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Enhancing DOA Estimation: MUSIC Algorithm and Comparative Analysis of TOA, TDOA, and AOA Techniques

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Abstract

DOA estimation plays an important role in array signal processing, and has a wide range of application. In this paper, I will describe what DOA (Direction of arrival) estimation is, and give a mathematical model of DOA estimation. Then estimate DOA based on the MUSIC algorithm, and give some simulations with MATLAB to simulate what factors can affect the accuracy and resolution of DOA estimation when using the MUSIC algorithm. An angle of arrival (AOA) estimator is presented. Many applications require accurate AOA estimates such as wireless positioning and signal enhancement using space-processing techniques. This paper examines methods of Direction-finding TOA, TDOA, and AOA. Experimental results confirm the methods success. Signal routing is very important in telecommunications and we have explained some methods in this article and specifically we have explained an AOA method. We described the results and simulations in the result section. We compared our method with other methods and our method shows a better result.

Keywords: Direction finding, TDOA, AOA, TOA, DOA.
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1. Introduction

Accurate estimation of the angle of arrival (AOA) of a received signal can significantly enhance signal reception [1, 2] and improve wireless positioning accuracy [3, 4]. If AOA estimates are inaccurate, wireless devices may not be located precisely, or the bit error rate (BER) of the received signal may increase. To address these challenges, researchers have developed and refined various AOA estimators to improve their accuracy. In [5], an iterative AOA estimation method is introduced, which relies on the generalized expectation-maximization algorithm. In [6], the authors propose an AOA estimator specifically designed for circular antenna arrays. Another widely used AOA estimation technique is the TLS-ESPRIT algorithm, presented in [7], which employs subspace-based methods to estimate the AOA. Additionally, [8] introduces an iterative approach for AOA estimation. In [9], the authors suggest estimating the AOA by analyzing the frequency domain of the received signals, which involves applying a Fourier transform. Direction finding has a wide range of applications in wireless communication, sensor networks, and military systems [10]. Array signal processing is a significant branch within the field of signal processing, having experienced substantial advancements in recent years. Its applications span various domains, including radar, communication systems, sonar, seismology, exploration, astronomy, and biomedicine.

In this context, digital processors implement algorithms such as least squares, cosine-function, and correlation-coefficient methods [11] to estimate the angle of arrival (AOA). In architectures utilizing digital receivers, the phase difference for each antenna pair can be conveniently calculated using a field-programmable gate array (FPGA) integrated into the digital receiver [12]. The paper is organized as follows: Section 2 investigates the system model, including methods for direction finding such as Time of Arrival (TOA), Time Difference of Arrival (TDOA), and Angle of Arrival (AOA). In Section 3, the MUSIC algorithm is discussed in detail. Section 4 presents the results of various simulations, where key parameters influencing the implementation of the proposed ideas are varied and analyzed. Finally, Section 5 provides the conclusions and outlines potential directions for future work.

2. Measurement Models

In this section, the models and assumptions for the TOA, TDOA, and AOA measurements are described. Let $\mathbf{x} = [x, y]$ *T be* the radar position to be determined and let the known coordinates of the ith BS be $\mathbf{x_i} = [\mathbf{x_i}, \mathbf{y_i}]^T$, $i = 1, 2, . . . ,M$, where the superscript T denotes the transpose operation and M is the total number of receiving ESs (Electronic Support). The distance between the radar and the ith ES, denoted by di, is given by :

$$d_i = \sqrt{(x - x_i)^2 + (y - y_i)^2} \quad i = 1.2...., M, \tag{1}$$

A) AOA measurement

The AOA of the transmitted signal from the radar at the ith ES, denoted by φ_i, is related to $\mathbf{x}$ and $\mathbf{x}_i$ by:

$$\tan(\Phi_i) = \frac{y - y_i}{x - x_i}, \qquad i = 1,2,..., M \tag{2}$$

Geometrically, φ_i is the angle between the LOB from the ith ES to the radar and the x-axis. The AOA measurements in the presence of angle errors, denoted by $\{r_{AOA,i}\}$, are modeled as [13]

$$r_{AOA,i} = \Phi_i + n_{AOA,i} = tan^{-1}\left(\frac{y - y_i}{x - x_i}\right) + n_{AOA,i}, \quad i = 1,2, \dots, M, \tag{3}$$

