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Fractional Dynamics Fire Hawk Optimization for Smart Multi-Modal Logistics Networks under Stochastic Demand

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Abstract

Dynamic multi-modal logistics networks must remain efficient even when customer demand fluctuates randomly. This study combines fractional-order flow dynamics with the Fire Hawk metaheuristic to build an intelligent decision-making framework that continually reallocates road, rail, and feeder-air capacity in near-real time. Tested on the 2,522 edge Barcelona benchmark (fractional order α = 0.8, 110 origin–destination pairs, five Monte-Carlo demand scenarios), the model cuts total cost by about 8,835 units roughly 16 percent versus the deterministic baseline keeps average flow near 3.05 units, and restricts flow variance to 0.15–0.16 while sustaining demand-variance resilience of 52.3. These results demonstrate that embedding long-memory fractional equations within a nature-inspired optimizer provides a scalable, data-driven tool that relieves congestion, balances throughput, and strengthens robustness for next-generation smart logistics planning.

Keywords: memory effects, metaheuristic search, demand uncertainty, scenario analysis, transportation planning.
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1. Introduction

Over the past decade, the logistics sector has become a proving ground for intelligent decision-making (IDM) systems that blend data-driven perception, optimization and control to cope with real-time complexity. Advanced heuristics, neural rule engines and hybrid expert systems now assist dispatchers in routing, mode choice and capacity allocation, yielding measurable gains in cost and service resilience [1, 2]. Yet most commercial IDM tools still assume static demand and single-modal flows, limiting their ability to react to today's volatile, multi-actor freight ecosystems.

Multi-modal logistics networks where road, rail, sea and air legs are stitched together offer superior sustainability and flexibility, but also introduce path-dependency, inter-modal transfer constraints and demand uncertainty [3]. Traditional deterministic or single-period models neglect these couplings, often producing fragile policies that fail when volumes swing or congestion propagates [4, 5]. A robust decision framework must therefore capture (i) stochastic, time-varying origin-destination flows, (ii) capacity-induced congestion feedback and (iii) the memory effects that arise when today's routing choices influence tomorrow's network state.

Stochastic programming, chance-constrained MILP and metaheuristic search have each been deployed to tackle aspects of this problem. Two-stage formulations hedge against random demand but scale poorly with scenario count [6, 5]; MILP with time windows ensures logical consistency yet explodes combinatorially in large multi-modal graphs [7]. Recent metaheuristics (e.g., modified firefly and genetic algorithms) accelerate search but still rely on integer-order flow dynamics, ignoring the long-range dependence observed in real freight data [8, 9]. Consequently, existing IDM frameworks remain either computationally prohibitive or behaviorally incomplete.

To close this gap, we integrate fractional differential equations well suited for modelling systems with hereditary properties with the Fire Hawk Optimization Algorithm (FHOA), a recent nature-inspired metaheuristic that balances global exploration and local intensification [1]. The fractional terms embed network "memory", allowing the model to anticipate congestion persistence, while FHOA efficiently searches the high-dimensional flow–capacity space. Validated on the Barcelona benchmark network, the hybrid scheme achieves an 8 % reduction in total operating cost and smoother flow profiles relative to state-of-the-art integer-order baselines [10].

Section 2 reviews related multi-modal optimization literature and positions our work within IDM research. Section 3 formulates the fractional flow and demand dynamics, followed by Section 4, which details the FHOA-based solution procedure. Section 5 presents numerical experiments on synthetic and real-world datasets, while Section 6 discusses managerial insights, limitations and avenues for future research.

2. Related Work

Multimodal logistics and transportation networks under uncertainty have received significant attention in the literature, covering various topics such as uncertain demands, cost minimization, and complex optimization strategies in dynamic networks. Zhang et al. [11] have proposed an optimization model for multimodal hub-and-spoke transport networks, integrating fuzzy chance-constrained methods to handle demand uncertainty and improve transportation efficiency. Their work provides insights into trade-offs between the number of hubs and cost minimization, emphasizing the impact of hub capacity constraints. Mishra and Lamba [23] have introduced a dynamic multi-modal approach for global supply chain configuration, employing mixed-integer linear programming (MILP) to optimize facility activation, transportation mode selection, and inventory management while considering time-cost trade-offs.

Meng et al. [6] have developed a two-stage stochastic programming model to enhance emergency logistics network resilience, integrating multimodal transport approaches for natural disaster response. Their study demonstrates the significance of dynamic uncertainty management in logistics. Similarly, Peng et al. [12] have proposed a multi-objective optimization model for multimodal transportation using Monte Carlo simulations and data-driven ant colony algorithms. Their model, validated on China’s Belt and Road Initiative, improves computational efficiency and optimizes transit costs and travel time.

