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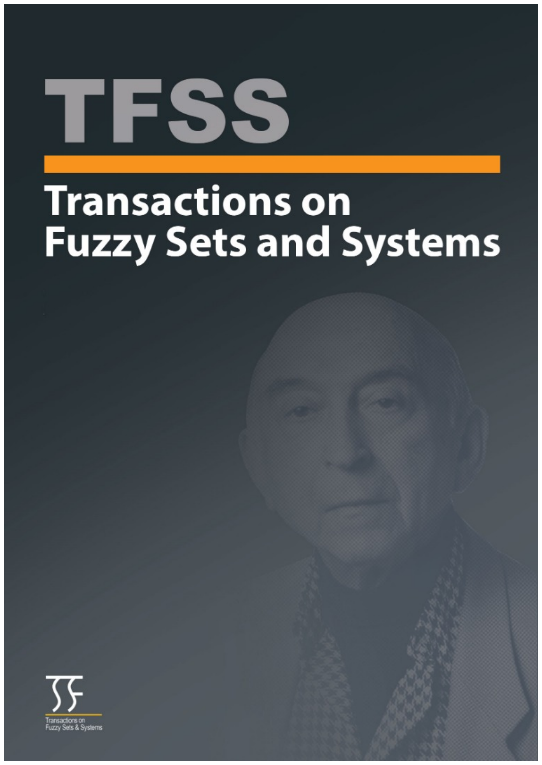

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Solution and Analysis of Coupled Homogeneous Linear Intuitionistic Fuzzy Difference Equation

Abdul Alamin, **Mostafijur Rahaman**, **Kamal Hossain Gazi**, **Shariful Alam**, **Sankar Prasad Mondal***

Abstract. In this paper, we have considered the initial valued coupled homogeneous linear difference equation in an intuitionistic fuzzy environment. We have given an outline of the general and analytical solutions at some fixed iteration levels and discussed the stability condition near the trivial intuitionistic equilibrium point of the system through a lemma. The proposed model is applied to check a predator-prey model and the graphical solutions are taken with taking different intuitionistic initial predator and prey population sizes to observe the dynamics in a short-sighted plan. Also, the graphical outcomes in an intuitionistic environment validate the theoretical result and the interrelated dynamical nature to fathom the significance of this work.

AMS Subject Classification 2020: 39A26; 39A60; 65Q10
Keywords and Phrases: Intuitionistic fuzzy set, Intuitionistic fuzzy difference equation, Stability analysis.

1 Introduction

The intuitionistic fuzzy set theory is essential in many areas of mathematics, including business, robotics, audiovisual systems, controlling complicated processes, and population and developmental biology. The general concept of the intuitionistic fuzzy set was introduced by Atanassov [1, 2] in the mid of 1980 century. Zadeh introduced the fuzzy set theory [3], considering only the indicator as the degree of belongingness of an arbitrary element of a universal set to its particular subset. Atanassov extends this concept by adding another indicator; the degree of non-belongingness [4, 5, 6] and the sum of both indicators is less than unity. Alamin et al. [7, 8, 9] have been used the fuzzy uncertainty environment and explained the stability analysis of the corresponding dynamical system. Numerous findings [10, 11, 12, 13] are also taken into account in the fuzzy uncertainty scenarios.

In the research field, the works on intuitionistic fuzzy difference equations need to be expanded, although there are several investigations on intuitionistic fuzzy differential equations. Differential and partial differential equations were explored using the intuitionistic ambiguous case by Melliani and Chadli [14, 15]. Tudu et al. [16] applied α-cut of the fuzzy set to solve differential equations. Abbasbandy and Allahviranloo [17] numerically solved the intuitionistic fuzzy differential equation using the Runge-Kutta technique. Mondal et al. [18] used triangular intuitionistic fuzzy numbers to study the differential equation system. Ettoussi et al. used the sequential approximation approach [19] to solve the intuitionistic fuzzy differential equation.

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Nirmala et al. [20] investigated the numerical solution of an intuitionistic fuzzy differential equation using the parametric representation of an intuitionistic fuzzy set and the Euler technique. The analytical solutions technique to solve the linear non-homogeneous fuzzy difference equation is discussed by Alamin et al. [21]. The generalization of the Hukuhara difference [22] in the intuitionistic fuzzy situation is introduced by Melliani et al. in [23]. Mondal et al. [24] discussed the one-dimensional linear intuitionistic fuzzy initial value problem using trapezoidal intuitionistic fuzzy numbers. Further, fuzzy sets, intuitionistic fuzzy sets and neutrosophic sets are used in numerous fields, including optimization [25, 26], differential equations [27], difference equations [28, 29] and so on.

In this study, we have taken into account a pair of initial valued homogeneous linear difference equations with positive coefficients in an intuitionistic fuzzy environment to solve it analytically and studied the stability situation of the trivial equilibrium point in this mentioned environment. We have applied this theoretical results into a prey-predator dynamical system to realize the extinction state of the dynamics. In the rest of the paper, in Section 2, we have stated some basic definitions involved inextricably with the intuitionistic fuzzy set and the stability criteria of the equilibrium points of the system of difference equations. In Section 3, the configuration of the general solutions and analytical solutions at some fixed iteration level of the coupled linear homogeneous difference equations in an intuitionistic fuzzy environment is discussed, and the stability conditions of only one intuitionistic equilibrium point are validated through proof of a lemma. The numerical examples and conclusions are drawn in Sections 4 and Section 5, respectively.

2 Preliminaries

This section presents the preliminary results of this study. It briefly discusses the definitions, theorems, and examples of intuitionistic fuzzy sets and difference equations.

2.1 Intuitionistic fuzzy set theory

This theory is considered to be an improvement on fuzzy set theory. The notions of membership function and non-membership function are considered in the intuitionistic fuzzy set framework. The intuitionistic fuzzy set theory was first proposed by the researcher Atanassov in 1986. In numerous fields of mathematics intuitionistic fuzzy set theory and the associated difference equations are essential. The second order linear intuitionistic difference equation can be applied to demonstrate the pressure dynamic of a gas cylinder. Now, some basic definitions and conceptual idea on intuitionistic fuzzy set theory are mentioned as below:

Definition 1. *The intuitionistic fuzzy set given by Atanassov on the crisp set* $\mathbb{X}$ *can be written as,* $\Gamma_{\tilde{A}^i} = \left\{\langle x, \mu_{\tilde{A}^i}(x), \nu_{\tilde{A}^i}(x)\rangle; x \in \mathbb{X}\right\}$, *where the functions* $\mu_{\tilde{A}^i}(x), \nu_{\tilde{A}^i}(x) : \mathbb{X} \to [0,1]$ *are the grade of belongingness and non belongingness, respectively of* $x \in \mathbb{X}$ *in* $\Gamma_{\tilde{A}^i}$ *and* $\mu_{\tilde{A}^i}(x) + \nu_{\tilde{A}^i}(x) \in [0,1]$.

