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## Research Paper

# Investigation of Population Transfer in The Two Coupled $\Lambda$ -Type Three-Level Systems Based on Stimulated Raman Adiabatic Passage

Ali Hatami<sup>1</sup>, Mehdi Hosseini\*<sup>1</sup>, Fatemeh Ahmadinouri<sup>2</sup>

<sup>1</sup> Department of Physics, Shiraz University of Technology, P.O. Box 313-71555 Shiraz, Iran.

<sup>2</sup> Department of Energy Engineering and Physics, Amirkabir University of Technology (Tehran Polytechnic), P.O. Box 15875-4413, Tehran, Iran.

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### Abstract

Efficient and coherent transfer of population between different quantum states. In this study, we delve into the investigation of population transfer in two coupled  $\Lambda$ -type three-level systems using the STIRAP technique. Our research focuses on understanding the dynamics and control of population transfer within these systems. The system Hamiltonian is constructed based on the physical condition of the coupled structure, then the respective time-dependent Schrodinger equation is solved numerically. By analyzing the adiabatic conditions, we explore the interaction between the two coupled  $\Lambda$ -type systems examine the role of various parameters, such as the time of the peak amplitude, and determine the pulse width. Furthermore, we observe the impact of interaction on the transition probability, comparing coupled systems to uncoupled systems. The findings of this study shed light on the underlying mechanisms of STIRAP and contribute to the development of advanced quantum control techniques.

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\*Corresponding author: Mehdi Hosseini

**Address:** Department of Physics, Shiraz University of Technology, P.O. Box 313-71555 Shiraz, Iran. **Email:** hosseini@sutech.ac.ir

## 1. INTRODUCTION

Since the advent of quantum physics, controlling and manipulating quantum systems have been of significant importance [1].

Among the most commonly employed techniques for population transfer, remarkable methods include Landau-Zener [2], chirped laser [3, 4], Raman stimulated transition [5],  $\pi$  pulse transition [6], and stimulated Raman adiabatic passage (STIRAP) [7].

STIRAP is the most prevalent method for population transfer in multi-state systems. Scientists have taken a keen interest in the model of STIRAP due to its diverse applications in the fields of chemistry and physics. These applications include laser cooling [8], the manipulation of atoms and molecules in optical traps and the generation of ultracold atoms [9-13], Bose-Einstein condensates [14, 15], quantum computing, and superconductors [16-22].

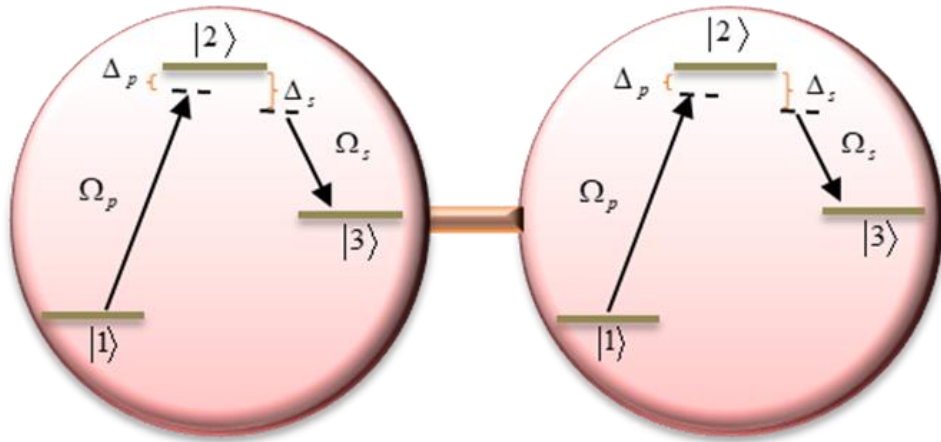
STIRAP is a three-level technique that transfers the population from the ground state to the target state without involving the intermediate state in population assignment [23, 24]. Compared to other techniques, STIRAP exhibits higher efficiency and is relatively less affected by noise [1].

Consideration of the interaction between particles is crucial for modeling experimental processes, such as artificial crystals, quantum computation, etc. [2]. STIRAP for the coupled  $\Lambda$ -type system is a simple and powerful method to investigate the interaction between pairs in composite nanostructures [25].