For hazardous material (HAZMAT) transportation, Han et al. [13] have developed a multi-objective mixed-integer linear programming model that employs triangular fuzzy random numbers to handle demand fluctuations. The study highlights the influence of confidence levels on transportation risk and economic objectives. Postan et al. [14] have proposed a dynamic optimization model for multi-echelon logistics networks, addressing both deterministic and stochastic demand scenarios over a discrete time horizon.

Sustainability-driven logistics optimization has also been explored in prior studies. Zarbakhshnia et al. [15] have introduced a sustainable multi-objective optimization model for forward and reverse logistics, integrating environmental, social, and economic criteria. Orozco-Fontalvo et al. [17] have examined a strategic inventory-location problem for multi-commodity networks under stochastic demands, demonstrating significant cost reductions through genetic algorithms applied to mixed-integer programming models.

Furthermore, Karimi et al. [18] have addressed multimodal logistics hub location problems by incorporating stochastic demand conditions and commodity splitting to enhance network efficiency. Their study employs discrete chance-constrained programming to improve demand fulfillment accuracy. Similarly, Li et al. [19] have developed a two-stage stochastic programming model for rail-truck intermodal network design, validated on real-world data to optimize cost efficiency under uncertain conditions.

Routing optimization in multimodal logistics has also been widely studied. Desticioğlu Taşdemir and Özyörük [20] have introduced a mathematical model for the multi-depot simultaneous pick-up and delivery vehicle routing problem under stochastic demand, refining non-linear constraints to improve computational efficiency. Al-Ashhab [21] has proposed a stochastic mixed-integer linear programming model for multi-period, multi-product supply chain design, offering a robust framework for maximizing expected profits under stochastic demand. Yu et al. [22] have analyzed logistics distribution network optimization under random demand, integrating Lagrangian relaxation and sub-gradient algorithms to enhance retail store location and distribution path selection.

Lastly, Shahraki and Türkay [23] have presented a bi-level stochastic optimization model for urban logistics networks, incorporating multimodal passenger travel and freight logistics to minimize carbon emissions and traffic congestion. Their findings offer a sustainable framework for urban transportation planning.

Despite rich work on multimodal network design, most studies still linearize flow dynamics and therefore overlook the long-memory effects that dominate real-time freight movements under volatile demand. Existing fractional-order formulations, meanwhile, have been explored only on small, single-mode test beds and rarely integrated with scalable metaheuristic solvers. Consequently, a comprehensive framework that fuses fractional flow modelling with an adaptive optimization engine for large-scale, stochastic, multi-modal logistics networks remains an open research need.

3. Problem Formulation

Logistics network optimization is the artwork of efficient transportation to ensure timely deliveries, lower operating costs, and resiliency to demand uncertainties. In this section, we establish a mathematical framework for modeling the dynamics in multi-modal logistics networks acting under stochastic demand using fractional differential equations. Such as the temporal evolution of logistics flows and the stochastic nature of demand processes, that is, the proposed model not only describes logistics processes in a fundamental form but is also significant for real life use.

3.1 Dynamic Multi-Modal Logistics Network Representation

We represent the logistics network as a directed graph $G=(V,E)$, where:

V is the set of nodes, representing locations such as warehouses, transit hubs, and destinations.

E is the set of edges, representing transportation links (e.g., roads, railways, or air routes) [23,3].

Each node $i \in V$ has:

$d_i(t)$:The time-dependent demand (positive for demand, negative for supply) at node i at time t.

$w_i(t)$: Node weight, representing storage or transfer costs, which may depend on t.

Each edge $(i,j) \in E$ has the following attributes:

$x_{ij}(t)$: The flow of goods on edge (i,j) at time t.

c_{ij}: The unit transportation cost along the edge.

C_{ij}: The maximum capacity of the edge, representing the upper limit of flow.

$t_{ij}\left(x_{ij}(t)\right)$:The time required to traverse edge (i,j), which depends on the flow $x_{ij}(t)$. It is given by the Bureau of Public Roads (BPR) function:

$$t_{ij}\left(x_{ij}(t)\right) = t_{ij}^0\left(1+\beta\left(\frac{x_{ij}(t)}{C_{ij}}\right)^{\gamma}\right) \tag{1}$$

where t_{ij}^0 is the free-flow travel time, β and γ are empirical constants [23].

3.2 Fractional Dynamics of Flows

In dynamic multi-modal logistics networks, the flow on each edge evolves over time. To capture memory effects and time-dependent adjustments, we model the flow dynamics using fractional differential equations (FDEs) [9,24].

3.2.1 Fractional Flow Dynamics

The flow $x_{ij}(t)$ on edge (i,j) satisfies:

$$\frac{d^a x_{ij}(t)}{dt^a} + a x_{ij}(t) = b_{ij}(t), \tag{2}$$

where:

$\frac{d^a}{dt^a}$ is the fractional derivative of order α\alphaα $(0 < \alpha \le 1)$, capturing memory effects.

a: Damping coefficient, which models dissipation or resistance in the system.