Definition 2. [30] *Let the real valued intuitionistic fuzzy number is denoted by the set* $\mathcal{F}_1 = \left\{\langle \zeta, \eta\rangle; \mathbb{R} \to [0,1]^2, \forall x \in \mathbb{R}; 0 \le \zeta(x) + \eta(x) \le 1\right\}$. *An element* ζ, η *in* $\mathcal{F}_1$ *to be an intuitionistic fuzzy number if the following conditions are maintained*

(i) $\langle \zeta, \eta\rangle$ *is normal that is, for some* t_0 *and* $t_1 \in \mathbb{R}$, $\zeta(t_0) = 1$ *and* $\eta(t_1) = 1$.

(ii) The membership functions of ζ *and* η *are fuzzy convex and fuzzy concave respectively.*

(iii) ζ *is a lower semi-continuous and* η *is a n upper semi-continuous function.*

(iv) Sup $\langle \zeta, \eta\rangle$ *is bounded.*

Definition 3. *A triangular intuitionistic fuzzy number $\widetilde{A}$ is defined with following membership function and non-membership function as*

$$\mu_{\tilde{A}^i}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1} & ;\ for\ a_1 \leq x \leq a_2 \\ \frac{a_3-x}{a_3-a_2} & ;\ for\ a_2 \leq b \leq a_3 \\ 0 & ;\ otherwise \end{cases} \tag{1}$$

and

$$\nu_{\tilde{A}^i}(x) = \begin{cases} \frac{a_2-x}{a_2-a_1\prime} & ;\ for\ a_1\prime \leq x \leq a_2 \\ \frac{x-a_2}{a_3\prime-a_2} & ;\ for\ a_2 \leq b \leq a_3\prime \\ 1 & ;\ otherwise \end{cases} \tag{2}$$

Where $a_1' \leq a_1 \leq a_2 \leq a_3 \leq a_3'$ and TIFN is denoted by $\widetilde{A}^i_{TIFN} = (a_1, a_2, a_3; a_1', a_2, a_3')$.

Definition 4. [23, 30] *If $\alpha, \beta \in [0, 1]$ and $\alpha + \beta = 1$, the parametric form of the α, β- cut of the intuitionistic fuzzy number $\langle \zeta, \eta \rangle$ is of the form $\langle \zeta, \eta \rangle_{(\alpha,\beta)} = [\zeta^1_L(\alpha), \zeta^1_R(\alpha); \eta^2_L(\beta), \eta^2_R(\beta)]$ where $[\zeta^1_L(\alpha), \zeta^1_R(\alpha)]$ and $[\eta^2_L(\beta), \eta^2_R(\beta)]$ are the parametric form of the α-cut of ζ and β-cut of η, respectively.*

Definition 5. [31] *The intuitionistic fuzzy number $\langle \zeta, \eta \rangle$ with the parametric form $\langle \zeta, \eta \rangle_{(\alpha,\beta)} = [\zeta^1_L(\alpha), \zeta^1_R(\alpha);$ $\eta^2_L(\beta), \eta^2_R(\beta)]$, $\alpha \in [0, 1]$ and $\beta = 1 - \alpha$. Then the functions satisfies the following conditions:*

(i) $\zeta^1_L(\alpha)$ *and* $\eta^2_L(\beta)$ *are bounded monotone increasing continuous functions.*

(ii) $\zeta^1_R(\alpha)$ *and* $\eta^2_R(\beta)$ *are bounded monotone decreasing continuous functions.*

(iii) $\zeta^1_L(\alpha) \leq \zeta^1_R(\alpha)$ *and* $\eta^2_L(\beta) \geq \eta^2_R(\beta)$.

Note 1. A difference equation or system of difference equations becomes an intuitionistic fuzzy difference equation if the coefficients and beginning conditions are assumed to be intuitionistic fuzzy numbers.

Theorem 2.1. [32] *If $A = \begin{bmatrix} P & Q \\ R & S \end{bmatrix}$, where $P, Q, R, S \in F^{n\times n}$ then*

$$det_F A = \begin{cases} det_F(PS - QR) & ;\ if\ RS = SR \\ det_F(PS - RQ) & ;\ if\ PR = RP \\ det_F(SP - QR) & ;\ if\ SQ = QS \\ det_F(SP - RQ) & ;\ if\ PQ = QP \end{cases} \tag{3}$$

2.2 Stability criteria of a homogeneous pair of crisp difference equation

Through this preliminary concept, we study about the method and stability criteria of a homogeneous pair of difference equation in the neighbourhood of equilibrium points, which are mentioned below:

2.2.1 Eigenvalue method: [33]

Let us consider a pair of homogeneous difference equations as:

$$\begin{cases} x_{n+1} = F(x_n, y_n) \\ y_{n+1} = G(x_n, y_n) \end{cases} \tag{4}$$

Also, let (u^*, v^*) be an equilibrium point of the System (4) and the corresponding reduced homogeneous linearize pair of difference equation in the neighbourhood of the equilibrium point is

$$U_{n+1} = BU_n \tag{5}$$

where $U_n = \begin{pmatrix} u_n \\ v_n \end{pmatrix}$, $B = (b_{ij})_{2\times 2}$, b_{ij} are the elements of the field of real numbers. If μ_1 and μ_2 are the characteristic roots of B then, the following stability conditions are true:

(a) If $|\mu_1| < 1$, $|\mu_2| < 1$ then the equilibrium point (u^*, v^*) is a stable node.

(b) If at least one of the values$|\mu_1| > 1$,$|\mu_2| > 1$ then the equilibrium point (u^*, v^*) is an unstable node.

(c) If $|\mu_1| > 1$ and $|\mu_2| < 1$ or the reverse condition holds, then, the critical point is a saddle point.

(d) If μ_1, μ_2 are complex conjugates then if

  (i) $|\mu_i| > 1$, the critical point is an unstable spiral point.

  (ii) $|\mu_i| < 1$, the critical point is a stable spiral point.

  (iii) $|\mu_i| = 1$, a marginally stable focus.

2.2.2 Jury conditions of stability: [34]

Let $\varphi(x)$ be the characteristic polynomial of the matrix B of the System (5). Then, all the eigenvalues are of modulus less than 1 if and only if $\varphi(1) > 0, \varphi(-1) > 0$ and $\det B < 1$ and hence the critical point (u^*, v^*) is stable.

2.2.3 Schur-Cohn stability criterion: [35]

Consider the System (5) and let the characteristic polynomial of the matrix B, the coefficient matrix of the linearized System (5) in the neighbourhood of the point (u^*, v^*) is

$$\varphi(x) = x^2 - trB\ x + det(B) \tag{6}$$

Then, by the well-known Schur-Cohn stability criterion, the modulus of all roots in Equation 6 is less than unity if and only if the following inequalities hold

$$\begin{cases} \varphi(1) < 0 \\ \varphi(-1) < 0 \\ |\varphi(0)| < 1 \end{cases} \tag{7}$$

Therefore, the equilibrium points (u^*, v^*) of the System (4) to be locally asymptotically stable if the inequalities in Equation (7) are satisfied.