The investigation of the population transfer in the STIRAP model, considering the effects of system parameters, has already been conducted [7]. In this article, the population control of the quantum systems population is investigated within two coupled  $\Lambda$ -type quantum systems by employing the STIRAP technique. Here, the distinctions between coupled and uncoupled quantum systems are discussed with an emphasis on how their interactions affect population transfer.

## 2. THEORY

Fig. 1 shows the scheme of the population complete transition of the coupled  $\Lambda$ -type three-level systems from the ground state to the excited state, where the ground, intermediate, and excited states are denoted by  $|1\rangle$ ,  $|2\rangle$ , and  $|3\rangle$ , respectively. The states  $|1\rangle$  and  $|2\rangle$  undergo coupling through a field with pump Rabi frequency ( $\Omega_p$ ), whereas the states  $|2\rangle$  and  $|3\rangle$  are linked through a field with Stokes Rabi frequency ( $\Omega_s$ ) [23].



**Fig. 1.** Scheme for two coupled  $\Lambda$ -type systems.

Here, The Hamiltonian is calculated for the coupled  $\Lambda$ -type systems by employing Eq. (1).

$$H = H_1 \otimes I + I \otimes H_2 + JL_z \otimes L_z$$

$$H_1 = H_2 = \frac{\hbar}{2} \begin{bmatrix} 0 & \Omega_p & 0 \\ \Omega_p & 2\Delta_p & \Omega_s \\ 0 & \Omega_s & 2\delta \end{bmatrix} \quad (1)$$

In Eq. (1),  $H_1$  and  $H_2$  are the Hamiltonian for the  $\Lambda$ -type systems in a STIRAP system[7],  $J$  is the interaction of the two systems,  $L_z$  is applied to represent the angular momentum along the z-axis [2],  $\hbar$  is Planck's reduced constant, and  $I$  is the unit matrix.  $\Omega_p$  is pump Rabi frequency between states  $|1\rangle$  and  $|2\rangle$  and  $\Omega_s$  is the Stokes Rabi frequency between states  $|2\rangle$  and  $|3\rangle$ . For simplicity of calculations, we introduced the following parameters:

$$\text{“The potential was calculated by using (1),” or “Using (1), we calculated the } \Delta_p = \omega_{21} - \omega_p, \Delta_s = \omega_{23} - \omega_s, \omega_{21} = \omega_2 - \omega_1, \omega_{23} = \omega_2 - \omega_3, \delta = \Delta_p - \Delta_s \quad (2)$$

$\Delta_p$  and  $\Delta_s$  are called “single-photon detuning” and introduced by Eq. (2);  $\delta$  is the difference between  $\Delta_p$  and  $\Delta_s$  which is named “two-photon detuning”. In those relations,  $\omega_{21}$  and  $\omega_{23}$  are the differences between the Bohr transition frequencies,  $\omega_p$  and  $\omega_s$  are the pump and Stokes laser frequencies. Furthermore,

the pump and Stokes pulses are distributed as a Gaussian function based on Eq. (3) [26].

$$\Omega_{p,s}(t) = \Omega_{\max p,s} \exp\left[-\frac{(t-t_0)^2}{2\tau}\right] \quad (3)$$

Here,  $\Omega_{\max p,s}$  is the normalized maximum of the pulse,  $t_0$  is the time of the peak amplitude, and the  $\tau$  determines the pulse width.

