Cooperative Controller Design for Nonlinear Multi-Agent Systems via Observer-Based Event-Triggered Scheme

S. Narges Mahdian Zadeh[1], Reza Ghasemi[2*]

Abstract –The Event-triggering protocol is addressed for nonlinear multi-agent systems bas ed on nonlinear observer. It is assumed that the agents have affine nonlinear dynamics. Th e proposed observer has nonlinear dynamics. Compared to other research that concentrates on the event-triggered controller for both linear multi-input multi-output and multi-agent sy stems, the proposed methodology deals with event-triggered controllers for nonlinear multi-agent systems.To decrease unnecessary information transportations, an event-triggered techni que with state-independent threshold is involved to update the control input law. The trigg ering mechanism guarantees both the acceptable performance and the stability of the overal l system. The observer is recommended 1) to decrease the hardware and sensors, 2) to ap proximate the relative full states in order to reach the overall leader-follower consensus. th e focal merits of the proposed methodology are 1) convergence of the observer error to ze ro, 2) convergence of the leader-follower consensus error to zero, 3) decreasing unnecessar y data in proposed topology, and 4) stability of the closed loop multi agent system. The p roposed methodology is applied on applicable dynamics and the simulation results elaborate the promising presentation of the methodological analysis.

Keywords: Event Triggered structure, Multi-Agent System, Consensus, Leader-Follower, Communication, Stability, Nonlinear Observer.

1. Introduction

This article gives you guidelines for preparing papers that, after thoroughly reviewed by the referees, have been decided to be published for JADSC. If you are using Microsoft Word 6.0 or later and reading a paper version of this document, please download the electronic file, Template.doc from the JADSC homepage so you can use this document as a template. A multi-agent system (MAS) is a hi-tech system collected of numerous networking intellectual systems. Multi-agent systems can solve problems that are challenging for an distinct agent to decide. Nowadays, the multi-agent systems has involved technologists' attentions due to the extensive usage in the different sciences.

The examples of their applications are in the UAV formations[1], triggered complex networks [2], and cooperative stewardship [3]. In MAS approach, one of the most important factor is the targets. [4] is deals with leader-follower approaches, [5] and [6] are discussed MAS using the virtual leader approaches and [7] demonstrates on leaderless consensus.

The Formation control is defined as a autonomous agents group which achieve a desired geometric form on their dynamics and retain it as formation[8]. In the MAS, every agent just has access information of its adjoining agents[9]. Distributed event-triggered consensus protocol is presented for the linear multi-agent systems in [9].

Each agent has usually the simple embedded microprocessors, actuation and sensor modules. They accomplish utilities as data gathering, collaborating with nearby agents, and do the agent task with the limited energy resources [9]. To solve this problem, Tabuada [10] proposed an event triggered control (ETC) to execute the task when a pre-defined error exceeds the threshold. In the other words, when the error magnitude reaches the prescribed threshold, an event is triggered [11] and consequently the agents connect each other. After tabuada, ETC has been extensively studied by many researchers such as Dimarogonas et al [12]. [12] designated ETC scheme for a single-integrator consensus problem and then

1 Department of Electrical Engineering, University of Qom, Qom, IRAN. Email:sn.mahdianzadeh@gmail.com

2* Corresponding Author : Department of Electrical Engineering, University of Qom, Qom, IRAN. Email:r.ghasemi@qom.ac.ir , reghasemi@gmail.com

Received: 2024.11.12; Accepted: 2024.12.24

expanded it to self-triggered control.

[13] through [15] show that event-triggered controller design in which satisfies both reduction task executions and retaining reasonable performance. Practically, triggering conditions depend on the thresholds that deal with: 1) comparative the state-dependent threshold condition [16], and 2) absolute state-independent threshold condition [17]. The state-dependent topology, [18] planned the stochastic ET consensus prototype for MAS. [17] is presented for the double-integrator under state-dependent topology. Also, [19] derived adaptive event-triggered control to develop for the strict feedback nonlinear system with the periodic disturbances.

The decentralized robust ETC is labelled for nonlinear MAS as leader–follower consensus in [20]. [21] has presented a Neuro-adaptive ETC for nonlinear MAS. Also, an ETC is deliberated for the second-order MAS systems in presence of loss sensors and the cyber-attacks in [22]. [23] suggested the ETC policies MAS to achieve average consensus. Design of event-triggered pinning control is one of the topics is discussed in [24] as a strategy of feedback control to synchronize all agents. A virtual leader is added to the interaction topology network called pinner in [25]. The event based finite-time consensus mechanism is used for unknown nonlinear heterogeneous MAS in [26]. [29] deals with adaptive intelligent FTC to reach containment for affine nonlinear MAS. Robust Neuro-fuzzy FTC is designated in [30] to improve the power quality. Both the event and self-triggered cooperative protocol is derived for linear time-invariant systems in [31]. The authors in [32] concentrates on adaptive event-triggered observer based controller design for discrete-time linear multi-agent systems under a switching topology. [33] reconstructs event-triggered finite-time consensus control for linear MAS.

Compare to the recent researches which concentrate on the event based controller, this paper deals with the observer-based Event-triggering consensus of nonlinear multi-agent systems. the reduction of unnecessary data communications, convergence of both the observer and tracking error to zero, Lyapunov stability of overall system, and the achievement of leader-follower consensus are all guaranteed in this approach. So, the novelties can be defined as follows:

1) In addition to ensuring the bound of settling time for any initial conditions, proposed protocols of consensus reduce the energy consumption of communications.

2) observer is designed to estimate the unknown states of systems.

3) The results of event-triggering consensus are robust against nonlinear dynamics.

The remainder of paper prearranged as: section 2 as the Notations, then in section 3, the problem statement is formulated. Section 4 introduces an event triggered cooperative control strategy. Section 5 demonstrates the simulation results. Section 6 follows by some concluding comments.

