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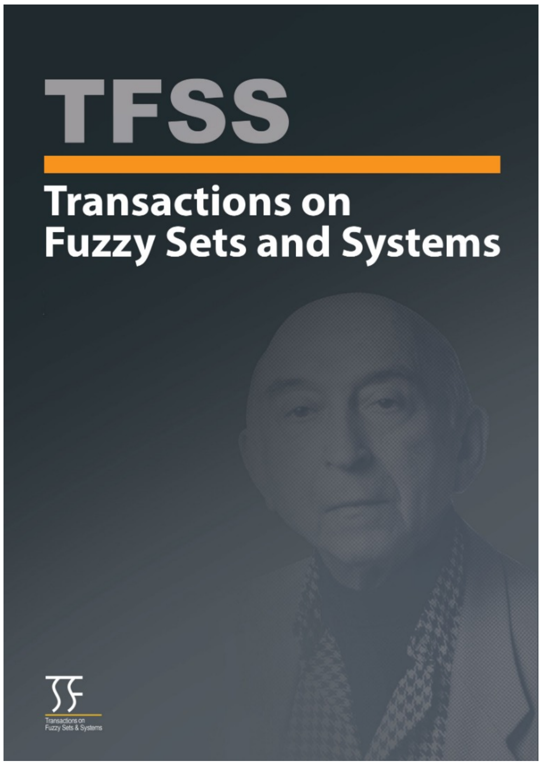

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Author(s):

Ajoy Kanti Das, Department of Mathematics, Tripura University, Agartala-799022, Tripura, India. E-mail: ajoykantidas@gmail.com

Rajat Das, Department of Mathematics, Tripura University, Agartala-799022, Tripura, India. E-mail: das.rajat666@gmail.com

Suman Das, Department of Education (ITEP), NIT Agartala, Jirania-799046, West Tripura, Tripura, India. E-mail: sumandas18842@gmail.com

Bijoy Krishna Debnath, Department of Applied Sciences, School of Engineering, Tezpur University, Sonitpur, Napaam 784028, Assam, India. E-mail: bijoy91@tezu.ernet.in

Carlos Granados, Estudiante de Doctorado en Matemticas, Magister en Ciencias Matemticas, Universidad de Antioquia, Medelln, Colombia. E-mail: carlosgranadosortiz@outlook.es

Bimal Shil, Department of Statistics, Tripura University, Agartala, 799022, Tripura, India. E-mail: bimalshil738@gmail.com

Rakhal Das, Department of Mathematics, The ICFAI University Tripura, Kamalghat, Mohanpur-799210, Tripura, India. E-mail: rakhaldas95@gmail.com, dasrakhaal@gmail.com

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A Comprehensive Study of Neutrosophic SuperHyper BCI-Semigroups and their Algebraic Significance

Ajoy Kanti Das*, **Rajat Das**, **Suman Das**, **Bijoy Krishna Debnath**, **Carlos Granados**, **Bimal Shil**, **Rakhal Das**

(This article is dedicated to Prof. Witold Pedrycz in recognition of his pioneering contributions to the field of Granular Computing.)

Abstract: This paper introduces and explores the concept of neutrosophic superhyper BCI-semigroups, an advanced algebraic structure integrating neutrosophic logic, superhyper operations, and BCI-algebras into a semigroup framework. Neutrosophic superhyper BCI-semigroups facilitate the handling of indeterminate and conflicting information by incorporating neutrosophic triplets for each element, denoting degrees of truth, indeterminacy, and falsity. The study defines essential properties such as closure, associativity, BCI identity, and neutrosophic membership and provides illustrative examples to demonstrate these properties in practical scenarios. Further, the paper delves into the characteristics of idempotent and commutative elements, homomorphisms, ideals, subsemigroups, and quotient sets within the context of neutrosophic superhyper BCI-semigroups. Key theorems are presented to establish foundational principles and behaviors of these structures, highlighting their theoretical implications and potential for practical applications.

AMS Subject Classification 2020: 17B35; 17B60; 17B65; 20M12; 20M05; 06F35
Keywords and Phrases: Neutrosophic, SuperHyperStructure, SuperHyperAlgebra, SuperHyper Operations, BCI-Algebras, Semigroups, Ideal.

1 Introduction

In recent years, the field of algebraic structures has experienced significant advancements with the introduction of novel concepts such as SuperHyperAlgebra, Neutrosophic SuperHyperAlgebra, Neutrosophic Algebras, and their various extensions and applications. These innovative ideas, pioneered by Smarandache and others, have opened new avenues for addressing complex and uncertain data in mathematical and scientific domains. Smarandache [1] laid the groundwork for Neutrosophy in his seminal work A Unifying Field in Logics: Neutrosophic Logic. This book introduces Neutrosophic Logic, encompassing Neutrosophic Sets and Neutrosophic Probability, aiming to unify various fields of logic under a single theoretical framework. This foundational work has been instrumental in developing advanced algebraic structures. In 2016, Smarandache [2] presented the ideas of SuperHyperAlgebra and Neutrosophic SuperHyperAlgebra. This work extends traditional algebraic structures by incorporating hyperoperations and neutrosophic logic, providing a robust

*∗***Corresponding Author:** Ajoy Kanti Das, Email: ajoykantidas@gmail.com, ORCID: 0000-0002-9326-1677
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framework for modeling systems characterized by indeterminate and inconsistent data. This pioneering effort has laid the foundation for further research and applications in the field of advanced algebraic structures. Smarandaches contributions continued with the publication of SuperHyperAlgebra and Neutrosophic SuperHyperAlgebra [2]. These works have sparked a surge of research interest, evidenced by the multitude of studies exploring related topics such as NeutroAlgebras, AntiAlgebras, HyperSoft Sets, and their implementations in decision support systems, information processing and system modeling. The innovative concepts introduced by Smarandache and other researchers in this domain have significantly expanded the scope and applicability of algebraic structures. They have provided powerful tools for tackling complex problems in various fields, demonstrating the potential of these advanced mathematical frameworks to address real-world challenges. In the recent study by Smarandache [3], the concepts of SuperHyperStructure and Neutrosophic SuperHyperStructure are thoroughly examined to address the complexities and uncertainties inherent in data analysis. The research introduces novel methodologies within neutrosophic logic, offering a robust model for addressing contradictory and imprecise information. Moreover, Smarandache [4] extends the concept of Soft Sets to HyperSoft Sets and subsequently to Plithogenic HyperSoft sets in his publication in Neutrosophic Sets and Systems. This study introduces new algebraic structures designed to address MCDM processes involving complex and inconsistent data. Further, the concepts of Hyperuncertain, Superuncertain, and SuperHyperUncertain Sets, Logics, Probabilities, and Statistics [5] provide additional tools for handling higher degrees of vagueness and imprecision. The n-SuperHyperGraph and Plithogenic n-SuperHyperGraph framework ([6]-[8]) further generalizes graph theory, allowing for multi-layered and hybrid uncertainty representations, crucial in fields such as network analysis, artificial intelligence, and decision science. Additionally, research on SuperHyperTopologies [9] extends topological structures to incorporate hyperconnectivity and multi-dimensional relations, demonstrating their potential in advanced computational models. Smarandache [10] explored the concepts of the SuperHyper Function and the Neutrosophic SuperHyper Function. This study extends the traditional function theory to encompass hyperoperations and neutrosophic logic, offering a novel approach to handling functions in systems with indeterminate and inconsistent data. In another study, Smarandache [11] introduced SuperHyperAlgebra and Neutrosophic SuperHyperAlgebra. This work provides a comprehensive overview of these advanced algebraic structures, detailing their theoretical properties and potential applications across various fields dealing with uncertainty and complexity. In 2023, Smarandache [12] laid the foundation for the SuperHyper Soft Set and its Fuzzy Extension, presenting a new vision in Neutrosophic Systems with Applications. This work explores the integration of hyperoperations and fuzzy logic within the SuperHyperSoft Set framework, highlighting its potential applications in areas requiring sophisticated data analysis techniques. In 2019, Smarandache [13] positioned the Neutrosophic Set as a generalization of several advanced fuzzy set theories [14], including intuitionistic fuzzy sets [15], Interval-valued fuzzy sets [16], Hesitant fuzzy sets [17], Pythagorean fuzzy sets [18], and spherical fuzzy sets [19]. He also generalized the concept of Neutrosophication, broadening its applicability in handling uncertainty and inconsistency. Additionally, Smarandache [20] discussed how NeutroAlgebra serves as a generalization of Partial Algebra. This study highlights the broader scope and applicability of NeutroAlgebra, emphasizing its capability to model and analyze systems where partial operations and neutrosophic elements are present. Agboola et al. [21] provided an elementary analysis of NeutroAlgebras and AntiAlgebras, comparing their properties with classical number systems. Their study contributes to understanding these unique characteristics and potential applications of these algebraic structures within the context of neutrosophic science. Xi [22] explored the structure and properties of Hyper BCI Algebras. This study provides foundational insights into the algebraic framework, enhancing the understanding of algebraic systems and their applications. Similarly, Kim and Kim [23] discussed BE-algebras, providing insights into the properties and applications of these algebraic structures. Their study contributes to the understanding of BE-algebras and their relevance in mathematical and scientific contexts. Furthermore, Rezaei and Smarandache [24] discussed Neutro-BE-Algebras and Anti-BE-Algebras. This work contributes to the exploration of algebraic structures that incorporate neutrosophic elements, shedding light on the properties and applications of Neutro-BE-Algebras and their counterparts in mathematical contexts. Ozturk and Jun [25] investigated commutative ideals of BCI-Algebras using MBJ-Neutrosophic Structures. This study explores the interplay between BCI-Algebras and MBJ-Neutrosophic Structures, providing insights into the properties and applications of commutative ideals within this algebraic framework. Hamidi [26] focused on exploring neutro-d-subalgebras, providing insights into these structural properties and applications of these algebraic entities within the context of neutrosophic systems. This study contributes to the ongoing research in algebraic hyperstructures and logical algebras, highlighting the significance of neutron-d-subalgebras in modeling uncertainty and indeterminacy. In a related work, Hamidi [27] delved into Superhyper BCK-Algebras, presenting a detailed analysis published in Neutrosophic Sets and Systems. This work extends the understanding of BCK-Algebras within the framework of superhyper structures, offering a deeper exploration of algebraic systems capable of handling complex and uncertain data scenarios.

