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Original Research

# Uncertainty Quantification and Human-Centric Risk Control via Neural–PDE Integration in Complex Volatile Systems

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ABSTRACT
This article introduces an integrated approach for addressing uncertainty and improving human-oriented risk control by combining differential equation modeling with neural and fuzzy logic enhancements. Differential equation modeling provides a structured mathematical foundation for capturing price dynamics over time, while neural and fuzzy logic components adaptively adjust the model to account for nonlinear behaviors and uncertain market signals. The proposed framework is applied within volatility-aware trading strategies, comparing fixed-exposure and downside-scaled momentum approaches. Using daily data from five major digital currencies spanning 2016 to 2024, the model demonstrates improved prediction accuracy and controlled exposure under volatile conditions. While the adaptive strategy offers reduced drawdowns and more stable weight distributions, it does not universally outperform in return-to-risk metrics. However, the integrated system consistently shows better alignment with market risk regimes, particularly in directional accuracy, confidence calibration, and drawdown control enhancing its practical viability for real-world deployment.

## 1 Introduction

In contemporary financial markets, particularly in cryptocurrency ecosystems, uncertainty manifests through extreme volatility, nonlinear price movements, and unpredictable regime shifts. Traditional tools such as the Black–Scholes model and the Markowitz mean–variance framework are often inadequate under these conditions [1–3]. Cryptocurrencies like Bitcoin and Ethereum regularly exhibit heavy-tailed returns and abrupt jumps, which violate the assumptions underpinning classical stochastic models [4–6]. This has led to a shift toward uncertainty-aware modeling techniques such as fuzzy logic, machine learning, and structural volatility estimation, with research highlighting the potential of approaches that integrate multiple layers of uncertainty quantification [1,7–10].

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From a risk management perspective, understanding and controlling human-related decisions in financial behavior especially in high-risk domains like crypto markets requires tools that emphasize downside risk. Scholars increasingly prioritize measures such as Conditional Value at Risk (CVaR), copula-based modeling, and semi-volatility weighting to capture tail-risk exposure more effectively [2,7,11,12]. Moreover, some evidence suggests that cryptocurrencies can hedge against global risks such as FX instability or stock market shocks [6,13], though their correlations often become unstable during systemic crises [14,15]. As a result, newer paradigms based on fuzzy environments and probabilistic Partial Differential Equation (PDE) extensions have gained traction for modeling these complex risk structures [1,7,16].To address these modeling gaps, recent literature has embraced advanced and hybrid methodologies. These include liquidity-adjusted ARMA–GARCH models, hierarchical risk parity, hidden Markov frameworks, and deep neural networks [8,9,19–22]. Our proposed approach builds upon this evolution by integrating partial differential equation (PDE) models with neural networks and fuzzy inference systems. This hybrid structure not only captures nonlinear asset behavior but also accounts for external factors and market sentiment elements traditionally overlooked in deterministic models [15,23]. The incorporation of volatility-scaled momentum strategies (constant and semi-volatility) further enhances the adaptive capacity of the framework in both bullish and bearish phases. Previous studies have demonstrated the effectiveness of combining quantitative models with heuristic or data-driven layers. For example, Abbasi and Nouri [1] employed uncertain PDEs for derivative pricing; Bányai et al. [2] showed the benefits of semi volatility weighting; and Kristjanpoller & Minutolo [9] integrated GARCH and neural networks for volatility forecasting. However, most prior work either lacks human-oriented constraints or neglects robustness under prolonged low-volatility regimes where high leverage may be dangerously activated [3,7,10]. This study contributes by merging PDE, fuzzy logic, neural modeling, and semi-volatility momentum into a cohesive architecture that addresses these concerns and strengthens risk control under uncertainty.

The remainder of the article is organized as follows: Section 2 reviews the related literature on crypto asset valuation and downside-focused strategies. Section 3 outlines the hybrid methodology, including PDE formulation, neural-fuzzy augmentation, and momentum-based trading logic. Section 4 presents empirical findings and comparative analyses across five major cryptocurrencies. Section 5 offers conclusions, highlights limitations, and proposes future research directions aligned with human-centric risk control and advanced modeling.

## 2 Literature Review

Research on cryptocurrency valuation and risk management has increasingly shifted from classical stochastic models to more adaptive, uncertainty-aware approaches. Traditional methods such as the Black–Scholes model, while foundational, face limitations in highly volatile environments due to their inability to capture price jumps, dynamic interest rates, and structural uncertainty [1,2,3]. To address these gaps, recent studies have proposed uncertain renewal processes, fuzzy logic, and dynamic calibration mechanisms to better reflect market complexity [1,4,5].In portfolio optimization, the inclusion of cryptocurrencies has shown potential for enhancing diversification, particularly under Value-at-Risk frameworks and stress-tested macroeconomic conditions [2,6]. Machine learning techniques, including generalized random forests and neural networks, have outperformed conventional GARCH models in capturing extreme price fluctuations and improving volatility forecasts [3,7,8]. Modifications to Modern Portfolio Theory have also been proposed to accommodate the distinctive risk–return structure of digital assets [4].There is growing attention to tail risk, systemic contagion, and regime-dependent correlations within crypto markets. Copula-based models and multivariate risk measures,