2. Notations

The $\mathbb{R}$ and $\mathbb{R}_{\geq 0}$ stand for the real numbers field and the non-negative real numbers, correspondingly. The n-dimensional Euclidean space and the set of all $n \times m$ real matrices are shown as $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$, consistently. I_n, $(.)^T$, $(.)^{-1}$ and $(.)^+$ designates as the $n \times n$ identity matrix, transpose, inverse, pseudo-inverse. $diag(.)$ symbolizes the diagonal matrix. $\lambda_{\min}(.)$ and $\lambda_{\min}(.)$ illustrate disparately the minimum and maximum eigenvalue of matrix. $\| A \|$ represents the norm of matrix A.

3. Problem Formulation

3-1-Graph Theory

A graph $\mathcal{G} = (\nu, \varepsilon)$ is a graph which is designated the information flow of the multi-agent systems in which $\nu = \{\upsilon_1, ..., \upsilon_N\}$ stands for the node set and $\varepsilon \subseteq \nu \times \nu$ labels the edge set [20]. If there exists a path between every pair of distinct nodes, the undirected graph is connected otherwise it is disconnected. In the directed graph, if there exists at least one node with directed path to all other nodes, it has a directed spanning tree [21]. The $\mathcal{A} = \left[a_{ij}\right] \in \mathbb{R}^{N\times N}$ is depicted as adjacency matrix. If $\left(\upsilon_j, \upsilon_i\right) \in \varepsilon$ then $a_{ij} = 1$, otherwise $a_{ij} = 0$. If the graph is undirected, $a_{ij} = a_{ji}$ and accordingly $\mathcal{A}$ is symmetric. The in-degree matrix is defined as $\mathcal{D} = \mathrm{diag}(d_1, d_2, \ldots, d_N)$ where $d_i = \sum_{j=1}^{N} a_{ij}$. Subsequently, $\mathcal{L} = \mathcal{D} - \mathcal{A}$ explains the Laplacian matrix. $G = \mathrm{diag}(g_1, g_2, \ldots, g_N)$ symbolizes the quality of the communication of N followers to a leader and furthermore the G is called connected if it does not include an isolated node [20].

3-2-Problem Statement

Consider the MAS with N followers and one leader that dynamics of the i^{th} follower as follows:

$$\begin{cases} \dot{x}_i(t) = Ax_i(t) + b[f(x_i(t)) + Bu_i(t)] \\ y_i(t) = Cx_i(t) \\ x_i(0) = x_{i0} \\ \quad i = 0,....,N \end{cases} \tag{1}$$

where $x_i(t) \in \mathbb{R}^n$, $u_i(t) \in \mathbb{R}^m$ and $y_i(t) \in \mathbb{R}^l$ demonstrate the state vector, control input and output vector, respectively. $A \in \mathbb{R}^{n\times n}, B \in \mathbb{R}^{1\times m}, b \in \mathbb{R}^{n\times 1}$, $C \in \mathbb{R}^{l\times n}$ are known constant matrices such that the system is controllable and observable. It is assumed $f(.): \mathbb{R}^n \to \mathbb{R}$ be a Lipchitz function respect to $x(t)$.

The compact form of the equation (1) is written as:

$$\begin{aligned} \dot{X}(t) &= (I_N \otimes A)X(t) \\ &+ (I_N \otimes b)[F(X) + (I_N \otimes B)U(t)] \end{aligned} \tag{2}$$

where $X(t) = \begin{bmatrix} x_1(t) & ... & x_N(t) \end{bmatrix}^T$, $F = \begin{bmatrix} f(x_1) & ... & f(x_N) \end{bmatrix}^T$ and $U(t) = \begin{bmatrix} u_1(t) & ... & u_N(t) \end{bmatrix}^T$.

The 0th agent is as the leader and N others express followers. The dynamics of leader is as:

$$\dot{x}_r = Ax_r + B'r, \tag{3}$$

where $x_r(t) \in \mathbb{R}^n$ as the leader state vector and $r(t)$ stands for the reference input and (A,B') is controllable.

The compact form of the equation (3) is derived as:

$$\dot{X}_r(t) = (I_N \otimes A)X_r(t) + (I_N \otimes B')R(t), \tag{4}$$

where $X_r(t) = \begin{bmatrix} x_{r1}(t) & x_{r2}(t) & ... & x_{rN}(t) \end{bmatrix}^T$ and $R(t) = \begin{bmatrix} r_1(t) & r_2(t) & ... & r_N(t) \end{bmatrix}^T$.

Due to the immeasurability of the agents' states, , the following observer dynamics is planned for each agent.

$$\begin{aligned} \dot{\hat{x}}_i(t) &= A\hat{x}_i(t) + b[f(\hat{x}_i(t) + Bu_i(t)] \\ &+ \ell_i c_i(\hat{x}_i(t) - x_i(t_k)), \end{aligned} \tag{5}$$

where ℓ_i is the gain of each observer. The compact form of the equation(5) can be constructed below.

$$\begin{aligned} \dot{\hat{X}}(t) &= (I_N \otimes A)\hat{X}(t) + (I_N \otimes b)[F(\hat{x}(t)) \\ &+ (I_N \otimes B)U(t)] + (I_N \otimes \ell c)(X_\Delta(t) - \bar{X}(t)) \end{aligned} \tag{6}$$

where $\hat{X}(t) = [\hat{x}_1(t)\,\hat{x}_2(t)....\hat{x}_N(t)]^T$, $F(\hat{x}(t)) = [f(\hat{x}_1(t))\,f(\hat{x}_2(t))....f(\hat{x}_N(t))]^T$, $\bar{x}_i(t) = x_i(t) - \hat{x}_i(t)$ and $x_{\Delta i} = x_i(t) - x_i(t_k)$.

The consensus error is candidate below.

$$e_i(t) = \sum_{j=1, j\neq i}^{N} a_{ij}(\hat{x}_j(t) - \hat{x}_i(t)) + g_i(x_r(t) - \hat{x}_i(t)) \tag{7}$$

where a_{ij} is related to the adjacency matrix $\mathcal{A}$ and g_i is a constant value and if the i^{th} agent has access directly to the leader, $g_i > 0$; otherwise $g_i = 0$.

