that carbon reduction technologies enhance mid-frequency IMF predictions, while carbon emission concerns exert nonlinear influences on long-term diesel price intervals. Moreover, the TFIDE approach demonstrates superior performance compared to alternative methods across various accuracy metrics [47]. Tan et al. (2024) introduced a multiscale time series decomposition learning framework aimed at forecasting crude oil prices. Their approach begins with the development of a multiscale time processing module designed to detect time series patterns across various frequencies in historical data at different scales. This is followed by the application of a multiscale decomposition method, which separates historical crude oil data into distinct temporal modes-comprising global shared information across multiple scales and localized details that vary at each scale. Finally, a multiscale aggregation mechanism integrates these components, providing inputs for constructing nonlinear and complex crude oil price forecasting models. Reviews conducted within the Shanghai crude oil market indicate that this method outperforms several econometric and machine learning models [40].

Based on the aforementioned points, the aim of this research is to utilize machine learning tools (including ANN, RNN, and LSTM) to predict Bitcoin cryptocurrency volatility. Additionally, the study employs a combination of the empirical mode decomposition method and machine learning tools for Bitcoin volatility forecasting. Finally, To assess the effectiveness of this combined approach, the results of the two methodologies are compared.

3 Methodology

The primary data used in this study is daily data, extracted from the investment website (*www.investing.com/crypto/bitcoin/*), spanning from the first working day of September 2011 to the last working day of December 2024. After gathering the data, the daily percentage changes in Bitcoin prices were calculated to determine its daily returns. Following Wang et al. (2024), the natural logarithm of the sum of the squared daily returns on each working day of the month was used as the measure of Bitcoin's volatility (VBTC) for that month [43]. Here, D represents the number of working days in each month, and Ret represents the daily returns of Bitcoin.

$$VBTC_t = Ln\left\{\sum_{i=1}^{D} Ret_i^2\right\} \tag{1}$$

The R software was employed to apply empirical mode decomposition and machine learning tools, including ANN, RNN, and LSTM. Notably, following Wang et al. (2024) and Chen et al. (2025), to eliminate the impact of dimensions, volatility values were normalized using the relationship $(X - X_{min})/(X_{max} - X_{min})$, where X_{min} and X_{max} represent the minimum and maximum values of each variable, respectively [8, 43]. Subsequently, the normalized series were first predicted out-of-sample solely using machine learning tools. Then, the normalized data were decomposed into intrinsic mode functions using empirical mode decomposition. These intrinsic mode functions were then predicted out-of-sample using machine learning tools, and through inverse decomposition, the predicted primary components were recombined to produce the main predicted series. The machine learning models were configured using a standardized architecture to ensure consistency in evaluation. Each model consists of two layers with six neurons per layer, maintaining a balance between complexity and computational efficiency. The learning rate was set at 0.001, allowing the models to converge effectively without excessive fluctuations. Training was conducted over 1000 epochs using the Adam optimization algorithm, which facilitates adaptive learning rate adjustments. The SeLU activation function was employed to enhance stability in neuron activations, ensuring proper gradient flow throughout the network. A batch size of 16 was used to optimize training efficiency while preserving generalizability. Additionally, the loss function was chosen to accurately capture prediction errors and enhance model robustness. The models were trained using data from September 2011 to April 2022 (128 observations), and the subsequent 32 months (from May 2022 to December 2024) were predicted. Comparing the results of non-decomposed and decomposed approaches can provide evidence of the usefulness of employing decomposition tools. Additionally, for comparison purposes, forecasts were also conducted using the time series regression model. In this regard, the results of the Augmented Dickey-Fuller (-5.5752) and Phillips-Perron (-8.4749) tests indicate the stationarity of the Bitcoin volatility series at the level of data. The attainment of the minimum values for the Akaike (2.8522), Schwarz (2.9281), and Hannan-Quinn (2.8830) information criteria in the second lag of the Bitcoin series led us to select the ARIMA(2,0,0) time series regression model as the baseline model. Each tool is briefly described below.

3.1 Machine Learning Tools (ANN, RNN and LSTM)

Artificial neural networks, designed as computer algorithms that mimic the biological functions of the human brain, are capable of learning, storing, and generating outputs. They are widely utilized in areas such as financial risk management and forecasting [5]. These networks excel at identifying solutions in multidimensional and nonlinear datasets, owing to their self-adaptive, self-learning, and error-correcting features. Typically, an artificial neural network consists of three layers: an input layer, hidden layers, and an output layer. Information from the external environment is received by the neurons in the input layer and passed to the hidden layer, where it is processed. The processed data is then sent to the output layer. Neural networks can take on various configurations by modifying the number of neurons, the structure of hidden layers, connection weights, learning algorithms, and activation functions. The optimal number of hidden layers is typically determined experimentally, with the network exhibiting the least error being selected [13]. Key applications of artificial neural networks include image recognition, natural language processing, and financial market forecasting. Their capacity to learn from data and enhance performance through experience positions them as powerful tools in data science and analytics. Figure 1 illustrates the structure of an artificial neural network.

