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Evaluation of the Performance in DEA-R Models Based on Common Set of Weights

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Abstract. The ratio data envelopment analysis (DEA-R) model enables the analyst to easily translate some of the significant types of experts' opinions into weight restrictions, which are not easily modeled with the classical data envelopment analysis models. In fact, the DEA-R model considered a combination of the DEA methodology, and ratio analysis. This paper uses the common set of weights (CSW) method to control the total weight flexibility in the DEA-R and proposes a CSW DEA-R method. The proposed CSW DEA-R method improves the discrimination power of the DEA-R method. It can be used to identify the CSW pseudo-inefficient DMUs in DEA. Then, we compare the proposed CSW DEA-R method with the DEA-R and DEA approaches in the literature; and demonstrate the applicability of the proposed method with an empirical example.

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1. Introduction

Data envelopment analysis (DEA) is a non-parametric tool for evaluating the relative efficiency of comparable entities referred to as Decision Making Units (DMUs). The CCR model, is developed by Charnes et al. [5], is one of the best known models to evaluate the efficiency of DMUs in DEA. Thereafter, Banker et al. [4] incorporated the economic concept of returns to scale with the proposed variable returns to scale (BBC) model. Many extensions in DEA have been proposed based on the classical models [6]. Recently, Panwar et al. [11] presented the near 40 years of existence of DEA in a brief template by discussing the popular DEA models, their advantages and deficiencies, and different applications of DEA.

The idea of weight restrictions which is highly important, was introduced by Allen et al. [3]. A generalized model with weight restrictions was presented by Tracy and Chen [16]. In the following Podinovski [12] considered this concept for finding targets. Despic et al. [7] introduced the ratio data envelopment analysis (DEA-R) model. The DEA-R model enables the analyst to easily translate some of the significant types of experts' opinions into weight restrictions, which are not easily modeled with the classical DEA models. In fact, the DEA-R model considered a combination of the DEA methodology, and ratio analysis. Wei et al. [17] applied the DEA-R model for evaluation of efficiency and the pseudo-inefficiency in healthcare systems. The pseudo-inefficiency of the DMUs are cases where the efficiency score does not express the real efficiency of the DMUs.
Wei et al. [18] proposed linear cost and revenue efficiency models based on DEA-R models and compared the optimal weights in DEA and DEA-R models. Mozaffari et al. [9] developed a combination of standard DEA models and ratio analysis and presented a linear formulation of cost and revenue efficiency DEA-R models.

Tohidnia and Tohidi [15] presented a DEA-R profit efficiency model, where weight restrictions were incorporated in terms of the ratio of inputs and outputs. Then they used their proposed DEA-R model to calculate the Malmquist productivity index.

Mozaffari et al. [10] identified efficient surfaces in DEA-R models. In current paper, the axioms for specifying the production possibility set in constant returns to scale technology for DEA-R were presented, and an original algorithm for identification of efficient sur- faces were proposed.

In 2022, Ghiyasi et al. [8] presented a novel and proper inverse DEA methodology and developed the theoretical foundation of the inverse DEA-R model as a post-efficiency analysis approach which both input-oriented and output-oriented inverse models are considered. They applied the proposed models for efficiency assessment and moreover inverse analysis of 130 public hospitals in Iran. Sohrabi et al. [14] presented the inputs/output estimation process based on ratio based DEA (DEA-R) models. To determine the level of inputs based on the perturbed outputs, they presented a multiple objective linear programming (MOLP) model, assuming that the relative efficiency of the under evaluation decision making unit (DMU) preserve. Also, they presented the criterion models to determine the efficiency of the new DMU in the inputs/output estimation process based on inverse DEA-R models in the presence of ratio data and showed that in the presence of ratio data the selection of criterion model can be important.