The event-triggering machinery is demonstrated as $W(E(t), X(t)) = 0$, where $E(t) = \begin{bmatrix} e_1(t) & e_2(t) & .... & e_N(t) \end{bmatrix}^T$, $X(t) = \begin{bmatrix} x_1(t) & x_2(t) & .... & x_N(t) \end{bmatrix}^T$ and $W(E(t), X(t))$ is the event-triggered function. The sequence of event-triggered implementations is represented as: $t_0, t_1, ...$ that each t_k is defined by $W(E(t_k), X(t_k))$, for $k = 0,1,2,...$.

The succeeding theories and lemma should be applied in next section.

Theorem 1: consider $\varphi: R^n \times R^m \to R^n$ as continuously differentiable function for $\forall a \in \rho \subset R^n$. Let $a,b \in \rho$ such that the line segment $L(a,b) \subset \rho$. Then $\exists x \in L(a,b)$ such that

$$\varphi(b) - \varphi(a) = \left.\frac{\partial \varphi}{\partial a}\right|_{a=x} (b-a)$$

Proof: The proof refers to [27].

Lemma 1 [27]: Let $\varphi: [x,y] \times D \to R^m$ be continuous. Assume $\partial\varphi/\partial a$ exists and be continuous on $[x,y] \times D$. If, $\forall\ \nu \subset D$ that ν is convex subset , then there is a constant $M \geq 0$ such that $\|\partial\varphi/\partial a(t,a)\| \leq M$ on $[x,y] \times \nu$ consequently, the following inequality holds $\|\varphi(t,a) - \varphi(t,b)\| \leq M\|a-b\|$ for all $t \in [x,y]$, $a \in \nu$ and $b \in \nu$.

Lemma 2[28]: For any $x, y \in \mathbb{R}^n$ and any positive definite matrix $p \in \mathbb{R}^{n\times n}$ the following holds

$$2x^T y \leq x^T p x + y^T p^{-1} y \, .$$

4. Event-Triggered Cooperative Control Design

The Fig. 1 shows the structure of the presented controller.

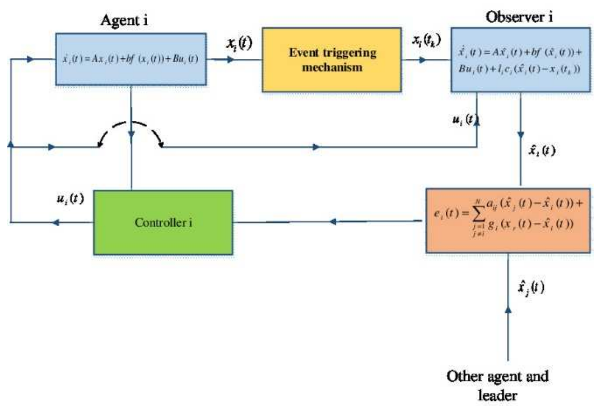

Fig. 1. Observer-based event-triggered control schematic.

For this purpose, the compact form of the equation (7) is constructed below.

$$E(t)=\begin{bmatrix}a_{i1} & .... & a_{iN}\end{bmatrix}\otimes I_n\hat{X}(t)-(D\otimes I_n)\hat{X}(t)$$
$$+\begin{bmatrix}g_1 & .. & 0\\ . & ... & .\\ 0 & .. & g_N\end{bmatrix}\begin{bmatrix}1\\ ..\\ 1\end{bmatrix}x_r(t)-\begin{bmatrix}g_1\otimes I_n & .. & 0\\ . & ... & .\\ 0 & .. & g_N\otimes I_n\end{bmatrix}\hat{X}(t) \quad (8)$$

Then, the equation (8) can be reestablished as:

$$\begin{aligned}E(t)&=(A^*\otimes I_n)\hat{X}(t)-(D\otimes I_n)\hat{X}(t)\\&+((L+G)\otimes I_n)X_r(t)-(G\otimes I_n)\hat{X}(t)\\&=-[(L+G)\otimes I_n](\hat{X}(t)-X_r(t))\end{aligned} \quad (9)$$

where $A^*=[a_{i1}\,a_{i2}....a_{iN}]$, $G=diag(g_1,g_2,...g_N)$. The in-degree matrix is depicted as $D=diag(d_1,d_2,...d_N)$ in which $d_i=\sum a_{ij}$. Correspondingly, The Laplacian matrix is shown as $L=D-A^*$. The time derivative of the error is calculated as follows:

$$\dot{E}(t)=-[(L+G)\otimes I_n](\dot{\hat{X}}(t)-\dot{X}_r) \quad (10)$$

By the equations (4) and (6), the equation (11) is obtained.

$$\begin{aligned}\dot{E}(t)&=-[(L+G)\otimes I_n](I_N\otimes A)\hat{X}(t)\\&-((L+G)\otimes I_n)(I_N\otimes b)F(\hat{x}(t))\\&+((L+G)\otimes I_n)(I_N\otimes lc)\bar{X}(t)\\&+((L+G)\otimes I_n)(I_N\otimes B')R(t)\\&-[(L+G)\otimes I_n](I_N\otimes lc)X_\Delta(t)\\&-((L+G)\otimes I_n)(I_N\otimes b)(I_N\otimes B)U(t)\\&+((L+G)\otimes I_n)(I_N\otimes A)X_r(t)\end{aligned} \quad (11)$$

Using Kronecker properties and some mathematical manipulations, The following equation is derived by:

$$\begin{aligned}\dot{E}(t)&=-[(L+G)\otimes A]\hat{X}(t)-[(L+G)\otimes b]F(\hat{x}(t))\\&-[(L+G)\otimes bB]U(t)-[(L+G)\otimes lc]X_\Delta(t)\\&+[(L+G)\otimes lc]\bar{X}(t)+[(L+G)\otimes A]X_r(t)\\&+((L+G)\otimes B')R(t)\end{aligned} \quad (12)$$

Now, the input control is considered as (13).

$$U(t)=(I_N\otimes B)^+(-F(\hat{x}(t))+k_1E(t)+k_2R(t)) \quad (13)$$

where k_2 is defined as follows.

$$k_2=-(I_N\otimes b)^+(I_N\otimes B') \quad (14)$$

and k_1 is specified during the calculation.

