

The Effect of Applying the PDCF Model on the Performance of the General Mathematical Problem-Solving

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Abstract. Problem-solving is a primary and fundamental aspect of mathematics education and learning. Problem-solving can strengthen the connection between mathematics and other branches of science and has been an essential and often controversial research topic for several decades. The present research aims to introduce the new PDCF model and examine its impact on improving the performance of general mathematics problem-solving among university students. The present research method is quasi-experimental. The target group for this purpose was the University students, who were randomly selected into two groups, including the experimental and the control groups. The proposed four-stage PDCF model was applied to the experimental group, but the control group was examined for general mathematics problem-solving performance without any educational intervention. The tools used included a researcher-made general mathematics test in three stages of the proposed model. Additionally, a researcher-made feedback form was provided after each test. The findings of the present research indicate that in the second cycle of the new PDCF model in the experimental group the average scores in the experimental group significantly increased compared to the control group, which did not undergo the model. Levene's test showed that at the beginning of the study and before any intervention, the two groups had equal variance. The results of the Shapiro-Wilk test indicated that the data had good normality. The results of the analysis of covariance indicated that after each cycle of the PDCF model in the experimental group, the post-test scores showed a significant difference from the pre-test of the same cycle.

Received: 18 February 2025; Revised: 23 April 2025; Accepted: 10 May 2025.

Keywords: Mathematics Education; Problem-Solving; Continuous Improvement Process.

AMS Subject Classification: 00A35; 97D50.

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1. Introduction

In the present era, reaching a society where citizens possess useful infrastructures of mathematical knowledge and skills, in addition to the capability to think and reason mathematically, is essential. Achieving this requires continuous improvement in mathematics education and learning, which in turn necessitates identifying and understanding the problems that hinder learners' progress. These problems may originate either within or outside the realm of mathematics. Extra-mathematical problems themselves can be studied in two sections: intra-personal or inter-personal. Those extra-mathematical problems that have intra-personal origins stem from the individual characteristics of learners in mental processing, learning, motivations, and attitudes. In

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contrast, inter-personal problems are influenced by cultural, social, educational factors, and the manner of teaching and interaction by educators.

Mathematics Education (Education Mathematics) is a branch of human knowledge and science that has received attention in scientific circles in recent years, especially in developed countries. As an interdisciplinary specialty, mathematics education addresses and responds to questions that require knowledge from other sciences. Therefore, the topics discussed in this field vary in quantity and quality, ranging from the most detailed to the most comprehensive issues, including the nature and content of mathematical knowledge, individual differences, learning and teaching styles, problem-solving (Problem Solving), assessment and evaluation. Modern views on mathematics education emphasize the importance of thinking and reasoning, understanding mathematical concepts, and how they are processed. The goal of a mathematics educator is to optimize the learner's mathematical learning experience both mentally and emotionally and to improve learners' mathematical learning, or to seek the roots of learners' inability to learn mathematics. Hence, anything related to mathematics education falls within the scope of mathematics education.

Mathematical Problem-solving is a primary and fundamental aspect of mathematics education and learning and has been a crucial topic of research and often controversial for several decades. According to Schoenfeld, Problem-solving is achieved through immersing oneself in the process of solving and the ability to correctly utilize knowledge resources [1]. Mathematical Problem-solving is a process that includes a set of factors and tasks to achieve a specific goal, and since it depends on various factors and skills, it is considered a challenge in both learning and teaching. The goal of a mathematics educator is always to improve the processes of mathematics education and learning, including mathematical Problem-solving. Evaluating and measuring learners' mathematical performance is an assessment of their reasoning ability and thinking power, problem-solving ability, and the ability to create meaningful connections between concepts and other mathematical categories. Administering exams throughout the educational period is a way to evaluate learners' understanding, comprehension, and learning of the course material. The mathematics test score is an educational action by the educator to inform about the learner's academic performance. Exam scores are a useful source of information for understanding learners' performance and can help the educator to make changes and improve the learner's learning and performance; however, scoring alone does not necessarily lead to subsequent learning or improvement [15]. One of the important outcomes of evaluating mathematical problem-solving performance is using the evaluation results to improve learners' learning and performance. Therefore, a mathematics educator should use the evaluation results to guide the learner towards continuous improvement in mathematical performance. Continuous improvement is equivalent to the Japanese term "Kaizen," which consists of two parts: "Kai," meaning change, and "Zen," meaning good, implying gradual and continuous improvement. Continuous improvement is based on the philosophy that to improve processes, there is no need for explosive or sudden changes; rather, any improvement or modification, as long as it is continuous and persistent, leads to performance enhancement and process growth. Continuous improvement involves identifying and making changes that result in better outcomes [24]. One of the effective methods presented in continuous process improvement is the Deming Cycle or PDCA cycle. It is a simple four-stage method, including planning, doing, checking, and acting, yet it is continuous and effective for making impactful changes in all processes [26]. In the Deming Cycle, the results obtained in each execution are compared with previous results, which is one of the advantages of this cycle, and in these repetitions, the improvement and progress achieved can be easily measured. The four stages of the Deming Cycle are as follows [14].