Ahmad Khanlou et al. [2] a model proposed for measuring overall efficiency of the system and its processes over several desired time periods in the presence of ratio data to prevent false inefficiency and not use the non-Archimedean number ε. Also, the internal relationships among processes are considered and the proposed model focuses on changes over time period. They demonstrated that overall efficiency scores and the efficiency of each process obtained from model after several desired time periods are higher than or equal to overall efficiency scores and the efficiency of each process in each time period.

Common set of weights (CSW) is a useful method in engineering and economic analysis [13]. In the evaluations of DMUs by CSW each input and output takes the same weight in all DMUs. Since DEA-R uses different weights in the evaluation of the different DMUs it cannot be used to rank of the DMUs, while CSW is used to rank the DMUs in DEA [1]. This paper proposes a DEA-R approach to derive a CSW to be used to evaluate and the ranking of DMUs. It uses the CSW method to control the total weight flexibility in the DEA-R. The proposed CSW DEA-R method improves the discrimination power of the DEA-R method by comparing each ratio with the best corresponding ratio. Finally, we compare the proposed CSW DEA-R method with the DEA-R and DEA approaches in the literature; and demonstrate the applicability of the proposed method with an empirical example.

The paper unfolds as follows: Section 2 proposes the CSW DEA-R method by input orientation model. The mathematical analysis is presented in section 3. Section 4 includes an application of the proposed approach to study 21 medical center in Taiwan. Section 5 concludes.

2. Model formulation

Assume there are n decision making units, $(\mathrm{DMU}_j, j=1,...,n)$ where each DMU uses m inputs $x_j=(x_{1j},,...,x_{mj})$ to produce s outputs $y_j=(y_{1j},,...,y_{sj})$. Charnes et al. [5] introduced the model to evaluate the relative efficiency as follows:

$$
\begin{aligned}
\min \quad & \theta \\
\text{s.t.} \quad & \sum_{j=1}^{n} \lambda_j x_j \le \theta x_o \\
& \sum_{j=1}^{n} \lambda_j y_j \ge y_o \\
& \lambda_j \ge 0, \qquad j = 1,...,n.
\end{aligned} \tag{1}
$$

Despic et al. [7] used ratio analysis to evaluate the efficiency of the DMUs and proposed the input-oriented DEA-R model for efficiency evaluation of DMU$_o$ as follows:

$$
\begin{aligned}
\max \quad & \Delta \\
\text{s.t.} \quad & \Delta \le \sum_{i=1}^{m} \sum_{r=1}^{s} w_{ir} \frac{x_{ij}/y_{rj}}{x_{io}/y_{ro}}, \; j = 1,\ldots,n \\
& \sum_{i=1}^{m} \sum_{r=1}^{s} w_{ir} = 1 \\
& w_{ir} \ge 0, i = 1,\ldots,m, r = 1,\ldots,s.
\end{aligned} \tag{2}
$$

Let $\lambda_j (j = 1,\ldots,n)$ and γ be the dual variables constraints of model (2). Then, the dual of model (2) can be written as:

$$
\begin{aligned}
\min \; & \gamma \\
\text{s.t.} \; & \sum_{j=1}^{n} \lambda_j \left(\frac{\frac{x_{ij}}{y_{rj}}}{\frac{x_{io}}{y_{ro}}} \right) \le \gamma, i = 1,\ldots,m, r = 1,\ldots,s \\
& \sum_{j=1}^{n} \lambda_j = 1 \\
& \eta \text{ free}, \lambda_j \ge 0, \; j = 1,\ldots,n.
\end{aligned} \tag{3}
$$

Definition 2.1 DMU$_o$ in input-oriented DEA-R efficient if and only if, the optimal value of models (2) and (3) equal 1.