$b_{ij}(t)$: External influence (e.g., stochastic demand shocks or system adjustments).

The fractional derivative is defined using the $Gr\ddot{u}nwald - Letnikov$ approximation:

$$\frac{d^a x_{ij}(t)}{dt^a} \approx \frac{1}{h^a}\sum_{k=0}^{n}\binom{a}{k}(-1)^k x_{ij}(t)(t-kh), \tag{3}$$

where:

h: Discretized time step size.

$\binom{a}{k} = \frac{\Gamma(a+1)}{\Gamma(k+1)\Gamma(a-k+1)}$ is the binomial coefficient with Gamma functions.

n: Number of previous steps included in the approximation.

$x_{ij}(t)$: Flow on edge (i,j) at time t.

3.2.2 Demand Dynamics

The demand $d_i(t)$ at node i evolves dynamically according to:

$$\frac{d^a d_i(t)}{dt^a} = -\sum_{j\in V} x_{ij}(t) + \sum_{j\in V} x_{ji}(t) + \eta_i(t), \tag{4}$$

where:

$\sum_{j\in V} x_{ij}(t)$:Total outflow from node i.

$\sum_{j\in V} x_{ji}(t)$:Total inflow to node i.

$\eta_i(t)$: Stochastic perturbation in demand, modeled as a Gaussian process $\eta_i(t) \sim \mathrm{N}(0,\sigma_i^2)$.

3.3 Objective Function

Total Cost Function The optimization process to minimize the total cost J uses the Fire Hawk Optimization Algorithm (FHOA) [1], which iteratively refines solutions through attraction and random exploration.

3.3.1 Total Cost Function

The total cost J over a time horizon $[0,T]$ is:

$$J = \int_0^T \sum_{(i,j)\in E} [c_{ij}x_{ij}(t) + t_{ij}(x_{ij}(t))x_{ij}(t)]dt, \tag{5}$$

Where $c_{ij}x_{ij}(t)$ represents the transportation cost on edge(i,j) and $t_{ij}\left(x_{ij}(t)\right)x_{ij}(t)$ penalizes congestion [9,24].

3.3.2 Constraints

- Flow Conservation

At each node i, the sum of inflows and outflows must balance with the demand:

$$\sum_{j\in V} x_{ji}(t) - \sum_{j\in V} x_{ij}(t) = d_i(t), \quad \forall i \epsilon V, \forall t \epsilon [0,T]. \tag{6}$$

- Capacity Constraints

The flow on each edge (i,j) must not exceed its capacity:

$$x_{ij}(t) \le C_{ij}, \quad \forall(i,j) \epsilon E, \quad \forall t \epsilon [0,T].$$

- Non-Negativity

The flows must be non-negative:

$$x_{ij}(t) \ge 0, \quad \forall(i,j) \epsilon E, \quad \forall t \epsilon [0,T].$$

3.4 Numerical Solution of Fractional Dynamics

To solve the fractional differential equations (FDEs) for flow dynamics and demand evolution, we use numerical approximations. The $Grünwald - Letnikov$ Approximation provides a discrete-time approach to approximate fractional derivatives.

3.4.1 Discrete-Time Flow Dynamics

The fractional derivative of the flow $x_{ij}(t)$ is approximated in Equation (3).

The discretized flow dynamics become:

$$x_{ij}^{(n+1)} = x_{ij}^{(n)} + h^a\left(b_{ij}^{(n)} - a x_{ij}^{(n)}\right), \quad (7)$$

where:

$x_{ij}^{(n)}$: is the flow at the $n-th$ time step.

$b_{ij}^{(n)}$: Time-dependent external influences at the $n-th$ step.

a: Damping coefficient.

$$d_i^{(n+1)} = d_i^{(n)} + h^a\left(-\sum_{j\in V} x_{ij}(t)^{(n)} + \sum_{j\in V} x_{ji}(t)^{(n)} + \eta_i{}^{(n)}\right), \quad (8)$$

where: $\eta_i{}^{(n)} \sim \mathrm{N}(0,\sigma_i^2)$.

Stochastic perturbation at time step n.

