

Temporal Nonlinear Dynamics of Plasmon-Solitons, a Duffing Oscillator-Based Approach

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Abstract: Plasmon-Solitons are quasi-particles resulting from coupling the plasmon modes and solitary solutions. This coupling can be intrinsically resonant in order to form plasmon-solitons with a high localization and large propagation length. This paper deals with the temporal nonlinear dynamics of plasmon-solitons in a plasmonic waveguide. Duffing equation is recognized as the temporal part of the nonlinear amplitude equation governing the plasmonic waveguide. Duffing equation is analytically solved for the low nonlinearity regime. It is shown that the Duffing oscillator waveforms stand for the temporal nonlinear dynamics of plasmon-solitons. The energy exchange of Lorentz-type bright and dark modes gives rise to a Fano resonance. It is thus shown that the interaction of solitons and the formation of plasmon-solitons is inherently nonlinear. It is accordingly indicated that the nonlinear modulation of the plasmon-solitons is achievable via tuning the nonlinearity of the plasmonic waveguide. The results can be appealing for the researchers intending to design the plasmonic waveguides with the high localization as well as the large propagation length. In particular, an all-plasmonic modulation method can be contemplated.

Keywords: Plasmon-Soliton, Duffing Oscillator, Nonlinear Optical Modulation, Temporal Dynamics, Fano Resonance

1. INTRODUCTION

Duffing equation is a mathematical tool to model a nonlinear dynamical system with damped and driven oscillations. Although it has been generally used for the mechanical systems, a variety of studies has addressed Duffing equation role in the nonlinear optical systems. K. Senthilnathan, et al developed an inharmonic oscillator equation for bright solitons in photorefractive media. Then, they showed that the equation can be reduced to undamped/unforced Duffing equation [1]. E Babourina-Brooks, et al. determined the noise response

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of a quantum nanomechanical resonator using a Duffing oscillator-based model due to the dynamical equivalency [2]. Heung-Ryoul Noh indicated that the Duffing oscillator can be realized in an intensity-modulated magneto-optical trap [3].

On the other hand, Surface Plasmons (SPs) are intrinsically electrons that oscillate at an interface (between two materials) or a surface (like a quantum dot surface or a 2D material). Their physical properties are closer to the bosons implying that SPs can intensely be localized beyond the diffraction limit of the light [4]. This is essential for the next generation nanophotonic devices [5-16]. Plasmonic waveguides have been investigated and developed aiming to increasing the propagation length and keeping the strong localization [15-19]. An exact study of SPs dynamics in a waveguide requires considering the effect of nonlinearity. Sergey Mikhailov showed that the influence of nonlinearity on plasmon resonances are the broadening and reducing the propagation length at special frequencies [20]. Considering the Kerr nonlinearity, Wiktor Walasik, et al. modeled the field profiles of plasmon-solitons in planar slot waveguide using the Jacobi elliptic special functions [21]. Jacobi elliptic functions are indeed the stationary solutions of the Duffing equation [22]. M. Scalora, et al. employed a Duffing oscillator-based model to investigate the third harmonic generation in metal nanostructures [23]. We have recently proposed Duffing oscillator model for delineation of dynamical states and chaotic maps in a surface plasmon laser [24]. As well, we found that unstable solutions and breather modes can be resulted for a plasmonic waveguide in consequence of the large nonlinear response [25].

Optical modulation is used in optical communication [26], optical computing and signal processing [27], sensing [28], pulse generation [29], cryptography [30] and integrated photonic circuits [31]. Meanwhile, nonlinear optical modulation has been recently contemplated for specific purposes including the tunable optical switching [32-36]. As well, SPs modulation is a novel study field which still needs further investigation due to the large nonlinear optical response appeared in nanostructure SPs' waveguides.

In this article, Duffing equation is re-considered to derive temporal stationary solutions in a plasmonic waveguide. However, only stable solutions obtained for the low nonlinearity regime are to be explored. It is shown that the nonlinearity is responsible for a Self-Amplitude Modulation (SAM) effect. It is also indicated that Lorentz-type soliton as well as Fano resonance can be obtained as the solitary solution if the absorption and nonlinear coefficients with complex values are proportionally tuned. The ratio of the nonlinear to absorption coefficient is recognized as a determining factor for modulating the temporal dynamics of the plasmon-solitons.