3 Main result

Consider the coupled linear homogeneous difference equations in an intuitionistic fuzzy environment as

$$\begin{cases} \widetilde{\mathcal{U}}_n + 1 = d_1\widetilde{\mathcal{U}}_n + \widetilde{d}_2\widetilde{\mathcal{V}}_n \\ \widetilde{\mathcal{V}}_n + 1 = d_3\widetilde{\mathcal{U}}_n + \widetilde{d}_4\widetilde{\mathcal{V}}_n \end{cases} \tag{8}$$

with the intuitionistic fuzzy initial conditions are

$$\begin{cases} \mathcal{U}_{n=0} = \widetilde{\mathcal{U}}_0 \\ \mathcal{V}_{n=0} = \widetilde{\mathcal{V}}_0 \end{cases} \tag{9}$$

The coefficients $\widetilde{d}_j, j = 1,2,3,4$ are all non-negative intuitionistic fuzzy numbers. Let the parametric α,β-cut representation of the intuitionistic fuzzy discrete variables $\widetilde{\mathcal{U}}_n, \widetilde{\mathcal{V}}_n$; intuitionistic fuzzy coefficients $\widetilde{d}_1, \widetilde{d}_2, \widetilde{d}_3$ and $\widetilde{d}_4$; the intuitionistic fuzzy initial conditions $\widetilde{\mathcal{U}}_0, \widetilde{\mathcal{V}}_0$ are given by

$$\begin{cases} \langle \widetilde{\mathcal{U}}_n \rangle_{\alpha,\beta} = \langle [\mathcal{U}_n^L(\alpha), \mathcal{U}_n^R(\alpha)]; [\mathcal{U}_n^L(\beta), \mathcal{U}_n^R(\beta)] \rangle \\ \langle \widetilde{\mathcal{V}}_n \rangle_{\alpha,\beta} = \langle [\mathcal{V}_n^L(\alpha), \mathcal{V}_n^R(\alpha)]; [\mathcal{V}_n^L(\beta), \mathcal{V}_n^R(\beta)] \rangle \\ \langle \widetilde{d}_j \rangle_{\alpha,\beta} = \langle [d_j^L(\alpha), d_j^R(\alpha)]; [d_j^L(\beta), d_j^R(\beta)] \rangle \end{cases} \tag{10}$$

Where $n = 0,1,2,\ldots$ and $j = 1,2,3,4$.

Now taking α,β- cut on both side of Equation (8) and using Equation (10) we have the following system of crisp difference equations:

$$\begin{cases} \mathcal{U}_{n+1}^L(\alpha) = d_1^L(\alpha)\mathcal{U}_n^L(\alpha) + d_2^L(\alpha)\mathcal{V}_n^L(\alpha) \\ \mathcal{U}_{n+1}^R(\alpha) = d_1^R(\alpha)\mathcal{U}_n^R(\alpha) + d_2^R(\alpha)\mathcal{V}_n^R(\alpha) \\ \mathcal{U}_{n+1}^L(\beta) = d_1^L(\beta)\mathcal{U}_n^L(\beta) + d_2^L(\beta)\mathcal{V}_n^L(\beta) \\ \mathcal{U}_{n+1}^R(\beta) = d_1^R(\beta)\mathcal{U}_n^R(\beta) + d_2^R(\beta)\mathcal{V}_n^R(\beta) \\ \mathcal{V}_{n+1}^L(\alpha) = d_3^L(\alpha)\mathcal{U}_n^L(\alpha) + d_4^L(\alpha)\mathcal{V}_n^L(\alpha) \\ \mathcal{V}_{n+1}^R(\alpha) = d_3^R(\alpha)\mathcal{U}_n^R(\alpha) + d_4^R(\alpha)\mathcal{V}_n^R(\alpha) \\ \mathcal{V}_{n+1}^L(\beta) = d_3^L(\beta)\mathcal{U}_n^L(\beta) + d_4^L(\beta)\mathcal{V}_n^L(\beta) \\ \mathcal{V}_{n+1}^R(\beta) = d_3^R(\beta)\mathcal{U}_n^R(\beta) + d_4^R(\beta)\mathcal{V}_n^R(\beta) \end{cases} \tag{11}$$

The Equation (11), in matrix representation, can be written as

$$U_{n+1}(\alpha,\beta) = \mathcal{M}(\alpha,\beta)\, U_n(\alpha,\beta) \tag{12}$$

with the initial condition $U_0(\alpha,\beta)$.

Where, $U_n(\alpha,\beta) = [\mathcal{U}_n^L(\alpha), \mathcal{U}_n^R(\alpha), \mathcal{U}_n^L(\beta), \mathcal{U}_n^R(\beta)\, \mathcal{V}_n^L(\alpha), \mathcal{V}_n^R(\alpha), \mathcal{V}_n^L(\beta), \mathcal{V}_n^R(\beta)]^T$ and $\mathcal{M}(\alpha,\beta) = \begin{bmatrix} D_1(\alpha,\beta) & D_2(\alpha,\beta) \\ D_3(\alpha,\beta) & D_4(\alpha,\beta) \end{bmatrix}$ is a block matrix of order 4×4 and each entry of this matrix is also the diagonal matrix of order 4, where $D_j(\alpha,\beta) = diag\left(d_j^L(\alpha), d_j^R(\alpha), d_j^L(\beta), d_j^R(\beta)\right)$, $j = 1,2,3,4$.

Solving Equation (12) through the iteration method, the solution at the n-th iteration by the principle of mathematical induction is found as

$$U_n(\alpha,\beta) = \mathcal{M}^n(\alpha,\beta)\, U_0(\alpha,\beta), n = 0,1,2,\ldots \tag{13}$$

This is the general solution in matrix form.

3.1 Intuitionistic fuzzy analytical solution

Considering the solutions of Equation (13), we try to investigate the analytical solution of the intuitionistic fuzzy difference Equation (8) for some different values of n. At first, we try to investigate analytical form for $n = 2$. Then, from Equation (13), the solution at the end of second iteration is $U_2(\alpha,\beta) = \mathcal{M}^2(\alpha,\beta)\, U_0(\alpha,\beta)$. The matrix,

$$\begin{aligned}\mathcal{M}^2(\alpha,\beta) &= \begin{bmatrix} D_1(\alpha,\beta) & D_2(\alpha,\beta) \\ D_3(\alpha,\beta) & D_4(\alpha,\beta)\end{bmatrix}\begin{bmatrix} D_1(\alpha,\beta) & D_2(\alpha,\beta) \\ D_3(\alpha,\beta) & D_4(\alpha,\beta)\end{bmatrix} \\ &= \begin{bmatrix} {D_1}^2(\alpha,\beta) + D_2(\alpha,\beta)D_3(\alpha,\beta) & D_1(\alpha,\beta)D_2(\alpha,\beta) + D_2(\alpha,\beta)D_4(\alpha,\beta) \\ D_3(\alpha,\beta)D_1(\alpha,\beta) + D_4(\alpha,\beta)D_3(\alpha,\beta) & D_3(\alpha,\beta)D_2(\alpha,\beta) + {D_4}^2(\alpha,\beta)\end{bmatrix}\end{aligned} \tag{14}$$