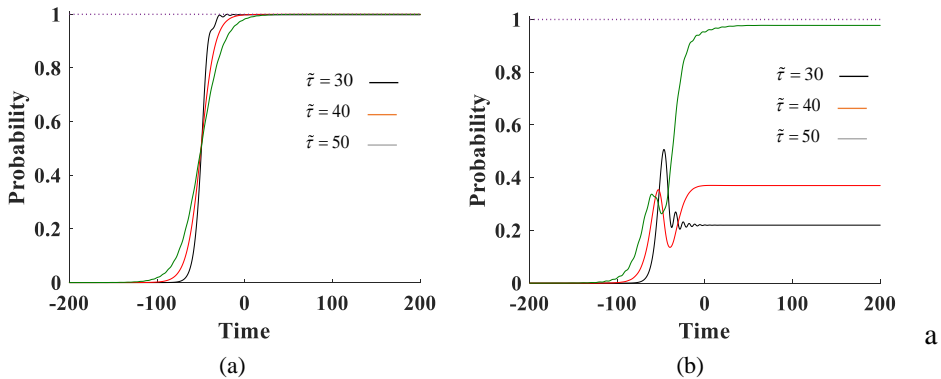
The probability of complete transitions from the ground state to the excited state in the quantum systems can be calculated by inserting the system's Hamiltonian into the time-dependent Schrödinger equation. As a result, the impact of interaction on the transition probability of coupled systems can be compared to that of uncoupled systems. To facilitate calculations and enhance understanding of equation solutions, the dimensionless system's unit is taken into account. Within this dimensionless system, the parameters are defined according to Eq. (4).

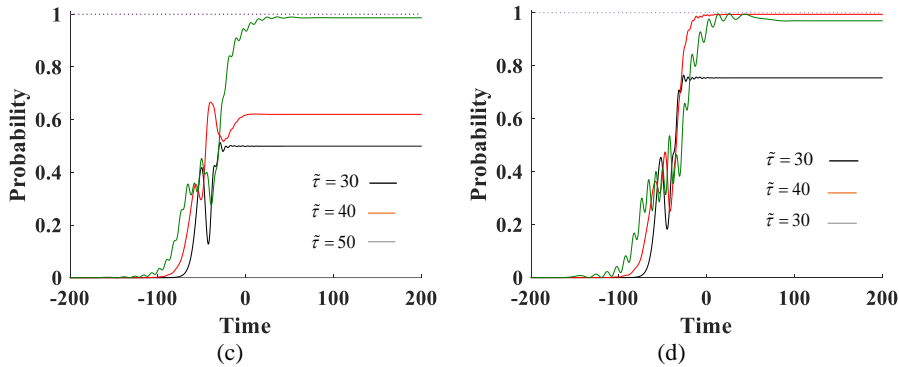
$$\tilde{\omega}_{23} = 1, \tilde{\omega}_{21} = \frac{\omega_{21}}{\omega_{23}}, \tilde{\Omega}_p = \frac{\Omega_p}{\omega_{23}}, \tilde{\Omega}_s = \frac{\Omega_s}{\omega_{23}}, \tilde{\omega}_p = \frac{\omega_p}{\omega_{23}}, \tilde{\omega}_s = \frac{\omega_s}{\omega_{23}}, \tilde{t} = t\omega_{23} \quad (4)$$

The time-dependent Schrödinger equation for the coupled system has been solved numerically. Here, it is assumed that the systems are in the ground states at distant times.

### 3. RESULTS AND DISCUSSION

Here, we have plotted the complete transition probability for the excited state in terms of different values of  $J$ ,  $t$ , and  $\tau$ .





**Fig. 2.** Probability of the population transition to the state  $|3\rangle$  for  $\tilde{t}_{0s} = -100, \Delta_{p,s} = 0, \Omega_{\max} = 1, \tilde{\tau} = 30$  (black line),  $\tilde{\tau} = 40$  (red line),  $\tilde{\tau} = 50$  (green line) (a)  $J=0$ , (b)  $J=0.1$ , (c)  $J=0.2$ , and (d)  $J=0.3$