2. THEORY

The nonlinear equation to describe the spatiotemporal SPs' amplitude Ψ in a plasmonic waveguide is given by Eq.(1)[37].

$$i \left(\frac{\partial \Psi}{\partial z} + D^{-1} \frac{\partial \Psi}{\partial t} \right) - \frac{k'}{2} \frac{\partial^2 \Psi}{\partial t^2} + \gamma |\Psi|^2 \Psi + i \eta \Psi = \delta \cos(\omega_d t), \quad (1)$$

where γ is the nonlinear coefficient; η is the linear absorption coefficient which characterizes the smoothness of oscillation; δ and ω_d are respectively, the amplitude and angular frequency of the driving force.

$D^{-1} = (dk/d\omega)_{\omega=\omega_0} = (v_g)^{-1}_{\omega=\omega_0}$; k is the wavenumber; ω is the angular frequency; $v_g = \frac{d\omega}{dk}$ is the group velocity; $k'' = (d^2k/d\omega^2)_{\omega=\omega_0}$. Assuming

$\delta = 0$, Eq.(1) can be separable and the spatiotemporal amplitude can be written as $\Psi(z, t) = Z(z)\lambda(t)$. In order to obtain $A(t)$, one should first assume the temporal part of Eq.(1) as in the form of Eq.(2).

$$\frac{k''}{2} \frac{d^2 A}{dt^2} + \gamma |A|^2 A + i \eta A = 0, \quad (2)$$

which can be further reduced to the form given in Eq.(3).

$$\frac{d^2 A}{dt^2} + \gamma' |A|^2 A + \eta' A = 0, \quad (3)$$

where it has been assumed that $\gamma' = 2\gamma/k''$ and $\eta' = -2i\eta/k''$. Considering A as an observable wave function, time-energy uncertainty principal will be written as Eq.(4). From the viewpoint of quantum theory, representation of A vs. the time also implicates the system energy eigenvalues explainable by Eq.(4) [38].

$$\sigma_E \frac{\sigma_A}{d\langle A \rangle} \geq \frac{\hbar}{2}, \quad (4)$$

where σ_E and σ_A are respectively, the energy and wave function eigenvalues; $\langle A \rangle$ is the expectation value and \hbar is the reduced Planck's constant. Analytical solution of Eq.(3) can be given by Eq.(5) [5].

$$A(t) = c_1 \operatorname{cn} \left((\eta' + c_1^2 \gamma')^{\frac{1}{2}} t + c_2, \left(\frac{c_1^2 \gamma'}{2(\eta' + c_1^2 \gamma')} \right)^{\frac{1}{2}} \right), \quad (5)$$

where cn , sn and dn are the Jacobi elliptic functions and c_1 and c_2 are to be obtained from the initial conditions given in Eq.(6) [5].

$$\begin{cases} c_1 \operatorname{cn}\left(c_2, \sqrt{\frac{c_1^2 \gamma'}{2(\eta' + c_1^2 \gamma')}}\right) = A_0 \\ -\sqrt{\eta' + c_1^2 \gamma'} \operatorname{sn}\left(c_2, \sqrt{\frac{c_1^2 \gamma'}{2(\eta' + c_1^2 \gamma')}}\right) \operatorname{dn}\left(c_2, \sqrt{\frac{c_1^2 \gamma'}{2(\eta' + c_1^2 \gamma')}}\right) = A_{z_0} \end{cases}, \quad (6)$$

where A_0 and A_{z_0} are the initial and boundary values of the wave amplitude A at $z = 0$ and $z = z_0$. Answers given in Eq.(5) are stationary solutions prior to unstable states. Transition to instability is beyond this study and has been investigated in our previous study [7]. If $\gamma' = -2\eta' / A_0^2$, solution of Eq.(5) will be reduced to Eq.(7)[5].

$$A(t) = A_0 \operatorname{sech}\left(\sqrt{\eta'} t\right). \quad (7)$$

Solitary solution given in Eq.(7) stands for negative values of γ' which is thus fulfilled for the real values of $\sqrt{\eta'}$. If otherwise, $\sqrt{\eta'}$ is generally a complex parameter, the solution of Eq.(7) will be accordingly modified.