Since, all the $D_j(\alpha,\beta)$ are diagonal matrices, hence we can easily calculate the polynomial matrices of the above matrix $\mathcal{M}^2(\alpha,\beta)$. So, the polynomial matrices

$$\begin{aligned}
&{D_1}^2(\alpha,\beta) + D_2(\alpha,\beta)D_3(\alpha,\beta) = \\
&\quad diag\left(\left(d_1^L(\alpha)\right)^2 + d_2^L(\alpha)d_3^L(\alpha), \left(d_1^R(\alpha)\right)^2 + d_2^R(\alpha)d_3^R(\alpha), \left(d_1^L(\beta)\right)^2 + d_2^L(\beta)d_3^L(\beta), \left(d_1^R(\beta)\right)^2 + d_2^R(\beta)d_3^R(\beta)\right) \\
&D_1(\alpha,\beta)D_2(\alpha,\beta) + D_2(\alpha,\beta)D_4(\alpha,\beta) = \\
&\quad diag\left(d_2^L(\alpha)\left(d_1^L(\alpha) + d_4^L(\alpha)\right), d_2^R(\alpha)\left(d_1^R(\alpha) + d_4^R(\alpha)\right), d_2^L(\beta)\left(d_1^L(\beta) + d_4^L(\beta)\right), d_2^R(\beta)\left(d_1^R(\beta) + d_4^R(\beta)\right)\right) \\
&D_3(\alpha,\beta)D_1(\alpha,\beta) + D_4(\alpha,\beta)D_3(\alpha,\beta) = \\
&\quad diag\left(d_3^L(\alpha)\left(d_1^L(\alpha) + d_4^L(\alpha)\right), d_3^R(\alpha)\left(d_1^R(\alpha) + d_4^R(\alpha)\right), d_3^L(\beta)\left(d_1^L(\beta) + d_4^L(\beta)\right), d_3^R(\beta)\left(d_1^R(\beta) + d_4^R(\beta)\right)\right) \\
&D_3(\alpha,\beta)D_2(\alpha,\beta) + {D_4}^2(\alpha,\beta) = \\
&\quad diag\left(\left(d_4^L(\alpha)\right)^2 + d_2^L(\alpha)d_3^L(\alpha), \left(d_4^R(\alpha)\right)^2 + d_2^R(\alpha)d_3^R(\alpha), \left(d_4^L(\beta)\right)^2 + d_2^L(\beta)d_3^L(\beta), \left(d_4^R(\beta)\right)^2 + d_2^R(\beta)d_3^R(\beta)\right)
\end{aligned}$$

Therefore, substituting the above polynomial matrices in the matrix $\mathcal{M}^2(\alpha,\beta)$, we have the solutions for $n = 2$ of the form

$$\begin{cases}
\mathcal{U}_2^L(\alpha) = \left\{\left(d_1^L(\alpha)\right)^2 + d_2^L(\alpha)d_3^L(\alpha)\right\}\mathcal{U}_0^L(\alpha) + \left\{d_2^L(\alpha)\left(d_1^L(\alpha) + d_4^L(\alpha)\right)\right\}\mathcal{V}_0^L(\alpha) \\
\mathcal{U}_2^R(\alpha) = \left\{\left(d_1^R(\alpha)\right)^2 + d_2^R(\alpha)d_3^R(\alpha)\right\}\mathcal{U}_0^R(\alpha) + \left\{d_2^R(\alpha)\left(d_1^R(\alpha) + d_4^R(\alpha)\right)\right\}\mathcal{V}_0^R(\alpha) \\
\mathcal{U}_2^L(\beta) = \left\{\left(d_1^L(\beta)\right)^2 + d_2^L(\beta)d_3^L(\beta)\right\}\mathcal{U}_0^L(\beta) + \left\{d_2^L(\beta)\left(d_1^L(\beta) + d_4^L(\beta)\right)\right\}\mathcal{V}_0^L(\beta) \\
\mathcal{U}_2^R(\beta) = \left\{\left(d_1^R(\beta)\right)^2 + d_2^R(\beta)d_3^R(\beta)\right\}\mathcal{U}_0^R(\beta) + \left\{d_2^R(\beta)\left(d_1^R(\beta) + d_4^R(\beta)\right)\right\}\mathcal{V}_0^R(\beta) \\
\mathcal{V}_2^L(\alpha) = \left\{d_3^L(\alpha)\left(d_1^L(\alpha) + d_4^L(\alpha)\right)\right\}\mathcal{U}_0^L(\alpha) + \left\{\left(d_4^L(\alpha)\right)^2 + d_2^L(\alpha)d_3^L(\alpha)\right\}\mathcal{V}_0^L(\alpha) \\
\mathcal{V}_2^R(\alpha) = \left\{d_3^R(\alpha)\left(d_1^R(\alpha) + d_4^R(\alpha)\right)\right\}\mathcal{U}_0^R(\alpha) + \left\{\left(d_4^R(\alpha)\right)^2 + d_2^R(\alpha)d_3^R(\alpha)\right\}\mathcal{V}_0^R(\alpha) \\
\mathcal{V}_2^L(\beta) = \left\{d_3^L(\beta)\left(d_1^L(\beta) + d_4^L(\beta)\right)\right\}\mathcal{U}_0^L(\beta) + \left\{\left(d_4^L(\beta)\right)^2 + d_2^L(\beta)d_3^L(\beta)\right\}\mathcal{V}_0^L(\beta) \\
\mathcal{V}_2^R(\beta) = \left\{d_3^R(\beta)\left(d_1^R(\beta) + d_4^R(\beta)\right)\right\}\mathcal{U}_0^R(\beta) + \left\{\left(d_4^R(\beta)\right)^2 + d_2^R(\beta)d_3^R(\beta)\right\}\mathcal{V}_0^R(\beta)
\end{cases} \tag{15}$$

In a similar way, although it is too laborious using paper and pencil, we can calculate the solutions for$n = 3, 4, 5\ldots$ etc., up to some desired iteration level.

Note 2. If the some or all the coefficients are negative intuitionistic fuzzy number of the Equation (8) then, utilizing the fuzzy arithmetic and the similar computational procedure which is discussed in this article, we can perform the general as well as analytical solution of the corresponding homogeneous pair of intuitionistic fuzzy difference equation up to some desired iteration level.

3.2 Intuitionistic fuzzy stability analysis

Solving Equation (8), $(\widetilde{0},\widetilde{0})$ is the only equilibrium point. Now, we are interested in the stability nature of the System (8) near the intuitionistic fuzzy equilibrium point $(\widetilde{0},\widetilde{0})$. This is why, we proceed with taking the corresponding crisp system of difference Equation (11) and find out the characteristic equation of the matrix $\mathcal{M}(\alpha,\beta)$. We now state a lemma,

Lemma 3.1. *Consider the System (8), then the intuitionistic equilibrium point $\left(\widetilde{0},\widetilde{0}\right)$ is locally asymptotically stable if*

$$\left|\left\{d_1^p\left(q\right)+d_4^p\left(q\right)\right\}\pm\sqrt{\left\{d_1^p\left(q\right)+d_4^p\left(q\right)\right\}^2-4\left\{d_1^p\left(q\right)d_4^p\left(q\right)-d_2^p\left(q\right)d_3^p\left(q\right)\right\}}\right|<2 \tag{16}$$

where $p=L,R$ and $q=\alpha,\beta$.