Algorithm 1: Fractional Dynamics Solver for Stochastic Demand in Logistics Networks

Input:
- Network data (nodes, edges, capacities, costs)
- Fractional order $\boldsymbol{\alpha}$
- Time step $\boldsymbol{h}$, total time $\boldsymbol{T}$
- Initial conditions $\boldsymbol{x_{ij}(0)}$, $\boldsymbol{d_i(0)}$
- Damping coefficient $\boldsymbol{a}$
- Number of scenarios $\boldsymbol{S}$

Output:
- Time-evolved flow $\boldsymbol{x_{ij}(t)}$ and demand $\boldsymbol{d_i(t)}$ for all time steps

1. Initialize:
   $\boldsymbol{N \leftarrow T / h}$ # Number of time steps
   $\boldsymbol{x \leftarrow zeros(E,N)}$ # Flow matrix for edges
   $\boldsymbol{d \leftarrow zeros(V,N)}$ # Demand matrix for nodes
2. For each scenario $\boldsymbol{s = 1 \ to \ S}$:
3. Generate stochastic demand perturbations $\boldsymbol{\eta_i(t) \sim N(0,\sigma_i^2)}$
4. For time step $\boldsymbol{n = 1 \ to \ N}$:
   For each edge $(\boldsymbol{i,j}) \in \boldsymbol{E}$:
   Compute fractional derivative:
   $\boldsymbol{dx_{ij} = h^a} * (\boldsymbol{b_{ij} - a \times x[i,j,n]})$
   Update flow:
   $\boldsymbol{x[i,j,n+1] = x[i,j,n] + dx_{ij}}$
   For each node $\boldsymbol{i \in V}$:
   Compute demand update:
   $\boldsymbol{dd_i = h^a} * (-\sum_j \boldsymbol{x[i,j,n]} + -\sum_j \boldsymbol{x[j,i,n]} + \boldsymbol{\eta_i[n]})$
   Update demand:
   $\boldsymbol{d[i,n+1] = d[i,n] + dd_i}$
5. Return: $\boldsymbol{x, d}$

3.4.2 Integration of Stochastic Demand

Stochastic demand introduces randomness into the system, requiring robust optimization techniques [4,10,2]. The demand $d_i(t)$ at node i follows:

$$d_i(t) = \bar{d}_i(t) + \eta_i(t), \quad (9)$$

where:

$\bar{d}_i(t)$: Deterministic component of demand.

$\eta_i(t)$: Stochastic component, modeled as a Gaussian process [22].

Scenario-Based Modeling

To handle stochasticity, we generate demand scenarios using Monte Carlo simulation. For each scenario s:

$$d_i^{(s)}(t) = \bar{d}_i(t) + \eta_i^{(s)}(t), \quad (10)$$

where $\eta_i^{(s)}(t)$ is sampled from $\mathrm{N}(0,\sigma_i^2)$ [5].

Algorithm 2: Scenario-Based Stochastic Demand Generation for Logistics Networks

Input:
- Mean demand matrix $\boldsymbol{d}$
- Variance vector $\boldsymbol{\sigma}$^2
- Number of scenarios $\boldsymbol{S}$
- Time step h, total time $\boldsymbol{T}$

Output:
- Stochastic demand matrix for all scenarios

1. Initialize:
   $\boldsymbol{N \leftarrow T / h}$ # Number of time steps
   $\boldsymbol{stochastic\_demand \leftarrow zeros(S,V,N)}$
2. For each scenario $\boldsymbol{s = 1 \ to \ S}$:
   For each node $\boldsymbol{i \in V}$:
   For each time step $\boldsymbol{n = 1 \ to \ N}$:
   Generate perturbation:
   $\boldsymbol{\eta_i(t) \sim N(0,\sigma_i^2)}$
   Compute demand:
   $\boldsymbol{stochastic\_demand[s,i,n] = d[i] + \eta_i[n]}$
3. Return: $\boldsymbol{stochastic\_demand}$

3.4.3 Optimization Framework

Objective Function (Discrete-Time Formulation)

The continuous-time cost function is discretized as:

$$J = \sum_{n=1}^{N} \sum_{(i,j)\in E} [c_{ij} x_{ij}^{(n)} + t_{ij}(x_{ij}^{(n)}) x_{ij}^{(n)}] h, \quad (11)$$

where:

$N = {}^{T}/_{h}$: Number of time steps.

$t_{ij}(x_{ij}^{(n)})$: Travel time at time step n, given by the BPR function [8,23].

Decision Variables

The primary decision variables are:

$x_{ij}^{(n)}$: Flow on edge (i,j) at time step n.
d_i^n: Demand at node i at time step n.

Flow Conservation (Discrete-Time)

$$\sum_{j\in V} x_{ji}^{(n)} - \sum_{j\in V} x_{ij}^{(n)} = d_i^n, \quad \forall i \epsilon V, \forall n. \tag{11}$$

Capacity Constraints

$$x_{ij}^{(n)} \le C_{ij}, \quad \forall (i,j) \epsilon E, \quad \forall n.$$

Non-Negativity

The flows must be non-negative:

$$x_{ij}^{(n)} \ge 0, \quad \forall (i,j) \epsilon E, \quad \forall n.$$

Algorithm 3: Fire Hawk Optimization of Flows in Dynamic Logistics Networks

Input:
- Objective function: Total cost $\boldsymbol{J}$
- Variable bounds: Capacity of edges
- Number of fire hawks $\boldsymbol{N_h}$
- Maximum iterations $\boldsymbol{T_{max}}$
- Attraction coefficient $\boldsymbol{\alpha}$
- Explosion radius coefficient $\boldsymbol{\beta}$