Step 1: Plan: The first step is to have a plan for improvement by identifying problems and offering ideas for their solutions.

Step 2: Do: Implementing the planned program and changes.

Step 3: Check: Results should be continuously reviewed to assess the impact of changes for improvement and to identify new problems.

Step 4: Act: Taking corrective actions to achieve better and more desirable results.

In a study conducted in 2019 at one of the universities in Florida, the Deming continuous improvement cycle and interdisciplinary dialogues (as tools) were used to improve student performance in introductory mathematics courses. This study provided opportunities for creating a shared vision for achieving a specific goal, utilizing the strengths and expertise of participants in dialogue, and reflecting on what needs to be emphasized in these courses [9].

In a 2018 study conducted at one of the universities in Bangladesh, the PDCA cycle was used to develop the skills of industrial engineering students. The results indicated that this cycle significantly contributed to the skill development of students during their undergraduate studies theoretically [8].

In another study conducted in 2011 at the School of Engineering at Borås University in Sweden, the Deming cycle was used to improve educational quality. This study used two assessment tools, including organizational excellence and student surveys, and the results from these assessments were used through the Deming cycle for continuous improvement in educational quality. The findings indicated that the Deming cycle was beneficial and effective in improving educational quality [7,12].

One of the common models for solving mathematical problems is George Polya's four-step model. Polya modelled the problem-solving thought process. This model is dependent on the learner and the learner has a direct role in it[24]. The four steps of George Polya's model are as follows:

1. Understanding the Problem (Understand): Identifying the data and requirements of the problem and the relationship between them.
2. Planning (Plan): Deciding on a strategy to solve the problem.
3. Solving (Solving): Acting to solve the problem based on the selected strategy and understanding of the problem.
4. Reviewing (Review): Revisiting the solution and the methods used to ensure that the obtained answer meets the problem's requirements.

Darash and colleagues conducted a study on the impact of teaching mathematics using George Polya's method on students' problem-solving skills and academic progress in mathematics. The findings of the research showed that teaching problem-solving skills to students positively affects their academic progress in mathematics [8]. Studies on mathematics education indicate that teaching mathematics has less contributed to developing problem-solving skills in students. Therefore, it is necessary to use specific strategies to enhance and improve the current situation. It is believed that increasing conceptual and procedural knowledge enhances students' problem-solving skills and boosts their confidence to engage in problem-solving activities.

Caprara conducted a study titled "Problem-Solving: The Goal and Tool of Learning Mathematics in School." The results showed that among all school subjects, teaching problem-solving skills and related concepts is fundamental to all school learning. In mathematics, Problem-solving represents an effective and usable concept for constructing and reconstructing concepts, transferring operational and general mathematical knowledge to ensure sustainable and meaningful learning [3]. Ersoy and colleagues stated that Problem-solving positively impacts improving teachers' mathematical Problem-solving skills and has a positive effect on mathematical thinking [10]. Marchis concludes that problem-solving activities can provide opportunities for students' autonomous learning, encouraging them to explore, seek the truth, develop their ideas, and discover the solution to the problem [19].