Proof. Let $\lambda(\alpha,\beta)$ is the intuitionistic fuzzy eigenvalue of $\mathcal{M}(\alpha,\beta)$ then we solve the equation

$$\det\left(\mathcal{M}\left(\alpha,\beta\right)-\lambda(\alpha,\beta)I_8\right)=0 \tag{17}$$

$$\mathcal{M}\left(\alpha,\beta\right)-\lambda\left(\alpha,\beta\right)I_8=\begin{bmatrix}D_1(\alpha,\beta)-\lambda\left(\alpha,\beta\right)I_4 & D_2(\alpha,\beta)\\ D_3(\alpha,\beta) & D_4(\alpha,\beta)-\lambda\left(\alpha,\beta\right)I_4\end{bmatrix} \tag{18}$$

Since, the matrix $D_3(\alpha,\beta)$ and $D_4(\alpha,\beta)$ are diagonal matrix, so commute with each other. Hence, the matrices $D_3(\alpha,\beta) and D_4(\alpha,\beta)-\lambda\left(\alpha,\beta\right)I_4$ are commutative. Therefore, by the Theorem 2.1, Equation (17) becomes

$$\det\left\{\left(D_1(\alpha,\beta)-\lambda\left(\alpha,\beta\right)I_4\right)\left(D_4(\alpha,\beta)-\lambda\left(\alpha,\beta\right)I_4\right)-D_2(\alpha,\beta)D_3(\alpha,\beta)\right\}=0$$
$$\text{or,}\det\left\{D_1\left(\alpha,\beta\right)D_4\left(\alpha,\beta\right)-\lambda\left(\alpha,\beta\right)\left(D_1\left(\alpha,\beta\right)+D_4\left(\alpha,\beta\right)\right)+\lambda^2\left(\alpha,\beta\right)I_4-D_2(\alpha,\beta)D_3(\alpha,\beta)\right\}=0$$

Which, gives the following system of equations in $\lambda\left(\alpha\right)$ and $\lambda\left(\beta\right)$ as

$$\begin{cases}\lambda^2\left(\alpha\right)-\lambda\left(\alpha\right)\left(d_1^L\left(\alpha\right)+d_4^L\left(\alpha\right)\right)+d_1^L\left(\alpha\right)d_4^L\left(\alpha\right)-d_2^L\left(\alpha\right)d_3^L\left(\alpha\right)=0\\ \lambda^2\left(\alpha\right)-\lambda\left(\alpha\right)\left(d_1^R\left(\alpha\right)+d_4^R\left(\alpha\right)\right)+d_1^R\left(\alpha\right)d_4^R\left(\alpha\right)-d_2^R\left(\alpha\right)d_3^R\left(\alpha\right)=0\\ \lambda^2\left(\beta\right)-\lambda\left(\beta\right)\left(d_1^L\left(\beta\right)+d_4^L\left(\beta\right)\right)+d_1^L\left(\beta\right)d_4^L\left(\beta\right)-d_2^L\left(\beta\right)d_3^L\left(\beta\right)=0\\ \lambda^2\left(\beta\right)-\lambda\left(\beta\right)\left(d_1^R\left(\beta\right)+d_4^R\left(\beta\right)\right)+d_1^R\left(\beta\right)d_4^R\left(\beta\right)-d_2^R\left(\beta\right)d_3^R\left(\beta\right)=0\end{cases} \tag{19}$$

Solving each quadratic Equation (19), we have obtained all the eigenvalues as

$$\lambda_p\left(q\right)=\frac{\left\{d_1^p\left(q\right)+d_4^p\left(q\right)\right\}\pm\sqrt{\left\{d_1^p\left(q\right)+d_4^p\left(q\right)\right\}^2-4\left\{d_1^p\left(q\right)d_4^p\left(q\right)-d_2^p\left(q\right)d_3^p\left(q\right)\right\}}}{2} \tag{20}$$

where, $p=L,R$ and $q=\alpha,\beta$.

Therefore, the intuitionistic equilibrium point $(\widetilde{0},\widetilde{0})$ is stable if $\left|\lambda_p\left(q\right)\right|<1$ that is

$$\left|\left\{d_1^p\left(q\right)+d_4^p\left(q\right)\right\}\pm\sqrt{\left\{d_1^p\left(q\right)+d_4^p\left(q\right)\right\}^2-4\left\{d_1^p\left(q\right)d_4^p\left(q\right)-d_2^p\left(q\right)d_3^p\left(q\right)\right\}}\right|<2 \tag{21}$$

where, $p=L,R$ and $q=\alpha,\beta$. □

4 Application of the model in prey-predator dynamics with numerical justification

We consider an ecological prey-predator model in an intuitionistic environment. Let, $\mathcal{U}_n$ and $\mathcal{V}_n$ represents the numbers of prey and predators, respectively at the n-th generation. The model is formulated under the following assumptions:

(i) $\mathcal{U}$ is the only one prey for the predator $\mathcal{V}$ and $\mathcal{U}$ is not consumed by the other predators except the predator $\mathcal{V}$.

(ii) The prey population increases in absence of the mentioned predator and extinct the predator without the existence of the prey populations.

Incorporated the above assumptions, the prey-predator model is obtained from [31],

$$\begin{cases} \delta\mathcal{U}_n = \mathcal{U}_{n+1} - \mathcal{U}_n = \varrho_1\mathcal{U}_n - \varrho_2\mathcal{V}_n \\ \delta\mathcal{V}_n = \mathcal{V}_{n+1} - \mathcal{V}_n = -\varrho_3\mathcal{V}_n + \varrho_4\mathcal{U}_n \end{cases} \tag{22}$$

Therefore, Equation (22) is manipulated as a coupled system of linear homogeneous difference equations as

$$\begin{cases} \mathcal{U}_{n+1} = (1+\varrho_1)\mathcal{U}_n - \varrho_2\mathcal{V}_n \\ \mathcal{V}_{n+1} = \varrho_4\mathcal{U}_n + (1-\varrho_3)\mathcal{V}_n \end{cases} \tag{23}$$

where, $\delta\mathcal{U}_n$ and $\delta\mathcal{V}_n$ represents the rate of changes of the prey and predator respectively. All coefficients are positive and $\varrho_1, \varrho_3 \in (0,1)$. The coefficients ϱ_1, the growth rate of the prey in the absence of the predator, ϱ_2 is the rate at which prey decreases in the presence of predator; ϱ_3, the mortality rate of the predator in the absence of prey and ϱ_4, the growth rate of the predator in the presence of sufficient numbers of prey.

Due to biological reason and uncertainty among the ecological factors, think the Model (23) in an intuitionistic fuzzy environment with introducing the new coefficients $(1+\varrho_1) = \varrho_5$ and $(1-\varrho_3) = \varrho_6$, the updated form of the model is

$$\begin{cases} \widetilde{\mathcal{U}}_{n+1} = \widetilde{\varrho}_5\mathcal{U}_n - \widetilde{\varrho}_2\mathcal{V}_n \\ \widetilde{\mathcal{V}}_{n+1} = \widetilde{\varrho}_4\mathcal{U}_n + \widetilde{\varrho}_6\mathcal{V}_n \end{cases} \tag{24}$$

Let, the values of the different intuitionistic coefficients of Equation (24) are,

$$\begin{cases} \widetilde{\varrho}_2 = \langle 0.17, 0.18, 0.19; 0.16, 0.18, 0.20\rangle \\ \widetilde{\varrho}_4 = \langle 0.41, 0.42, 0.43; 0.40, 0.42, 0.44\rangle \\ \widetilde{\varrho}_5 = \langle 1.09, 1.10, 1.11; 1.08, 1.10, 1.12\rangle \\ \widetilde{\varrho}_6 = \langle 0.51, 0.52, 0.53; 0.50, 0.52, 0.54\rangle \end{cases} \tag{25}$$