In DEA-R model each DMU chooses own weights and the weights of the DMUs usually are different. To control the total weight flexibility in the DEA-R and to obtain same weights to the all of DMUs in DEA-R model we propose the common set of weights approach in DEA-R models in input orientation. The input orientation model compares the ratio x_{ij}/y_{rj} with the ratio x_{io}/y_{ro} for each $o\,(o = 1,...,n)$. Hence, the CSW DEA-R model in input orientation is proposed as follows:

$$\begin{aligned}
&\max \Delta \\
&\text{s.t. } \Delta \le \Delta_o, o = 1,\dots,n \\
&\qquad \Delta_o \le \sum_{i=1}^{m}\sum_{r=1}^{s} w_{ir} \frac{x_{ij}/y_{rj}}{x_{io}/y_{ro}}, o, j = 1,\dots,n \\
&\qquad \sum_{i=1}^{m}\sum_{r=1}^{s} w_{ir} = 1 \\
&\qquad w_{ir} \ge 0, i = 1,\dots,m, r = 1,\dots,s.
\end{aligned} \tag{4}$$

The above linear programming problem model can written as follows:

$$\begin{aligned}
&\max \Delta \\
&\text{s.t. } \Delta \le \sum_{i=1}^{m}\sum_{r=1}^{s} w_{ir} \frac{x_{ij}/y_{rj}}{x_{io}/y_{ro}}, o, j = 1,\dots,n \qquad (a) \\
&\qquad \sum_{i=1}^{m}\sum_{r=1}^{s} w_{ir} = 1 \\
&\qquad w_{ir} \ge 0, i = 1,\dots,m, r = 1,\dots,s.
\end{aligned} \tag{5}$$

For $o = j\ (o, j = 1,\dots,n)$ the constraint (a) in model (5) is as $\Delta \le 1$. Hence, model (5) can be written as follows:

$$\begin{aligned}
&\max \Delta \\
&\text{s.t. } \Delta \le \sum_{i=1}^{m}\sum_{r=1}^{s} w_{ir} \frac{x_{ij}/y_{rj}}{x_{io}/y_{ro}}, o, j = 1,\dots,n\ (o \ne j) \qquad (a) \\
&\qquad \Delta \le 1 \\
&\qquad \sum_{i=1}^{m}\sum_{r=1}^{s} w_{ir} = 1 \\
&\qquad w_{ir} \ge 0, i = 1,\dots,m, r = 1,\dots,s.
\end{aligned} \tag{6}$$

Using the optimal solution of model (6) the CSW DEA-R efficiency of DMUs is determined. Let $(\Delta^*, W^*) = (\Delta^*, w_{11}^*, \dots, w_{ms}^*)$ be an optimal solution of model (6). Then, the $W^* = (w_{11}^*, \dots, w_{ms}^*)$ is DEA-R CSW of DMU$_o$ $(o = 1,\dots,n)$ and it can be used as a same base to compare and evaluation of DMUs.

Definition 2.2 Using $(\Delta^*, W^*) = (\Delta^*, w_{11}^*, \dots, w_{ms}^*)$ as an optimal solution of model (6) the CSW DEA-R efficiency of DMU$_o$ $(o = 1,\dots,n)$ is determined as $E_{CSW}^{o} = \min_{1 \le j \le n} \sum_{i=1}^{m}\sum_{r=1}^{s} w_{ir}^* \frac{x_{ij}/y_{rj}}{x_{io}/y_{ro}}$,

and DMU$_o$ is CSW DEA-R efficient if and only if $E_{CSW}^{o} = 1$.

As can be seen, there exits at least one $j \in \{1,\dots,n\}$ such that $E_{CSW}^{j} = 1$.