Neural networks can adopt various configurations by adjusting elements such as the number of neurons, the architecture of hidden layers, connection weights, learning methods, and activation functions. Empirical testing is used to identify the optimal number of hidden layers, with the selection criteria based on achieving the highest coefficient of determination and minimizing error. Artificial neural networks lack feedback between layers and the ability to retain historical information. In contrast, recurrent neural networks (RNNs) are specifically designed for sequential data processing, utilizing internal memory to integrate both past and present inputs. This capability makes them highly effective for handling financial data with time-based correlations [43]. This capability allows RNNs to identify temporal and sequential patterns more efficiently, making them ideal for applications such as speech recognition, machine translation, and time series forecasting. Despite their advantages, RNNs can encounter difficulties with information retention over extended sequences, a challenge that advanced models like LSTM help address.

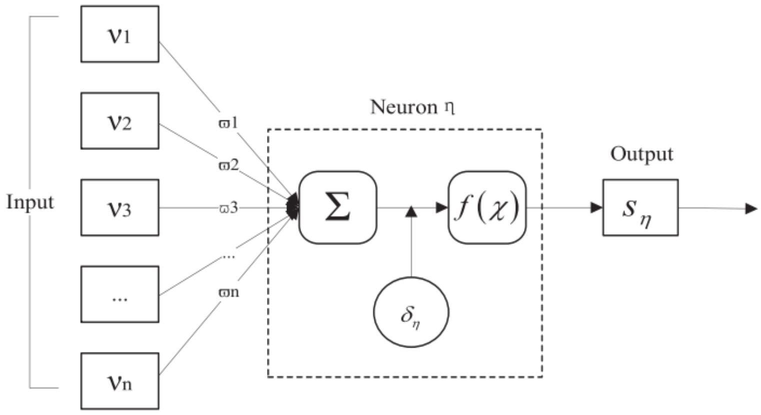

Fig. 1: An overview of an artificial neural network (ANN) – (Source: [43])

Figure 2 illustrates the structure of a recurrent neural network.

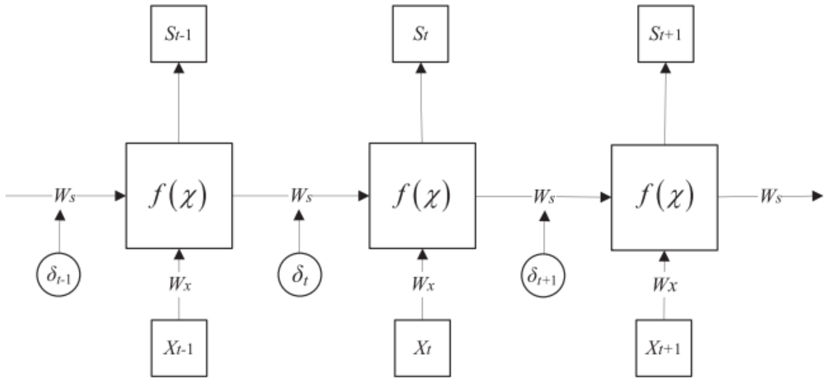

Fig. 2: An overview of an recurrent neural network (RNN) – (Source: [43])

One of the drawbacks of simple recurrent neural networks (RNNs) is gradient disappearance and gradient explosion problems, which makes working with time series data that have long-term correlations challenging. In such situations, Long Short-Term Memory (LSTM) networks, with the inclusion of a forget gate on the simple recurrent neural network, can effectively resolve this issue [29]. Figure 3 provides an overview of a Long Short-Term Memory network:

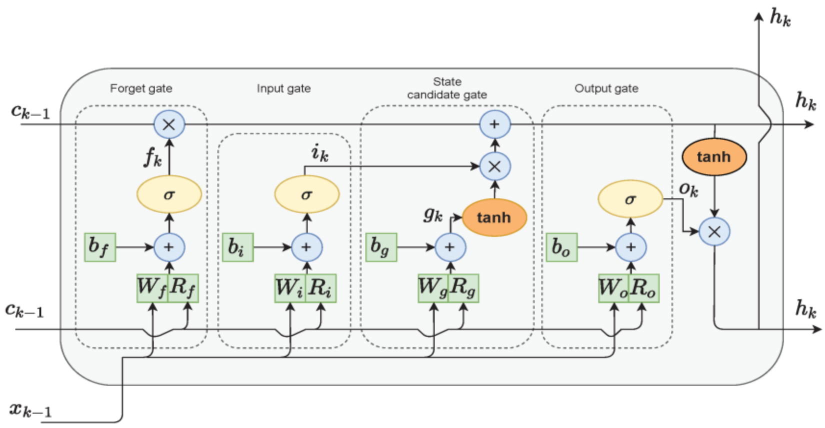

Fig. 3: An overview of a Long short-term memory network (LSTM) - (Source: [51])

In other words, Long Short-Term Memory networks are an advanced type of recurrent neural network (RNN) designed to address common issues in long-term learning with sequential data. LSTMs use memory cells and gating mechanisms such as input, output, and forget gates, which allow them to selectively retain or forget information. This feature enables LSTMs to effectively handle problems such as gradient instability and information loss in long sequences. The main applications of LSTMs include natural language processing, machine translation, speech recognition, and time series analysis. Due to their strong ability to learn long-term relationships, LSTMs have become one of the key tools in the field of deep learning [25].