Fanoo-Pichat and colleagues found in their research that the main issue for students in the problem-solving process lies in understanding mathematical problems. The research results indicated that:

1. Students cannot identify the keywords in the problem statement.
2. Students cannot determine what the problem is asking of them.
3. When students cannot find a solution, they resort to guessing without thinking.
4. Students are impatient and do not read the problem statement carefully and thoroughly.
5. Students do not read long problem statements [23].

Progress and improvement in learning are influenced by both knowledge structures and information processing processes. One of the key factors in information processing is feedback [12]. According to behaviourist theories, including those of Thorndike and Skinner, feedback is one of the essential components of learning [4]. Feedback is a type of commentary based on assessment or evaluation results [5].

Feedback is necessary as an action by the mathematics educator after each test or exam because learners need the educator to tell them why they made mistakes and the reasons behind them. Research on feedback shows that teachers' written explanations on assignments, including problem-solving, lead to improved performance [19]. Mitrovica, Olson, and Brova [20] believe that teacher feedback can significantly and directly impact students' perceptions of their abilities and efforts, thereby serving as a strategy to enhance academic performance.

Manoochehri and colleagues [21] found in their study that learners' weaknesses in mathematics at all levels are related to their weaknesses in Problem-solving, and problem-solving weaknesses are related to the lack of knowledge and processing of problem-solving strategies. Another study found that among all school subjects, teaching problem-solving skills and related concepts is fundamental to all school learning [7]. Another study showed that information processing and cognitive variables affect understanding problem-solving models [6]. In another research, it was found that the main issue for students in the problem-solving process is understanding mathematical problems [2]. Some studies indicate that success in mathematical Problem-solving depends on a combination of strong subject knowledge, knowledge of mathematical problem-solving strategies, and confidence.

According to the research by Gowash [18], feedback and its proper use result in improved student performance and talent development, leading to academic progress. The studies by Hattie and Timperley [17] on the effectiveness of feedback methods indicate that providing feedback to students is effective if it involves not only the outcome but also the awareness of errors, error correction, and how to engage with tasks and processing methods. Carless, Salter, Yang, and Lam [16] believe that effective feedback should place less emphasis on conventional techniques and more on enhancing students' autonomy and self-regulation capacities.

Butler and Nissan [4] examined the effects and scores of descriptive feedback versus no feedback on learning. The results indicated that students who received descriptive suggestions as feedback in the first session performed better in the final session. In another study, it was found that written feedback from the teacher improves self-efficacy in academics, including mathematical problem-solving [28]. The results regarding the impact of written feedback from the teacher in the classroom on self-efficacy and mathematical problem-solving among middle school students showed a significant difference in the problem-solving performance of the experimental group [4].

Most of the presented mathematical education models, including classical models, models based on experiential learning and feedback, as well as some modern approaches based on modeling and adaptive learning, focus only on problem solving or teaching. However, in the PDCF model presented in this study, we have designed a personalized and

cyclical feedback that plays an important role in the gradual improvement of problem solving performance. This model also goes beyond one-step learning and targets continuous improvement of problem solving. In this method, attention is paid to the human learning cycle with continuous feedback, and by involving the student in each cycle, this model increases the student's cognitive self-awareness of his/her weaknesses and strengths in problem solving. This model designs a feedback framework with an emphasis on gradual growth, and compared to models based on one-step education, this model focuses on sustainable and measurable human learning.

In recent years, many studies have been conducted on using mathematical modeling and problem-solving training with artificial intelligence. Some authors [9, 29] considered using language models combined with symbolic solvers for mathematical problem solving, and other researchers [22, 25, 27, 30] studied new methods for mathematical problem solving using mathematical modeling and machine learning.

The PDCF model is located in the gap between classical education and modern machine learning systems and can be considered as a bridge between interactive human learning and the process of continuous improvement. The PDCF model is located at the intersection of two approaches. On the one hand, it is similar to the Deming quality management model, and on the other hand, it is aligned with modern educational models that emphasize feedback and correction, such as modeling-based learning. However, its distinguishing point is the design of an iterative educational process with step-by-step feedback, whose main focus is active human learning and personalization, not just achieving a solution to the problem.