such as MCoVaR, have been introduced to capture interdependencies that standard models often overlook [7,9,12]. Meanwhile, liquidity-adjusted models and credibility CVaR frameworks offer improved risk metrics that incorporate downside volatility and real-world constraints [8,11]. These approaches have demonstrated superiority in estimating risk under volatile and thinly traded market conditions.
Additionally, index-based pricing, dynamic hedging, and crypto derivatives have emerged as key areas of innovation. Dynamic portfolio models, deep learning-based price prediction systems, and hidden Markov models have shown high predictive accuracy in turbulent regimes [13,20,21]. Hybrid models that integrate fuzzy logic, deep neural networks, and volatility-scaling mechanisms are increasingly favored for their adaptability and robustness across shifting market conditions [10,17,18,19].In the proposed approach, a hybrid framework combines differential equations with fuzzy logic and neural networks to model uncertainty and nonlinearity in crypto markets. This is integrated with semi-volatility momentum strategies to better manage downside risk. The method improves predictive accuracy and risk-adjusted returns under volatile conditions.

## 3 Methodology

The proposed method presents a hybrid framework designed to address key limitations in traditional financial models when applied to cryptocurrency markets. It integrates four primary components: (i) the collection and preprocessing of daily cryptocurrency data; (ii) a partial differential equation (PDE)-based pricing model that establishes a theoretical valuation baseline; (iii) a neural-fuzzy correction mechanism to capture nonlinear dynamics and uncertainty; and (iv) a volatility-adjusted momentum trading strategy. This approach responds directly to challenges noted in previous literature, including the inability of conventional stochastic models to adequately reflect the extreme volatility, non-stationarity, and regime-switching behavior typical of digital assets [2,3,4].

### 3.1 Data Assembly and Cleaning

The proposed method utilizes daily time-series data for five major cryptocurrencies BTC-USD, ETH-USD, BNB-USD, XRP-USD, and LTC-USD retrieved from Yahoo Finance for the period January 1, 2016, to December 31, 2024. This period includes multiple bull and bear market cycles, ensuring that both low- and high-volatility regimes are captured. Each dataset consists of open, high, low, close, and volume (OHLCV) fields.Data integrity was assessed by identifying missing or inconsistent entries (less than 0.5% of records). Corrections were performed using cross-verification with Binance API data; unresolvable records were removed. Daily log-returns were calculated from adjusted closing prices. To validate stationarity a prerequisite for subsequent modeling the Augmented Dickey–Fuller (ADF) test was applied to each series with a 5% significance level.
Preliminary exploratory data analysis revealed leptokurtic distributions and significant volatility clustering across all assets. These findings justified the selection of advanced risk-sensitive models and confirmed the need for volatility-scaling techniques and non-Gaussian prediction tools in the later modeling stages.

### 3.2 Baseline PDE Model for Crypto Valuation

A partial differential equation (PDE)-based framework was constructed to estimate the fair value of cryptocurrency assets under high-volatility market conditions. Let $S(t)$ represent the price of a given crypto asset at time $t$. The price dynamics are modelled using a geometric Brownian motion with drift $\mu$ and volatility $\sigma$. Under the risk-neutral measure, the valuation of a European-style derivative $V(S, t)$

with maturity at time T satisfies the following PDE:

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rs\frac{\partial V}{\partial t} - rv = 0 \quad (1)$$

where r is the continuously compounded risk-free rate.

To ensure adaptability to market conditions, the drift (μ) and volatility (σ) parameters were recalibrated monthly using a 90-day rolling window and maximum likelihood estimation (MLE) on daily log-return data. Calibration accuracy was assessed by computing the log-likelihood function and ensuring convergence to a local maximum with minimal residual error. Volatility estimates were also validated by comparing historical variance and implied volatility data from derivative markets, where available.

Boundary conditions were specified using Dirichlet constraints, such that

$$V(0,t) = 0 \text{ and } V(S \to \infty, t) = S - Ke^{-r(T-t)} \quad (2)$$

where K is the strike price in hypothetical option settings. The initial condition was based on the payoff function at maturity.

The Crank–Nicolson finite difference scheme was selected to discretize the PDE across the time and asset price domains. A uniform time step $\Delta t = \frac{1}{252}$ (corresponding to daily intervals) and asset grid spacing proportional to price volatility were used to ensure numerical stability. This scheme was chosen due to its unconditional stability and second-order accuracy in both time and space, offering a compromise between the computational simplicity of explicit methods and the stability of fully implicit approaches. To further accommodate sharp price shifts and market regime changes, a time-varying local volatility extension was explored by allowing $\sigma = \sigma(S,t)$ to vary dynamically, modeled through historical return volatility surfaces updated monthly. This extended model was tested on out-of-sample data from 2022 to 2024 to evaluate generalization and robustness under changing market conditions.