3.2 Empirical Decomposition Mode (EMD)

Empirical Mode Decomposition (EMD) is a signal analysis method that decomposes complex and non-stationary signals into a set of intrinsic mode functions (IMFs). Each intrinsic mode function uniquely contains a specific oscillation at different time scales. This method was introduced by Norbert Huang in 1998 and is highly efficient for analyzing data with temporal variations and nonlinear oscillations [16]. Important applications of EMD include signal processing, biological data analysis, and mechanical vibration analysis. In signal processing, EMD is used for pattern recognition, noise removal, and analysis of complex data. In biological data analysis, this method is employed to analyze signals, helping identify anomalies and hidden patterns. In mechanical vibration analysis, EMD is used to analyze vibrations in various structures, such as bridges and buildings, aiding in the detection of failures and weaknesses. A notable feature of EMD is that it does not require the pre-selection of mathematical bases, but instead empirically identifies and extracts components. EMD's applications have expanded into various fields, including signal processing, biological data analysis, mechanical vibrations, and even finance and economics. This method is particularly suitable for analyzing complex and combined data with varying temporal oscillations, providing high accuracy and interpretability [34, 35]. Empirical Mode Decomposition identifies and extracts components empirically without the need for initial assumptions or predefined mathematical bases. This makes EMD a powerful tool for signal analysis in fields such as signal processing, biological data analysis, and mechanical vibration analysis [16]. However, due to its high sensitivity to noise, EMD sometimes does not provide stable and accurate results. Figure 4 provides an overview of what EMD accomplishes.

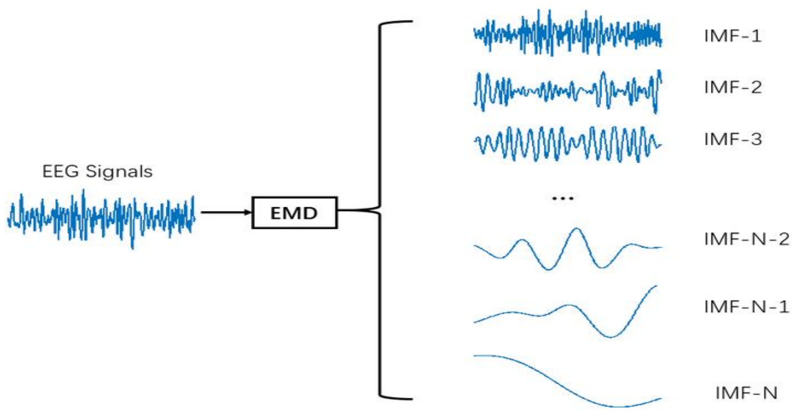

Fig. 4: An overview of empirical mode decomposition (EMD) – (Source: [55])

As previously stated, EMD decomposes complex signals into simpler components called Intrinsic Mode Functions (IMFs). In this method, all local minimum and maximum points of the signal $x(t)$ are first identified. Using these points and applying an interpolation method (such as Spline), the upper envelope $u(t)$, the lower envelope $l(t)$, and the mean of these two envelopes $m(t)$ are created for the original signal. By subtracting this mean from the original signal, the initial IMF component is obtained ($h(t) = x(t) - m(t)$). At this stage, the number of times the signal crosses the zero axis and the number of local maxima and minima should be equal or differ by one. If this is not the case, the previous steps are repeated until this condition (which is the stopping criterion for the process) is met. After extracting one intrinsic component, its value is subtracted from the original signal ($r(t) = x(t) - h(t)$) to obtain the residual signal $r(t)$. The previous steps are then repeated until $r(t)$ becomes a constant or monotonic function. Finally, the original signal $x(t)$ can be expressed as the sum of the IMFs and the residual term $r_n(t)$ [55]:

$$x(t) = \sum_{i=1}^{n} IMF_i(t) + r_n(t) \tag{2}$$

3.3 Combining EMD and ML

Combining machine learning tools and Empirical Mode Decomposition (EMD) offers numerous advantages that are highly effective in analyzing complex and nonlinear data. EMD, as a signal analysis method, has unique capabilities in isolating intrinsic components and analyzing temporal oscillations. These features make EMD highly effective in identifying and separating noise and hidden patterns within data. On the other hand, machine learning, with its ability to learn from data and predict complex patterns, can lead to more precise analyses and better predictions [6]. Combining these two tools can offer benefits such as improved model accuracy, enhanced data interpretability, and increased data processing efficiency. For instance, EMD can help remove noise and identify key components, thereby improving the quality of data fed into machine learning models. This noise removal process ensures that the machine learning models are trained on cleaner and more relevant data, leading to better performance and more reliable results. Additionally, using EMD in conjunction with machine learning algorithms can help detect hidden and complex patterns in time-series data, enabling more accurate and reliable predictions. Time-series data often contain intricate and overlapping patterns that can be challenging to discern. EMD's ability to decompose these data into simpler intrinsic mode functions (IMFs) allows machine learning models to focus on the most relevant features, thereby enhancing their predictive accuracy [30].