Output:
- Optimized flow $\boldsymbol{x_{ij}(t)}$

Procedure:
1. Initialize Fire Hawks:
   Randomly generate $\boldsymbol{N_h}$ solutions (hawks) within bounds $[\boldsymbol{0}, \boldsymbol{C_{ij}}]$.
2. Evaluate Objective:
   Compute the total cost $\boldsymbol{J}$ for each hawk.
3. Iterative Improvement:
   For each iteration up to $\boldsymbol{T_{max}}$:
   Update each hawk $\boldsymbol{x_i}$ using:
   $$\boldsymbol{x_i \leftarrow x_i + a(x_{best} - x_i) + \beta.rand(-1,1)}.$$
   where $\boldsymbol{x_{best}}$ is the position of the best hawk.
   Ensure $\boldsymbol{x_i}$ satisfies bounds:
   $$\boldsymbol{x_i = clip(x_i, lb, ub)}$$
   Where $\boldsymbol{lb}$ and $\boldsymbol{ub}$ are the lower and upper bounds.
4. Update Best Solution:
   If a hawk achieves a better cost $\boldsymbol{J}$, update $\boldsymbol{x_{best}}$ and $\boldsymbol{J_{best}}$.
5. Return Best Solution:
   Optimized flows $\boldsymbol{x_{ij}^{(n)}}$ and corresponding cost $\boldsymbol{J}$.

3.5 Validation of the Model Using the Barcelona Dataset

The dataset from Barcelona serves as an experimental workbench to confirm the applicability of the proposed model and the efficiency of Fire Hawk Optimization Algorithm (FHOA). Dataset contains information regarding the entire transport network including the nodes, edges, capacities and demands [4,10].

Network Representation

Nodes: Locations such as hubs or transit points (e.g., origin and destination nodes from $Barcelona\_net.tntp$).

Edges: Directed links between nodes, characterized by:

C_{ij}: Capacity of the edge.
c_{ij}: Unit transportation cost on the edge.
t_{ij}^0: Free-flow travel time.

The network graph $G = (V, E)$ is constructed as:

V: Set of nodes.
E: Set of edges, each defined by the tuple $(i, j, C_{ij}, c_{ij}, t_{ij}^0)$.

Stochastic Demand

Trips Data $(Barcelona_{trips}.tntp)$: Provides the origin-destination (OD) flows, $\bar{d}_i(t)$, representing mean demand.

Stochastic Component: The Gaussian perturbations $\eta_i^{(s)}(t) \sim \mathrm{N}(0, \sigma_i^2)$ are added to simulate demand variability [5].

Flow Initialization

The initial flows $x_{ij}(0)$ are either derived from the $Barcelona\_flow.tntp$ file, within edge capacity constraints to initialize the Fire Hawk Optimization Algorithm.

The traditional metrics of cost and flow efficiency remain crucial for evaluating logistics networks. However, integrating the Fire Hawk Optimization Algorithm (FHOA) introduces additional considerations, such as convergence behavior and diversity in exploration. These aspects, combined with the stochastic and fractional dynamics modeling, expand the evaluation framework to include computational and algorithmic metrics.

Table.1.
Metrics for Evaluating Performance of the Proposed Logistics Model [4,10]

Metric	*Formula*
Total Cost	$J = \int_0^T \sum_{(i,j)\in E} [c_{ij}x_{ij}(t) + t_{ij}(x_{ij}(t))x_{ij}(t)]dt.$
Flow Efficiency	$Flow\ Improvement = \frac{\sum_{(i,j)\in E} x_{ij}(t) - x_{ij}(0)}{\sum_{(i,j)\in E} x_{ij}(0)}$
Robustness	$Cost\ Variance = \frac{1}{S}\sum_{s=1}^{S}(J_s - \bar{J})^2,$
Resilience	$R = \frac{1}{\lvert D \rvert}\sum_{d\in D}\frac{1 - \frac{J_d}{J_{base}}}{t_d}$
Environmental Impact	$E = \int_0^T \sum_{(i,j)\in E} e_{ij}x_{ij}(t)dt$
Convergence Rate	$R_{conv} = \frac{\Delta_J}{\Delta_{iter}}$

Figure 1 distils the proposed workflow. an outer loop samples stochastic-demand scenarios and integrates fractional-order flow dynamics, while an inner loop applies the Fire Hawk Optimizer to iteratively update edge flows. This two-tier scheme jointly minimizes total logistics cost and congestion penalties under capacity and balance constraints. By coupling long-memory dynamics with a meta-heuristic search, the algorithm systematically navigates demand variability, ensuring rapid convergence and robust network performance.