In 2022, Das and Pramanik [28] delved into the intricacies of NeutroAlgebra and NeutroGroup, presenting a comprehensive analysis within the framework of Neutroalgebras as extensions of classical algebras. Their work, detailed in the book Theory and Applications of Neutroalgebras as Generalizations of Classical Algebras [29], contributes significantly to the theoretical understanding and practical applications of these algebraic structures. Similarly, Das et al. [30] explore the Lie-algebra properties of single-valued pentapartitioned neutrosophic sets, offering insights into the algebraic structures underlying such complex neutrosophic systems. This study enriches the understanding of neutrosophic algebra and its relevance in analyzing real-world data with inherent uncertainty. Furthermore, Das et al. [31] investigated the Neutrosophic d-filter of d-Algebra, contributing to the ongoing research on algebraic structures in the context of neutrosophic systems. Their work sheds light on the interplay between neutrosophic d-algebras and filters, offering valuable insights into manipulating and analyzing uncertain data within algebraic frameworks. On the other hand, Muhiuddin et al. [32] applied hyperstructure theory to BF-Algebras. This study explores the interactions between hyperstructures and BF-Algebras, offering a novel perspective on the algebraic properties and potential applications of BF-Algebras within the framework of hyperstructure theory. Additionally, Muhiuddin et al. [33] investigated the properties of neutrosophic N-ideals in subtraction semigroups. This research contributes to understanding neutrosophic algebraic frameworks and their behavior in the context of subtraction semigroups, providing valuable insights into the manipulation and analysis of uncertain data within algebraic frameworks. Santhakumar, Sumathi, and Mahalakshmi [34] offer a novel approach to the algebraic framework of Neutrosophic SuperHyperAlgebra. Their work explores the theoretical properties and potential applications of this advanced algebraic structure in managing complex and uncertain data. Finally, in 2023, Smarandache and Bordbar [35] applied Neutrosophic N-structures to BCK/BCI-Algebras, as detailed in their publication in information. This study integrates neutrosophic logic with BCK/BCI-Algebras, offering new insights into the manipulation and analysis of algebraic systems characterized by uncertainty and indeterminacy. Yang, Roh, and Jun [36] investigate Single Valued Neutrosophic Ordered Subalgebras of Ordered BCI-Algebras in their study published in Neutrosophic Sets and Systems. This research explores the properties and applications of ordered subalgebras within the framework of single-valued neutrosophic logic, providing valuable insights into how these algebraic structures can be utilized to handle uncertainty and order in mathematical systems. Rahmati and Hamidi [37] extend G Algebras to SuperHyper G Algebras. This research contributes to the development of SuperHyper G Algebras, offering a deeper understanding of algebraic structures capable of handling complex and uncertain data scenarios. The development of neutrosophic sets and their extensions and applications were documented in the studies ([38],[39]). Broumi et al. [40] provided a comprehensive overview of neutrosophic sets, highlighting their significance in decision-making processes. Pramanik et al. [41] contributed significantly to MADM in neutrosophic environments, showcasing the practical utility of this theory. Otay and Kahraman [42] conducted a state-of-the-art review, emphasizing the evolving nature of neutrosophic sets and their implementations in fuzzy MCDM. Additionally, Peng and Dai [43] performed a bibliometric analysis spanning two decades, illustrating the growth and impact of neutrosophic set theory from 1998 to 2017. Building on this foundation, Pramanik delved into specific aspects such as rough neutrosophic sets [44] and single-valued neutrosophic sets [45], providing comprehensive overviews that contribute significantly to the theoretical understanding and practical implementation of neutrosophic concepts. Delcea et al. [46] further quantified the impact of neutrosophic research through a bibliometric study, shedding light on the trajectory and scope of this evolving field. The research highlights the importance of Neutrosophic SuperHyper BCI-Algebra in addressing real-world problems across various disciplines. Challenges like data availability variations and integration complexities persist, necessitating ongoing refinement and development for comprehensive decision-making in practical scenarios. In this study, we introduce neutrosophic superhyper BCI-semigroups, an advanced algebraic structure that integrates neutrosophic logic, superhyper operations, and BCI-algebras into a semigroup framework. These semigroups help manage uncertain and conflicting information by using neutrosophic triplets for each element, representing truth, indeterminacy, and falsity degrees. We define key properties like closure, associativity, BCI identity, and neutrosophic membership, showcasing their practical application through examples. Additionally, we explore idempotent elements, homomorphisms, ideals, subsemi groups, and quotient sets within this context, establishing foundational principles and theoretical implications for practical use.

2 Historical Development of Related Algebraic Structures

The development of algebraic structures has followed a systematic progression, beginning with classical set theory and algebraic systems and advancing toward more generalized frameworks, such as fuzzy sets, neutrosophic sets, and hyperstructure-based extensions. These mathematical models have evolved to better handle uncertainty, vagueness, and imprecision in real-world decision-making and computational applications.

2.1 Classical Set Theory and Algebraic Structures

The foundation of modern algebraic structures originates from set theory, which provides a fundamental basis for mathematical reasoning. Extensions such as group theory and semigroup theory have introduced operations with associative and closure properties, while lattice theory and Boolean algebra [47] have been instrumental in logic, order theory, and computational applications.

2.2 Fuzzy and Neutrosophic Generalizations

To address the limitations of classical sets in handling uncertainty, fuzzy set theory [14] was introduced as a generalization of crisp sets. Over time, various extensions have been developed:

- Intuitionistic fuzzy sets [15] extend fuzzy sets by incorporating both membership and non-membership degrees, offering a more flexible approach to representing uncertainty ([48],[49],[50]).
- Interval-valued fuzzy sets [16] express membership degrees as intervals rather than precise values, thereby accommodating additional vagueness in data representation.
- Hesitant fuzzy sets [17] enable decision-makers to assign multiple possible membership values when uncertainty or hesitation exists, making them particularly valuable in multi-criteria decision-making.
- Pythagorean fuzzy sets [18] and spherical fuzzy sets [19] introduce higher-dimensional representations of fuzziness, enhancing modeling capabilities for complex systems.

- Neutrosophic sets [1] generalize fuzzy and intuitionistic fuzzy sets by incorporating truth, indeterminacy, and falsity values, making them highly effective in handling incomplete, inconsistent, and conflicting information.

Over the years, researchers have explored and extended fuzzy and neutrosophic concepts in various domains, including multi-criteria decision-making , water pollution assessment engineering optimization, and multi-expert evaluation processes ([51],[52],[53]). These advancements have contributed to more effective decision-making and problem-solving in complex systems.

2.3 Hyperstructure-Based Extensions

With the increasing complexity of mathematical models, hyperstructure-based algebraic systems have gained prominence. These include:

- BCI-Algebras and BCI-Semigroups [22]structures that generalize algebraic operations with weakened assumptions.
- Hyper BCIs and Hyper Algebras [22]extending BCI-algebra to hyperstructures, where elements can belong to multiple sets simultaneously.
- SuperHyperAlgebra [34]introducing multi-element operations within hyperstructures.
- Neutrosophic SuperHyperAlgebra [34]integrating neutrosophic logic into superhyperstructures for enhanced uncertainty modeling.