4 Findings

4.1 Year-by-year mean of Log(BTC) and VBTC and Unit Root Test

The results of Figure 5 and Table 1 indicate that the logarithmic value of Bitcoin prices (Log(BTC)) shows a general upward trend from 2011 to 2024, with significant price increases observed in 2013 (127.58%) and 2017 (22.55%). These spikes likely reflect major market events or shifts in investor sentiment. Despite the overall increase, the %Change data reveals high volatility, with substantial fluctuations in both positive and negative directions. For example, after a substantial increase in 2013, there were declines in 2014 (-11.53%) and 2015 (-11.53%), followed by another rise in 2017. The volatility of Bitcoin prices (VBTC) demonstrates significant fluctuations over the years from 2011 to 2024. In 2013, VBTC increased dramatically by 66.63%, and in 2017, it reached an even higher peak with an increase of 119.70%, indicating periods of heightened market instability and uncertainty. These peaks highlight the challenges faced by analysts and traders in predicting market behavior. Conversely, certain years saw notable decreases in volatility, such as 2012 (-50.04%) and 2023 (-36.12%), suggesting periods of relative market stability. Overall, the data reveals a highly unpredictable and unstable Bitcoin market. The combination of significant price changes and variable volatility underscores the complexities and challenges involved in forecasting VBTC. Additionally, the results of the ADF and PP unit root tests indicate that while Log(BTC) is non-stationary at the level data (ADF statistic: -2.655547, p-value: 0.2567; PP statistic: -1.533877, p-value: 0.8139), VBTC is stationary at the level data (ADF statistic: -5.614416, p-value: 0.0000; PP statistic: -8.288536, p-value: 0.0000), emphasizing the importance of using advanced analytical tools for accurate prediction and analysis.

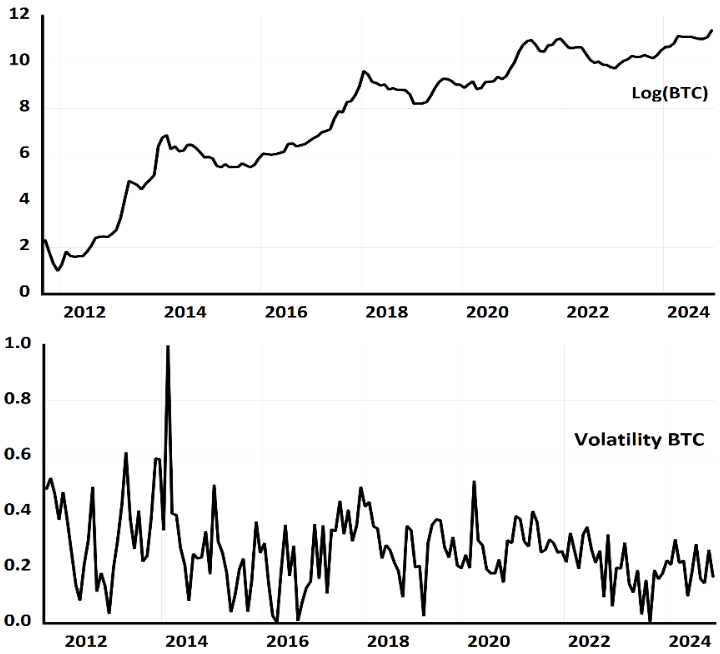

Fig. 5: Graph of Log(BTC) and VBTC from 2011M09 to 2024M12 - (Source: Research findings)

Table 1: Year-by-year mean of Log(BTC) and VBTC, and unit root test

Panel A. Year-by-year mean of Log(BTC) and VBTC				
Year/Variable	**Log(BTC)**	**%Change**	**VBTC**	**%Change**
2011	1.590533		0.460009	
2012	1.927837	21.21	0.229803	-50.04
2013	4.387425	127.58	0.382928	66.63
2014	6.295012	43.48	0.324114	-15.36
2015	5.569505	-11.53	0.216137	-33.31
2016	6.255863	12.32	0.149356	-30.90
2017	7.666560	22.55	0.328138	119.70
2018	9.004570	17.45	0.290735	-11.40
2019	8.774830	-2.55	0.252543	-13.14
2020	9.167580	4.48	0.252738	0.08
2021	10.682000	16.52	0.308729	22.15
2022	10.277610	-3.79	0.238829	-22.64
2023	10.174400	-1.00	0.152557	-36.12
2024	11.004960	8.16	0.205534	34.73
Total	7.628875		0.261411	

Panel B. Unit Root Test		
	Log(BTC)	**VBTC**
ADF test (Sig.)	-2.655547 (0.2567)	-5.614416 (0.0000)
PP test (Sig.)	-1.533877 (0.8139)	-8.288536 (0.0000)

4.2 Forecasting Using the Time Series Regression

Figure 6 presents the out-of-sample forecast results of VBTC using the time series method. The ARIMA(2,0,0) model was employed to forecast the volatility of Bitcoin prices (VBTC) from May 2022 to December 2024, using data from September 2011 to April 2022 for estimation.