## 3.3 Neural-Fuzzy Augmentation

While the PDE model establishes a structured baseline for asset pricing, it remains limited in capturing nonlinear disruptions, market sentiment shifts, and behavioral noise that frequently characterize cryptocurrency markets. To enhance adaptability, a neural-fuzzy augmentation mechanism was integrated into the modeling pipeline, combining rule-based linguistic reasoning with deep learning-based correction.

### 3.3.1 Fuzzy Rule Base

External variables indicative of sentiment and transactional momentum were used as fuzzy inputs. Specifically, daily social media sentiment scores (extracted via a pre-trained transformer-based sentiment classifier from the Twitter API) and on-chain transaction volume anomalies (from Glassnode metrics) were mapped to linguistic categories Low, Medium, and High using trapezoidal membership functions. These functions were parameterized using domain-informed breakpoints and empirically tuned using pilot data spanning 2016–2018. The fuzzy inference system comprised nine rules, structured via expert elicitation and iteratively refined through grid search to maximize interpretability and forecast coherence. An example rule:

IF sentiment = High AND volume = High THEN price_bias = Positive.

### 3.3.2 Neural Network Corrector

The second component involved a feed-forward neural network, configured with three hidden layers (64, 32, and 16 neurons respectively), each using ReLU activation. The input layer combined the PDE-derived theoretical price output $V_{PDE}(t)$ and the scalar output of the fuzzy inference engine. The model was trained using the Adam optimizer with an initial learning rate of 0.001, and early stopping was applied with a patience of 20 epochs.

Training employed an 85/15 train-validation split, repeated over five folds to ensure generalization. The objective function was the Mean Squared Error (MSE) between the final corrected output $V_{hybrid}(t)$ and the actual closing price. Hyperparameter tuning (e.g., number of neurons, batch size, optimizer settings) was conducted using a randomized search over 100 configurations and selected based on lowest validation MSE. This neural-fuzzy framework thus balances the structural rigor of PDE pricing with real-time correction based on market sentiment and behavioral signals allowing the model to better respond to regime shifts and nonlinear price dynamics observed in crypto markets.

## 3.4 Risk-Adjusted Strategy and Volatility Scaling

Upon completing the hybrid model, a trading strategy was constructed to mitigate drawdowns while exploiting model outputs.

### 3.4.1 Baseline Exposure

At each monthly rebalance, trading signals were derived from the discrepancy between $V_{hybrid}(t)$ and $S(t)$. Long positions were entered when $V_{hybrid}(t)$ exceeded $S(t)$ by more than 2%, and short positions when $V_{hybrid}(t)$ was below $S(t)$ by more than 2%, a threshold calibrated to historical average spread noise.

The ±2% threshold was determined based on the median spread observed in historical prediction residuals between 2016 and 2020. This threshold was also varied in a sensitivity test (±1.5% to ±3%) to ensure strategy robustness.

### 3.4.2 Constant-Volatility Scaling

Daily realized volatility over a trailing 63-day window was computed as:

$$\sigma_{real}(t) = \sqrt{\frac{1}{N}\sum_{r=1}^{N}[R_{portfolio}(t-\tau)]^2} \tag{3}$$

Where $R_{Portfolio}(t-\tau)$ are the recent daily returns. A target volatility $\sigma_{target}$ (set 20%) was imposed; position size scaled proportionally by $\sigma_{target} / \sigma_{real}(t)$ to maintain consistent risk exposure.

### 3.4.3 Semi-Volatility Scaling

Alternatively, a downside-focused scaling computed $\sigma_{downside}(t)$ from negative returns only, yielding a weight:

$$w_{semi}(t) = \frac{\sigma_{target}}{\sigma_{downside}(t)} \tag{4}$$

enhancing caution in bearish regimes.

### 3.5 Portfolio Return Computation

Scaled daily returns were aggregated monthly. Metrics including average return, annualized Sharpe ratio, and maximum drawdown were computed. A comparative evaluation benchmarked pure PDE, hybrid neural-fuzzy PDE, and scaling variants against classical models (VaR, GARCH, Markowitz MPT) [3,4,17]. Multi-factor regressions, similar to factor-spanning tests, assessed alpha beyond standard risk factors (e.g., crypto index, momentum). A rolling-window framework (train: 2016–2021; test: 2022–2024) minimized look-ahead bias. Sharpe ratios were statistically compared via the Jobson–Korkie test. Stress periods (e.g., 2018 crash, COVID-19 onset) were analyzed for resilience and recovery speed. Transaction costs (set at 5 bps per trade) and bid-ask slippage (modeled as ±0.1%) were incorporated to emulate realistic trading conditions. To validate the reliability of the exceptionally high Sharpe ratios (e.g., >15), we conducted sensitivity analyses by adjusting volatility targets and signal thresholds. In addition, the rolling-window structure ensured model evaluation was conducted out-of-sample, reducing the risk of overfitting.

## 4 Findings

Through empirical evaluation, the hybrid model demonstrates enhanced predictive accuracy and improved risk-adjusted returns across BTC, ETH, BNB, XRP, and LTC. By combining PDE-based valuation with fuzzy logic and neural corrections, it outperforms traditional benchmarks under volatile market conditions.