The innovation and novelty of this research are important in the following ways: 1. Combination of feedback and repetition: Unlike traditional models that consider learning as linear and single-stage, PDCF is based on a repetitive cycle where in each iteration, the level of learning is improved based on the data of the previous stage. 2. Dynamics and personalization: This model has systematic planning on the one hand and provides precise individual and personalized feedback on the other hand that is adjusted to the specific performance of each student. 3. Learning based on increasing cognitive self-awareness: By focusing on cycles of error analysis, correction, and retraining, this model helps strengthen the skill of thinking about thinking (metacognition).

Logothetis believes that there is a clear lack of providing education compatible with the real needs in the context of continuous improvement. Addressing each of the following questions can have effective results in laying the groundwork for achieving continuous improvement [18]:

1. How can we consciously structure students' activities to best develop the identified mathematical problem-solving capabilities?
2. How can we evaluate the degree to which students' performance in mathematical Problem-solving is enhanced?

In this research, we first introduce the new PDCF model and then answer this question: Does the implementation of the newly presented PDCF model in this article affect the general mathematical problem-solving performance of students?

2. New Model and Implementation Method

In this research, we introduce the PDCF model, which is an extension of George Polya's model, combined with the four steps of the Deming Cycle. The four steps of the PDCF model used in this research are as follows:

1. Step 1: Plan: Planning for improvement by identifying problems.
2. Step 2: Do: Executing or implementing the plan.
3. Step 3: Check: Continuously reviewing the results to assess the impact of changes for improvement.

4. Step4: Feedback: Providing feedback as an action and strategy for continuous improvement.

Figure 1 illustrates the overall view of the model used.

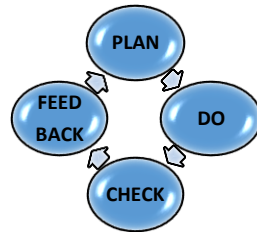


Figure 1. Overview of the PDCF model.

The purpose of the feedback in this research is aligned with the structure of the model used and is aimed at improving the learning process and mathematical problem-solving performance through effective feedback. Effective feedback is the feedback that is provided immediately after observing performance. The benefit of immediate feedback is that if the performance is correct, it reinforces the learner's motivation; if the performance is incorrect, it helps the learner to prevent repeating the mistake [19]. Therefore, if the educator answers the following questions, the feedback and comments will be effective:

- What was the main mistake?
- What is the probable reason for the learner's mistake?
- How can we guide the learner so that the mistake is not repeated?

In this research, we will use the PDCF model to reflect periodic improvement tools and periodic thought corrections for students throughout the process. During this process, we need to pay attention to how changes impact students' problem-solving performance and their attitudes towards the changes.

The statistical population of this study consisted of 300 students of Mathematics I at Islamic Azad University, who were in 10 classes with 30 students under the same conditions, and 74 of them were selected to participate in this study by cluster sampling. In this way, each class was considered as a cluster, the clusters were numbered from one to ten, then three clusters were randomly selected from these ten clusters, which included 90 people. A list of these people was prepared, numbered from 1 to 90, and then 74 people were randomly selected from the 90 people by generating random numbers. Finally, these people were randomly divided into two independent groups of 37 people, including the experimental group and the control group.

The research tools included a researcher-made general mathematics test, designed and prepared by the researcher to determine the students' general mathematics problem-solving ability for three test stages. Each test comprised 4 questions across 5 sections covering topics such as limits, continuity, and derivatives. Each question was worth 5 points, with the total score for each test being 20 points.

Another tool used was a researcher-made feedback report form, designed according to the PDCF model by the researcher, which was used only for the experimental group in this research. This form included not only the status of responses to each question but also the probable reasons for errors and mistakes in answering each question, along with strategies and improvement recommendations to address and prevent potential errors.