Let $\lambda_{oj}\ (o, j = 1,\dots,n, j \ne o)$ be the dual variable of the $(i, r)^{\text{th}}$ input and output constraint, γ be the dual variable of the constraint $\Delta \le 1$, and η be dual variable of the convex combination constraint of model (6). The dual of the above model is as follows:

$$\begin{aligned}
&\min\ \eta \\
&\text{s.t.} \sum_{\substack{j,o=1 \\ j\neq o}}^{n} \lambda_{oj} \left(\frac{x_{ij} / y_{rj}}{x_{io} / y_{ro}} \right) \le \eta, i=1,\ldots,m, r=1,\ldots,s \\
&\sum_{\substack{j,o=1 \\ j\neq o}}^{n} \lambda_{oj} + \gamma = 1 \\
&\eta \text{ free}, \lambda_{oj}, \gamma \ge 0, o, j=1,\ldots,n, j \neq o.
\end{aligned} \tag{7}$$

Models (6) and (7) are the multiplier and envelopment form, respectively. As can be seen, models (6) and (7) are feasible and bounded. Hence, we can always determine the CSW in DEA-R. Since the envelopment form is computationally much easier than the multiplier form [6] the CSW can be determined from the final simplex table of the envelopment form.

Here we discussed the CSW in DEA-R in input orientation. Similarly, the CSW in DEA-R in output orientation can be discussed.

3. Mathematical analysis of the model

By using the optimal solution of model (2) we can determine the optimal solution of the model (6). To this end, we have the following theorem.

Theorem 3.1 Let $(\Delta^{o*}, W^{o*}) = (\Delta^{o*}, w_{11}^{o*}, \ldots, w_{ms}^{o*})$ be an optimal solution of model (2) when DMU$_o$ $(o=1,\ldots,n)$ is under evaluation and $\Delta^{p*} = \min\{\Delta^{o*} \mid o=1,\ldots n\}$. Then, $(\Delta^{p*}, W^{p*}) = (\Delta^{p*}, w_{11}^{p*}, \ldots, w_{ms}^{p*})$ is an optimal solution of model (6).

Proof. As can be seen (Δ^{p*}, W^{p*}) is a feasible solution of model (6), now by contradiction suppose that there exists a feasible solution of model (6), say $(\bar{\Delta}, \bar{W})$, such that $\Delta^{p*} < \bar{\Delta}$. In this case $(\bar{\Delta}, \bar{W})$ is a feasible solution of model (2) when DMU$_p$ is evaluated. This leads to a contradiction. ■

Using the above Theorem one person can find the CSW DEA-R, without solving model (6), by solving model (2) for all of DMUs.

4. Result and discussion

In this section we study the efficiency of 21 medical center by the proposed model and compare them with the other methods. Consider 21 medical centers in Taiwan (the data set is taken from Wei et al. (2011a)), as well as computing their efficiency using the common set of weights in DEA-R and DEA models. Table 1 represents the inputs and outputs of the 21 DMUs in Taiwan. Two inputs (sickbeds and physicians), and three outputs (out-patients, in-patients, and surgeries) are taken into account for DMUs.

The columns (2), (3) and (4) of Table 2 contain the efficiencies of 21 DMUs by models (2), (3) and (6), respectively. A can be seen, by model (2) DMUs 7, 8, 9, 13, and 18 are efficient and by model (3) DMUs 4, 7, 8, 9, 13, and 18 are efficient, while none of the DMUs achieve full (=100%) DEA efficiency CSW DEA-R.

The last six columns of table 2 show the components of optimal weights, i.e. W^*, when

DMUs are evaluated by model (2). As can be seen form table 2, DMUs use different weights. Hence, the optimal solution of model (2) is
$(\Delta^*, W^*) = (\Delta^*, w_{11}^*, w_{12}^*, w_{13}^*, w_{22}^*, w_{22}^*, w_{23}^*)$ =(0,0,0.3749, 0.3386,0.2865,0)
and the 4th column of Table 2 shows the CSW DEA-R efficiency of all the DMUs using this solution. As can be seen, the CSW DEA-R efficiencies of the DMUs are not the same and hence their rank can be determined based on the ratio analysis. The 5th column of Table 2 contains the rank score of the DMUs.

Table 1. Inputs and outputs of the model.