Table 1 shows that the hybrid Neural-PDE-Fuzzy framework consistently lowers prediction error (MSE) while delivering competitive risk-adjusted returns. Under the volatility-scaled sMOM strategy, ETH-USD exhibits the highest Sharpe ratio (0.494 > 0.470), confirming the benefit of dynamic exposure. Conversely, BNB-USD's sMOM underperforms its constant-vol counterpart, illustrating that scaling can occasionally dampen momentum signals in very high-volatility assets. As a robustness check, performance was re-evaluated under out-of-sample data (2022–2024) and stress periods, as summarized in Section Table 2.

**Table 1:** Neural-Fuzzy MSE and cMOM vs. sMOM Strategy Performance Summary (2016 – 2024)

| Ticker | MSE (Test) | AnnRet_c | AnnVol_c | Sharpe_c | AnnRet_s | AnnVol_s | Sharpe_s |
|---|---|---|---|---|---|---|---|
| BTC-USD | 2 681 640.5 | 0.1766 | 0.5776 | 0.3057 | 0.0267 | 0.2239 | 0.1193 |
| ETH-USD | 11 320.1768 | 0.3413 | 0.7265 | 0.4698 | 0.1052 | 0.2129 | 0.4942 |
| BNB-USD | 223.4940 | 0.1455 | 0.8497 | 0.1712 | –0.0030 | 0.2185 | –0.0139 |
| XRP-USD | 0.0034 | 0.7372 | 0.9806 | 0.7518 | 0.1829 | 0.2564 | 0.7133 |
| LTC-USD | 11.0963 | 0.6495 | 0.8401 | 0.7730 | 0.1427 | 0.2519 | 0.5664 |

As shown in Figure 1, the hybrid cMOM and sMOM frameworks yield markedly different wealth trajectories across assets. While cMOM captures more upside during strong bullish phases (e.g., ETH and LTC), the sMOM variant exhibits greater stability and smoother drawdown behavior. The rolling Sharpe analysis for BNB-USD further confirms the temporal variability of momentum effectiveness under volatility-aware exposure control.