Theorem 2: Consider the multi-agent systems given by (1)for each agent and the dynamics of consensus error mentioned (12). The control input presented in equations (13) - (14) makes the overall multi-agent system uniformly ultimately bounded in $\|E\|_2/\|\bar{X}\|_2<\lambda$ that λ is scalar value in which $k_1\in\mathbb{R}^{1\times n}$ is gain for the nominal system and the $p_1\in\mathbb{R}^{n\times n}$ and $p_2\in\mathbb{R}^{n\times n}$ are the positive definite matrices satisfied the following inequalities:

$$-Q_1=A^Tp_1+p_1A<0, \quad (15)$$
$$-Q_2=A^Tp_2+p_2A<0. \quad (16)$$

proof: The following Lyapunov function is candidate to prove the stability of the closed-loop system.

$$V(t)=\frac{1}{2}\begin{bmatrix}E^T & \bar{X}^T\end{bmatrix}\left(I_N\otimes\begin{bmatrix}p_1 & 0\\ 0 & p_2\end{bmatrix}\right)\begin{bmatrix}E\\ \bar{X}\end{bmatrix} \quad (17)$$

where $p_1,p_2\in\mathbb{R}^{n\times n}$ are positive definite constant matrices.

The time derivative of the equation(17) is as (18).

$$\begin{aligned}\dot{V}(t)&=\frac{1}{2}\dot{E}^T(t)(I_N\otimes p_1)E(t)+\frac{1}{2}E^T(t)(I_N\otimes p_1)\dot{E}(t)\\&+\frac{1}{2}\dot{\bar{X}}^T(t)(I_N\otimes p_2)\bar{X}(t)+\frac{1}{2}\bar{X}^T(t)(I_N\otimes p_2)\dot{\bar{X}}(t)\end{aligned} \quad (18)$$

Based on the equations (2),(6) and (12), the following equation is derived.

$$\begin{aligned}\dot{V}(t)&=-E^T(t)(I_N\otimes p_1)[(L+G)\otimes A]\hat{X}(t)\\&-E^T(t)(I_N\otimes p_1)[(L+G)\otimes b]F(\hat{x}(t))\\&-E^T(t)(I_N\otimes p_1)[(L+G)\otimes bB]U(t)\\&-0.5X_\Delta^T[(L+G)\otimes lc]^T(I_N\otimes p_1)E(t)\\&-0.5E^T(t)(I_N\otimes p_1)[(L+G)\otimes lc]X_\Delta(t)\\&+0.5\bar{X}^T[(L+G)\otimes lc]^T(I_N\otimes p_1)E(t)\\&+0.5E^T(t)(I_N\otimes p_1)[(L+G)\otimes lc]\bar{X}(t)\\&+0.5E^T(t)(I_N\otimes p_1)[(L+G)\otimes A]X_r(t)\\&+E^T(t)(I_N\otimes p_1)[(L+G)\otimes B']R(t)\\&+0.5\bar{X}^T[I_N\otimes A]^T(I_N\otimes p_2)\bar{X}(t)\\&+0.5\bar{X}^T(I_N\otimes p_2)(I_N\otimes A)\bar{X}(t)\\&+\bar{X}^T(I_N\otimes p_2)(I_N\otimes b)[F(x(t))-F(\hat{x}(t))]\\&-0.5X_\Delta^T(I_N\otimes lc)^T(I_N\otimes p_2)\bar{X}(t)\\&-0.5\bar{X}^T(I_N\otimes p_2)(I_N\otimes lc)X_\Delta(t)\\&+0.5\bar{X}^T(I_N\otimes lc)^T(I_N\otimes p_2)\bar{X}(t)\\&+0.5\bar{X}^T(I_N\otimes p_2)(I_N\otimes lc)\bar{X}(t)\end{aligned} \quad (19)$$

The equation(20) is obtained by equations (9), (13), (14) and lemma2.

$$\begin{aligned}\dot{V}(t)&=\begin{bmatrix}E^T(t) & \bar{X}^T(t)\end{bmatrix}Z\begin{bmatrix}E(t)\\ \bar{X}(t)\end{bmatrix}\\&+\frac{1}{2}E^T(t)((I_N\otimes p_1A)+(I_N\otimes A^Tp_1))E(t)\\&+\frac{1}{2}\bar{X}^T(t)((I_N\otimes p_2A)+(I_N\otimes A^Tp_2))\bar{X}(t)\\&-\frac{1}{2}X_\Delta^T(t)((L+G)\otimes lc)^T(I_N\otimes p_1)E(t)\\&-\frac{1}{2}E^T(t)(I_N\otimes p_1)((L+G)\otimes lc)X_\Delta(t)\end{aligned}$$

$$-\frac{1}{2}X_{\Delta}^{T}(t)(I_N\otimes lc)^{T}(I_N\otimes p_2)\bar{X}(t)$$
$$-\frac{1}{2}\bar{X}^{T}(t)(I_N\otimes p_2)(I_N\otimes lc)X_{\Delta}(t) \quad (20)$$

where Z defines as below.

$$Z=\begin{bmatrix} 0.5p_3-((L+G)\otimes p_1 b)k_1 & 0.5((L+G)\otimes(p_1 lc) \\ & 0.5(I_N\otimes(p_2 lc+(lc)^T p_2))+ \\ 0.5((L+G)\otimes(lc))^T(I_N\otimes p_1) & 0.5(I_{nN}\|I_N\otimes p_2 b\|)p_3^{-1} \\ & \times(I_{nN}\|I_N\otimes p_2 b\|) \end{bmatrix}$$

The equation(21) can be rewritten in the following form.

$$\dot{V}(t)=\left[E^T(t)\quad \bar{X}^T(t)\right]Z\left[E^T(t)\quad \bar{X}^T(t)\right]^T$$
$$-\frac{1}{2}E^T(t)Q_1E(t)-\frac{1}{2}\bar{X}^T(t)Q_2\bar{X}(t)$$
$$-\frac{1}{2}X_{\Delta}^T(t)A_{31}E(t)-\frac{1}{2}E^T(t)A_{13}X_{\Delta}(t)$$
$$-\frac{1}{2}X_{\Delta}^T(t)A_{32}\bar{X}(t)-\frac{1}{2}\bar{X}^T(t)A_{23}X_{\Delta}(t). \quad (21)$$