3. SIMULATION RESULTS AND DISCUSSION

A. Ansatz I. Real positive absorption coefficient

Fig.1 shows the solitary solution given in Eq.(7) for different values of the nonlinear coefficient γ' . As the latter increases, the soliton width decreases.

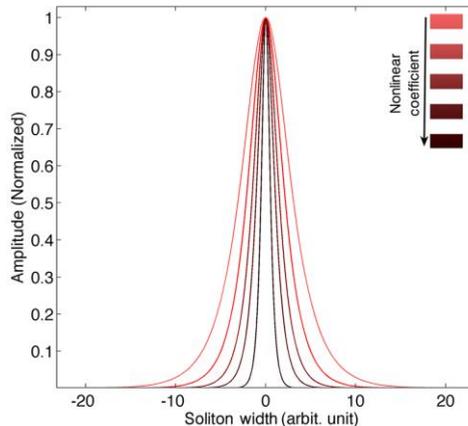


Fig 1. soliton width for different values of nonlinear coefficient

However, solution of Eq.(7) is insufficient to deduce a generalized result. Once introducing the parameter γ' , the effects of carrier wave frequency and dispersion will be latent. A proper investigation can be instead carried out by

introducing the independent parameter γ' / η' . General solutions of Eq.(6) are very sensitive to the ratio of the nonlinear coefficient to absorption coefficient γ' / η' . Oscillatory behavior in Fig.2 unveils a SAM. This modulation feature is to be broadened for the lower nonlinear coefficients as the nonlinear coefficient increases (Fig.2(a)) and vice versa (Fig.2(b)). The broadening also means increasing the number of pulses per wave envelop. This result may also explain the broadening of the plasmonic waves for special frequencies when the nonlinearity increases [24]. In better word, one can deduce that plasmon-solitons influenced by the nonlinearity may show both the widening and contracting depending on the plasmonic waveguide absorption coefficient.

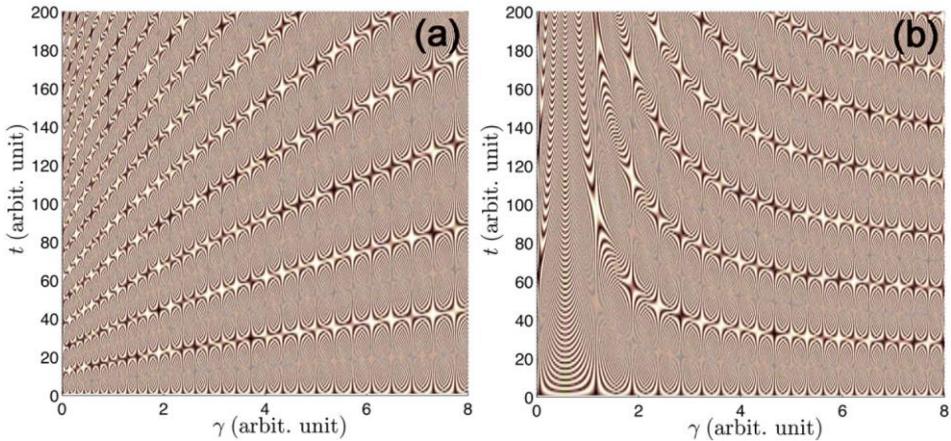


Fig 2. Oscillatory Duffing waveforms for different values of nonlinear coefficient γ when (a) $\gamma/\eta \sim 10^{-8}$; (b) $\gamma/\eta \sim 10^{-5}$.

Time evolution of the Duffing oscillator is depicted in Fig. 3 for different values of γ' / η' . The amplitude evolves from the single periodic state (Fig. 3(a)) to quasi-period state (Fig.3(b), (c) & (d)). The phase and group velocities are interpreted to be identical in Fig.3(a). Then, a difference in phase and group velocities leads to SAM as shown in Fig.3(b), (c) & (d). Modulation depth raises up to 0.89 for Fig.3(c) and 0.97 for Fig.3(d) while the modulation rate significantly increases. This triggers an idea for all-plamonic modulation, tunable with the nonlinearity and absorption. The presented theory in this study can truly support the corresponding experimental observations [41].