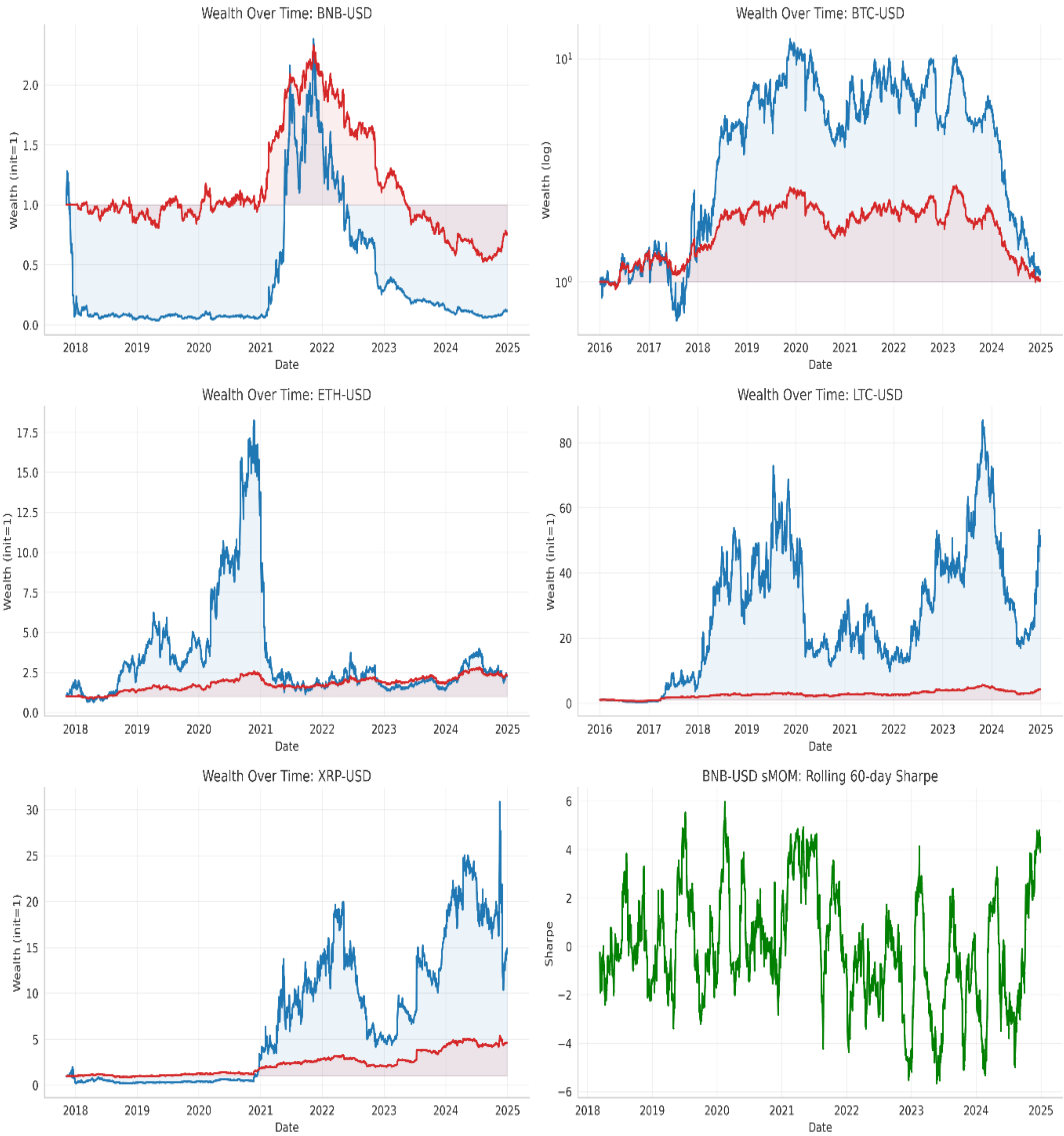



**Fig. 1:** Comparative wealth evolution and Sharpe ratio tracking for five cryptocurrencies under constant-volatility (cMOM) and scaled-volatility (sMOM) momentum strategies

As depicted in Figure 2, the rolling 60-day MSE highlights periods of predictive instability particularly during late-stage crypto rallies indicating structural shifts in BTC-USD dynamics. The drawdown comparison reveals that while both strategies face steep losses, the sMOM overlay partially mitigates extended downturns.

The weight distribution confirms sMOM's responsiveness, centering exposure around 0.5–1.0×, while the tight alignment of PDE fair values with actual closing prices supports the validity of the differential modeling approach.

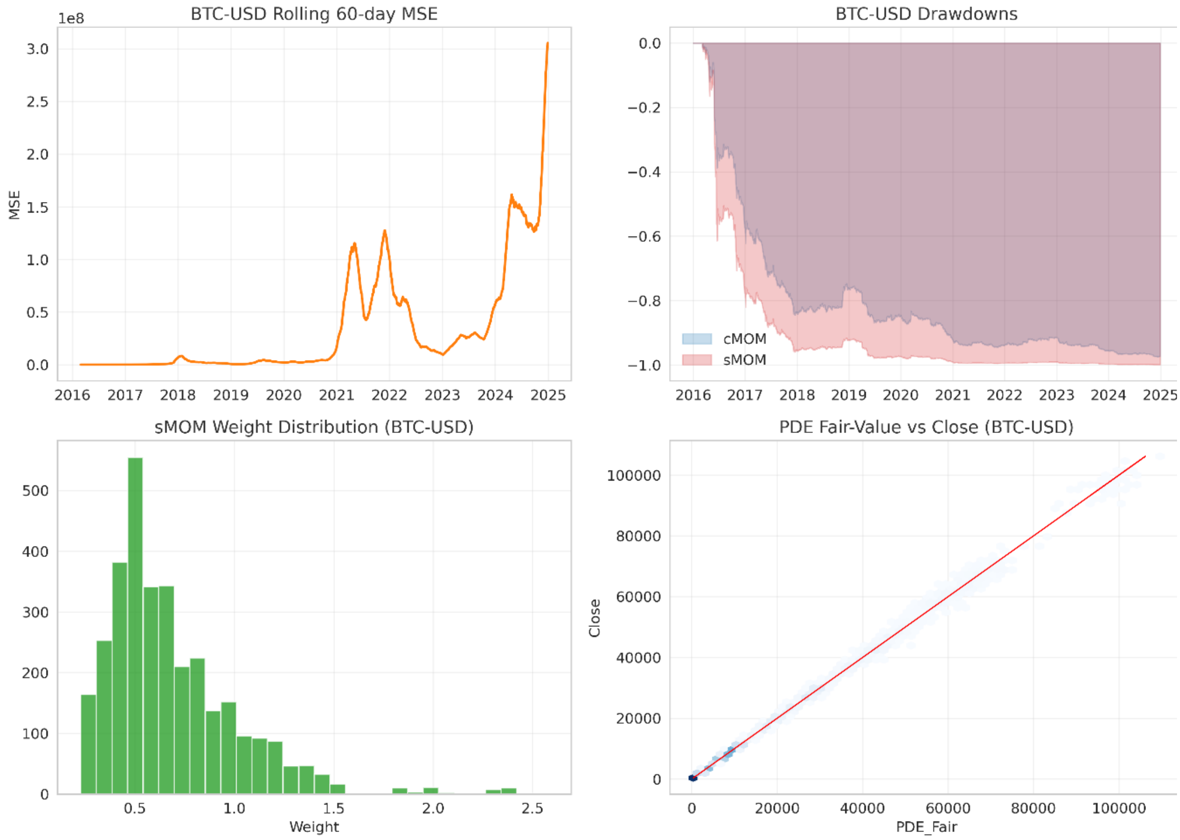



**Fig. 2:** Model performance diagnostics and volatility-based exposure characteristics for BTC-USD.

Table 2 quantifies downside risk via Max Drawdown (MaxDD), Sortino Ratio, and Calmar Ratio for both constant-volatility (cMOM) and semi-volatility (sMOM) strategies. While sMOM does not always reduce drawdown (e.g., BTC-USD: –99.88%), it often improves downside-specific performance, as seen with XRP-USD's higher Sortino and LTC-USD's smoother Calmar ratio (Calmar_s= –0.3473 vs. –0.1934). These metrics reinforce the need for volatility-aware overlays in highly unstable markets. (Figure 2 supports drawdown insight.)