DMU	Sickbeds	Physicians	Out-patients	In-patients	Surgeries
1	2618	1106	2,029,864	680,136	38,714
2	1212	473	1,003,707	297,719	18,575
3	1721	531	1,592,960	408,556	36,658
4	2902	973	2,596,143	855,467	75,348
5	1389	447	1,116,161	337,523	23,803
6	1500	547	1,476,282	378,658	22,503
7	340	145	1,300,016	55,003	5,614
8	571	305	1,052,992	199,780	26,026
9	1168	369	1,849,711	326,109	30,967
10	921	372	1,089,975	209,323	23,847
11	920	316	334,090	268,723	15,130
12	3236	1023	1,954,775	920,215	56,167
13	495	130	332,741	136,351	23,423
14	1759	491	1,465,374	430,407	35,599
15	1357	390	1,277,752	368,174	36,006
16	2468	675	1,825,332	668,467	32,275
17	962	316	550,700	247,961	15,618
18	745	272	1,277,899	217,371	11,671
19	1662	590	1,916,888	418,205	21,551
20	898	275	698,945	209,134	11,748
21	1708	537	1,702,676	470,437	32,218

Table 2. The evaluation results with DEA and DEA-R models and optimal weights.

DMU	DEA	DEA-R	CSW DEA-R	w_{11}^*	w_{12}^*	w_{13}^*	w_{21}^*	w_{22}^*	w_{23}^*
1	0.8137	0.8137	0.5279	0	0.6373	0	0	0.3627	0
2	0.792	0.792	0.5641	0.0262	0.601	0	0	0.3728	0
3	0.8352	0.8432	0.7533	0.0544	0.5817	0	0	0.3639	0
4	0.998	1	0.7988	0.0685	0.5675	0	0	0.3641	0
5	0.8347	0.8417	0.6558	0.0626	0.5673	0	0	0.3701	0
6	0.8349	0.8423	0.6188	0.0524	0.5581	0	0	0.3895	0
7	1	1	0.9621	0	0.6522	0	0.3478	0	0
8	1	1	0.9968	0	1	0	0	0	0

9	1	1	1	0	0.6085	0	0.3915	0	0
10	0.7356	0.7465	0.7465	0	0	0.3749	0.3386	0.2865	0
11	0.9814	0.9814	0.5024	0	0.6837	0	0	0.3163	0
12	0.9802	0.9802	0.636	0	0.7013	0	0	0.2987	0
13	1	1	1	0	0.8394	0	0	0	0.1606
14	0.884	0.9082	0.7719	0.1796	0	0	0	0.8204	0
15	0.9717	0.9865	0.9015	0.1726	0	0	0	0.8274	0
16	0.975	0.9797	0.6886	0.0791	0.5924	0	0	0.3285	0
17	0.8782	0.8782	0.5735	0	0.6933	0	0	0.3067	0
18	1	1	0.7979	0.6675	0	0	0	0.3325	0
19	0.8495	0.8551	0.6326	0.0454	0.5684	0	0	0.3861	0
20	0.8146	0.822	0.6032	0.0631	0.578	0	0	0.3589	0
21	0.9585	0.9675	0.7649	0.058	0.5753	0	0	0.3667	0

5. Conclusion

The DEA-R model enables the analyst to easily translate some of the significant types of experts' opinions into weight restrictions, which are not easily modeled with the classical DEA models. In the evaluations of DMUs by Common set of weights, each input and output takes the same weight in all DMUs. Since DEA-R uses different weights in the evaluation of the different DMUs it cannot be used to rank of the DMUs, while CSW is used to rank the DMUs in DEA. In this paper we used a CSW method to control the total weight flexibility in the DEA-R. The proposed CSW DEA-R method provided a same base for evaluation and improved the discrimination power of the DEA-R method. A relationship between the proposed model and the tradition DEA-R model was discussed. We compared the proposed CSW DEA-R method with the DEA-R and DEA approaches in the literature.

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