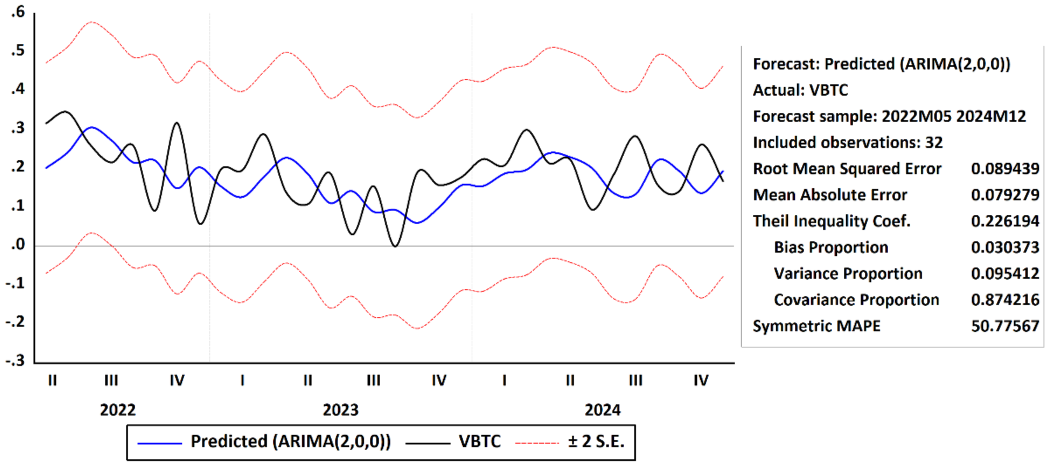

Fig. 6: Graph of VBTC, Forecasted VBTC, and forecast performance metrics - (Source: Research findings)

The performance of the model was evaluated using several metrics. The Root Mean Squared Error (RMSE) is 0.089439, indicating the average magnitude of the forecast errors. The Mean Absolute Error (MAE) is 0.079279, showing the average absolute difference between the predicted and actual values. Both metrics suggest that the model provides reasonably accurate predictions. The Theil Inequality Coefficient is 0.226194, which is relatively low and implies good forecasting performance, as values closer to zero indicate more accurate forecasts. The Bias Proportion of 0.030373 shows that only a small portion of the forecast error is due to systematic bias, while the Variance Proportion of 0.095412 indicates that the error due to differences in the variability of the predicted and actual series is also low. The Covariance Proportion is high at 0.874216, signifying that the majority of the forecast error is unsystematic and due to random fluctuations. The Symmetric Mean Absolute Percentage Error (SMAPE) is 50.77567, reflecting the average percentage error of the forecasts. This relatively high value highlights the inherent limitations of traditional time series models in predicting Bitcoin volatility. Given the highly dynamic and complex nature of Bitcoin's price movements, time series approaches struggle to capture rapid fluctuations effectively. To address these shortcomings, data-driven models were employed, leveraging advanced machine learning techniques to enhance predictive accuracy. Overall, the ARIMA(2,0,0) model provides a reasonable forecast of Bitcoin price volatility, with most errors stemming from random fluctuations rather than systematic biases, indicating its utility in predicting VBTC. In the next section, the forecast results using machine learning methods and the combination of machine learning with Empirical Decomposition Mode will be presented.

4.3 Decomposition of VBTC into Intrinsic Mode Functions (IMFs)

Figure 7 presents the decomposition results of the VBTC series into Intrinsic Mode Functions (IMFs). The decomposition of VBTC into its Intrinsic Mode Functions (IMFs) and residual provides a detailed insight into the underlying components contributing to the volatility of Bitcoin prices. The first IMF (IMF 1) captures the highest frequency fluctuations, which are often associated with short-term market noise or rapid changes in market sentiment. Subsequent IMFs (IMF 2, IMF 3, IMF 4, and IMF 5) reflect progressively lower frequency components, indicating medium to long-term trends and cycles. For instance, IMF 2 and IMF 3 display moderate fluctuations that can be linked to monthly or quarterly market trends, while IMF 4 and IMF 5 capture longer-term trends that might be driven by significant economic or geopolitical events influencing Bitcoin volatility. The residual component represents the long-term trend, capturing the overall stable pattern of VBTC that remains after removing all the oscillatory components represented by the IMFs. This residual trend is crucial as it highlights the inherent stability or long-term behavior of Bitcoin volatility that is not influenced by short-term market fluctuations.