2.4 Neutrosophic SuperHyper BCI Structures

The integration of neutrosophic sets with superhyperstructures has led to advanced theoretical models:

- Neutrosophic BCI-Semigroups [34] unify neutrosophic sets and BCI-semigroups, allowing better handling of uncertainty and logical contradictions.
- Superhyper BCI-Semigroups [34] introduce superhyper operations, expanding the versatility of these algebraic systems.
- Neutrosophic Superhyper BCI-semigroups (proposed) provides a unified framework that:
  - Combines neutrosophic logic, superhyper operations, and BCI-semigroups.
  - Handles indeterminacy, inconsistencies, and conflicting data in a structured manner.
  - Offers potential applications in decision-making, AI, and uncertainty modeling.

Recent studies, including those by Smarandache ([3]-[10]), have further established SuperHyperStructures as an essential component of mathematical frameworks, extending to SuperHyperGraphs, SuperHyperTopologies, SuperHyperNeutrosophic, and SuperHyperFuzzy systems. These advancements strengthen the importance of hyperstructure-based algebra in handling complex real-world problems. The evolution from classical algebraic structures to neutrosophic superhyper BCI-semigroups represents a significant paradigm shift in mathematical modeling. These advanced structures serve as powerful tools for decision-making, AI, uncertainty analysis, and knowledge representation, paving the way for future innovations in computational intelligence and applied mathematics.

The structure of this research work is as follows: Section 3 provides an in-depth discussion of foundational concepts such as neutrosophic sets, BCI-algebra, and neutrosophic super hyper BCI-algebra, along with their

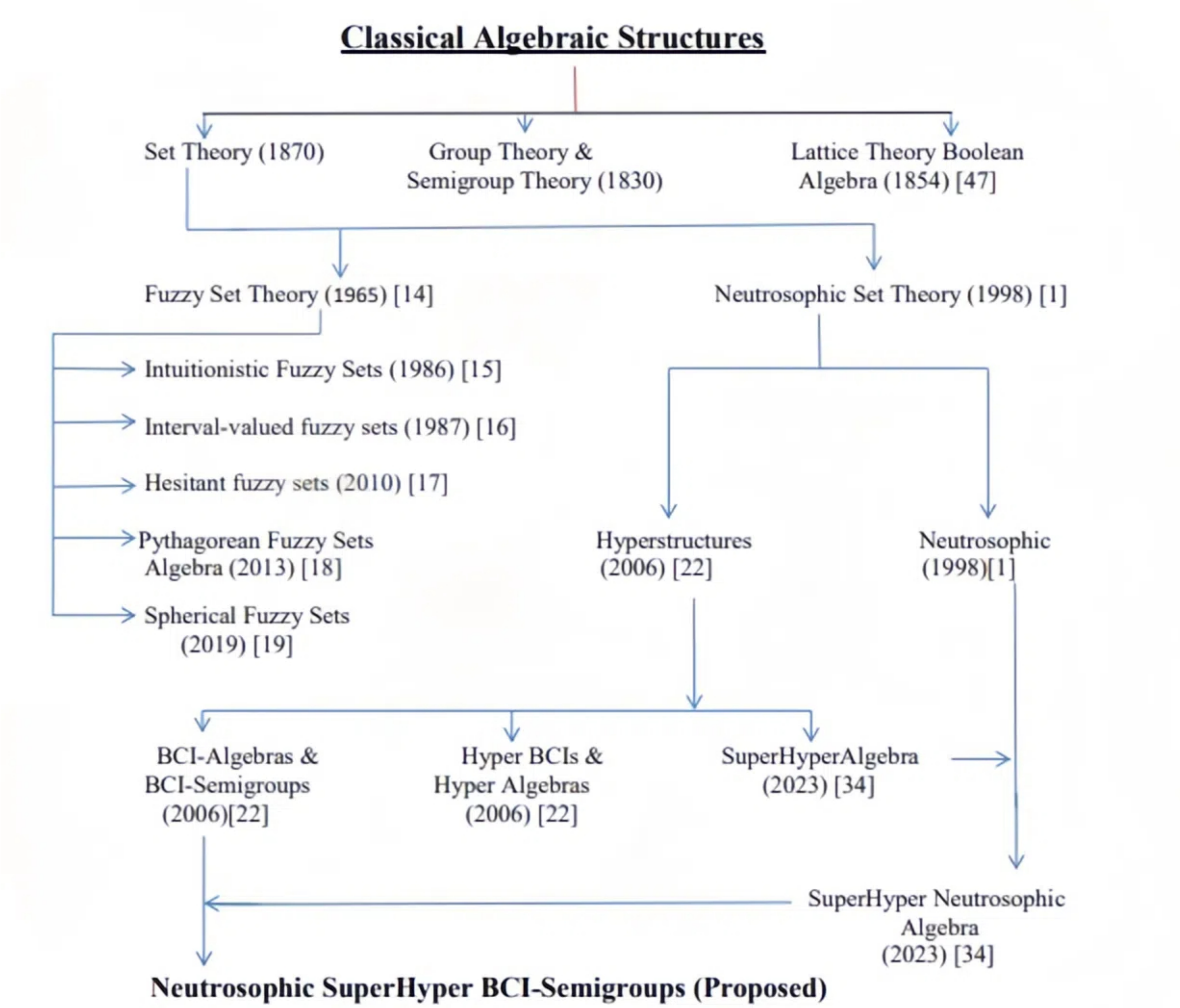

Figure 1: Flowchart depicting the stepwise methodology for water quality assessment using the GIVNRS set-based MCDM

properties. In Section 4, we introduce and explore the concept of neutrosophic superhyper BCI-semigroups, an advanced algebraic structure integrating neutrosophic logic, superhyper operations, and BCI-algebras into a semigroup framework. Further, the paper delves into the characteristics of idempotent and commutative elements, and homomorphisms within the context of neutrosophic superhyper BCI-semigroups. Moving to Section 5, we define the characteristics of ideals, subsemigroups, and quotient sets within the context of neutrosophic superhyper BCI-semigroups. Key theorems are presented to establish foundational principles and behaviors of these structures, highlighting their theoretical implications and potential for practical applications. Finally, Section 6 serves as the conclusion of our study, summarizing key findings and proposing potential directions for future research endeavors.

3 Preliminaries

In this section, we recall some basic notions relevant to neutrosophic sets, BCI-algebra, and neutrosophic super hyper BCI-algebra.

Definition 3.1. [1] *For any subset X of S, a neutrosophic set X on S is of the form:*

$$X = \{(\alpha, T_X(\alpha), I_X(\alpha), F_X(\alpha)) \mid \alpha \in S\}$$

where $T_X, I_X, F_X : S \to [0,1]$ represents the truth, indeterminacy, and falsity membership functions respectively. These functions must satisfy:

$$0 \leq T_X(\alpha) + I_X(\alpha) + F_X(\alpha) \leq 3, \quad \forall \alpha \in S.$$

Definition 3.2. [29] *Consider a non-empty set B with a binary operation Θ and a constant θ. If the following axioms hold, then (B, Θ, θ) is known as a BCI-algebra:*

1. $((a\Theta b)\Theta(a\Theta c))\Theta(c\Theta b) = \theta$
2. $(a\Theta(a\Theta b))\Theta b = \theta$
3. $a\Theta a = \theta$
4. *If $a\Theta b = \theta$ and $b\Theta a = \theta$, then $a = b$, for all $a, b, c \in B$.*

Definition 3.3. [30] *If the hypergroupoid $(H, \odot)$ satisfies the following axioms, it is known to be a hyper BCI-algebra.*

1. $(v_1 \odot v_2) \odot (v_2 \odot v_3) \preceq v_1 \odot v_2$
2. $(v_1 \odot v_2) \odot v_3 = (v_1 \odot v_3) \odot v_2$
3. $v_1 \preceq v_1$
4. $v_1 \preceq v_2$ *and* $v_2 \preceq v_1$ *implies* $v_1 = v_2$
5. $0 \odot (0 \odot v_1) \preceq v_1$ *for all* $v_1, v_2, v_3 \in H$.