Where $n_{AOA,i}$ is the noise in $r_{AOA,i}$. Equation (3) can also be expressed in vector form as:

$$r_{AOA} = f_{AOA}(x) + n_{AOA},$$
$$r_{AOA} = [r_{AOA,1} r_{AOA,2} \cdots r_{AOA,M}]^T$$
$$n_{AOA} = [n_{AOA,1} n_{AOA,2} \cdots n_{AOA,M}]^T,$$

$$f_{AOA}(x) = \begin{bmatrix} \tan^{-1}\left(\frac{y - y_1}{x - x_1}\right) \\ \tan^{-1}\left(\frac{y - y_2}{x - x_2}\right) \\ \vdots \\ \tan^{-1}\left(\frac{y - y_M}{x - x_M}\right) \end{bmatrix} \tag{4}$$

To facilitate the development and analysis of the proposed location algorithms, we make the following assumptions for the TOA, TDOA, RSS, and AOA measurements. [13]. (A1) All measurement errors, namely, $\{n_{TOA,i}\}$, $\{n_{TDOA,i}\}$, $\{n_{RSS,i}\}$, and $\{n_{AOA,i}\}$ are sufficiently small and are modeled as zero-mean Gaussian random variables with known covariance matrices, denoted by **Cn**, TOA, **Cn**, TDOA, **Cn**, RSS, and **Cn**, AOA, respectively. The zeromean error assumption implies that multipath and non-line-of-sight (NLOS) errors have been mitigated.

(A2) For RSS-based location, the propagation parameter a is known and has a constant value for all RSS measurements.

The basic formula for the RSS (Received Signal Strength) model is as follows: $(\mathrm{P(d)} = \mathrm{P}(d_0) - 10 \cdot \mathrm{a} \cdot log_{10}\left(\frac{d}{d_0}\right) + X_\sigma)$ where:

- $P(d)$: received power (in dB) at distance d
- $P(d_0)$: reference received power at reference distance d_0 (usually 1 meter)
- a: path loss exponent (propagation parameter)
- d: distance between transmitter and receiver
- d_0: reference distance (e.g., 1 meter)
- X_σ: a random noise term (normally distributed) to model environmental noise and fading

This formula models the relationship between received signal strength and distance in RSS-based positioning systems.

(A3) The numbers of BSs for location using the TOA, TDOA, RSS, and AOA measurements are at least 3, 4, 3, and 2, respectively.

B) TOA measurement

The TOA is the one-way propagation time taken for the signal to travel from the radar to a ES. In the absence of disturbance, the TOA measured at the ith ES, denoted by t_i, is

$$t_i = \frac{d_i}{c}, \quad i = 1,2, \dots, M, \tag{5}$$

Where c is the speed of light. The range measurement based on ti in the presence of disturbance, denoted by $r_{TOA,i}$, is modeled as:

$$r_{TOA,i} = d_i + n_{TOA,i}$$
$$= \sqrt{(x - x_i)^2 + (y - y_i)^2} + n_{TOA,i} \quad i = 1.2...., M, \tag{6}$$

where $n_{TOA,i}$ is the range error in $r_{TOA,i}$. Equation (6) can also be expressed in vector form as

$$r_{TOA} = f_{TOA}(x) + n_{TOA},$$
$$r_{AOA} = [r_{TOA,1} r_{TOA,2} \dots r_{TOA,M}]^T \tag{7}$$
$$n_{AOA} = [n_{TOA,1} n_{TOA,2} \cdots n_{TOA,M}]^T,$$

$$f_{AOA}(x) = \begin{bmatrix} \sqrt{(x - x_1)^2 + (y - y_1)^2} \\ \sqrt{(x - x_2)^2 + (y - y_2)^2} \\ \vdots \\ \sqrt{(x - x_M)^2 + (y - y_M)^2} \end{bmatrix} \tag{8}$$

C) TDOA measurement

The Time Difference of Arrival (TDOA) refers to the difference in the Times of Arrival (TOAs) of a radar signal at a pair of Electronic Stations (ESs). By assigning the first ES as the reference point, it can be readily deduced that the range measurements derived from the TDOAs take the following form [13]:

$$r_{TDOA,i} = (d_i - d_1) + n_{TDOA,i}$$
$$= \sqrt{(x - x_i)^2 + (y - y_i)^2} - \sqrt{(x - x_1)^2 + (y - y_1)^2} + n_{TDOA,i} \qquad i = 2,3,...,M, \tag{9}$$