The implementation stages were in accordance with the proposed PDCF model, including planning, test administration, reviewing results, and implementing improvement actions (providing feedback based on the test results). These stages were explained to the students in the experimental group before the research began, but the control group students were not informed. During the course of this research, three tests were administered to both groups. Before each test, the date, time, and topics of the test were announced to the

participants (Step 1 of the model). In the first stage, a pre-test was administered (Step 2 of the model). In the next stage, the test results were reviewed (Step 3 of the model). Then, feedback was provided to the participants in the experimental group as an improvement action using the feedback form (Step 4 of the model), while no feedback was given to the control group. Considering that repeating the four steps of the model is an inherent characteristic of the model, the second test was administered again after two weeks, and the results were reviewed and compared with the first stage, and feedback was provided to the experimental group participants. The third test was then administered two weeks after the second test, and the results were reviewed and compared with the second stage, and the third feedback report was provided to the experimental group participants. The proposed PDCF model cycle in this research was repeated twice.

During the data collection stages, all students participated in all three tests, so there was no missing data in this study, and all analyses were performed based on the complete data set. Based on the power analysis for the independent t-test with a significance level of 5%, medium effect size, and statistical power of 0.85, it can be said that this sample size is sufficient to detect a statistically significant difference. This means that the selected sample size has adequate power to test the hypotheses of this study.

To evaluate content validity, the test questions were reviewed and approved by five Mathematics I course instructors before the test was administered. Since the test questions were descriptive, the correction key with details was also reviewed by another specialist, and a Pearson correlation was performed between the scores given by the researcher and the secondary scorer, resulting in a value of 0.86, indicating the reliability of the test [11, 13].

3. Findings

The findings of this section were conducted using R software, and the "pwr" and "dplyr" packages, and the "rand" function in Excel was used to generate random numbers. Table 1 shows that the average pre-test scores in the experimental and control groups were 6.11 and 6.16, respectively, indicating a very small difference of 0.05, which suggests similar performance in both groups. After implementing one cycle of the PDCF model, the average scores in the experimental and control groups were 15.84 and 5.95, respectively, showing a significant difference of 9.89. Therefore, with the implementation of one cycle of the new PDCF model, the average score in the experimental group increased significantly compared to the control group, and a remarkable improvement in problem-solving performance was observed. Additionally, the standard deviation (SD), first quartile (Q1), median (Q2), third quartile (Q3), minimum (Min), and maximum (Max) values related to the experimental and control groups in the pre-test and post-test emphasize the improvement in scores for the experimental group after one cycle of the PDCF model.

Table 1. Descriptive statistics indicators of general mathematical problem solving performance after two cycles of PDCF model.

Group	Total Number	Pre-Test							Post-Test						
		Mean	SD	Q1	Q2	Q3	Min	Max	Mean	SD	Q1	Q2	Q3	Min	Max
Experiment	37	6.11	2.77	3.00	6.00	8.00	2.00	11.00	15.84	2.30	15.00	16.00	17.00	10.00	19.00
Control	37	6.16	2.81	4.00	6.00	8.00	2.00	12.00	5.95	2.74	4.00	6.00	8.00	2.00	11.00

Table 2 shows that after completing one cycle of the PDCF model, the average post-test scores in the experimental and control groups were 15.84 and 5.95, respectively, indicating a significant difference of 9.89 points. After implementing the second cycle of the PDCF model, the average scores in the experimental and control groups were 18.03 and 6.05,

respectively, showing a considerable difference of 11.98 points. Thus, with the implementation of the second cycle of the new PDCF model, the average score in the experimental group increased significantly compared to the control group, and a remarkable improvement in problem-solving performance was observed. Additionally, the standard deviation (SD), first quartile (Q1), median (Q2), third quartile (Q3), minimum (Min), and maximum (Max) values related to the experimental and control groups in the pre-test and post-test emphasize the improvement in scores for the experimental group after two cycles of the PDCF model.

Table 2. Descriptive statistics indicators of general mathematical problem solving performance after two cycles of PDCF model.