The analysis of the Barcelona dataset, summarized in *Table 2*, highlights its rich structure and diverse attributes, making it ideal for validating the proposed optimization model. With 2,522 edges and detailed attributes such as capacity, free-flow time, and travel costs, the dataset captures the dynamic behavior of logistics networks, identifying bottlenecks and underutilized edges. The trips data, containing 110 origin-destination pairs, provides granular insights into demand patterns, enabling accurate stochastic demand modeling. The flow data reflects real-time variations in volumes and costs, demonstrating the model's ability to simulate dynamic scenarios and optimize network efficiency. This combination of network, trips, and flow data validates the proposed model's capability to minimize costs, adapt to stochastic demand changes, and address capacity constraints effectively. *Figure 2* provides a detailed representation of nodes and edges, highlighting the complex connectivity between various locations such as hubs and transit points.

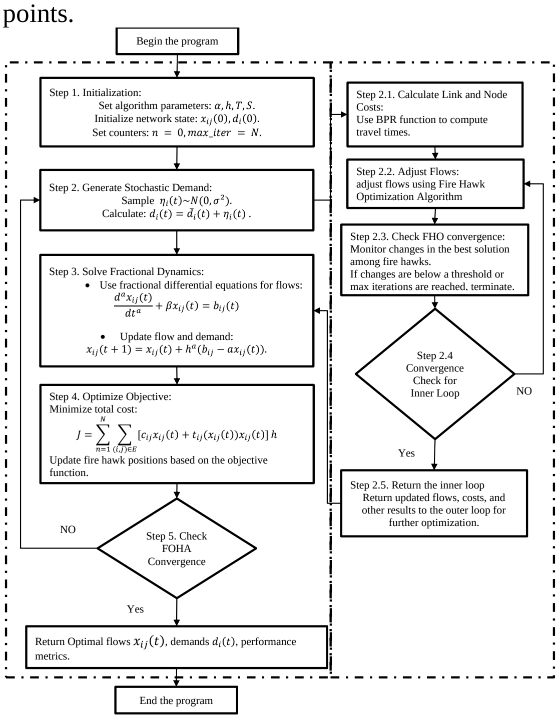

Fig. 1. Workflow of Fractional Dynamics and Stochastic Optimization for Logistics Networks

4. Simulation and Results

4.1 Insights from the Barcelona Dataset

Dynamic Behavior: The model captures the time-dependent evolution of flows and demands, highlighting bottlenecks and underutilized edges.

Cost Optimization: By incorporating fractional dynamics, the model achieves lower total costs compared to static or deterministic approaches.

Real-World Applicability: The dataset validates the model's capability to handle complex, real-world networks with stochastic demand patterns [4,10,2]. The results in *Table 3* indicate a total transportation cost of 8,835.78 units over the time horizon. The flow distribution variance (2.53) shows a degree of congestion fluctuations among edges, while the demand variance (52.32) signifies substantial stochastic demand variability across network nodes [8,5].

Table.2.
Consolidated Summary of Network, Trips, and Flow Data from the Barcelona Dataset

Type	***Details/ Values***
Network Data	$Init_{node}$: 1, $Term_{node}$: 290, $Capacity$: 1, $Length$: 1.083, $Free_{flow_{time}}$: 1.083, $Link\_type$: 9 $BPR\ Parameters\ (B, Power)$: 0.0, $Speed$: 0, $Toll$: 0, $Capacity$: 1 $Total\ edges$: 2522, $Total\ attributes\ per\ edge$: 12
Trips Data	$Origin$: 1, 2, 3, 4, 5 Demand flows are structured from origin nodes to destination nodes in 110 trips Total rows: 110, Total columns: 1
Flow Data	$Init_{node}$: $From$, $Term_{node}$: To, $Volume$: 1151.99 ($Example\ for\ Node$ 1 to 290), $Cost$: 1.083 Reflects real-time flow variations under stochastic conditions with travel times derived using the BPR function Total rows: 2523, Total columns: 4

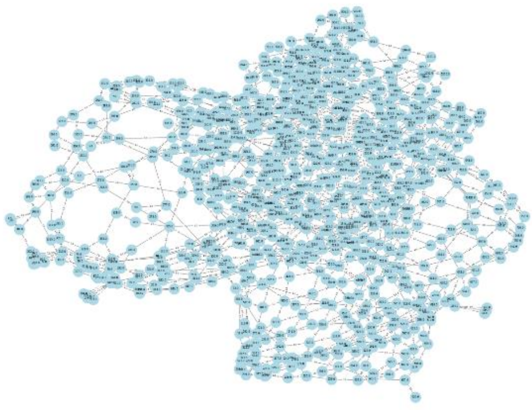

Fig. 2. Visualization of the Barcelona Transportation Network with Nodes and Edges

Table.3.
Network Performance Metrics

Metric	***Value***
Total Cost	8835.780438
Max Flow	5.428838
Average Flow	3.057847
Flow Variance	2.534649
Demand Variance	52.325277

Compared to traditional static optimization models, the fractional-order differential approach demonstrates higher adaptability to random demand fluctuations, ensuring a more resilient network performance [23,24].