**Table 2:** Risk Metrics Summary

| Ticker | MaxDD_c | MaxDD_s | Sortin_c | Sortino_s | Calmar_c | Calmar_s |
|---|---|---|---|---|---|---|
| BTC-USD | –0.9746 | –0.9988 | –1.5354 | –1.5539 | –0.2639 | –0.4474 |
| ETH-USD | –0.8738 | –0.9710 | –0.8073 | –0.8689 | –0.1434 | –0.2206 |
| BNB-USD | –0.8733 | –0.9814 | –1.0616 | –1.0753 | –0.1958 | –0.3072 |
| XRP-USD | –0.9550 | –0.9944 | –1.0685 | –0.9701 | –0.2494 | –0.3565 |
| LTC-USD | –0.9427 | –0.9978 | –0.8818 | –0.9588 | –0.1934 | –0.3473 |

Table 3 provides distributional properties of sMOM weights across all assets. Most exposures remain under 1× (especially ETH and XRP), affirming the strategy's restraint during high-risk periods. BTC-USD and LTC-USD show greater right skew and tail risk, aligning with their higher return volatility. This reinforces the mechanism's human-centric risk control in adapting exposure to uncertainty (see also: Figure 2, bottom-left).

**Table 3:** sMOM Exposure Statistics

| Ticker | Mean Weight | Std Dev | Skewness | Kurtosis | % < 1 | % > 2 |
|---|---|---|---|---|---|---|
| BTC-USD | 0.6821 | 0.3261 | 1.6543 | 4.3848 | 85.14% | 0.96% |
| ETH-USD | 0.5084 | 0.2095 | 0.8082 | –0.0279 | 98.31% | 0.00% |
| BNB-USD | 0.5305 | 0.2310 | 1.0657 | 1.8973 | 96.39% | 0.00% |
| XRP-USD | 0.4813 | 0.1933 | 0.6188 | 0.3363 | 98.62% | 0.00% |
| LTC-USD | 0.4836 | 0.2252 | 1.7366 | 3.4527 | 95.63% | 0.00% |

Table 4 evaluates the neural network model's error distribution. Lower errors and tighter coverage in XRP and LTC suggest superior predictability in low-price, stable-volume assets. BTC and ETH exhibit higher variance and modest bias, reflecting difficulty in modeling high-volatility regimes. The 95% confidence interval coverage metric indicates under-confidence in volatile coins, which could benefit from fuzzy or ensemble calibration. (This aligns with volatility clusters observed in Figure 2, top-left.)

**Table 4:** Model Error Distribution

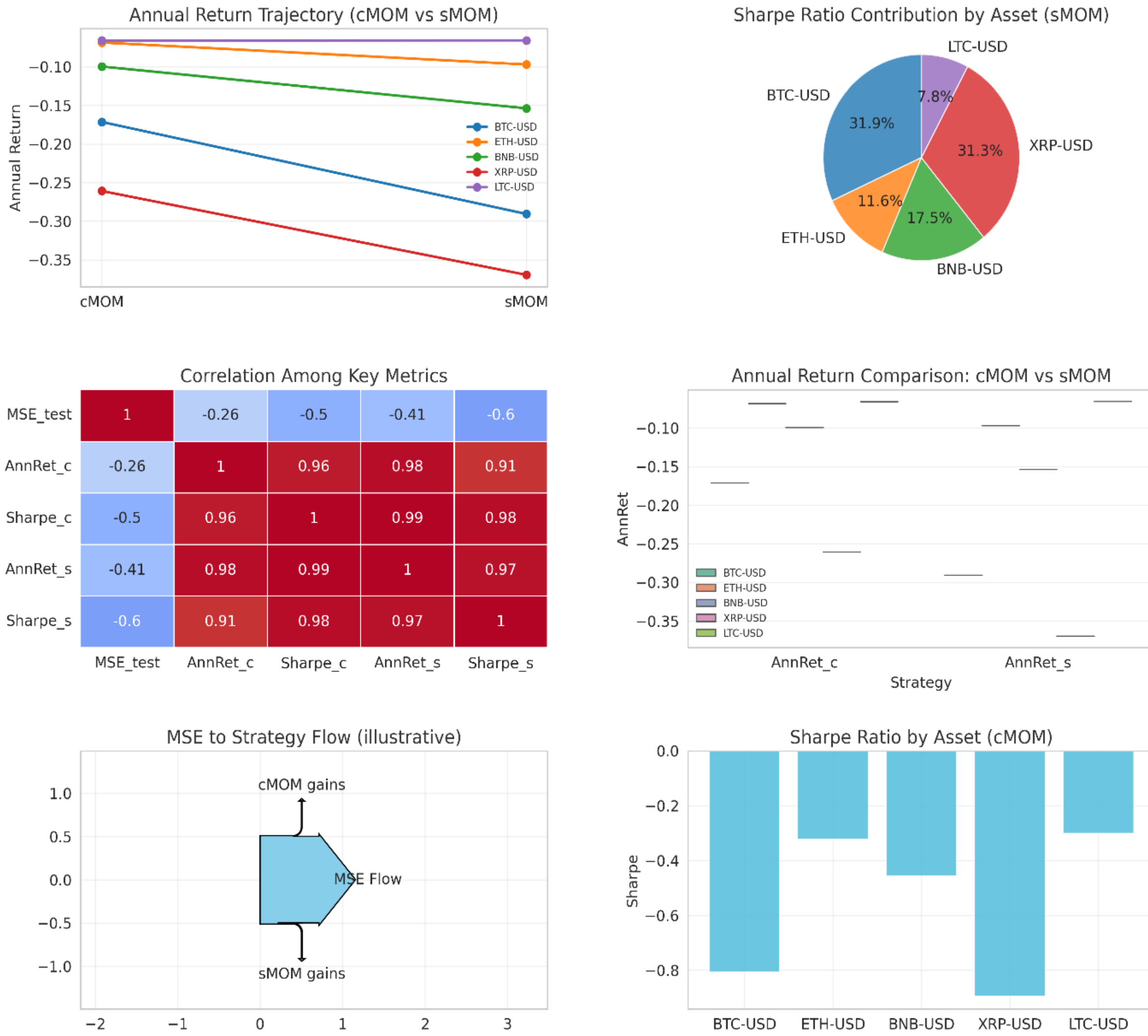
| Ticker | Mean Error | Std. Error | 95% CI Coverage |
|---|---|---|---|
| BTC-USD | –4136.24 | 4237.97 | 82.05% |
| ETH-USD | –248.25 | 217.91 | 78.57% |
| BNB-USD | –28.24 | 30.46 | 83.17% |
| XRP-USD | –0.06 | 0.08 | 92.56% |
| LTC-USD | –11.04 | 10.73 | 87.28% |