Group	Total Number	Pre-Test							Post-Test						
		Mean	SD	Q1	Q2	Q3	Min	Max	Mean	SD	Q1	Q2	Q3	Min	Max
Experiment	37	15.84	2.30	15.00	16.00	17.00	10.00	19.00	18.03	1.36	17.00	18.00	19.00	14.00	20.00
Control	37	5.95	2.74	4.00	6.00	8.00	2.00	11.00	6.05	2.73	4.00	6.00	8.00	2.00	12.00

To examine the homogeneity of the experimental group before implementing the PDCF model and the control group, Levene's test was used, and the significance level value of 0.21 was obtained, which is greater than 0.05; thus, the null hypothesis indicating the equality of variance between the two groups is not rejected. This means that at the beginning of the study and before any intervention, the two groups had equal variance. To assess the normality of the data, we used the Shapiro-Wilk test, and the results are shown in Table 3.

Table 3 shows that in the first cycle of the PDCF model, the data used in the model for both the experimental and control groups are normal, as the p-value for the experimental group is 0.31 and for the control group is 0.15, both of which are greater than 0.05. Thus, the null hypothesis is not rejected, and the data are normally distributed. Similarly, in the second cycle of the PDCF model, the data used in the model for both the experimental and control groups are also normal, as the p-value for the experimental group is 0.81 and for the control group is 0.06, both of which are greater than 0.05, indicating that the null hypothesis of normality is not rejected.

Table 3. The results of the normality test (Shapiro Wilk) in the data of the first and second cycles of the PDCF model.

period	first cycle of PDCF		second cycle of PDCF	
Index	statistic	p-value	statistic	p-value
experimental group	0.9376	0.31	0.8384	0.81
control group	0.9556	0.15	0.9369	0.06

To investigate the effect of the new educational model on problem-solving performance, analysis of covariance (ANCOVA) was used because ANCOVA controls for student differences in the pretest and only shows the net effect of the PDCF model. In this analysis, pretest scores were entered as the covariate variable, first-cycle posttest scores as the dependent variable, and experimental and control groups as the independent variables. The results are shown in Table 4.

Table 4 shows that after the first cycle of the PDCF model in the experimental group, the scores in the post-test are significantly different from the pre-test of the same cycle because the significance level of the test (p-value) is less than 0.05, indicating that after applying the PDCF model, the problem-solving performance of the students in the experimental group has improved significantly. Also, the large effect (eta) size indicates the significant

effect of the PDCF model on improving the performance of the experimental group compared to the control group.

Table 4. Results of analysis of covariance on pre-test-post-test scores of academic achievement in the first cycle of PDCF.

Source of variation	Sum of Squares	df	Mean of Squares	F	Significance level	eta
Group	1824.75	1	1825.75	850.91	0.00	0.92
Pre-test	308.66	1	308.66	143.93	0.00	0.79
Residual error	152.26	71	2.14			
Total	2285.67	73				

Table 5. Results of analysis of covariance on pre-test-post-test scores of academic achievement in the second cycle of PDCF.

Source of variation	Sum of Squares	df	Mean of Squares	F	Significance level	eta
Group	2665.08	1	2666.08	1170.18	0.00	0.94
Pre-test	173.16	1	173.16	76.07	0.00	0.88
Residual error	161.70	71	2.28			
Total	2999.94	73				

Table 5 shows that after the second cycle of the PDCF model in the experimental group, the scores in the post-test are significantly different from the pre-test of the same cycle because the significance level of the test (p-value) is less than 0.05, indicating that after applying the PDCF model, the problem-solving performance of the students in the experimental group has improved significantly, which indicates the statistical significance of the study. Also, the large effect size indicates the significant effect of the PDCF model on improving the performance of the experimental group compared to the control group, which indicates the practical significance of the study.

The box plot of academic progress by applying the PDCF model to students in the experimental group and students in the control group is shown in Figure 2.