where $n_{TDOA,i}$ is the range error in $r_{TDOA,i}$. Notice that if the TDOA measurements are directly obtained from the TOA data, then $n_{TDOA,i} = n_{TOA,i} - n_{TOA,1}$, $i = 2, 3, \ldots, M$. In vector form, (9) becomes

$$r_{TDOA} = r_{TDOA}(x) + n_{TDOA}$$
$$r_{TDOA} = [r_{TDOA,2} r_{TDOA,3} r_{TDOA,M}]^T \tag{10}$$
$$n_{TDOA} = [n_{TDOA,2} n_{TDOA,3} n_{TDOA,M}]^T$$

$$f_{TDOA}(x) = \begin{bmatrix} \sqrt{(x - x_2)^2 + (y - y_2)^2} - \sqrt{(x - x_1)^2 + (y - y_1)^2} \\ \sqrt{(x - x_3)^2 + (y - y_3)^2} - \sqrt{(x - x_1)^2 + (y - y_1)^2} \\ \vdots \\ \sqrt{(x - x_M)^2 + (y - y_M)^2} - \sqrt{(x - x_1)^2 + (y - y_1)^2} \end{bmatrix} \tag{11}$$

D) DOA estimation

Spatial spectrum estimation is a specialized signal processing technology that employs spatial arrays to estimate parameters of signals in space. The entire spatial spectrum estimation system consists of three key components: the incident signal space, the spatial array receiver, and parameter estimation. These components can be conceptualized as corresponding to three stages: the target stage, the observation stage, and the estimation stage [14].

In array signal processing, when an antenna array receives multiple signals from unknown sources, both the signal source and the transmission channel may be completely unknown and time-varying. This uncertainty in the transmission channel is one of the primary factors that limit high-resolution DOA estimation. To address this challenge, researchers have introduced the concept of blind DOA estimation [15].

DOA (Direction of Arrival) refers to the direction of incoming radio waves relative to the orientation of the array antenna. If the received radio waves satisfy the far-field narrowband condition, the wavefronts can be approximated as planar. The angle between the normal vector of the array and the direction vector of the plane wave is defined as the Direction of Arrival (DOA).

The objective of DOA estimation is to estimate the DOA values (θ) of multiple signals using N snapshots of data: $X(1),\ldots,X(N)$. For far-field narrowband signals, there is a propagation delay when the same signal reaches different array elements. This delay results in a phase difference between the signals received at the array elements. By analyzing these phase differences, the azimuth of the incoming signal can be estimated, which forms the fundamental principle of DOA estimation [16].

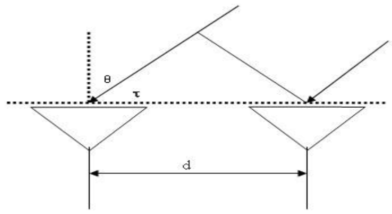

Fig. 1. The principle of DOA estimation

For instance, Fig. 1 considers two array elements,

d is the distance between the array elements,

c is the speed of light,

θ is the incident angle of the far field signal,

τ is the time delay of the array element.

The signal received by the antenna due to the path difference is

$$\tau = \frac{d \sin\theta}{c}, \tag{12}$$

Thus, one can obtain the phase difference between the array elements as

$$\phi = e^{-jw\tau} = e^{-jw\frac{d \sin\theta}{c}} = e^{-j2\pi\frac{d \sin\theta}{\lambda f_0}f}, \tag{13}$$

Where f_0 is the center frequency. For narrow band signals, the phase difference is

$$\phi = e^{-j2\pi\frac{d \sin\theta}{\lambda}} \tag{14}$$

Where λ represents the wavelength of the signal. Therefore, if the time delay of the signal is known, the direction of the signal can be determined using (12). This forms the fundamental principle underlying spatial spectrum estimation techniques [19].

3. MUSIC estimation

MUSIC is a technique used to determine the parameters of multiple wave fronts arriving at an antenna array from measurements made on the signals received at the individual elements [17].