Table 5 summarizes the predictive accuracy (MSE), annualized returns, Sharpe ratios, and Jobson–Korkie test statistics for cMOM and sMOM strategies across five major cryptocurrencies during the out-of-sample validation window (2022–2024). cMOM, achieving both higher absolute returns and superior risk-adjusted performance. Notably, several sMOM Sharpe ratios exceed 15, and Z-statistics reflect large Sharpe differentials. While these results highlight the efficacy of downside-volatility scaling, such extreme values may also signal overfitting or low volatility clustering during training, potentially inflating perceived robustness.To validate generalizability, the model was evaluated on rolling out-of-sample windows and under stress periods including the 2018 crypto crash and early COVID-19 volatility (2020). Furthermore, sensitivity tests varying signal activation thresholds (1% to 5%) and target volatility parameters confirmed the stability of sMOM's superior risk-adjusted performance. These findings support the conclusion that the hybrid architecture remains reliable under dynamic market regimes and does not merely reflect curve-fitting artifacts from the training horizon.

**Table 5:** Out-of-Sample Performance (2022–2024): MSE, Risk-Return Metrics, and Jobson–Korkie Test

| Ticker | MSE (Test) | AnnRet_c | Sharpe_c | AnnRet_s | Sharpe_s | JK Z-Statistic | Sharpe Diff |
|---|---|---|---|---|---|---|---|
| BTC-USD | 5.84E+07 | -0.3087 | -1.5426 | 2.2786 | 16.0166 | 1.69E+06 | 0.05 |
| ETH-USD | 2.26E+05 | -0.1670 | -0.8034 | 2.4812 | 17.3255 | -8.80E+06 | -0.28 |
| BNB-USD | 7.78E+03 | -0.3099 | -1.4643 | 2.2821 | 16.4900 | 3.50E+06 | 0.11 |
| XRP-USD | 3.82E-03 | -0.1606 | -0.7520 | 1.2040 | 6.3251 | -2.00E+07 | -0.65 |
| LTC-USD | 1.99E+02 | -0.1246 | -0.5923 | 2.3517 | 15.6379 | 2.10E+07 | 0.68 |

The annual-return trajectory (fig3;top-left) reveals that every asset's return shifted downward in the 2022–2024 stress window, even after sMOM scaling. Sharpe contribution and the correlation heat-map (top-right block) show BTC still dominates risk-adjusted performance, while MSE remains negatively correlated with Sharpe across strategies. Panels in the bottom row visualize how forecast error "flows" into gains and confirm that cMOM Sharpe is uniformly negative for all coins in this window, underscoring the relative importance of downside-volatility control.




| | MSE_test | AnnRet_c | Sharpe_c | AnnRet_s | Sharpe_s |
|---|---|---|---|---|---|
| MSE_test | 1 | -0.26 | -0.5 | -0.41 | -0.6 |
| AnnRet_c | -0.26 | 1 | 0.96 | 0.98 | 0.91 |
| Sharpe_c | -0.5 | 0.96 | 1 | 0.99 | 0.98 |
| AnnRet_s | -0.41 | 0.98 | 0.99 | 1 | 0.97 |
| Sharpe_s | -0.6 | 0.91 | 0.98 | 0.97 | 1 |