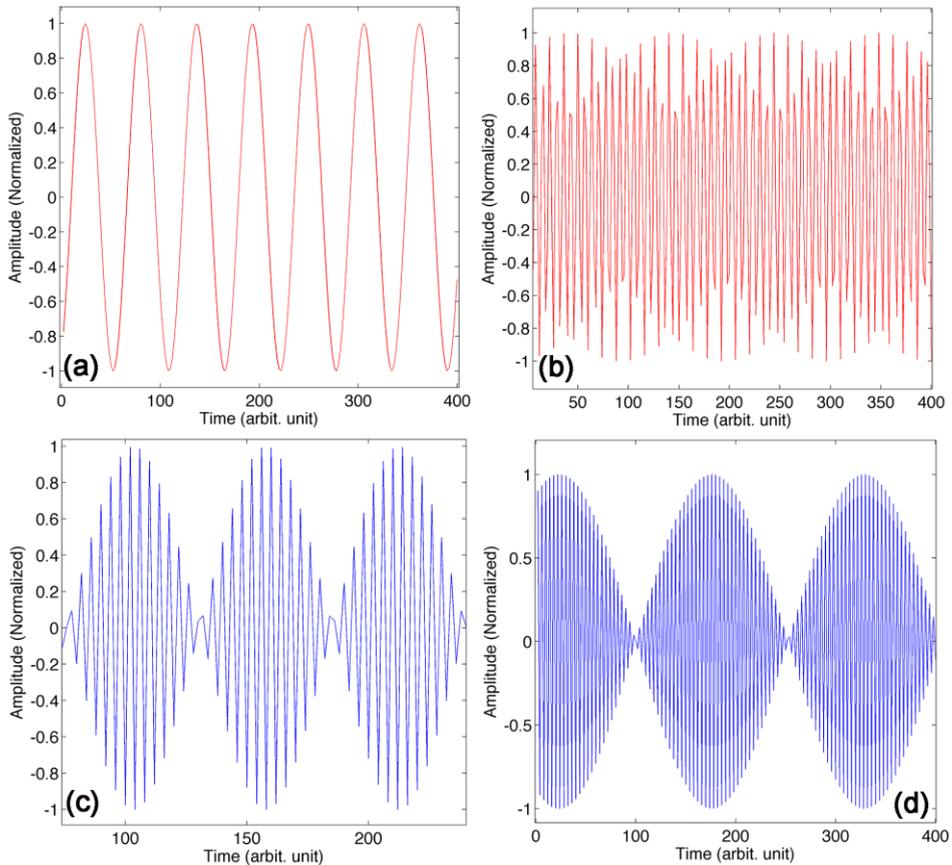


Fig 3. Time evolution of Duffing waveforms for (a) $\gamma'_1/\eta'_1=1.675\times 10^{-6}$; (b) $\gamma'_2/\eta'_2=3.75\times 10^{-5}$, $\gamma'_2=0.22 \gamma'_1$, $\eta'_2=1/100 \eta'_1$; (c) $\gamma'_3/\eta'_3=7.5\times 10^{-6}$, $\gamma'_3=1/5\gamma'_2$, $\eta'_3= \eta'_2$ and (d) $\gamma'_4/\eta'_4=7.5\times 10^{-8}$, $\gamma'_4=1/5\gamma'_3$, $\eta'_4= \eta'_3$.

B. Ansatz II. Complex absorption coefficient, plasmon-soliton modes

For a complex absorption coefficient, if $\gamma' = -2\eta'/A_0^2$, a solitary shape can be obtained as depicted in Fig.4 for different values of the nonlinear coefficient γ and initial amplitude A_0 . An important issue is the formation of flat-top plasmon-solitons in comparison with Fig.1. The larger nonlinear coefficients as well as the lower initial amplitudes (Fig.4(a) compared to Fig.(b)) still lead to decreasing the soliton width.