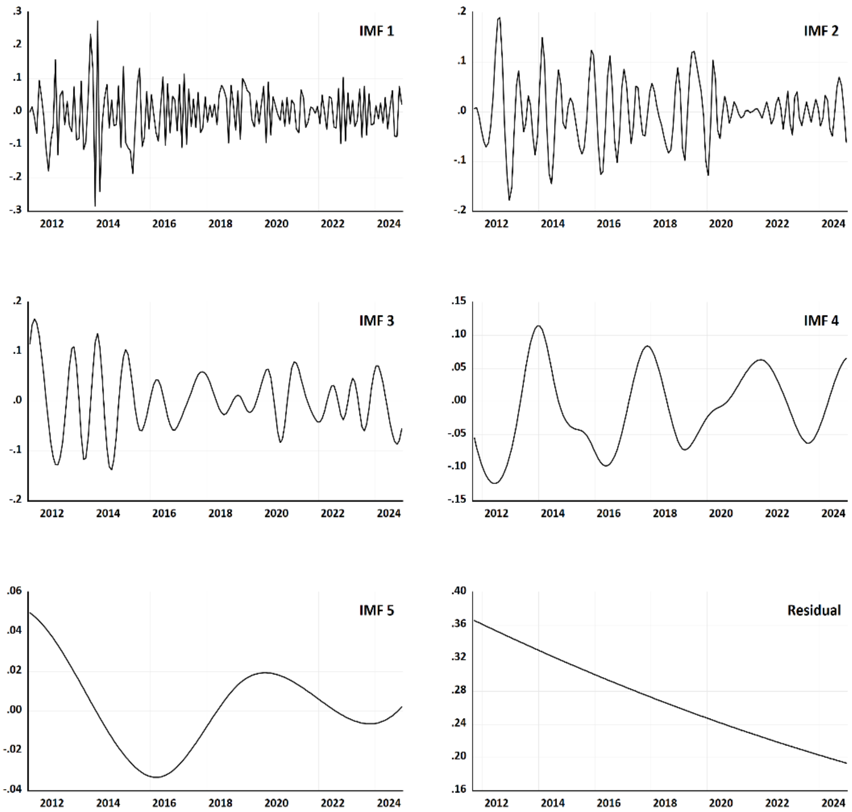

Fig. 7: Results of VBTC decomposition into its intrinsic mode functions - (Source: Research findings)

In summary, the Empirical Mode Decomposition provides a comprehensive breakdown of Bitcoin volatility, isolating various frequency components that contribute to the overall behavior of VBTC. This multi-scale analysis can be valuable for market analysts and traders to understand and predict the different factors affecting Bitcoin volatility over different time horizons.

4.4 Forecasting Using ANN and the Combined EMD-ANN Model

Figure 8 shows the graph of VBTC along with its predicted values, obtained using both the Artificial Neural Network (ANN) method and the combined Empirical Mode Decomposition and Artificial Neural Network (EMD-ANN) model. Furthermore, Table 2 provides a comparison of the forecast performance metrics between the ANN model and the EMD-ANN model. The Mean Absolute Error (MAE) for the ANN model is 0.077141, while the EMD-ANN model achieves a significantly lower MAE of 0.004142, resulting in a substantial reduction of 94.63%. This improvement is statistically significant, with a t-value of 7.0958 and a p-value of 0.0000. Similarly, the Root Mean Squared Error

(RMSE) for the ANN model is 0.096068, compared to the much lower RMSE of 0.004433 for the EMD-ANN model, indicating a 95.39% reduction in error. This result is also statistically significant, with a t-value of 4.2692 and a p-value of 0.0000.

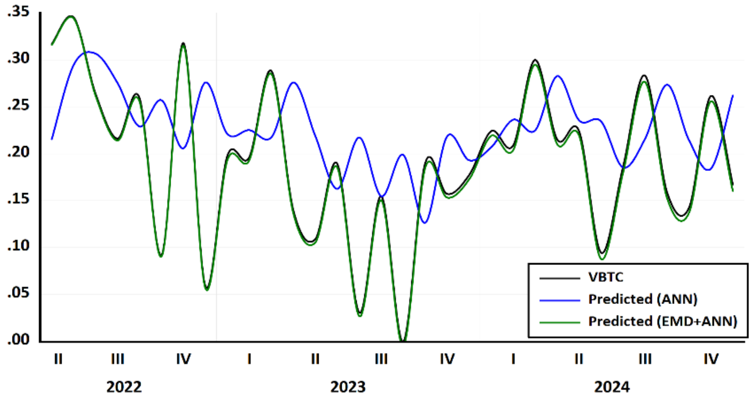

Fig. 8: Graph of VBTC and its forecasted values using ANN and EMD-ANN - (Source: Research findings)

Table 2: Comparison of forecast performance metrics for ANN and EMD-ANN

Forecast performance metrics	ANN	EMD-ANN	%Change	T-Value	Sig.
Mean Absolute Error (MAE)	0.077141	0.004142	-94.63	7.0958	0.0000
Root Mean Squared Error (RMSE)	0.096068	0.004433	-95.39	4.2692	0.0000
Diebold-Mariano test				3.9435	0.0004

The significant decreases in both MAE and RMSE for the EMD-ANN model highlight its superior accuracy and predictive performance over the standalone ANN model. Moreover, the significance of the Diebold-Mariano test statistic (3.9435) indicates that the predictive accuracy of the combined EMD-ANN model is significantly higher than that of the ANN model. These findings underscore the effectiveness of incorporating Empirical Mode Decomposition with ANN to enhance the precision of volatility forecasts for Bitcoin prices.