By the equations (15) and (16), the following inequality can be obtained from the above equation.

$$\dot{V}(t)\le\left[E^T(t)\quad \bar{X}^T(t)\right]Z\left[E^T(t)\quad \bar{X}^T(t)\right]^T$$
$$-\frac{1}{2}E^T(t)Q_1E(t)-0.5\bar{X}^T(t)Q_2\bar{X}(t)$$
$$+\lambda_{\max 31}\|E(t)\|\|X_{\Delta}(t)\|+\lambda_{\max 32}\|\bar{X}(t)\|\|X_{\Delta}(t)\|$$
$$+\lambda_{\max 13}\|X_{\Delta}(t)\|\|E(t)\|+\lambda_{\max 23}\|X_{\Delta}(t)\|\|\bar{X}(t)\| \quad (22)$$

Using mathematical formulas, the following inequality is obtained:

$$\dot{V}(t)\le\left[E^T(t)\quad \bar{X}^T(t)\right]Z\left[E^T(t)\quad \bar{X}^T(t)\right]^T$$
$$-\frac{1}{2}E^T(t)Q_1E(t)-\frac{1}{2}\bar{X}^T(t)Q_2\bar{X}(t)$$
$$+\lambda_1\|E(t)\|\|X_{\Delta}(t)\|+\lambda_2\|\bar{X}(t)\|\|X_{\Delta}(t)\|, \quad (23)$$

where $\lambda_1=-Max\{\lambda_{\max 13},\lambda_{\max 31}\}$ $\lambda_2=Max\{\lambda_{\max 23},\lambda_{\max 32}\}$ and $\lambda=\lambda_2/\lambda_1$. $\|E(t)\|\le\lambda\|\bar{X}(t)\|$ and k_1 is designed in such a way that the matrix of Z be negative definite. Consequently, inequality (24) holds and furthermore the uniformly ultimately boundedness of the overall MAS is guaranteed.

$$\dot{\mathrm{V}}(t)\le 0 \quad (24)$$

Also, the events are triggered when:

$$W(E,\bar{X})\triangleq\|E(t)\|+\lambda\|\bar{X}(t)\|=0 \quad (25)$$

Theorem 3: Consider the multi-agent systems mentioned in equation (1), the consensus error cited in (12) and the control input derived in equations (13) through (14). Then for any initial condition, the interval times $\{t_{k+1}-t_k\}$ tacitly depicted by equation (25) with lower bound τ which is set by $\tau=\alpha^{-1}\ln|\alpha y+\beta|$. y is attained below.

$$y=\|E(t)\|/\|\bar{X}(t)\| \quad (26)$$

proof*:* The time derivative of (26), is as follows.

$$y^2=\frac{E^T(t)E(t)}{\bar{X}^T(t)\bar{X}(t)}\Rightarrow$$
$$2y\dot{y}=\frac{(\dot{E}^T(t)E(t)+E^T(t)\dot{E}(t))(\bar{X}^T(t)\bar{X}(t))}{(\bar{X}^T(t)\bar{X}(t))^2}$$
$$-\frac{\dot{\bar{X}}^T(t)\bar{X}(t)+\bar{X}^T(t)\dot{\bar{X}}(t))E^T(t)E(t)}{(\bar{X}^T(t)\bar{X}(t))^2}. \quad (27)$$

After some manipulations of the mathematical, the equation(28) is achieved:

$$2y\dot{y}=\frac{\dot{E}^T(t)E(t)+E^T(t)\dot{E}(t)}{(\bar{X}^T(t)\bar{X}(t))}$$
$$-\frac{\dot{\bar{X}}^T(t)\bar{X}(t)+\bar{X}^T(t)\dot{\bar{X}}(t)}{\bar{X}^T(t)\bar{X}(t)}\frac{E^T(t)E(t)}{\bar{X}^T(t)\bar{X}(t)}. \quad (28)$$

Then, the following inequality holds.

$$2y\dot{y}\le\frac{2\|E(t)\|\|\dot{E}(t)\|}{\|\bar{X}(t)\|^2}+\frac{2\|\dot{\bar{X}}(t)\|}{\|\bar{X}\|}y^2. \quad (29)$$

Finally, the inequality (30) is achieved.

$$\dot{y}\le\frac{\|\dot{E}(t)\|}{\|\bar{X}(t)\|}+\frac{\|\dot{\bar{X}}(t)\|}{\|\bar{X}\|}y. \quad (30)$$

The following inequality is obtained from equations (2) and (6).

$$\|\dot{\bar{X}}(t)\|\le\|I_N\otimes A\|\|\bar{X}(t)\|+\|I_N\otimes b\|\|F(x(t))-F(\hat{x}(t))\|$$
$$+\|I_N\otimes lc\|\|X_{\Delta}(t)-\bar{X}(t)\| \quad (31)$$

With lemma 1, the following inequality is derived.

$$\|\dot{\bar{X}}(t)\|\le\|I_N\otimes A\|\|\bar{X}(t)\|+\|I_N\otimes b\|\|\bar{X}(t)\|$$
$$+\|I_N\otimes lc\|\|X_{\Delta}(t)\|+\|I_N\otimes lc\|\|\bar{X}(t)\| \quad (32)$$

By the equations (12) through (14), the following inequality can be constructed:

$$\|\dot{E}(t)\|\le\|I_N\otimes A\|\|E(t)\|+\|(L+G)\otimes bB\|\|k_1\|\|E(t)\|$$
$$+\|(L+G)\otimes(lc)\|\|X_{\Delta}(t)\|+\|(L+G)\otimes(lc)\|\|\bar{X}(t)\| \quad (33)$$

The inequality (30) is rewritten as (34) by using (32) and (33).