Therefore, the corresponding parametric values are,

$$\begin{cases} \langle\widetilde{\varrho}_2\rangle_{\alpha,\beta} = \langle [0.17+0.01\alpha, 0.19-0.01\alpha]\,; [0.18-0.02\beta, 0.18+0.02\beta]\rangle \\ \langle\widetilde{\varrho}_4\rangle_{\alpha,\beta} = \langle [0.41+0.01\alpha, 0.43-0.01\alpha]\,; [0.42-0.02\beta, 0.42+0.02\beta]\rangle \\ \langle\widetilde{\varrho}_5\rangle_{\alpha,\beta} = \langle [1.09+0.01\alpha, 1.11-0.01\alpha]\,; [1.10-0.02\beta, 1.10+0.02\beta]\rangle \\ \langle\widetilde{\varrho}_6\rangle_{\alpha,\beta} = \langle [0.51+0.01\alpha, 0.53-0.01\alpha]\,; [0.52-0.02\beta, 0.52+0.02\beta]\rangle \end{cases} \tag{26}$$

Using the parametric values of System (26), the intuitionistic System (24) can be written in a similar way as Equation (25) in terms of parametric condition turns into the system as

$$\begin{cases} \mathcal{U}_{n+1}^{L}(\alpha) = (1.09 + 0.01\alpha)\,\mathcal{U}_{n}^{L}(\alpha) - (0.19 - 0.01\alpha)\,\mathcal{V}_{n}^{R}(\alpha) \\ \mathcal{U}_{n+1}^{R}(\alpha) = (1.11 - 0.01\alpha)\,\mathcal{U}_{n}^{R}(\alpha) - (0.17 + 0.01\alpha)\,\mathcal{V}_{n}^{L}(\alpha) \\ \mathcal{U}_{n+1}^{L}(\beta) = (1.10 - 0.02\beta)\,\mathcal{U}_{n}^{L}(\beta) - (0.18 + 0.0\beta)\,\mathcal{V}_{n}^{R}(\beta) \\ \mathcal{U}_{n+1}^{R}(\beta) = (1.10 - 0.02\beta)\,\mathcal{U}_{n}^{R}(\beta) - (0.18 - 0.02\alpha)\,\mathcal{V}_{n}^{L}(\alpha) \\ \mathcal{V}_{n+1}^{L}(\alpha) = (0.41 + 0.01\alpha)\,\mathcal{U}_{n}^{L}(\alpha) - (0.51 + 0.01\alpha)\,\mathcal{V}_{n}^{L}(\alpha) \\ \mathcal{V}_{n+1}^{R}(\alpha) = (0.43 - 0.01\alpha)\,\mathcal{U}_{n}^{R}(\alpha) + (0.53 - 0.01\alpha)\,\mathcal{V}_{n}^{R}(\alpha) \\ \mathcal{V}_{n+1}^{L}(\beta) = (1.10 - 0.02\beta)\,\mathcal{U}_{n}^{L}(\beta) - (0.18 + 0.0\beta)\,\mathcal{V}_{n}^{L}(\beta) \\ \mathcal{V}_{n+1}^{R}(\beta) = (0.42 + 0.02\beta)\,\mathcal{U}_{n}^{R}(\beta) + (0.52 + 0.02\alpha)\,\mathcal{V}_{n}^{R}(\alpha) \end{cases} \tag{27}$$

We now investigate the dynamics of the Model (27) through the graphical solutions considering the following cases:

Case 1: Suppose, the fixed initial intuitionistic prey population size

$$\widetilde{u}_0 = (245, 250, 255;\ 240, 250, 260) \tag{28}$$

and vary the initial intuitionistic predator population size to investigate the prey-predator dynamical interaction of the Model (27). Based on the graphical outlook, the given examples fit with the example and counter example.

01.png 01.bb

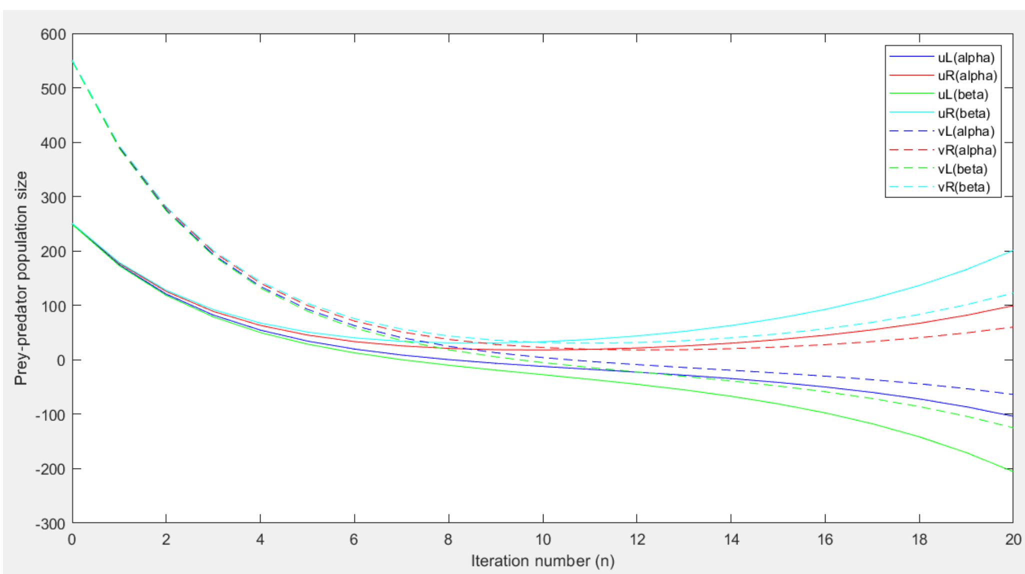


Figure 1: The prey-predator scabbard dynamics of the Model (27) for the initial prey and predator population size $\widetilde{u}_0$ as in Model (28) and $\widetilde{v}_0 = (545, 550, 555; 540, 550, 560)$ respectively with $\alpha = 0.9$ and $\beta = 0.1$

Remark 4.1. *From the Figure 1, we observe that till the 8th generation, both the populations gradually decrease but after this the graphical scenario shows that some intuitionistic components continue their decreasing nature and other some components approache unbounded positive growth. As a consequence, there is a tendency of extinction as well as survive each population with a good number of population size while both the population extinct surely in crisp environment.*