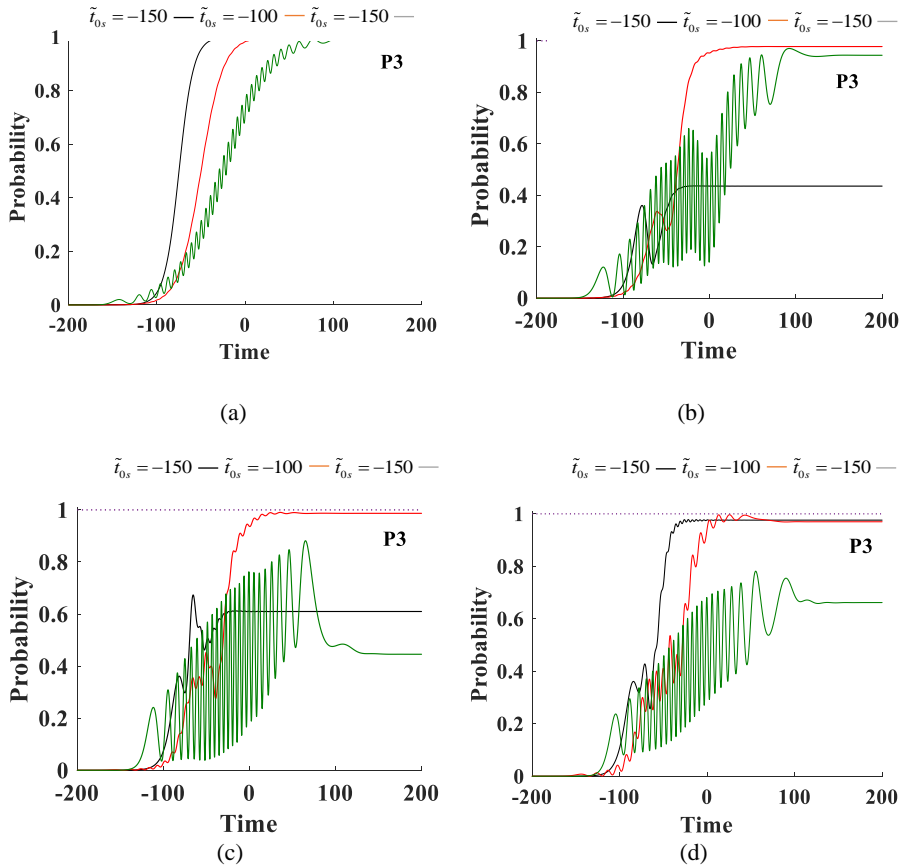
Fig. 2 shows the probability of the population transition to the state  $|3\rangle$  for  $\tilde{t}_{0s} = -100, \Delta_{p,s} = 0, \Omega_{\max} = 1, \tilde{\tau} = 30$  (black line),  $\tilde{\tau} = 40$  (red line),  $\tilde{\tau} = 50$  (green line).

The diagrams depict different values of 'J'. Fig. 2(a) is drawn for a system without interaction. As previously mentioned, introducing the interaction parameter to a coupled system results decrease in the probability of complete transition. This phenomenon is evident in the black and red lines displayed in Fig. 2(a) and Fig. 2(b). Surprisingly, even with the inclusion of system interactions,

complete transitions can still be observed in Fig. 2(b), Fig. 2(c), and Fig. 2(d) for  $\tilde{\tau} = 50$  (green line). This figure reveals that by increasing the pulse width, the transition time increases. As the amount of coupling increases, the probability of complete transmission and its stability decrease. This is while for pulse width 40, in addition to increasing the probability of transmission in higher coupling (0.3), its stability is also increased. However, the pulse width of 30 decreases both the probability and the stability.

Fig. 3 shows the probability of the population transition to the state  $|3\rangle$  for  $\tilde{\tau} = 50, \Delta_{p,s} = 0, \Omega_{\max} = 1, \tilde{t}_{0s} = -150$  (black line),  $\tilde{t}_{0s} = -100$  (red line),  $\tilde{t}_{0s} = -50$  (green line), (a)  $J=0$ , (b)  $J=0.1$ , (c)  $J=0.2$ , (d)  $J=0.3$ . The changes in the interaction and the variations in time  $t_{0s}$  are depicted in this figure. Similarly, the diagram effectively illustrates that the interaction parameter complicates the overall transition (represented by the black line). Within this graph, a complete

transition also occurs, which is more pronounced and distinct compared to the previous state (indicated by the red line). The increase in  $t_{0s}$  has led to an escalation in fluctuations (illustrated by the green line).