4.5 Forecasting Using RNN and the Combined EMD-RNN Model

Figure 9 displays the graph of VBTC alongside its forecasted values, derived through both the RecurrentNeural Network (RNN) method and the combined Empirical Mode Decomposition and RecurrentNeural Network (EMD-RNN) model. Additionally, Table 3 compares the forecast performance metrics for the ANN model and the EMD-RNN model. The Mean Absolute Error (MAE) for the RNN model stands at 0.076955, while the EMD-RNN model records a significantly lower MAE of 0.003263, showcasing a remarkable reduction of 95.76%. This improvement is statistically significant, as indicated by a t-value of 7.2550 and a p-value of 0.0000. Likewise, the Root Mean Squared Error (RMSE) for the RNN model is 0.095494, in contrast to the much lower RMSE of 0.003488 for the EMD-RNN model, signifying a 96.35% decrease in error. This result is also statistically significant, with a t-value of 4.2085 and a p-value of 0.0000.

The substantial reductions in both MAE and RMSE for the EMD-RNN model emphasize its superior accuracy and predictive performance compared to the standalone RNN model. Furthermore, the significance of the Diebold-Mariano test statistic (3.2191) indicates that the predictive accuracy of the combined EMD-RNN model is significantly higher than that of the RNN model These results highlight the effectiveness of integrating Empirical Mode Decomposition with RNN to enhance the accuracy of Bitcoin volatility forecasts.

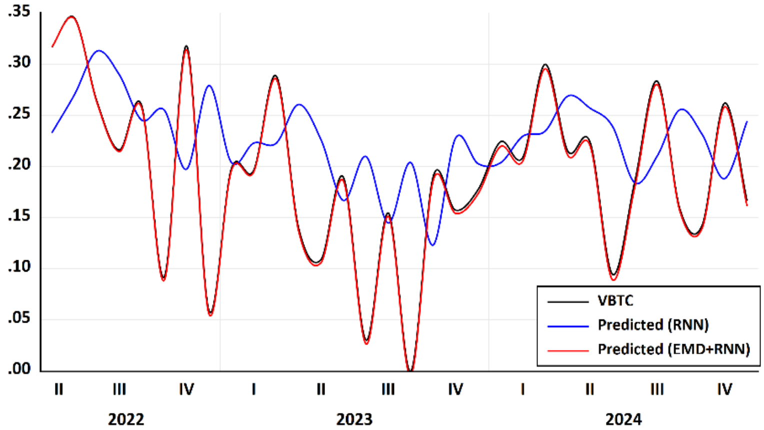

Fig. 9: Graph of VBTC and its forecasted values using RNN and EMD-RNN - (Source: Research findings)

Table 3: Comparison of forecast performance metrics for RNN and EMD-RNN

Forecast performance metrics	RNN	EMD-RNN	%Change	T-Value	Sig.
Mean Absolute Error (MAE)	0.076955	0.003263	-95.76	7.2550	0.0000
Root Mean Squared Error (RMSE)	0.095494	0.003488	-96.35	4.2085	0.0000
Diebold-Mariano test				3.2191	0.0030

4.6 Forecasting Using LSTM and the Combined EMD-LSTM Model

Figure 10 presents the graph of VBTC along with its forecasted values, obtained using both the Long Short-Term Memory (LSTM) method and the combined Empirical Mode Decomposition and Long Short-Term Memory (EMD-LSTM) model. Additionally, Table 4 compares the forecast performance metrics between the LSTM model and the EMD-LSTM model. Furthermore, Table 4 provides a comparative analysis of the forecast performance metrics between the Long Short-Term Memory (LSTM) model and the combined Empirical Mode Decomposition and Long Short-Term Memory (EMD-LSTM) model. The Mean Absolute Error (MAE) for the LSTM model is 0.075012, while the EMD-LSTM model significantly reduces this error to 0.008422, reflecting an 88.77% decrease. This reduction is statistically significant with a t-value of 6.3114 and a p-value of 0.0000.Similarly, the Root Mean Squared Error (RMSE) for the LSTM model is 0.095267, in contrast to a much lower RMSE of 0.008534 for the EMD-LSTM model, marking a 91.04% reduction. This result is also statistically significant, as indicated by a t-value of 4.1466 and a p-value of 0.0000. The significant declines in both MAE and RMSE underscore the enhanced accuracy and predictive power of the EMD-LSTM model compared to the standalone LSTM model. In addition, the significance of the Diebold-Mariano test statistic (2.9796) indicates that the predictive accuracy of the combined EMD-LSTM model is significantly higher than that of the LSTM model.