Definition 3.4. [34] *The following criteria characterizes a SuperHyper groupoid $(S, \star)$ with a constant 0 as a SuperHyper BCI algebra.*

- ***(SH1)*** $(\wp \star c) \star (a \star c) \leq \wp \star a$
- ***(SH2)*** $(\wp \star a) \star c = (\wp \star c) \star a$
- ***(SH3)*** $\wp \leq \wp$
- ***(SH4)*** $\wp \leq a$ *and* $a \leq \wp$ *implies* $\wp = a$
- ***(SH5)*** $0 \star (0 \star \wp) \leq \wp$ *for every* $\wp, a, c \in S$

Let $\mathcal{X} = \{(\alpha, T_{\mathcal{X}}(\alpha), I_{\mathcal{X}}(\alpha), F_{\mathcal{X}}(\alpha)) \mid \alpha \in S\}$ be a neutrosophic set in S, and Y be a subset of S. Then

$$T_Y = \inf\{T_{\mathcal{X}}(\tau) \mid \tau \in Y\}, \quad I_Y = \inf\{I_{\mathcal{X}}(\tau) \mid \tau \in Y\}, \quad F_Y = \inf\{F_{\mathcal{X}}(\tau) \mid \tau \in Y\}.$$

Example 3.5. Let $S = \{a, b, c\}$ be a set, and consider the following neutrosophic set $\mathcal{X}$ over S:

$$\mathcal{X} = \{(a, 0.8, 0.4, 0.2), (b, 0.6, 0.5, 0.3), (c, 0.7, 0.6, 0.1)\}.$$

Now, let $Y = \{b, c\}$ be a subset of S. Using Definition 2.3, the neutrosophic membership values for Y are computed as follows:

- **Truth Membership for Y:**

$$T_Y = \inf\{T_{\mathcal{X}}(b), T_{\mathcal{X}}(c)\} = \inf\{0.6, 0.7\} = 0.6$$

- **Indeterminacy Membership for Y:**

$$I_Y = \inf\{I_{\mathcal{X}}(b), I_{\mathcal{X}}(c)\} = \inf\{0.5, 0.6\} = 0.5$$

- **Falsity Membership for Y:**

$$F_Y = \inf\{F_{\mathcal{X}}(b), F_{\mathcal{X}}(c)\} = \inf\{0.3, 0.1\} = 0.1$$

Thus, the neutrosophic membership values for subset Y are:

$$T_Y = 0.6, \quad I_Y = 0.5, \quad F_Y = 0.1.$$

Definition 3.6. [34] *Let $\mathcal{X} = \{(\alpha, T_{\mathcal{X}}(\alpha), I_{\mathcal{X}}(\alpha), F_{\mathcal{X}}(\alpha)) \mid \alpha \in S\}$ be a neutrosophic set in S. Then $\mathcal{X}$ is said to be a neutrosophic super hyper BCI-algebra of S when it adheres to the ensuing conditions for all elements τ and ρ in S:*

$$T_{\mathcal{X}}(\tau \star \rho) \geq \min(T_{\mathcal{X}}(\tau), T_{\mathcal{X}}(\rho)), \quad I_{\mathcal{X}}(\tau \star \rho) \geq \min(I_{\mathcal{X}}(\tau), I_{\mathcal{X}}(\rho)), \quad F_{\mathcal{X}}(\tau \star \rho) \leq \max(F_{\mathcal{X}}(\tau), F_{\mathcal{X}}(\rho)).$$

Example 3.7. Let $S = \{a, b\}$ and define a neutrosophic set X as:

$$X = \{(a, 0.8, 0.4, 0.2), (b, 0.6, 0.5, 0.3)\}$$

Consider the binary hyperoperation Θ, defined by:

$$a\Theta b = c, \text{ where } c \text{ is an element in } S.$$

The neutrosophic membership values for $a\Theta b$ should satisfy:

Truth Membership Condition:

$$T_X(a\Theta b) \geq \min(T_X(a), T_X(b))$$

Substituting values:

$$T_X(c) \geq \min(0.8, 0.6) = 0.6$$

Indeterminacy Membership Condition:

$$I_X(a\Theta b) \geq \min(I_X(a), I_X(b))$$

Substituting values:

$$I_X(c) \geq \min(0.4, 0.5) = 0.4$$

Falsity Membership Condition:

$$F_X(a\Theta b) \leq \max(F_X(a), F_X(b))$$

Substituting values:

$$F_X(c) \leq \max(0.2, 0.3) = 0.3$$

Thus, for X to be a Neutrosophic SuperHyper BCI-Algebra, the membership values for c should satisfy:

$$T_X(c) \geq 0.6, \quad I_X(c) \geq 0.4, \quad F_X(c) \leq 0.3$$

If we define $c = (0.7, 0.5, 0.2)$, it meets the required conditions, confirming that X forms a Neutrosophic SuperHyper BCI-Algebra.

4 Neutrosophic Superhyper BCI-Semigroup

A Neutrosophic superhyper BCI-semigroup is a generalized algebraic structure that combines the principles of neutrosophic logic, superhyper operations, and BCI-algebras into a semigroup framework. This structure allows for the exploration of associative properties and the handling of indeterminate and conflicting information.

Let S be a non-empty set. Each element $x \in S$ is associated with a neutrosophic triplet (T_x, I_x, F_x), where T_x, I_x, and F_x represent the truth, indeterminacy, and falsity membership degrees respectively, with values in the interval $[0, 1]$. Let $*$ and $\circ$ denote the binary operation and binary hyperoperation on S. A superhyper operation Θ on S is such that for any $x, y \in S$, $x\Theta y$ is a subset of S, considering the superhyper structure. It incorporates the axioms of BCI-algebras, adapted to the neutrosophic and superhyper context.

Definition 4.1. *A BCI-semigroup is a set $(S, *)$ equipped with a binary operation $*$ satisfying the following conditions:*

i) ***Closure***: *For all $x, y \in S$, $x * y \subseteq S$.*

ii) ***Associativity***: *For all $x, y, z \in S$, there exists a set $w \subseteq S$, such that $(x * y) * z = x * (y * z) = w$.*

Definition 4.2. *A hyper BCI-semigroup is a set $(S, \circ)$ equipped with a binary hyper operation $\circ$ satisfying the following conditions:*

i) ***Closure***: *For all $x, y \in S$, $x \circ y \subseteq S$.*

ii) ***Associativity***: *For all $x, y, z \in S$, there exists a set $w \subseteq S$, such that $(x \circ y) \circ z = x \circ (y \circ z) = w$.*

Definition 4.3. *A superhyper BCI-semigroup is a set (S, Θ) equipped with a binary superhyper operation Θ satisfying the following conditions:*

i) ***Closure***: *For all $x, y \in S$, $x\Theta y \subseteq S$.*

ii) ***Associativity***: *For all $x, y, z \in S$, there exists a set $w \subseteq S$, such that $(x\Theta y)\Theta z = x\Theta(y\Theta z) = w$.*

Definition 4.4. *A neutrosophic superhyper BCI-semigroup is a set (S, Θ) equipped with a binary superhyper operation Θ satisfying the following conditions:*

i) ***Closure***: *For all $x, y \in S$, $x\Theta y \subseteq S$.*

ii) ***Associativity***: *For all $x, y, z \in S$, there exists a set $w \subseteq S$ such that $(x\Theta y)\Theta z = x\Theta(y\Theta z) = w$.*

iii) ***Neutrosophic Membership***: *For all $x, y \in S$,*

$$T(x\Theta y) \geq \min\{T(x), T(y)\}, \quad I(x\Theta y) \geq \min\{I(x), I(y)\}, \quad F(x\Theta y) \leq \max\{F(x), F(y)\}.$$

Example 4.5. Consider a set $S = \{a, b, c\}$ with the neutrosophic triplets

$$(T(a), I(a), F(a)) = (0.7, 0.5, 0.2), \quad (T(b), I(b), F(b)) = (0.6, 0.3, 0.1), \quad (T(c), I(c), F(c)) = (0.4, 0.5, 0.1).$$

Let's define the binary superhyper operation Θ as follows (Table 1):

Table 1: The binary superhyper operation Θ on neutrosophic superhyper BCI-semigroup (S, Θ)

Θ	$a, (0.7, 0.5, 0.2)$	$b, (0.6, 0.3, 0.1)$	$c, (0.4, 0.5, 0.1)$
$a, (0.7, 0.5, 0.2)$	$\{a\}, (0.7, 0.5, 0.2)$	$\{a, b\}, (0.6, 0.3, 0.1)$	$\{a, c\}, (0.4, 0.5, 0.1)$
$b, (0.6, 0.3, 0.1)$	$\{a, b\}, (0.6, 0.3, 0.1)$	$\{b\}, (0.6, 0.3, 0.1)$	$\{b, c\}, (0.4, 0.3, 0.1)$
$c, (0.4, 0.5, 0.1)$	$\{a, c\}, (0.4, 0.5, 0.1)$	$\{b, c\}, (0.4, 0.3, 0.1)$	$\{c\}, (0.4, 0.5, 0.1)$

Then:

1. **Closure**: For all $x, y \in S$, $x\Theta y \subseteq S$. From the table, each result of $x\Theta y$ is either $\{a\}$, $\{b\}$, $\{c\}$, $\{a, b\}$, $\{a, c\}$, or $\{b, c\}$. All these subsets are contained within S, thus satisfying closure.