$$\dot{y}\le\|I_N\otimes A\|\frac{\|E(t)\|}{\|\bar{X}(t)\|}+\|k_1\|\|(L+G)\otimes bB\|\frac{\|E(t)\|}{\|\bar{X}(t)\|}+$$
$$\|(L+G)\otimes bB\|\frac{\|X_{\Delta}(t)\|}{\|\bar{X}(t)\|}+\|(L+G)\otimes(lc)\|+\|I_N\otimes(lc)\|y \quad (34)$$
$$\|I_N\otimes A\|y+\|I_N\otimes b\|y+\|I_N\otimes(lc)\|\frac{\|X_{\Delta}(t)\|}{\|\bar{X}(t)\|}y$$

Based on the equation (26), the above inequality is rewritten as inequality (35).

$$\begin{aligned}\dot{y} &\le (\|I_N \otimes A\| + \|k_1\| \|(L+G)\otimes bB\| + \|I_N \otimes A\| \\ &+ \|I_N \otimes b\| + \|I_N \otimes (lc)\|) y + \|(L+G)\otimes (lc)\| \\ &+ \|(L+G)\otimes bB\| \frac{\|X_\Delta(t)\|}{\|\bar{X}(t)\|} + \|I_N \otimes (lc)\| \frac{\|X_\Delta(t)\|}{\|\bar{X}(t)\|} y\end{aligned} \tag{35}$$

Because of $\|X_\Delta(t)\| / \|\bar{X}(t)\| < \gamma$, the following holds:

$$\begin{aligned}\dot{y} &\le (\|I_N \otimes A\| + \|k_1\| \|(L+G)\otimes bB\| + \|I_N \otimes A\| \\ &+ \|I_N \otimes b\| + \|I_N \otimes (lc)\| + \|I_N \otimes (lc)\|\gamma) y \\ &+ \|(L+G)\otimes (lc)\| + \|(L+G)\otimes bB\|\gamma\end{aligned} \tag{36}$$

So, the general form of the above inequality can be reconstructed as (37).

$$\dot{y} \le \alpha y + \beta \tag{37}$$

The event time is obtained by the solving the above differential equation below.

$$t = \frac{1}{\alpha} \ln|\alpha y + \beta| \tag{38}$$

5. Simulation Result

In this paper, each dynamical agents are considered as follows.

$$\begin{cases} \dot{x}_i(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix} x_i(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} [f(x_i(t) + u_i(t)] \\ y_i(t) = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} x_i(t) \end{cases} \tag{39}$$

$i = 1, \ldots, 4$

where $f(x_i(t)) = 0.05\sin(x_{i3}(t))$ and $B = 1$. The dynamical model of the leader is presented as follows, too.

$$\dot{x}_r(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix} x_r(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r(t), \tag{40}$$

where $r(t)$ is a reference input and it can be the arbitrarily function.

Also, the dynamics of the observer for each agent is considered as equation (41)

$$\begin{aligned}\dot{\hat{x}}_i(t) &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix} \hat{x}_i(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} [f(\hat{x}_i(t) + u_i(t)] \\ &+ lc(\hat{x}_i(t) - x_i(t_k))\end{aligned} \tag{41}$$

where l and c are as follows:

$$c = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}, \ l = \begin{bmatrix} 4 & 2 & -3 \end{bmatrix}^T \tag{42}$$

The communication topology of this multi-agent system according to $\mathcal{A}$ is considered as $\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$ and $G = diag(1,1,1,1)$. Matrices p_1 and p_2 according to equations (15) and (16), are as follows, too.

$$p_1 = p_2 = \begin{bmatrix} 2.3000 & 2.1000 & 0.5000 \\ 2.1000 & 4.6000 & 1.3000 \\ 0.5000 & 1.3000 & 0.6000 \end{bmatrix}, \tag{43}$$

Figure 2 shows the states of the leader agent.

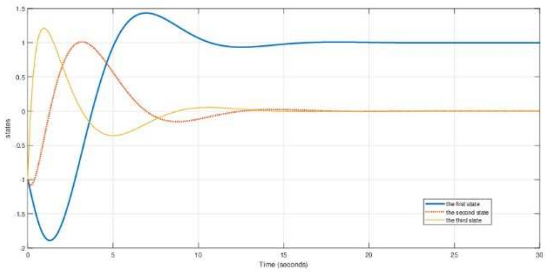

Fig. 2. The output and states of the leader system.

Then, the error between the followers and the leader are shown in Fig. 3 through Fig. 6.

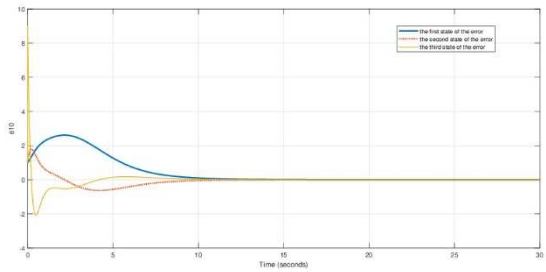

Fig. 3. the error between the 1th follower and leader.

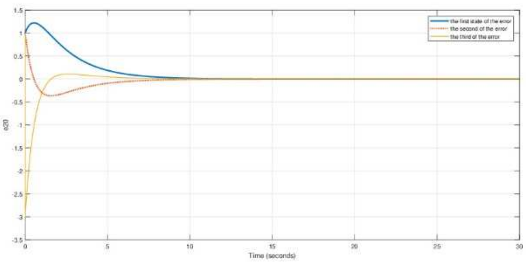

Fig. 4. the error between the 2nd follower and leader.

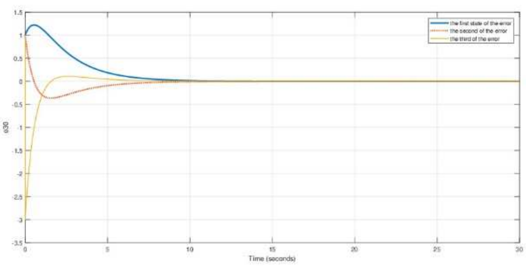

Fig. 5. the error between the 3rd follower and leader.