Figure 2(a) shows the scores of the experimental group in three stages: 1) pre-test, 2) after applying the first cycle of the model, and 3) after applying the second cycle of the model. Before applying the PDCF model to the experimental group, the scores of the experimental group according to Figure 1 had an average score of 6 and a minimum and maximum of 2 and 11, respectively. The first and third quartile values were also 3 and 8, respectively. However, after applying the model under the first cycle of the PDCF model for the experimental group, it can be seen that the average, minimum, and maximum values have made good progress, which indicates that by applying only one cycle of the PDCF model, an increase in performance can be observed. Also, these indicators have grown again after the second cycle of the PDCF model, the results of which can be seen in Figure 2(a). The box plot of the control group's performance without applying the PDCF model is shown in Figure 2(b), which shows that without applying the PDCF model to the control group, no significant progress was achieved in the problem-solving performance of this

group in the first, second, and third stages of the test, and the various measurement indicators were almost at the same level.

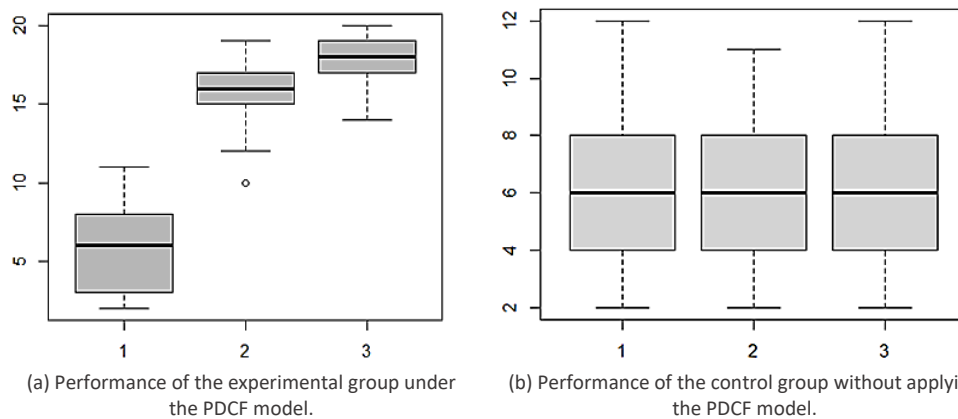


Figure 2. (a) Box plot of the performance of the experimental group under the PDCF model and (b) the performance of the control group without applying the PDCF model.

4. Conclusion

The findings of this research indicate the effectiveness of the PDCF model in improving general mathematics problem-solving performance among students. This means that using this model has been able to improve both general mathematics problem-solving performance and students' mathematics learning. Evaluating mathematics problem-solving performance essentially evaluates the ability to learn concepts, definitions, and other mathematical categories and their interrelations. This is very important in mathematics education because, according to the definition, mathematics education relates to learning mathematics. The purpose of conducting a mathematics test or exam by the educator is to become aware of the learner's ability and success in understanding mathematical concepts and topics. It is evident that simply announcing the test or exam score by the educator cannot alone guarantee improved learning and mathematics problem-solving performance.

According to learning methods emphasized in mathematics education, learners need guidance and direction to address their gaps in mathematical knowledge at every stage of learning or evaluation processes, including during the mathematics test process. Currently, the outcome of the mathematics test process during education is often in the form of the test score announcement by the educator, which is necessary but not sufficient for improving learning and problem-solving performance. This research aimed to ensure the sufficiency of improved learning and problem-solving performance in mathematics. What distinguishes this research from other similar studies is the use of a four-stage scientific model for continuous improvement in the learning process and mathematics problem-solving, which is teacher/mentor-centered. It uses the mathematics test tool to structurally identify weaknesses and root causes of learners' failure to understand mathematical concepts and improve problem-solving performance through effective and timely feedback based on the PDCF model.

Processing and analyzing information (test results and question responses) is targeted and follows the scientific model of the research, characterized by the repetition of the model's four stages to achieve continuous improvement.

One of the drawbacks of this research is the time-consuming nature of its implementation stages. To continuously improve learners' mathematics learning and problem-solving performance, the following recommendations are made:

1. Given the model's effectiveness in this research, mathematics educators should use the applied model at all educational levels.

2. To validate and ensure the reliability of the model used and the results obtained in this research, this model should be used in studies with larger samples and different educational levels.

Acknowledgments

The authors thankfully acknowledge the critical suggestions and comments from the associate Editor and referees, which greatly helped us in the improvement of the paper.

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