2. **Associativity**: For all $x, y, z \in S$, there exists a set $w \subseteq S$ such that $(x\Theta y)\Theta z = x\Theta(y\Theta z) = w$. Let's verify a few cases:

$$(a\Theta b)\Theta c = \{b\}\Theta c = \{b, c\}, \quad a\Theta(b\Theta c) = a\Theta\{b, c\} = \{a, b, c\}.$$

   This needs to hold for all combinations, ensuring the associativity property. For our set of three elements, we need to check more combinations to be thorough.

3. **Neutrosophic Membership**: From Table 1, we can see that the provided example satisfies the neutrosophic membership condition. The operations defined in the table adhere to the required neutrosophic membership values, fulfilling the conditions for a neutrosophic superhyper BCI-semigroup.

This satisfies all the conditions required for a neutrosophic superhyper BCI-semigroup. Hence, (S, Θ) with the defined operations and membership values is an example of such a semigroup.

Theorem 4.6. *For any $x, y \in S$ in a neutrosophic superhyper BCI-semigroup (S, Θ), the neutrosophic triplet of the binary superhyper operation result $x\Theta y$ is preserved within the neutrosophic constraints.*

Proof. Given $x, y \in S$ with neutrosophic triplets (T_x, I_x, F_x) and (T_y, I_y, F_y), the binary superhyper operation $x\Theta y$ results in a set whose elements also have neutrosophic triplets. By the closure property, $x\Theta y \subseteq S$, and the definition of neutrosophic membership, these resulting triplets will satisfy the conditions:

$$0 \leq T_z + I_z + F_z \leq 3 \quad \text{and} \quad 0 \leq T_z, I_z, F_z \leq 1 \quad \text{for all} \quad z \in x\Theta y \subseteq S.$$

□

These theorems provide a foundational understanding of the properties and behavior of neutrosophic superhyper BCI-semigroups, paving the way for further theoretical exploration and practical applications.

Definition 4.7. *An element $x \in S$ in a neutrosophic superhyper BCI-semigroup (S, Θ) is idempotent if $x\Theta x = \{x\}$.*

Example 4.8. Consider the neutrosophic superhyper BCI-semigroup $S = \{a, b, c\}$ with the neutrosophic triplets

$$a = (0.7, 0.3, 0.4), \quad b = (0.5, 0.2, 0.2), \quad c = (0.6, 0.3, 0.3),$$

and we define the binary superhyper operation Θ as in Table 2.

Table 2: The binary superhyper operation Θ on S

Θ	a, (0.5, 0.3, 0.4)	b, (0.7, 0.2, 0.2)	c, (0.6, 0.3, 0.3)
a, (0.5, 0.3, 0.4)	{ a }, (0.5, 0.3, 0.4)	{ a, b }, (0.5, 0.2, 0.2)	{ a, c }, (0.5, 0.3, 0.3)
b, (0.7, 0.2, 0.2)	{ a, b }, (0.5, 0.2, 0.2)	{ b }, (0.7, 0.2, 0.2)	{ b, c }, (0.6, 0.2, 0.2)
c, (0.6, 0.3, 0.3)	{ a, c }, (0.5, 0.3, 0.3)	{ b, c }, (0.6, 0.2, 0.2)	{ a, b }, (0.6, 0.3, 0.3)

Here, all elements a and b are idempotent because:

- $a\Theta a = \{a\}$ which satisfies $a\Theta a = \{a\}$,
- $b\Theta b = \{b\}$ which satisfies $b\Theta b = \{b\}$.

Thus, a and b are idempotent elements in the neutrosophic superhyper BCI-semigroup (S, Θ).

Definition 4.9. *The neutrosophic superhyper BCI-semigroup (S, Θ) is commutative if for all $x, y \in S$, there exists a set $w \subseteq S$, such that $x\Theta y = y\Theta x = w$.*

Example 4.10. Consider the neutrosophic superhyper BCI-semigroup $S = \{a, b\}$ with the neutrosophic triplets as $a = (0.5, 0.3, 0.4)$, $b = (0.7, 0.2, 0.2)$, and $c = (0.6, 0.3, 0.3)$, and we define the binary superhyper operation Θ as in Table 3.

Table 3: The binary superhyper operation Θ on S

Θ	a, (0.5, 0.3, 0.4)	b, (0.7, 0.2, 0.2)	c, (0.6, 0.3, 0.3)
a, (0.5, 0.3, 0.4)	{a}, (0.5, 0.3, 0.4)	{b}, (0.5, 0.2, 0.2)	{c}, (0.5, 0.3, 0.3)
b, (0.7, 0.2, 0.2)	{b}, (0.5, 0.2, 0.2)	{b}, (0.7, 0.2, 0.2)	{b, c}, (0.6, 0.2, 0.2)
c, (0.6, 0.3, 0.3)	{c}, (0.5, 0.3, 0.3)	{b, c}, (0.6, 0.2, 0.2)	{c}, (0.6, 0.3, 0.3)

This satisfies the condition required for a commutative neutrosophic superhyper BCI-semigroup. Hence, (S, Θ) with the defined binary superhyper operation Θ and membership values is an example of such a commutative neutrosophic superhyper BCI-semigroup.

Proposition 4.11. *In a neutrosophic superhyper BCI-semigroup (S, Θ), the BCI identity element θ is unique.*

Proof. Assume there are two BCI identity elements θ and θ' in S. By the definition of the BCI identity element, for all $x \in S$, $x\Theta\theta = \{x\}$ and $x\Theta\theta' = \{x\}$. Consider $\theta\Theta\theta'$. Since both are BCI identity elements, $\theta\Theta\theta' = \{\theta'\}$ and $\theta\Theta\theta' = \{\theta\}$, which implies $\{\theta\} = \{\theta'\}$, hence $\theta = \theta'$. Thus, the BCI identity element is unique. □

Definition 4.12. *A function $f : S_1 \to S_2$ between two neutrosophic superhyper BCI-semigroups (S_1, Θ_1) and (S_2, Θ_2) is called a homomorphism if for all $x, y \in S_1$, $f(x\Theta_1 y) \subseteq f(x)\Theta_2 f(y)$ and the neutrosophic triplets are preserved, i.e., $T_f(x) = T_x$, $I_f(x) = I_x$, $F_f(x) = F_x$.*

Example 4.13. Let $S_1 = \{a, b\}$ and $S_2 = \{c, d\}$ be two neutrosophic superhyper BCI-semigroups with the neutrosophic triplets $a = (0.8, 0.1, 0.1)$, $b = (0.6, 0.2, 0.2)$, and $c = (0.8, 0.1, 0.1)$, $d = (0.6, 0.2, 0.2)$. We define the binary superhyper operation Θ_1 and Θ_2 as in Tables 4 and 5, respectively.

Now, define a function $f : S_1 \to S_2$ as $f(a) = c$, $f(b) = d$. Then the function f is a homomorphism between the two neutrosophic superhyper BCI-semigroups (S_1, Θ_1) and (S_2, Θ_2), since for all $x, y \in S_1$,

$$f(x\Theta_1 y) \subseteq f(x)\Theta_2 f(y)$$

and the neutrosophic triplets are preserved.

Table 4: The binary superhyper operation Θ_1 on S_1

Θ_1	a, (0.8, 0.1, 0.1)	b, (0.6, 0.2, 0.2)
a, (0.8, 0.1, 0.1)	{a}, (0.8, 0.1, 0.1)	{b}, (0.6, 0.1, 0.1)
b, (0.6, 0.2, 0.2)	{b}, (0.6, 0.1, 0.1)	{b}, (0.6, 0.2, 0.2)

Table 5: The binary superhyper operation Θ_2 on S_2

Θ_2	c, (0.8, 0.1, 0.1)	d, (0.6, 0.2, 0.2)
c, (0.8, 0.1, 0.1)	{c}, (0.8, 0.1, 0.1)	{d}, (0.6, 0.1, 0.1)
d, (0.6, 0.2, 0.2)	{d}, (0.6, 0.1, 0.1)	{d}, (0.6, 0.2, 0.2)

Theorem 4.14. *If $f : S_1 \to S_2$ and $g : S_2 \to S_3$ are homomorphisms between neutrosophic superhyper BCI-semigroups $(S_1, \Theta_1), (S_2, \Theta_2)$, and (S_3, Θ_3), then the composition $g \circ f : S_1 \to S_3$ is also a homomorphism.*

Proof. For any $x, y \in S_1$, we have

$$f(x\Theta_1 y) \subseteq f(x)\Theta_2 f(y).$$

Applying g,

$$g(f(x\Theta_1 y)) \subseteq g(f(x))\Theta_3 g(f(y)).$$

Since g is a homomorphism,

$$g(f(x)\Theta_2 f(y)) \subseteq g(f(x))\Theta_3 g(f(y)).$$

This implies

$$g(f(x\Theta_1 y)) \subseteq g(f(x))\Theta_3 g(f(y)),$$

so $g \circ f$ is a homomorphism. □

5 Neutrosophic SuperHyper BCI-ideal

Neutrosophic superhyper BCI-ideal is a generalized algebraic structure that combines the principles of neutrosophic logic, superhyper operations, and BCI-algebras into an ideal framework.