The waveforms received at the N array elements are linear combinations of the incident wave fronts from M narrowband signals and noise. Thus, the multiple signal classification approach begins with the following model for characterizing the receive vector G as

$$G = AF + V \tag{15}$$

Where V a is noise vector. Expanding Eq. (15);

$$\begin{bmatrix} g_1 \\ g_2 \\ \cdot \\ \vdots \\ g_N \end{bmatrix} = \begin{bmatrix} a_1(\theta_1) & a_1(\theta_2) & \Lambda & a_1(\theta_M) \\ a_2(\theta_1) & a_2(\theta_2) & \Lambda & a_2(\theta_M) \\ \Lambda & \Lambda & \Lambda & \ldots \\ \Lambda & \Lambda & \Lambda & a_N(\theta_M) \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ \cdot \\ \vdots \\ F_N \end{bmatrix} + \begin{bmatrix} V_1 \\ V_2 \\ \cdot \\ \vdots \\ V_N \end{bmatrix} \tag{16}$$

The size of each matrix is as follows: *G:* vector of element outputs, *N* by 1, *A:* matrix of mode vectors, *N* by *M*, *F:* vector of signals, *M* by 1, *V:* noise vector, JV by 1, The incident signals are represented in amplitude and phase at some arbitrary reference point, for instance, the origin of the coordinate system, by the complex vector *F*. The elements of *G* and *A* are also complex in general. The $a_N(\theta_M)$ are functions of the signal arrival angles

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and the array element locations. That is, *a,,(ßm)* depends on the nth array element, its position relative to the origin of the coordinate system, and its response to a signal incident from the direction of the *mth* source. The *mth* column of A can be considered as a 'mode' vector of responses to the direction -of arrival *dm* of the *mth* signal. This JV by 1 mode vector will be denoted by $a(\theta)$. The M by M covariance matrix of the G vector is

$$S \triangleq E[GG^*] = \overline{GG^*} = A\overline{FF^*}A^* + \overline{VV^*} \quad (17)$$

Where E is expectation operator and the asterisk denotes complex conjugation. Eq. (17) Can be rewritten as *S = APA* +AS0,* where *P=FF* is* a diagonal *M-by-M* matrix and S0 is the noise covariance matrix. When the number of incident wavefronts M is less than the number of array elements JV, then *APA** is singular. It has a rank less than *N*; therefore, *APA = \S-XSo\ = 0.'* This equation is only satisfied with A (not to be confused with wavelength) equal to one of the. Eigenvalues, of *S* in the matrix of *S0.* Therefore, A can only be the minimum eigenvalue, *Amin.* Observe that any vector orthogonal to A is an eigenvector of S with the eigenvalue *a2*, the noise variance. Hence, we can write

$$S = U\Lambda U^H = U_S\Lambda_S U_S^H + U_N\Lambda_N U_N^H \quad (18)$$

Where $A = diag\,\{\lambda_1.\lambda_1.\ldots.\lambda_M\}$ is a diagonal matrix of real eigenvalues. The superscript H represents the Hermitian, U_s is the signal eigenvectors and U_N is the noise eigenvectors. The noise eigenvectors can be used to form an estimator for the spatial spectrum

$$p_{MU}(\theta) = \frac{1}{a(\theta)^H U_N U_N^H a(\theta)} \quad (19)$$

Although MUSIC is highly robust it requires characterization of the array response and a spatial search through all possible angles of arrival.

Multiple Signal Classification (MUSIC) algorithm was proposed by Schmidt and is colleagues in 1979 [18]. It has created a new era for spatial spectrum estimation algorithms. The promotion of the structure algorithm characterized rise and development, and it has become a crucial algorithm for theoretical system of spatial spectrum. Before this algorithm was presented, some relevant algorithms directly processed data received from array covariance matrices. The basic idea of MUSIC algorithm is to conduct characteristic decomposition for the covariance matrix of any array output data, resulting in a signal subspace orthogonal with a noise subspace corresponding to the signal components. Then these two orthogonal subspaces are used to constitute a spectrum function, be got though by spectral peak search and detect DOA signals. It is because MUSIC algorithm has a high resolution, accuracy and stability under certain conditions that it attracts a large number of scholars to conduct in-depth research and analyses. In general, it has the following advantages when it is used to estimate a signal's DOA. The ability to simultaneously measure multiple signals.

- High precision measurement.
- High resolution for antenna beam signals.
- Applicable to short data circumstances.
- It can achieve real-time processing after using high-speed processing technology [19].

Direction of Arrival methods, also called Angle of Arrival (AOA), attempt to estimate a position based on comparing multiple readings of the direction that the signal came from. The most common method of obtaining a DOA reading utilizes the TDOA measurements of individual antennas in a single antenna array to determine the most likely direction that a signal arrived from. However, this method usually involves a larger antenna array than is practical for a single small vehicle. In this case, a simpler and older method is used, which involves rotating an antenna to locate the direction from which a signal provides the largest amplitude response [20].