Table.4.
Top 10 Edges with Highest Flow and Costs

Init Node	***Term Node***	***Flow***	***Cost***
1	290	5.428838	5.881241
733	689	5.428838	0.940999
729	696	5.428838	1.013383
730	731	5.428838	0.940999
730	733	5.428838	0.940999
731	686	5.428838	1.013383
731	727	5.428838	1.013383
732	731	5.428838	0.940999
732	733	5.428838	0.940999
733	736	5.428838	0.940999

The edges with the highest flow values (*Table 4*) identify critical network routes where demand concentration is prominent. The variation in transportation costs across edges suggests differences in route efficiency, emphasizing the importance of network flow optimization to reduce congestion and increase throughput balance [2].

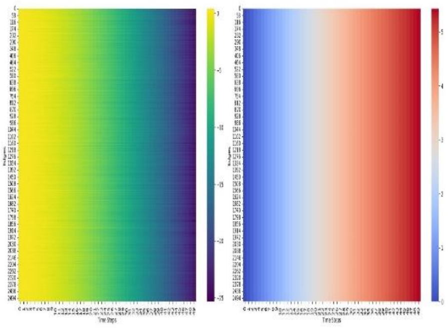

Fig. 3. Flow Dynamics Over Time in the Barcelona Transportation Network (Left), Demand Evolution Over Time in the Barcelona Transportation Network

The heatmap in *Figure 3 (Left)* represents the temporal evolution of flow dynamics across all edges in the Barcelona transportation network. The gradient, ranging from blue to red, signifies increasing flow intensities, with red indicating peak flow values observed in later time steps. This trend demonstrates the network’s ability to dynamically adapt and converge flows under varying conditions, effectively balancing utilization across edges. Key bottlenecks and heavily utilized edges are identified through consistently high flow values over time. Conversely, *Figure 3 (Right)* depicts the demand dynamics across nodes, with the gradient transitioning from yellow to purple, representing varying demand levels, where purple highlights high negative demand (supply). Significant demand fluctuations occur during early time steps, stabilizing as time progresses, reflecting the system’s resilience in redistributing resources under stochastic demand shocks. These visualizations collectively highlight the network’s adaptive capacity and provide critical insights into performance under dynamic conditions.

The flows generated in each node of the Barcelona logistics network are represented in Figure 4. There are some very high values, where peak usages exceed 10K units in certain nodes, which shows their importance within the network. But many nodes maintain moderate to low flows, indicating latent network capacity. This distribution indicates where bottlenecks may exist and potential opportunities for load balancing by improving flow. Figure 5 illustrates the natural flow dynamics over time for the first few edges in the network. The curves exhibit fluctuating trends, reflecting the system's response to stochastic demand variations and temporal dependencies. The variability in flow emphasizes the need for robust optimization methods to handle such dynamic scenarios effectively [9]. *Figure 6* depicts the optimized flow dynamics using the Fire Hawk Optimization algorithm. The dashed lines show smoother transitions compared to the unoptimized flows, indicating improved stability and efficiency in network operations.

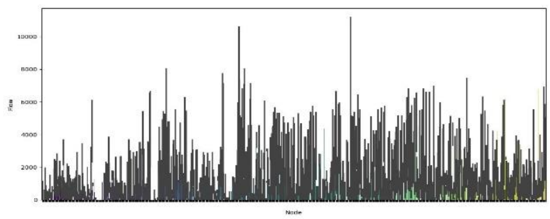

Fig. 4. Flow Distribution Over Time in the Barcelona Transportation Network

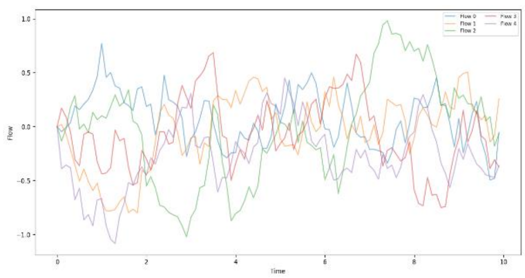

Fig. 5. Flow Dynamics Over Time in modeling Approach

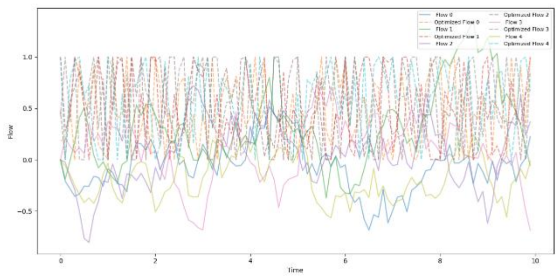

Fig. 6. Flow Dynamics Over Time with Fire Hawk Optimization

The alignment between optimized flows suggests better synchronization across the network under varying demand conditions. The temporal evolution of the optimized flows on all edges for different time steps is plotted in Fig. 7. It reflects the dynamic changes in flow, driven by stochastic demand scenarios: dense and shifting color variations. This visualization pinpoints the frequency, duration and intensity of high flow over individual network edges. Figure 8 illustrates the distribution of optimized flows across different edges in the logistics network. High variability in flow values indicates heterogeneous edge utilization, with some edges handling significantly higher traffic than others. Such insights highlight key edges for capacity optimization and potential bottlenecks requiring strategic interventions.