Let S be a non-empty set. Each element $x \in S$ is associated with a neutrosophic triplet (T_x, I_x, F_x), where T_x, I_x, F_x represent the truth, indeterminacy, and falsity membership degrees, respectively, with values in the interval $[0, 1]$. Let $*$ denote the binary operation and binary hyperoperation on S. A superhyper operation Θ on S is such that for any $x, y \in S$, $x\Theta y$ is a subset of S, considering the superhyper structure.

Definition 5.1. *A non-empty subset $I \subseteq S$ of a BCI-semigroup $(S, *)$ is called a BCI-ideal, if*

1. *For all $x \in I$ and $y \in S$, $y * x \subseteq I$.*
2. *For all $x, y \in I$, $x * y \subseteq I$.*

Definition 5.2. *A non-empty subset $I \subseteq S$ of a neutrosophic superhyper BCI-semigroup (S, Θ) is called an ideal, if*

1. *For all $x \in I$ and $y \in S$, $y\Theta x \subseteq I$.*
2. *For all $x, y \in I$, $x\Theta y \subseteq I$.*

Example 5.3. Consider the same set $S = \{a, b, c\}$ with the binary superhyper operation Θ defined as in Table 6.

Table 6: The binary superhyper operation Θ on S

Θ	a, (0.5,0.3,0.4)	b, (0.7,0.2,0.2)	c, (0.6,0.3,0.3)
a, (0.5,0.3,0.4)	{a}, (0.5,0.3,0.4)	{b}, (0.5,0.2,0.2)	{c}, (0.5,0.3,0.3)
b, (0.7,0.2,0.2)	{b}, (0.5,0.2,0.2)	{b}, (0.7,0.2,0.2)	{b, c}, (0.6,0.2,0.2)
c, (0.6,0.3,0.3)	{c}, (0.5,0.3,0.3)	{b, c}, (0.6,0.2,0.2)	{c}, (0.6,0.3,0.3)

Let $I = \{b, c\}$. Then I is an ideal on the neutrosophic superhyper BCI-semigroup (S, Θ), since for $x = b$ and $y \in S$,

$$b\Theta a = \{b\} \subseteq I, \quad b\Theta b = \{b\} \subseteq I, \quad b\Theta c = \{b, c\} \subseteq I,$$

and for $x = c$ and $y \in S$,

$$c\Theta a = \{c\} \subseteq I, \quad c\Theta b = \{b, c\} \subseteq I, \quad c\Theta c = \{c\} \subseteq I.$$

Since all conditions are satisfied, $I = \{b, c\}$ is a neutrosophic superhyper BCI-ideal in (S, Θ).

Theorem 5.4. *The intersection of any two neutrosophic superhyper BCI-ideals in a neutrosophic superhyper BCI-semigroup is also ideal.*

Proof. Let I_1 and I_2 be two ideals in (S, Θ). Consider $I = I_1 \cap I_2$. For $x, y \in I$, we have $x, y \in I_1$ and $x, y \in I_2$. Thus, $x\Theta y \subseteq I_1$ and $x\Theta y \subseteq I_2$, implying $x\Theta y \subseteq I_1 \cap I_2 = I$. Therefore, I is an ideal in (S, Θ). □

Theorem 5.5. *If $f : S_1 \to S_2$ is a homomorphism between two neutrosophic superhyper BCI-semigroups (S_1, Θ_1) and (S_2, Θ_2), and $I \subseteq S_1$ is an ideal, then $f(I) \subseteq S_2$ is also an ideal.*

Proof. By the definition of a homomorphism, if $I \subseteq S_1$ is an ideal, for any $x \in S_1$ and $y \in I$, $x\Theta_1 y \subseteq I$. Applying f to both sides, $f(x\Theta_1 y) \subseteq f(I)$. Since f is a homomorphism, $f(x\Theta_1 y) \subseteq f(x)\Theta_2 f(y)$. Thus, $f(I)$ is an ideal in (S_2, Θ_2). □

Definition 5.6. *Let (S, Θ) be a neutrosophic superhyper BCI-semigroup with a binary superhyper operation Θ. A non-empty subset $T \subseteq S$ is said to be a neutrosophic superhyper BCI-subsemigroup of (S, Θ) if T is also a semigroup under the binary superhyper operation Θ.*

Example 5.7. Consider the same set $S = \{a, b, c, d\}$ with the neutrosophic triplets as $a = (0.5, 0.3, 0.4)$, $b = (0.7, 0.2, 0.2)$, $c = (0.6, 0.3, 0.3)$, and the binary superhyper operation Θ defined as in Table 7. Let $T = \{a, b, c\}$. Then $T = \{a, b, c\}$ is a neutrosophic superhyper BCI-subsemigroup of (S, Θ).

Table 7: The binary superhyper operation Θ on S

Θ	a, $(0.5, 0.3, 0.4)$	b, $(0.7, 0.2, 0.2)$	c, $(0.6, 0.3, 0.3)$	d, $(0.6, 0.3, 0.3)$
a, $(0.5, 0.3, 0.4)$	$\{a\}$, $(0.5, 0.3, 0.4)$	$\{b\}$, $(0.5, 0.2, 0.2)$	$\{c\}$, $(0.5, 0.3, 0.3)$	$\{d\}$, $(0.5, 0.3, 0.3)$
b, $(0.7, 0.2, 0.2)$	$\{b\}$, $(0.5, 0.2, 0.2)$	$\{b\}$, $(0.7, 0.2, 0.2)$	$\{b, c\}$, $(0.6, 0.2, 0.2)$	$\{b, c\}$, $(0.6, 0.2, 0.2)$
c, $(0.6, 0.3, 0.3)$	$\{c\}$, $(0.5, 0.3, 0.3)$	$\{b, c\}$, $(0.6, 0.2, 0.2)$	$\{c\}$, $(0.6, 0.3, 0.3)$	$\{b, c\}$, $(0.6, 0.3, 0.3)$
d, $(0.6, 0.3, 0.3)$	$\{d\}$, $(0.5, 0.3, 0.3)$	$\{b, c\}$, $(0.6, 0.2, 0.2)$	$\{b, c\}$, $(0.6, 0.3, 0.3)$	$\{a, d\}$, $(0.6, 0.3, 0.3)$

Theorem 5.8. *If T is a neutrosophic superhyper BCI-subsemigroup of S, and I is an ideal of S, then $T \cap I$ is an ideal of T.*

Proof. Let $x \in T \cap I$ and $y \in T$. Since $x \in I$ and I is ideal, we have $y\Theta x \subseteq I$. Also, since $x, y \in T$ and T are closed under Θ, we have $y\Theta x \subseteq T$. Therefore, $y\Theta x \subseteq T \cap I$, proving that $T \cap I$ is an ideal of T. □

Theorem 5.9. *In a neutrosophic superhyper BCI-semigroup (S, Θ), if an element $x \in S$ has an inverse $y \in S$ such that $x\Theta y = \{\theta\}$ and $y\Theta x = \{\theta\}$, where θ is a BCI identity, then y is unique.*

Proof. Suppose y and y' are two inverses of x. Then $x\Theta y = \{\theta\}$, $y\Theta x = \{\theta\}$, $x\Theta y' = \{\theta\}$, and $y'\Theta x = \{\theta\}$. By the properties of BCI-algebra, we have $y = y'$, proving the uniqueness of the inverse. □

Theorem 5.10. *In a neutrosophic superhyper BCI-semigroup (S, Θ), if $x\Theta y = x\Theta z$ or $y\Theta x = z\Theta x$ for $x, y, z \in S$, then $y = z$.*

Proof. Assume $x\Theta y = x\Theta z$. If there exists $w \in S$ such that $w \in x\Theta y$ and $w \in x\Theta z$, then by the closure property and the definition of the binary superhyper operation, it must hold that $y = z$. Similarly, if $y\Theta x = z\Theta x$, then $y = z$. □

Theorem 5.11. *If (S, Θ) is a commutative neutrosophic superhyper BCI-semigroup and (T, Θ) is a neutrosophic superhyper BCI-subsemigroup of (S, Θ), then (T, Θ) is also commutative.*

Proof. Since (S, Θ) is commutative, for any $x, y \in S$, we have $x\Theta y = y\Theta x$. Since T is a subsemigroup, for any $x, y \in T$, both $x\Theta y$ and $y\Theta x$ are in T. Therefore, (T, Θ) is commutative. □

Definition 5.12. *Let S be a neutrosophic superhyper BCI-semigroup and I be an ideal of S. The neutrosophic superhyper BCI-quotient set S/I consists of equivalence classes of S modulo the ideal I. Each equivalence class $x+I$ contains all elements in S that are related to x via the ideal I with the binary superhyper operation:*

$$(x + I)\Theta(y + I) = (x\Theta y) + I$$

and neutrosophic triplets for all $x \in S$, $T(x + I) = T(x)$, $I(x + I) = I(x)$, and $F(x + I) = F(x)$.