02.png 02.bb

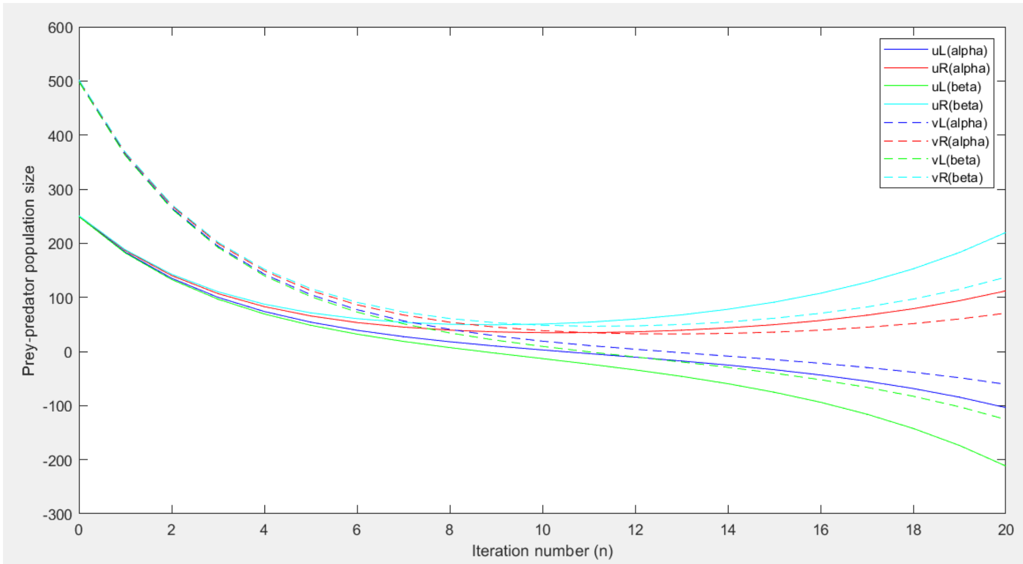


Figure 2: The prey-predator scabbard dynamics of the Model (27) for the initial prey and predator population size $\widetilde{u}_0$ as in Model (28) and $\widetilde{v}_0 = (490, 500, 510; 480, 500, 520)$ respectively with $\alpha = 0.9$ and $\beta = 0.1$.

Remark 4.2. *The graphical scenarios (from Figure 2) shows that the dynamics is in unstable nature. There is a possibility of extinction of both the population. The overall intuitionistic branch curve indicates the unbounded positive growth of the system.*

03.png 03.bb

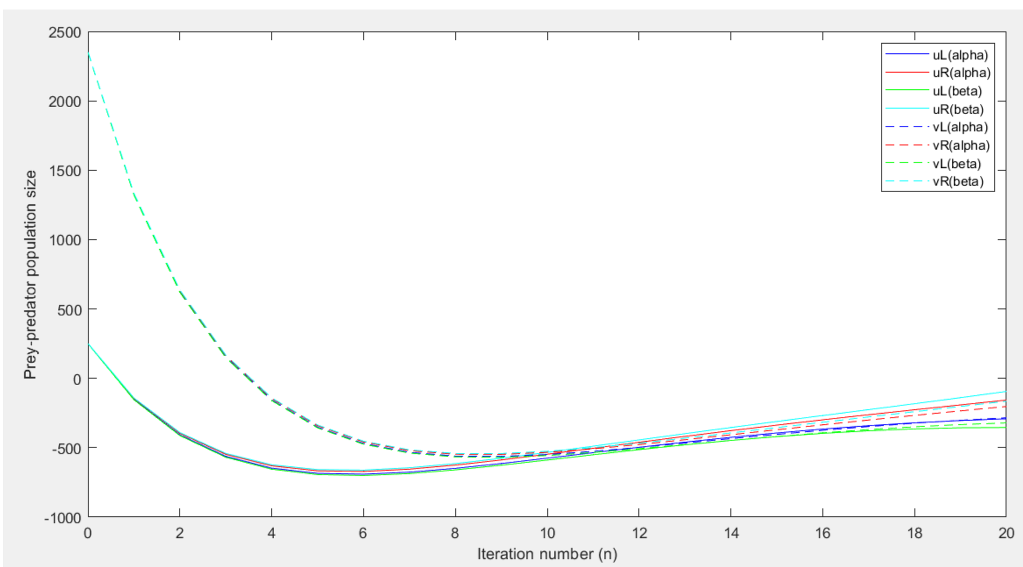


Figure 3: The prey-predator scabbard dynamics of the Model (27) for the initial prey and predator population size $\widetilde{u}_0$ as in Model (28) and $\widetilde{v}_0 = (2340, 2350, 2360; 2335, 2350, 2365)$ respectively with $\alpha = 0.9$ and $\beta = 0.1$.

Remark 4.3. *Figure 3 indicates that the dynamics represent stable solutions. For such values of intuitionistic initial prey and predator population size, both the species were extinct biologically while the prey dynamics collapsed immediate time. However, the predators try to struggle to survive near about the fourth generation.*

Case 2: Suppose the fixed initial intuitionistic predator population size

$$\widetilde{v}_0 = (590, 600, 610; 580, 600, 620) \tag{29}$$

and vary the initial intuitionistic prey population size to investigate the prey-predator dynamical interaction of the Model (27).

04.png 04.bb

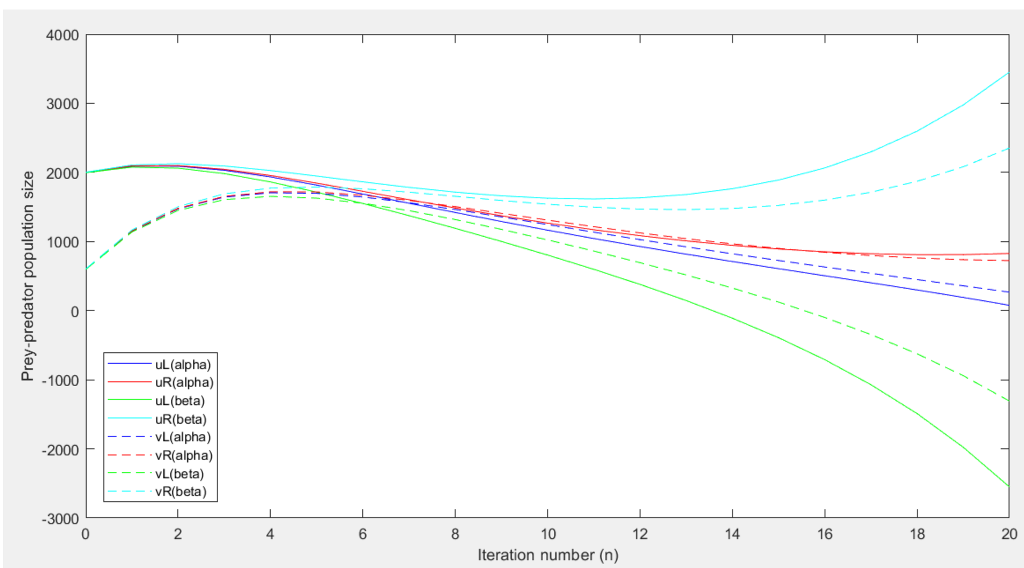


Figure 4: The prey-predator scabbard dynamics of the Model (27) for the initial prey and predator population size $\widetilde{v}_0$ as in Model (29) and $\widetilde{u}_0 = (1990, 2000, 2010; 1980, 2000, 2020)$ respectively with $\alpha = 0.95$ and $\beta = 0.2$.