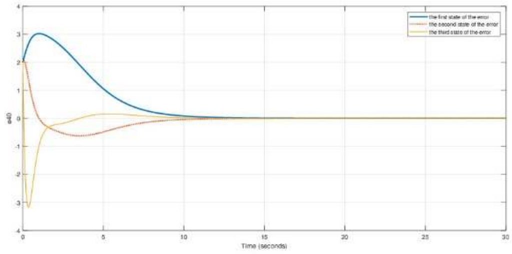

Fig. 6. the error between the 4th follower and leader.

As shown in figures 3 through 6, the followers are tracking the leader and consequently the tracking error achieves to the neighborhood of zero. The observer errors are shown in figures 7 to 10, which show the states are well estimated by observers.

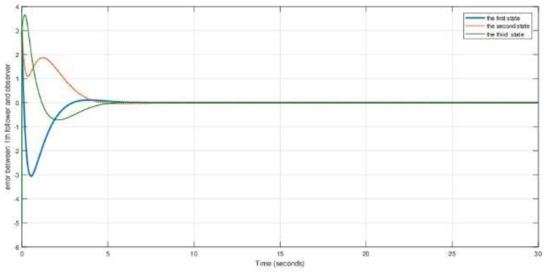

Fig. 7. the error between the states of the 1st follower and its observer.

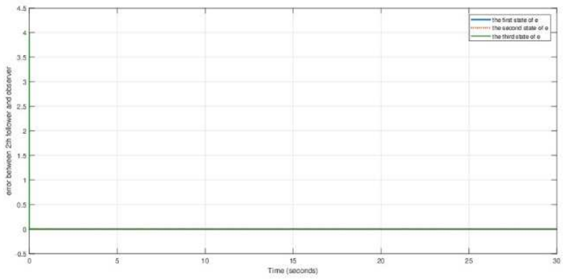

Fig. 8. the error between the states of the 2nd follower and its observer.

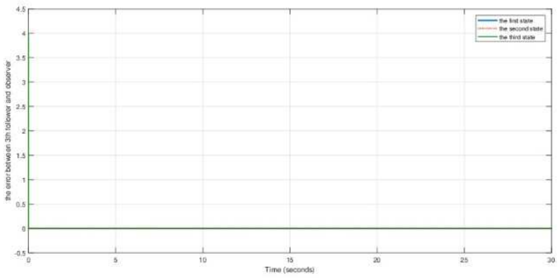

Fig. 9. the error between the states of the 3rd follower and its observer.

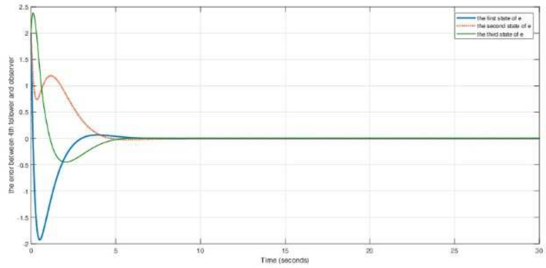

Fig. 10. the error between the states of the 4th follower and its observer.

Figure 11 shows the control input.

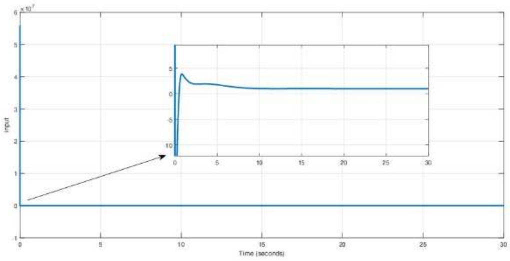

Fig. 11. The control input

Also, updating of the event triggered is shown in Fig. 12.

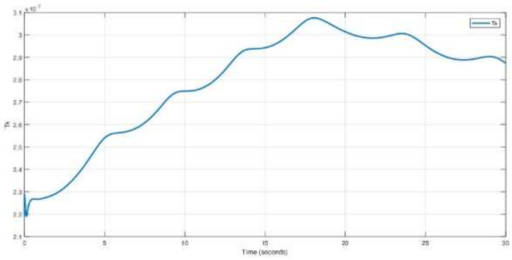

Fig. 12. the Updating of the event triggered Time

Based on the shown figures, the promising performance of the observer, the closed loop system stability and high performance of the proposed controller are all obvious.

6. Conclusion

This paper deals with an event-triggering consensus protocol for nonlinear multi-agent system based on nonlinear observer. The event based controller is summarized to reduce the unnecessary data transition in the planned topology. Merging of the agreement error to neighborhood of zero, stability of the compact form of the MAS in consensus emission, decreasing both size and time of the mathematical manipulation because of the cloud computation are all the focal characteristics of the presented methodology. Designing of the fault tolerant control of the nonlinear systems based on the cloud computation can be considered as the future research of the authors.

Funding

There is no funding for this paper. All the data are included in this paper in the simulation results section.

Conflict of Interest

The authors declare that they have no conflict of interest. There is no fund for this research.

Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

References

[1] R. Olfati-Saber, J. A. Fax, and R. M. Murray, "Consensus and cooperation in networked multi-agent systems", Proc. IEEE, 95(1), pp. 215–233, 2007.

[2] Z. Chen, Q.-L. Han, Y. Yan, and Z.-G. Wu, "How often should one update control and estimation: Review of networked triggering techniques", Sci. China Inf. Sci. 63(5), 2020.

[3] R. Olfati-Saber, "Flocking for multi-agent dynamic systems: Algorithms and theory", IEEE Trans. Autom. Control 51(3), pp. 401-420, 2006.

[4] X. Dong, Y.Zhou, Z. Ren, and Y. Zhong, "Time-varying formation tracking for second-order multi-agent systems subjected to switching topologies with application to quad-rotor formation flying", IEEE Trans. Ind. Electron, vol. 64(6), pp. 5014-5024, 2017.

[5] P. Mill´an, L. Orihuela, I. Jurado, and F. Rodriguez Rubio, "Formation control of autonomous underwater vehicles subject to communication delays", IEEE Trans. Control System Technology 22(2), pp. 770-777, 2014.

[6] T. Liu and Z.P. Jiang, "Distributed formation control of non-holonomic mobile robots without global position measurements", Automatica 49(1), pp. 592-600, 2013.