Theorem 5.13. *If S is a neutrosophic superhyper BCI-semigroup and I is an ideal of S, then the quotient set S/I with the binary superhyper operation $(x + I)\Theta(y + I) = (x\Theta y) + I$ forms a neutrosophic superhyper BCI-semigroup.*

Proof. To prove that S/I with the operation defined forms a neutrosophic superhyper BCI-semigroup, we need to check that the following properties are preserved:

1. **Closure:**

   - For all $x, y \in S$, we have $x\Theta y \in S$.
   - Since I is an ideal, $(x\Theta y) + I$ is in the quotient set S/I.

   Thus, the operation $(x + I)\Theta(y + I)$ results in an element of S/I, ensuring closure.

2. **Associativity:**

   - The operation Θ in S is associative: $(x\Theta y)\Theta z = x\Theta(y\Theta z)$ for all $x, y, z \in S$.
   - In the quotient set, we have: $((x+I)\Theta(y+I))\Theta(z+I) = ((x\Theta y)+I)\Theta(z+I) = ((x\Theta y)\Theta z)+I = (x\Theta(y\Theta z)) + I = (x + I)\Theta((y + I)\Theta(z + I))$.

   Therefore, associativity is preserved.

3. **BCI Identity:**

Table 8: The binary superhyper operation Θ on S

Θ	a, (0.5,0.3,0.4)	b, (0.7,0.2,0.2)	c, (0.6,0.3,0.3)	d, (0.6,0.3,0.3)
a, (0.5,0.3,0.4)	{a}, (0.5,0.3,0.4)	{b}, (0.5,0.2,0.2)	{c}, (0.5,0.3,0.3)	{d}, (0.5,0.3,0.3)
b, (0.7,0.2,0.2)	{b}, (0.5,0.2,0.2)	{b}, (0.7,0.2,0.2)	{b, c}, (0.6,0.2,0.2)	{b, c}, (0.6,0.2,0.2)
c, (0.6,0.3,0.3)	{c}, (0.5,0.3,0.3)	{b, c}, (0.6,0.2,0.2)	{c}, (0.6,0.3,0.3)	{b, c}, (0.6,0.3,0.3)
d, (0.6,0.3,0.3)	{d}, (0.5,0.3,0.3)	{b, c}, (0.6,0.2,0.2)	{b, c}, (0.6,0.3,0.3)	{a, d}, (0.6,0.3,0.3)

- If θ is the identity element in S (i.e., $\theta\Theta x = x\Theta\theta = x$ for all $x \in S$), then in the quotient set, the identity element will be $\theta + I$.
- For any $x + I \in S/I$: $(\theta + I)\Theta(x + I) = (\theta\Theta x) + I = x + I$.

Thus, $\theta + I$ acts as the BCI identity element in S/I.

4. **Neutrosophic Membership:** From the definition of neutrosophic superhyper BCI-quotient set S/I, $(S/I, \Theta)$ preserves all the neutrosophic membership values. The binary superhyper operation Θ in the neutrosophic superhyper BCI-quotient set S/I respects these values because the ideal I does not alter the inherent neutrosophic membership properties of elements in S.

□

Definition 5.14. *Let (S, Θ) be a neutrosophic superhyper BCI-semigroup. A neutrosophic superhyper BCI-ideal I is said to be maximal if there does not exist any proper neutrosophic superhyper BCI-ideal J such that $I \subset J$. In simpler terms, a maximal neutrosophic superhyper BCI-ideal is an ideal within the semigroup for which no larger ideal is contained within it. Any additional element or subset that could be added to the maximal ideal would either break closure under the binary superhyper operation or violate the properties of an ideal. Maximal ideals are important because they represent the largest possible ideals within a semigroup that maintain the desired algebraic properties, making them a focus of study in the context of semigroup theory and algebraic structures.*

Example 5.15. Consider the same set $S = \{a, b, c, d\}$ with the binary superhyper operation Θ defined as in Table 8. Let $I = \{b, c\}$. Then I is an ideal on the neutrosophic superhyper BCI-semigroup (S, Θ), since for $x = b$ and $y \in S$, $b\Theta a = \{b\} \subseteq I$, $b\Theta b = \{b\} \subseteq I$, $b\Theta c = \{b, c\} \subseteq I$, $b\Theta d = \{b, c\} \subseteq I$, and for $x = c$ and $y \in S$, $c\Theta a = \{c\} \subseteq I$, $c\Theta b = \{b, c\} \subseteq I$, $c\Theta c = \{c\} \subseteq I$, $c\Theta d = \{b, c\} \subseteq I$. Since all conditions are satisfied, $I = \{b, c\}$ is a neutrosophic superhyper BCI-ideal in (S, Θ). Now, let's check if I is maximal. We need to verify if there exists a proper neutrosophic superhyper BCI-ideal J such that $I \subset J$. Let's consider $J = \{b, c, d\}$ and $K = \{a, b, c\}$.

Case-1: I is a proper subset of J because d is in J but not in I. However, J is not a neutrosophic superhyper BCI-ideal because $d \circ d = \{a, d\}$ is not in J, violating the closure under the binary superhyper operation property.

Case-2: I is a proper subset of K because a is in K but not in I. However, K is not a neutrosophic superhyper BCI-ideal because $a \circ d = \{a, d\}$ is not a subset of K, violating the closure under the binary superhyper operation property. Therefore, $I = \{b, c\}$ is a maximal neutrosophic superhyper BCI-ideal.

Theorem 5.16. *Let (S, Θ) be a neutrosophic superhyper BCI-semigroup. Let (T, Θ) be a neutrosophic superhyper BCI-semisubgroup of (S, Θ) and a proper neutrosophic superhyper BCI-ideal I of S, the extension $T \cup I$ with the binary superhyper operation Θ defined as $x\Theta y = z$ for $x, y \in I$ and $z \subset T$ forms a neutrosophic superhyper BCI-semigroup.*

Proof. To show that $(T \cup I, \Theta)$ forms a neutrosophic superhyper BCI-semigroup, we need to verify that the structure maintains the properties of a neutrosophic superhyper BCI-semigroup under the given extension operation.

1. **Closure:** For any $a, b \in T \cup I$, we need to show that $a \circ b \in T \cup I$.
   - **Case-1:** If $a, b \in T$, then $a\Theta b \in T$ because (T, Θ) is a neutrosophic superhyper BCI-semigroup and thus closed under Θ.
   - **Case-2:** If $a, b \in I$, then $a\Theta b \subset T$ by the definition of the binary superhyper operation in the extension, which implies that $a\Theta b$ is a subset of $T \cup I$.
   - **Case-3:** If $a \in T$ and $b \in I$ (or vice versa), then $a\Theta b \subset I$ by the definition of the ideal I, it follows that $a\Theta b$ is a subset of $T \cup I$. Thus, the closure property holds.
2. **Associativity:** For any $a, b, c \in T \cup I$, we need to show that $(a\Theta b)\Theta c = a\Theta(b\Theta c)$.
   - **Case-1:** If $a, b, c \in T$, associativity holds because (T, Θ) is a neutrosophic superhyper BCI-semigroup.
   - **Case-2:** If $a, b, c \in I$, associativity holds by the extended definition: $a\Theta(b\Theta c) = (a\Theta b)\Theta c$, both resulting in subsets of S.
   - **Case-3:** For mixed cases, $a, b \in T$ and $c \in I$ (or permutations), we extend the operation to respect associativity, preserving the semigroup structure. Thus, the associativity property holds.
3. **BCI Identity:** $(T \cup I, \Theta)$ satisfies BCI identity because (T, Θ) is a neutrosophic superhyper BCI-semigroup. Thus, the BCI identity properties hold.
4. **Neutrosophic Membership:** From the definition of neutrosophic superhyper BCI-quotient set S/I, $(S/I, \Theta)$ preserved all the neutrosophic membership values. The binary superhyper operation Θ in the neutrosophic superhyper BCI-quotient set S/I respects these values because the ideal I does not alter the inherent neutrosophic membership properties of elements in S.