These findings highlight the substantial benefits of incorporating Empirical Mode Decomposition with LSTM in achieving more precise and reliable forecasts for Bitcoin volatility. The EMD-LSTM model's superior performance demonstrates its potential as a robust tool for market analysts and traders seeking to better predict and navigate the complexities of Bitcoin price movements.

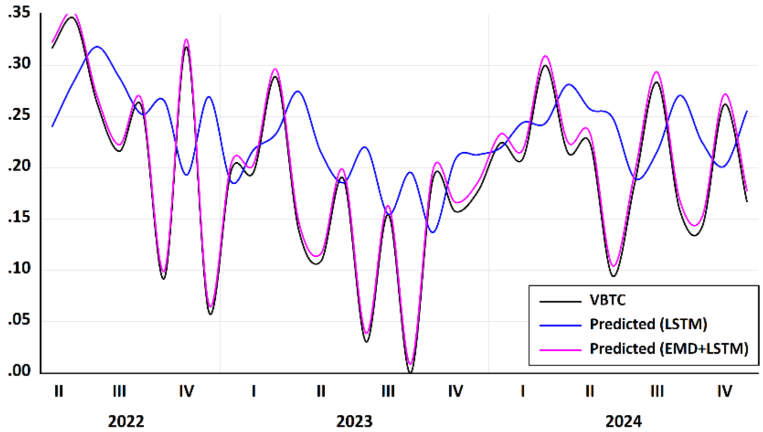

Fig. 10: Graph of VBTC and its forecasted values using LSTM and EMD-LSTM – (Source: Research findings)

Table 4: Comparison of forecast performance metrics for LSTM and EMD-LSTM

Forecast performance metrics	LSTM	EMD-LSTM	%Change	T-Value	Sig.
Mean Absolute Error (MAE)	0.075012	0.008422	-88.77	6.3114	0.0000
Root Mean Squared Error (RMSE)	0.095267	0.008534	-91.04	4.1466	0.0000
Diebold-Mariano test				2.9796	0.0056

5 Discussion and Conclusions

The Bitcoin market has emerged as one of the most dynamic and influential sectors in the financial world. As a decentralized digital currency, Bitcoin has revolutionized the way transactions are conducted, offering a level of anonymity, security, and efficiency that traditional financial systems cannot match. Its market capitalization and global adoption have grown exponentially, making it a critical asset for investors, traders, and financial analysts. The volatility of Bitcoin, while presenting substantial profit opportunities, also poses significant risks and challenges. Understanding and predicting Bitcoin's price movements and volatility are essential for informed decision-making and effective risk management.This study has provided a comprehensive analysis of Bitcoin's volatility and its underlying components using advanced analytical tools and decomposition techniques. The descriptive statistics and unit root test results revealed significant trends and fluctuations in Bitcoin prices over the years. The time series forecasting models, such as ARIMA, demonstrated reasonable accuracy in capturing the general trend and volatility of Bitcoin. The decomposition of Bitcoin's volatility into Intrinsic Mode Functions (IMFs) provided deeper insights into the short-term and long-term components driving market behavior. Furthermore, the comparison of various forecast performance metrics highlighted the superior accuracy of combining Empirical Mode Decomposition (EMD) with neural network models, such as ANN, LSTM, and RNN, over standalone models. In conclusion, this research underscores the importance of employing advanced analytical techniques to understand and predict Bitcoin's volatility. The integration of Empirical Mode Decomposition with neural network models has proven to enhance the precision and reliability of volatility forecasts, providing valuable insights for market participants. The ability to accurately forecast Bitcoin volatility can significantly improve risk management strategies and investment decisions in this highly volatile market. As the Bitcoin market continues to evolve, ongoing research and the development of more sophisticated models will be crucial in maintaining accurate and reliable forecasts. Our results aligns with prior research, including Risse (2019) [36], Lin et al. (2022) [28], Tan et al. (2024) [40], and Yan et al. (2024) [47], which also highlight the advantages of hybrid approaches in financial forecasting.

The findings of this study have several practical implications for market analysts, traders, and investors. By incorporating advanced analytical tools and decomposition techniques, stakeholders can achieve more accurate forecasts, enabling better risk management and strategic planning. The enhanced predictive performance of EMD-Machine Learning Tools offers a robust tool for navigating the complexities of Bitcoin price movements, ultimately contributing to more informed and effective decision-making. Future research should focus on exploring other advanced analytical techniques and machine learning models to further enhance the accuracy of Bitcoin volatility forecasts. Investigating the impact of external factors, such as regulatory changes, technological advancements, and macroeconomic indicators, on Bitcoin volatility could provide additional insights into market behavior. Moreover, expanding the scope of research to include other cryptocurrencies and digital assets will help in developing a comprehensive understanding of the broader cryptocurrency market. Continuous innovation and refinement of forecasting models will be essential in keeping pace with the rapidly evolving landscape of digital currencies.

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