In order analyses and derive more conveniently, now assume the following conditions for the ideal mathematical model of DOA problems.

- Each test signal source has the same but unrelated polarization. Generally, consider that the signal sources are narrow bands, and each source has the same center frequency ω0. The number of testing signal source is D.
- Antenna array is a spaced linear array which consists of M (M>D) array elements; each element has the same characteristics, and it is isotropic in each direction.
- The spacing is d, and the array element interval is not larger than half the wavelength of the highest signal frequency.
- Each antenna element in the far field source, namely, an antenna array receiving the signals coming from the signal source is a plane wave.
- Both array elements and test signals are uncorrelated; variance σ^2 is zero-mean Gaussian noise $n_m(t)$.
- Each receiving branch has the same characteristics.

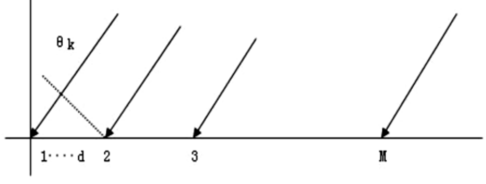

Fig. 2. The model of DOA estimation

DOA estimation results can be affected by many factors, not only related to the source of the incoming signal, but also related to the actual application environment [21].

A) Number of array elements

The number of array elements in basic arrays can affect the estimation performance for super resolution algorithm. Generally speaking, if array parameters are the same, the greater number of array elements, the better estimation performance for super resolution algorithm.

B) Snapshots

In the time domain, the number of snapshots is defined as the number of samples. In the frequency domain, the number of snapshots is defined as the number of time sub-segments of discrete Fourier transform (DFT).

C) SNR

Assuming the signal and noise have a flat pass band power spectral density, and the power of signal source is σ2p, noise power isσ2n, and then in this case SNR can be defined as

$$SNR = 20\, log \left(\frac{\sigma_p}{\sigma_n}\right) \quad (20)$$

SNR directly affects the performance of super-resolution DOA estimation algorithm. At a low SNR, super-resolution algorithm performance would drop dramatically. As thus, how to improve the algorithm under a low SNR is the research focus for the sup-resolution DOA algorithm [22].

D) The coherence of the signal source

The problem involving coherent sources is a fatal problem for subspace algorithms. When there is a coherent signal in the signal source, the signal covariance matrix is no longer for the non-singular matrix. In this case, the original super-resolution algorithm will fail. Therefore, it will greatly affect the performance of DOA estimation. In addition to the factors mentioned above, many other factors can affect the performance of DOA estimation in practical applications, such as the array element amplitude and phase inconsistencies, mutual coupling between array elements, and the wrong position of sensors.

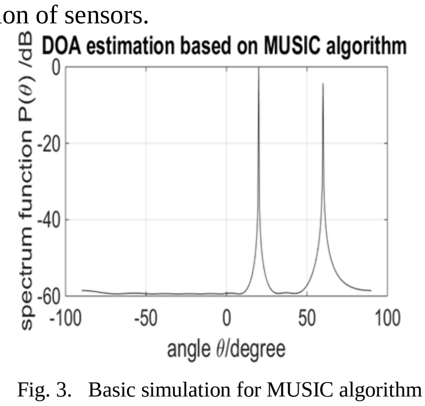

Fig. 3. Basic simulation for MUSIC algorithm

4. Simulation

A) Basic simulation of the MUSIC algorithm for DOA estimation

The first simulation demonstrates the ability of the MUSIC algorithm to distinguish between two signals. The scenario involves two independent narrowband signals with incident angles of 20° and 60°, respectively. These signals are uncorrelated, and the noise is modeled as ideal Gaussian white noise. The signal-to-noise ratio (SNR) is set to 20 dB, and the element spacing is configured to be half the wavelength of the input signal. The array consists of 10 elements, and the number of snapshots is fixed at 200. The simulation results are presented in Figure 3.

As shown in Figure 3, for a hypothetical scenario involving two independent signals, the MUSIC algorithm can construct a needle-like spectrum peak. This allows for accurate estimation of both the number and direction of the incident signals, effectively determining the DOA (Direction of Arrival) of independent signal sources. Under an accurate model, the DOA estimation can achieve arbitrarily high precision, overcoming the traditional limitations of low-resolution methods. Consequently, the MUSIC algorithm provides high-resolution and high-precision solutions for direction-finding problems in environments with multiple signals. The high-resolution MUSIC algorithm exhibits characteristics such as high accuracy, high sensitivity, and the potential to handle multi-resolution signals. With its superior performance and efficiency, it delivers high-resolution and asymptotically unbiased DOA estimates, which hold significant importance for practical applications [19].