□

Definition 5.17. *Let (S, Θ) be a neutrosophic superhyper BCI-semigroup. A neutrosophic superhyper BCI-ideal I is said to be maximal if there does not exist any proper neutrosophic superhyper BCI-ideal J such that $I \subsetneq J$. In simpler terms, a maximal neutrosophic superhyper BCI-ideal is an ideal within the semigroup for which no larger ideal is contained within it. Any additional element or subset that could be added to the maximal ideal would either break closure under the binary superhyper operation Θ or violate the properties of an ideal.*

Maximal ideals are important because they represent the largest possible ideals within a semigroup that maintain the desired algebraic properties, making them a focus of study in the context of semigroup theory and algebraic structures.

Example 5.18. Consider the same set $S = \{a, b, c, d\}$ with the binary superhyper operation Θ defined as in Table 9. Let $I = \{b, c\}$. Then I is an ideal on the neutrosophic superhyper BCI-semigroup (S, Θ), since for $x = b$ and $y \in S$:

$$b\Theta a = \{b\} \subseteq I, \quad b\Theta b = \{b\} \subseteq I, \quad b\Theta c = \{b, c\} \subseteq I, \quad b\Theta d = \{b, c\} \subseteq I,$$

Table 9: The binary superhyper operation Θ on S

Θ	$a,(0.5,0.3,0.4)$	$b,(0.7,0.2,0.2)$	$c,(0.6,0.3,0.3)$	$d,(0.6,0.3,0.3)$
$a,(0.5,0.3,0.4)$	$\{a\},(0.5,0.3,0.4)$	$\{b\},(0.5,0.2,0.2)$	$\{c\},(0.5,0.3,0.3)$	$\{d\},(0.5,0.3,0.3)$
$b,(0.7,0.2,0.2)$	$\{b\},(0.5,0.2,0.2)$	$\{b\},(0.7,0.2,0.2)$	$\{b,c\},(0.6,0.2,0.2)$	$\{b,c\},(0.6,0.2,0.2)$
$c,(0.6,0.3,0.3)$	$\{c\},(0.5,0.3,0.3)$	$\{b,c\},(0.6,0.2,0.2)$	$\{c\},(0.6,0.3,0.3)$	$\{b,c\},(0.6,0.3,0.3)$
$d,(0.6,0.3,0.3)$	$\{d\},(0.5,0.3,0.3)$	$\{b,c\},(0.6,0.2,0.2)$	$\{b,c\},(0.6,0.3,0.3)$	$\{a,d\},(0.6,0.3,0.3)$

and for $x = c$ and $y \in S$:

$$c\Theta a = \{c\} \subseteq I, \quad c\Theta b = \{b,c\} \subseteq I, \quad c\Theta c = \{c\} \subseteq I, \quad c\Theta d = \{b,c\} \subseteq I.$$

Since all conditions are satisfied, $I = \{b,c\}$ is a neutrosophic superhyper BCI-ideal in (S,Θ).

Now, let's check if I is maximal. We need to verify if there exists a proper neutrosophic superhyper BCI-ideal J such that $I \subsetneq J$.

Let's consider $J = \{b,c,d\}$ and $K = \{a,b,c\}$:

Case 1: I is a proper subset of J because d is in J but not in I. However, J is not a neutrosophic superhyper BCI-ideal because $d \circ d = \{a,d\}$ is not in J, violating the closure under the binary superhyper operation property.

Case 2: I is a proper subset of K because a is in K but not in I. However, K is not a neutrosophic superhyper BCI-ideal because $a \circ d = \{a,d\}$ is not a subset of K, violating the closure under the binary superhyper operation property.

Therefore, $I = \{b,c\}$ is a maximal neutrosophic superhyper BCI-ideal.

Theorem 5.19. *Let (S,Θ) be a neutrosophic superhyper BCI-semigroup. Let (T,Θ) be a neutrosophic superhyper BCI-semisubgroup of (S,Θ) and let I be a proper neutrosophic superhyper BCI-ideal of S. The extension $T \cup I$ with the binary superhyper operation Θ defined as $x\Theta y = z$ for $x,y \in I$ and $z \subseteq T$ forms a neutrosophic superhyper BCI-semigroup.*

Proof. To show that $(T \cup I,\Theta)$ forms a neutrosophic superhyper BCI-semigroup, we need to verify that the structure maintains the properties of a neutrosophic superhyper BCI-semigroup under the given extension operation.

(i) **Closure:** For any $a,b \in T \cup I$, we need to show that $a\Theta b \in T \cup I$.

Case 1: If $a,b \in T$, then $a\Theta b \in T$ because (T,Θ) is a neutrosophic superhyper BCI-semigroup and thus closed under Θ.

Case 2: If $a,b \in I$, then $a\Theta b \subseteq T$ by the definition of the binary superhyper operation in the extension, which implies that $a\Theta b$ is a subset of $T \cup I$.

Case 3: If $a \in T$ and $b \in I$ (or vice versa), then $a\Theta b \subseteq I$ by the definition of the ideal I, it follows that $a\Theta b$ is a subset of $T \cup I$. Thus, the closure property holds.

(ii) **Associativity:** For any $a,b,c \in T \cup I$, we need to show that $(a\Theta b)\Theta c = a\Theta(b\Theta c)$.

Case 1: If $a,b,c \in T$, associativity holds because (T,Θ) is a neutrosophic superhyper BCI-semigroup.

Case 2: If $a,b,c \in I$, associativity holds by the extended definition: $a\Theta(b\Theta c) = (a\Theta b)\Theta c$, both resulting in subsets of S.

Case 3: For mixed cases, $a,b \in T$ and $c \in I$ (or permutations), we extend the operation to respect associativity, preserving the semigroup structure. Thus, the associativity property holds.

(*iii*) **BCI Identity:** $(T \cup I, \Theta)$ satisfies the BCI identity, because (T, Θ) is a neutrosophic superhyper BCI-semigroup. Thus, the BCI identity properties hold.

(*iv*) **Neutrosophic Membership:** $(T \cup I, \Theta)$ preserves all the neutrosophic membership values, since (S, Θ) is a neutrosophic superhyper BCI-semigroup and $T \cup I \subseteq S$.

□

6 Conclusion

The exploration of neutrosophic superhyper BCI-semigroups offers a robust framework for addressing and manipulating uncertain, indeterminate, and conflicting information. By extending traditional BCI-algebraic structures with neutrosophic logic and superhyper operations, these semigroups provide a versatile toolset for complex information systems. The defined properties and illustrated examples confirm that neutrosophic superhyper BCI-semigroups adhere to crucial algebraic principles, such as closure, associativity, and the existence of a unique BCI identity. Additionally, the introduction of idempotent and commutative elements, homomorphisms, ideals, subsemigroups, and quotient sets enriches the theoretical landscape, enabling further research and practical applications. The presented theorems substantiate the consistency and viability of these structures, suggesting significant potential for their integration into various domains requiring nuanced information processing.

Future research on neutrosophic superhyper BCI-semigroups could focus on developing more sophisticated algorithms for their application in complex decision-making systems and real-world scenarios. The feasibility of such applications can be supported by referencing existing studies on SuperHyper BCI-Semigroups, Neutrosophic Sets, Neutrosophic BCI-Semigroups, and Fuzzy BCI-Semigroups, which have demonstrated their effectiveness in handling uncertainty and imprecise information [34]-[35]. Additionally, integrating these algebraic structures with emerging technologies like artificial intelligence and machine learning could further enhance their practical utility, particularly in dealing with indeterminate and conflicting data. To strengthen the theoretical foundation and broaden the applicability of these structures, future research could explore extending the framework of BCI- Semigroups by incorporating Plithogenic Sets, HyperNeutrosophic Sets, and SuperHyperNeutrosophic Sets ([1]-[13]). Standardizing methodologies and addressing regional data variations will also be critical for ensuring wider adoption across various disciplines.

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Ajoy Kanti Das
Department of Mathematics
Tripura University
Agartala, India
E-mail: ajoykantidas@gmail.com

Rajat Das
Department of Mathematics
Tripura University
Agartala,Tripura, India
E-mail: das.rajat666@gmail.com

Suman Das
Department of Education (ITEP)
NIT Agartala
Agartala,Tripura, India
E-mail: sumandas18842@gmail.com

Bijoy Krishna Debnath
Department of Applied Sciences, School of Engineering
Tezpur University
Assam, India
E-mail: bijoy91@tezu.ernet.in

Carlos Granados
Department of Mathematics
Universidad de Antioquia
Medellin, Colombia
E-mail: carlosgranadosortiz@outlook.es

Bimal Shil
Department of Statistics
Tripura University
Agartala, India
E-mail: bimalshil738@gmail.com

Rakhal Das
Department of Mathematics
The ICFAI University Tripura
Agartala, India
E-mail: rakhaldas95@gmail.com