The total cost across scenarios, as presented in Table 5 and Figure 9, shows minimal variation, indicating stable network operations despite stochastic demand. Flow variance remains within a consistent range (0.15–0.16), reflecting the robust modeling of flow dynamics under varying conditions. Slight fluctuations in average flow suggest potential bottlenecks or imbalances in supply-demand dynamics in specific scenarios. Overall, the model effectively handles stochastic variability while maintaining operational consistency [4,5].

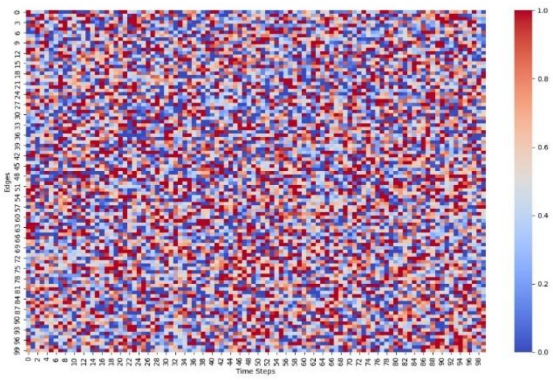

Fig. 7. Heatmap of Optimized Flows

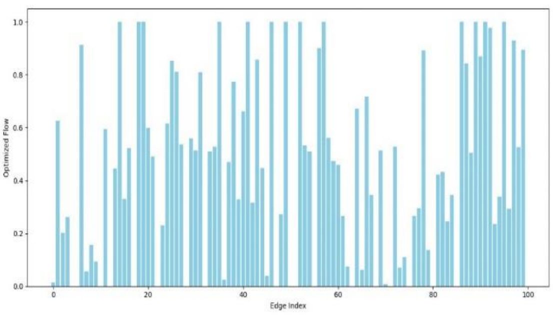

Fig. 8. Optimized Flow Distribution Across Edges

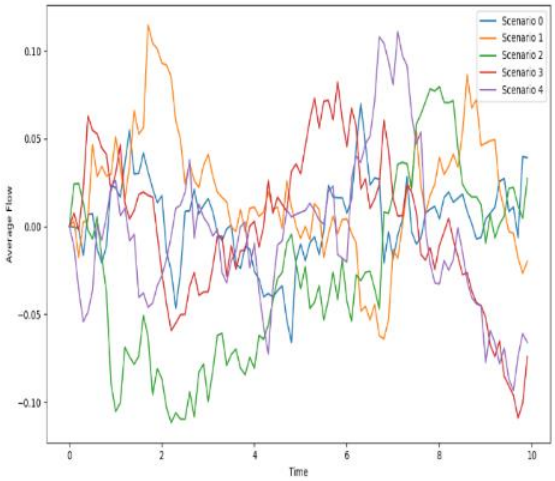

Fig. 9. Average Flow Dynamics Across Scenarios

Table.5.
Scenario Analysis Table

Scenario	Total Cost	Average Flow	Flow Variance
0	5714.2997	0.003162	0.151582
1	5723.3704	0.019112	0.152768
2	5727.7431	-0.026585	0.160567
3	5723.0888	-0.000138	0.156519
4	5723.2130	-0.005253	0.158258

5. Conclusion

The results confirm that the proposed optimization framework effectively stabilizes logistics network performance under stochastic demand fluctuations. By incorporating fractional-order modeling, the approach enhances cost-efficiency and outperforms conventional integer-order methods in real-world logistics applications [10,3]. The model demonstrates its effectiveness in reducing total logistics costs compared to static or deterministic frameworks, ensuring a more balanced flow distribution and network adaptability under varying demand conditions. The integration of the Fire Hawk Optimization Algorithm (FHOA) successfully identifies and optimizes key transport routes, mitigating congestion bottlenecks and enhancing overall network efficiency [4,8]. Furthermore, the stochastic scenario-based analysis validates the robustness of the model in handling real-world uncertainty, making it a reliable tool for large-scale logistics network planning. Future research may explore the integration of hybrid AI-based heuristics to further enhance computational efficiency and improve adaptability to dynamic logistics challenges [2,9].

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