B) The relationship between DOA estimation and the number of array elements

The second simulation involves two independent narrowband signals with incident angles of 20° and 60°, respectively. These signals are uncorrelated, and the noise is modeled as ideal Gaussian white noise. The signal-to-noise ratio (SNR) is set to 20 dB, and the element spacing is configured to be half the wavelength of the input signal. The number of array elements is varied, with simulations conducted for 10, 50, and 100 elements, while the number of snapshots is fixed at 200. The simulation results are presented in Figure 4.

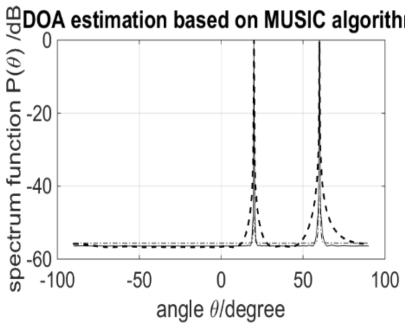

Fig. 4. Simulation for the relationship between MUSIC algorithm and the number of array elements

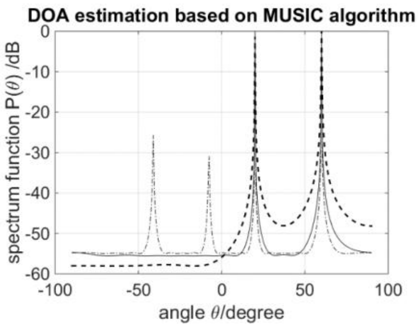

Fig. 5. Simulation for the relationship between MUSIC algorithm and array element spacing

As shown in Figure 4, the dashed line represents the case where the number of array elements is 10, the solid line corresponds to 50 elements, and the dash-dotted line indicates 100 elements. Under otherwise identical conditions, increasing the number of array elements results in a narrower beam width in the DOA estimation spectrum, improving the directivity of the array. This enhancement strengthens the ability to distinguish spatial signals, thereby improving the resolution of the DOA estimation. However, increasing the number of array elements also increases the volume of data that needs to be processed, leading to higher computational complexity and slower processing speeds. From the figure, it can be observed that when the number of array elements reaches 50 or 100, their beam widths are very similar. Therefore, in practical applications, the number of array elements can be appropriately selected based on specific requirements, ensuring the accuracy of the estimated spectrum while minimizing resource waste and accelerating computational speed. By optimizing these factors, operational efficiency can be significantly improved [19].

C) The relationship between DOA estimation and the array element spacing

The third simulation shows there are two independent narrow band signals, the incident angle is 20° and 60 respectively, those two signals are not correlated, the noise is ideal Gaussian white noise, the SNR is 20dB, array element number is 10, the number of snapshots is 200, the array spacing is λ,$\lambda/2$, $\lambda/6$. The simulation results are shown in Figure 5:

As shown in Figure 4.3, the dashed line represents an array element spacing of $\lambda/6$, the solid line corresponds to a spacing of $\lambda/2$, and the dash-dotted line indicates a spacing of λ. Under identical conditions, when the array element spacing does not exceed half the wavelength ($\lambda/2$), increasing the spacing results in a narrower beam width in the DOA estimation spectrum. This improvement enhances the resolution of the MUSIC algorithm. However, when the spacing exceeds half the wavelength, false peaks appear in the estimated spectrum outside the signal source direction, leading to a significant loss in estimation accuracy.

Therefore, in practical applications, careful attention must be paid to the spacing of array elements. While increasing the spacing can improve resolution, it should not exceed half the wavelength, as this is a critical constraint. Ideally, the array element spacing should be set to $\lambda/2$ to achieve optimal performance.

5. Conclusion

This paper begins with an introduction to direction-finding methods, including Time of Arrival (TOA), Time Difference of Arrival (TDOA), and Angle of Arrival (AOA), and describes each method individually. Subsequently, simulations are performed using the MUSIC algorithm for Direction of Arrival (DOA) estimation. These simulations explore the relationship between the MUSIC algorithm and two key parameters: the number of array elements and the spacing between array elements. The simulation results are presented in the accompanying figures.

The methods of direction finding discussed in this paper are widely used in modern systems. For high-accuracy applications, hybrid direction-finding methods, such as combining AOA and TDOA, are often employed to achieve superior performance.

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