[7] H. Rezaee and F. Abdollahi, "Pursuit formation of double-integrator dynamics using consensus control approach", IEEE Trans. Ind. Electron. 62(7), pp. 4249-4256, 2015.

[8] X. Ge, and Q. Han, "Distributed formation control of networked multi-agent systems using a dynamic event-triggered communication mechanism", IEEE Transactions on Industrial Electronics 64(10), pp. 8118-8127, 2017.

[9] W. Hu, L. Liu, and G. Feng, "Consensus of linear multi-agent systems by distributed event-triggered strategy", IEEE transactions on cybernetics 46(1), pp. 148-157, 2015.

[10] P. Tabuada, "Event-triggered real-time scheduling of stabilizing control tasks", IEEE TransactionAutomic Control 52(9), pp. 1680-1685, 2007.

[11] H.Zhang, G. Feng, H. Yan, Q. Chen, "Observer-based output feedback event-triggered control for consensus of multi-agent systems", IEEE Transactions on Industrial Electronics 61(9), pp. 4885-4894, 2013.

[12] D.V. Dimarogonas, E. Frazzoli, and K. H. Johansson, "Distributed event-triggered control for multi-agent systems", IEEE Transaction Automic Control 57(5), pp. 1291-1297, 2012.

[13] C. Peng, Q. L. Han, and D. Yue, "To transmit or not to transmit: A discrete event-triggered communication scheme for networked Takagi–Sugeno fuzzy systems", IEEE Trans. Fuzzy Syst 21(1), pp. 164-170, 2013.

[14] K.J.Astrom and B. M. Bernhardsson, "Comparison of periodic and event based sampling for first order stochastic system", Proc. IFAC World Conference, pp. 301-306, 1999.

[15] J. Lunze and D. Lehmann, "A state-feedback approach to event-based control", Automatica 46(1), pp. 211-215, 2010.

[16] H. Dong, Z. Wang, F. E. Alsaadi, and B. Ahmad, "Event-triggered robust distributed state estimation for sensor networks with state-dependent noises", Int. J. Gen. System 44(2), pp. 254-266, 2015.

[17] G. S. Seyboth, D. V. Dimarogonas, and K. H. Johansson, "Event-based broadcasting for multi-agent average consensus", Automatica 49(1), pp. 245-252, 2013.

[18] C. Yi, J. Feng, C. Xu, J. Wang, and Y. Zhao, "Event-triggered consensus control for stochastic multi-agent systems under state-dependent topology". International Journal of Control 94(9), pp. 2379-2387, 2021.

[19] H. Ma, H. Li, R. Lu, and T. Huang, "Adaptive event-triggered control for a class of nonlinear systems with periodic disturbances", Science China Information Sciences, 63(5), pp. 1-15, 2020.

[20] D. Yao, L. Hongyi, L. Renquan and Sh. Yang, "Distributed sliding-mode tracking control of second-order nonlinear multi-agent systems: an event-triggered approach", IEEE transactions on cybernetics 50(9), pp. 3892-3902, 2020.

[21] H. Liang, G. Liu, H. Zhang, and T. Huang, "Neural-network-based event-triggered adaptive control of non-affine nonlinear multi-agent systems with dynamic uncertainties", IEEE Transactions on Neural Networks and Learning Systems 32(5), pp. 2239-2250, 2020.

[22] D. Ding, Z. Wang, D. WC Ho, and G. Wei, "Observer-based event-triggering consensus control for multi-agent systems with lossless sensors and cyber-attacks",

IEEE Transactions on Cybernetics 47(8), pp. 1936-1947, 2016.
[23] G. S. Seyboth, D. V. Dimarogonas, and K. H. Johansson, “Event-based broadcasting for multi-agent average consensus”, Automatica 49(1), pp. 245-252, 2013.
[24] ZH.Wu, Y. Xu, Y.J. Pan, P. Shi, and Q. Wang, "Event-triggered pinning control for consensus of multi-agent systems with quantized information", IEEE Transactions on Systems, Man, and Cybernetics: Systems 48(11), pp. 1929-1938, 2017.
[25] J.L. Wang, H.N. Wu, T. Huang, S.Y. Ren, and J. Wu, "Pinning control for synchronization of coupled reaction-diffusion neural networks with directed topologies", IEEE Trans. Syst., Man, Cybernetic, System, 46(8), pp. 1109-1120, 2017.
[26] M. Zamanian, F. Abdollahi, and S. K. Yadavar Nikravesh, "Finite-time consensus of heterogeneous unknown nonlinear multi-agent systems with external disturbances via event-triggered control". Journal of Vibration and Control, 27(15-16), pp. 1806-1823, 2021.
[27] H. Khalil, "Nonlinear systems", 3 edition, 2001.
[28] M.J. Khosrowjerdi, S. Barzegary, "Fault tolerant control using virtual actuator for continuous-time Lipchitz nonlinear systems", Int. J. Robust Nonlinear Control 10(1), pp. 2597-2607, 2014.
[29] C. Deng, G. H. Yang, “Distributed adaptive fault-tolerant containment control for a class of multi-agent systems with non-identical matching non-linear functions”, IET Control Theory & Applications 10(3), pp. 273-281, 2016.
[30] S. Hou, Y. Chu, J. Fei, “Modified fuzzy neural network control using sliding mode technique for power quality improvement system with experimental verification”, IET Control Theory & Applications 14(19), pp. 3029-3037, 2020.
[31] J. Zhang; H. Zhang; Y. Lu; S. Sun, “Event\self-triggered leader-following consensus of multi-agent systems with general dynamics”, IET Control Theory & Applications 14(9), pp. 1209-1219, 2020.
[32] M. Hou, H. Shi, “The Adaptive Distributed Event-Triggered Observer Approach to Cooperative Output Regulation of Discrete-Time Linear Multi-Agent Systems under Directed Switching Networks”, IEEE Access 8(1), pp. 221568 – 221579, 2020.
[33] P. Xu, X. Wang, G. Xie, J. Tao, M, Xu, Quan Zhou, “Distributed Finite-time Bipartite Consensus of Multi-agent Systems via Event-triggered Control”, IFAC-Papers On Line, 53(2), pp. 2927-2932, 2020.