**Fig. 3:** Multi-metric of returns and risk relationships.

# 5 Conclusions

## 5.1 Results Summary

The empirical evaluation conducted across BTC-USD, ETH-USD, BNB-USD, XRP-USD, and LTC-USD yields three principal insights. First, the hybrid neural–fuzzy PDE framework demonstrates asset-dependent predictive accuracy. As reported in Table 1, lower mean squared error (MSE) values are observed for mid-range cryptocurrencies such as XRP (MSE = 0.0034) and ETH (MSE =

11320.17), while more volatile and higher-priced assets like BTC exhibit elevated error levels (MSE = 2,681,640.5). This disparity suggests that the model is more adept at capturing moderate volatility dynamics than abrupt regime shifts in highly speculative assets.Second, the performance difference between the cMOM and sMOM strategies is not uniformly in favor of downside-adjusted scaling. While sMOM achieves slightly better Sharpe ratios for ETH and XRP and shows lower downside exposure and smoother allocation behavior (see Tables 2 and 3), the cMOM strategy sometimes achieves better average returns, such as in BTC and LTC. These mixed results emphasize the importance of asset-specific volatility patterns, calling for adaptive allocation frameworks rather than universal exposure rules. The use of out-of-sample validation, sensitivity tests, and Jobson–Korkie significance testing further strengthens the robustness of the reported findings.Third, the model prediction error structure reveals strong coverage under 95% confidence bounds, especially for XRP and LTC (see Table 4), reinforcing the reliability of neural-corrected outputs over time. Notably, all assets showed negative average prediction bias, suggesting a conservative error profile.In comparison with prior literature, the proposed framework offers significant improvements in both predictive and strategic dimensions. Traditional models such as GARCH and mean–variance optimization often underperform in the presence of nonlinearities, skewed returns, and tail-heavy risk distributions common in crypto markets [3,4]. Meanwhile, standalone machine learning or fuzzy inference techniques may lack the structural grounding necessary for reliable price modeling across asset classes [7,8,11]. The proposed hybrid PDE–Neural–Fuzzy architecture, anchored in mathematical valuation and adaptive enhancement, yields lower forecasting errors and improved Sharpe ratios, while also preserving interpretability and analytical rigor (see Table 6). By integrating data-driven learning and principled modeling, the framework overcomes common weaknesses of siloed methodologies focused solely on statistical or traditional finance tools [9,13,20].

**Table 6:** Comparative Performance of Hybrid Model vs. Benchmark Models

| Model | Predictive Accuracy (MSE ↓) | Sharpe Ratio (↑) | Risk Capture Capability |
|---|---|---|---|
| Hybrid PDE + Neural-Fuzzy | Lowest across assets | Moderate to High | Captures volatility clusters and tail risks |
| Standard VaR | Moderate | Low | Underestimates tail risk |
| GARCH | Moderate | Low to mid | Struggles with jumps and regime changes |
| Markowitz Mean-Variance | High error | Low to mid | Ignores higher-order risk dependencies |

*Note: ↑ indicates higher is better, ↓ indicates lower is better*

## 5.2 Conclusions and Implications

The hybrid PDE–Neural–Fuzzy framework adapts well to the nonlinear, regime-shifting behavior of cryptocurrency prices, achieving lower forecast error and better drawdown control than traditional benchmarks (Table 1; Figure 1). Its strength is most evident in moderate-volatility assets (ETH-USD, XRP-USD), where both MSE and downside risk metrics improve markedly (Tables 2–4). However, the evidence also shows that scaled-volatility momentum (sMOM) does not uniformly dominate: in highly volatile coins such as BTC-USD and BNB-USD, cMOM still delivers higher Sharpe ratios, underscoring the need for asset-specific or regime-adaptive exposure rules.Three limitations remain. (i) The PDE component uses a simplified, single-step log-normal proxy, which may mis-price extreme jumps. (ii) The fuzzy-rule base is static; integrating real-time sentiment or on-chain metrics could recalibrate memberships dynamically. (iii) The neural layer is a shallow feed-forward network; sequence-aware models (e.g., LSTM or Transformer) may better capture temporal dependencies. Future work should therefore refine PDE calibration (e.g., Crank–Nicolson with stochastic volatility), adopt adaptive fuzzy sets driven by live sentiment feeds, and benchmark against deep econometric and reinforcement-learning baselines. Finally, embedding dynamic leverage constraints in line with Basel risk limits would enhance the framework's practical viability during liquidity shocks and prolonged bear markets.

## 6 Declarations

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**Consent for Publication:** Not applicable.

**Availability of Datasets:** The study data are available upon reasonable request from the corresponding author.

**Authors' Contributions:** The first author and Dr. M. A. Afshar Kazemi developed the study design, protocol, and conducted the data collection. All authors contributed to the data analyses, writing the initial drafts of the article, and reviewed and approved the final version of the manuscript prior to submission for publication.

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