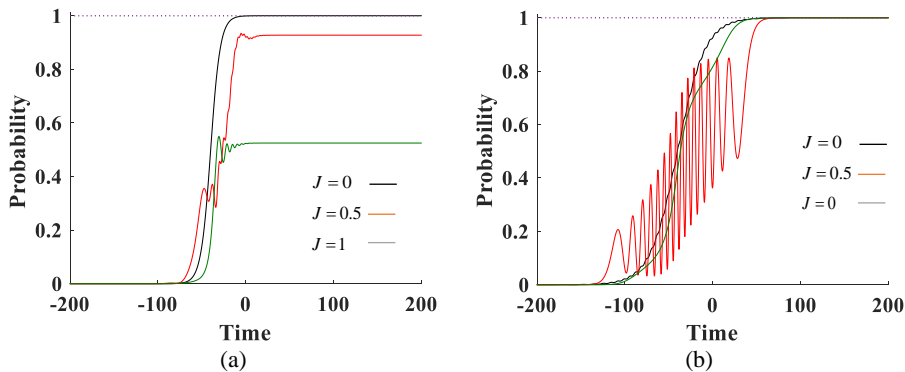
**Fig. 3.** Probability of the population transition to the state  $|3\rangle$  for  $\tilde{\tau} = 50, \Delta_{p,s} = 0, \Omega_{\max} = 1, \tilde{t}_{0s} = -150$  (black line),  $\tilde{t}_{0s} = -100$  (red line),  $\tilde{t}_{0s} = -50$  (green line) (a)  $J=0$ , (b)  $J=0.1$ , (c)  $J=0.2$ , (d)  $J=0.3$ .

In Fig. 3(a) with the increase of  $t_{0s}$ , although the population is completely transferred, its stability decreases. Fig. 3(b) illustrates that as the coupling increases; the stability decreases in three modes. The greatest instability is related to  $t_{0s} = -50$ . Although at  $t_{0s} = -50$  and  $-100$ , the entire population is transferred to the target level, but at lower  $t_{0s}$ , despite greater stability, the

probability of transition decreases. Fig. 3(c) By increasing coupling, the transition probability decreases to -50 and increases for -150. In Fig. 3(d), the probability of all three predicates increases and reaches full transition for  $t_{0,s} = -150$  and -100.

For all three graphs (b-d), stability decreases with increasing coupling, but the best stability is lower  $t_{0,s}$  (-150). It should be noted that in different coupling (a-d) at  $t_{0,s}$  equal to -100, complete population transition occurs, which shows that by choosing the optimal  $t_{0,s}$ , complete transition can be achieved even by increasing the coupling.

Fig. 4 displays the probability of the population transition to the state  $|3\rangle$  for,  $J=0$  (black line),  $J=0.5$  (red line),  $J=1$  (green line), (a)  $\tilde{\tau} = 30$  and, (b)  $\tilde{\tau} = 60$ . This figure illustrates the impact of  $J$  on the probability of a complete transition. In Fig. 4(a), as  $J$  increases, there is a decrease in the likelihood of a complete transition, As predicted and expected, it will decrease. However, Fig. 4(b) demonstrates that even in a coupled system with an increased  $J$  value, it is still possible to achieve a complete transition.



**Fig. 4.** Probability of the population transition to the state  $|3\rangle$  for  $\tilde{t}_{0,s} = -80, \Delta_{p,s} = 0, \Omega_{\max} = 1, J=0$  (black line),  $J=0.5$  (red line),  $J=1$  (green line), (a)  $\tilde{\tau} = 30$ , (b)  $\tilde{\tau} = 50$ .

#### 4. CONCLUSION

Our investigation into population transfer in the two coupled  $\Lambda$ -type three-level systems based on STIRAP has yielded significant insights. By carefully controlling the characteristics, including the time of the peak amplitude, and determining the pulse width. You can see the optimal conditions to achieve the

probability of a complete transition. In addition, we investigated the effect of interaction in two  $\Lambda$ -type coupled systems on the probability of population transfer using the STIRAP method. In general, the probability of transmission decreases when the interaction of the system increases, sometimes this probability approaches zero. However, in certain scenarios, by adjusting the parameters time of the peak amplitude, and determining the pulse width and despite the increase in the interaction parameter, the probability of population transfer increases, which is an attractive phenomenon in its way. This research not only deepens our understanding of STIRAP mechanisms but also provides valuable knowledge for the advancement of quantum control techniques. Going forward, further exploration and experimentation in this field can lead to the development of novel applications in quantum information processing, quantum computing, and quantum communication.

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