Remark 4.4. *From Figure 4, there is no chance of extinction of both species in intuitionistic environment as well as in crisp environment till the 20th generations.*

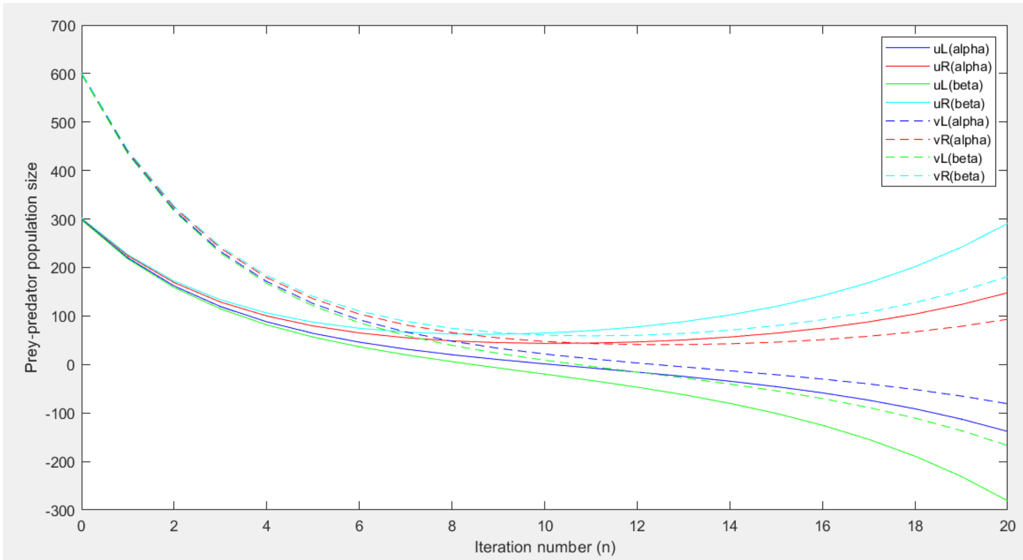
05.png 05.bb 

Figure 5: The prey-predator scabbard dynamics of the Model (27) for the initial prey and predator population size $\widetilde{v}_0$ as in Model (29) and $\widetilde{u}_0 = (290, 300, 310; 280, 300, 320)$ respectively with $\alpha = 0.95$ and $\beta = 0.2$.

Remark 4.5. *In Figure 5, the prey-predator dynamics with such intuitionistic starting values, the dynamical state is unstable but there is an extinction possibility. Although, the curve tendency indicates the unbounded positive growth in crisp situation.*

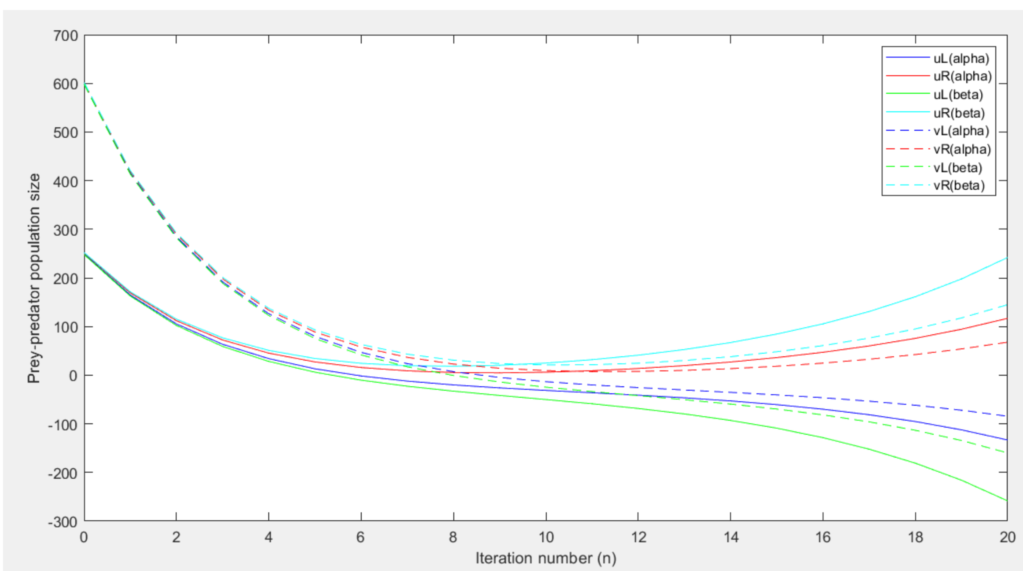
06.png 06.bb 

Figure 6: The prey-predator scabbard dynamics of the Model (27) for the initial prey and predator population size $\widetilde{v}_0$ as in Model (29) and $\widetilde{u}_0 = (240, 250, 260; 230, 250, 270)$ respectively with $\alpha = 0.95$ and $\beta = 0.2$.

Remark 4.6. *In this scenario, of the Figure 6 the dynamical nature is almost similar with the observation of the Figure 5.*

5 Conclusion

Through this article, we have studied the solutions and stability analysis of the pair of homogeneous linear difference equations by intuitionistic fuzzy numbers. All the coefficients, together with the initial conditions, are taken as positive intuitionistic fuzzy numbers in the main discussion of this article, although some or all coefficients may be negative. Applying the parametric intuitionistic fuzzy numbers, the System (8) is transformed into a system of eight difference Equation (11). The only trivial intuitionistic equilibrium point is either stable or unstable depending on the eigenvalues which lies within the unit disc or outside the unit disc. We have applied this model in the two dimensional linear prey-predator Model (24), considering one of the coefficients as negative intuitionistic fuzzy numbers in reality as well as incorporating the ecological phenomena. We validate the Model (27) depending on the various initial intuitionistic prey-predator starting population information. The dynamical behavioural changes, the tendency of extinction of both the populations and stability situations are observed by taking different sets of intuitionistic prey-predator initial population sizes are detected through the graphical representation (see Figures 1 to 6). The most notable outcomes of this article are found from the prey-predator dynamics about the caution of extinction of the population in an intuitionistic environment whenever in the same scenarios in a crisp environment show positive unbounded growth.

This solution technique and approach may be an inspiration as well as helpful among the researchers who are interested in working on the fractional system of linear difference equation, a system of p-linear difference equation in q-numbers of unknown variables and in higher order linear difference equations in an intuitionistic fuzzy environment. Also, anyone can be extended in different types of linear mathematical models in a neutrosophic environment.

Symbols and Abbreviation

Symbols and abbreviations used in this study are presented as follows

Symbols or abbreviations	**:**	**Full form**
$\mu_{\tilde{A}^i}(x)$	:	Membership function of intuitionistic fuzzy set
$\nu_{\tilde{A}^i}(x)$	:	Non-membership function of intuitionistic fuzzy set
$\mathbb{R}$	:	Set of real numbers
$\mathcal{U}^L(\alpha)$	:	Lower bound of α-cut of intuitionistic fuzzy set $\mathcal{U}$
$\mathcal{U}^R(\alpha)$	:	Upper bound of α-cut of intuitionistic fuzzy set $\mathcal{U}$
$\mathcal{U}^L(\beta)$	:	Lower bound of β-cut of intuitionistic fuzzy set $\mathcal{U}$
$\mathcal{U}^R(\beta)$	:	Upper bound of β-cut of intuitionistic fuzzy set $\mathcal{U}$
$\mathcal{U}^L(\gamma)$	:	Lower bound of γ-cut of intuitionistic fuzzy set $\mathcal{U}$
$\mathcal{U}^R(\gamma)$	:	Upper bound of γ-cut of intuitionistic fuzzy set $\mathcal{U}$
$det(A)$	:	Detiermentet of matrix A
$diag(A)$	:	Diagonal element of matrix A

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