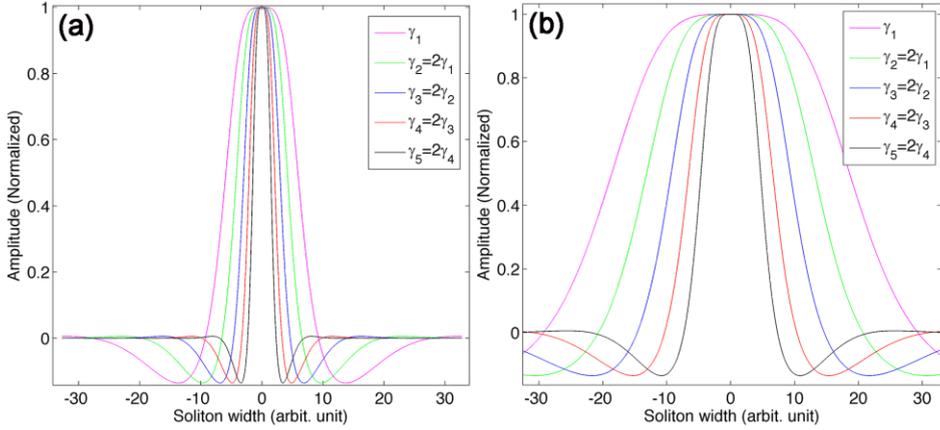


Fig 4. (a) Soliton width for different values of nonlinear coefficient, (b) the same for 10 times higher initial amplitude A_0 .

Furthermore, temporal dynamics obtained for some assumed values for γ' / η' are shown in Fig.5 which are in accordance with the modified form of the general solutions in Eq. (6) provided for the complex values considered for the absorption coefficient. The results include the Lorentz-type solitons [40] (Fig.5(b) & Fig.5(d)), Fano resonance [42-44] (Fig.5(c)) and a combination of both (Fig.5(a) & Fig.5(d)).

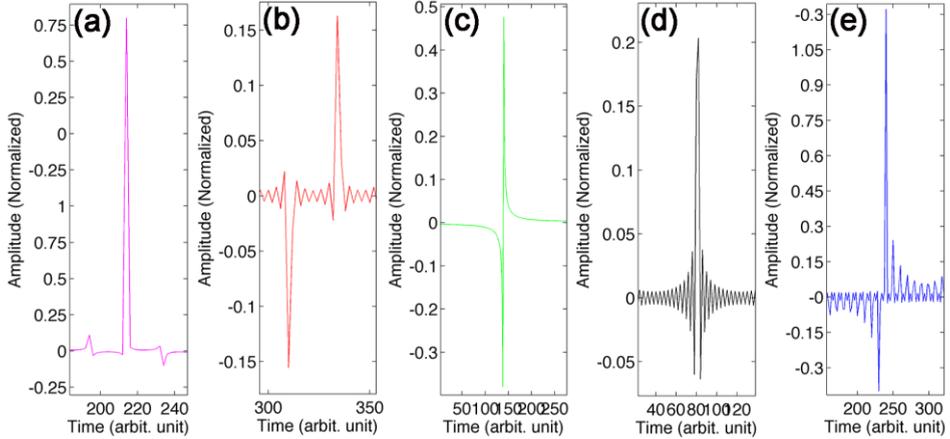


Fig 5. Time evolution of Duffing waveforms for (a) $\gamma'_1 / \eta'_1 = 0.6 \times 10^{-4}$; (b) $\gamma'_2 / \eta'_2 = 2.5 \times 10^{-7}$, $\gamma'_2 = 0.01 \gamma'_1$, $\eta'_2 = 10 \eta'_1$; (c) $\gamma'_3 / \eta'_3 = 5 \times 10^{-6}$, $\gamma'_3 = 2\gamma'_2$, $\eta'_3 = 0.1 \eta'_2$, (d) $\gamma'_4 / \eta'_4 = 1 \times 10^{-6}$, $\gamma'_4 = 2\gamma'_3$, $\eta'_4 = \eta'_3$ and (e) $\gamma'_5 / \eta'_5 = 8 \times 10^{-5}$, $\gamma'_5 = 8\gamma'_4$, $\eta'_5 = 0.1 \eta'_4$.

Finally, the spatiotemporal solutions of Eq.(1) can be obtained in accordance with the wave function $\Psi = A(t)Z(z)$ if one considers that the spatial part to be conventionally given by Eq.(8)[37].

$$Z(z) \propto \begin{cases} e^{-X_{\perp}} & X_{\perp} > 0 \\ e^{X_{\perp}} & X_{\perp} < 0 \end{cases}, \quad (8)$$

where X_{\perp} is the localization coordinate perpendicular to the propagation direction[5]. Some spatiotemporal plasmon-soliton modes are shown in Fig.6 for different values assumed for γ' / η' .

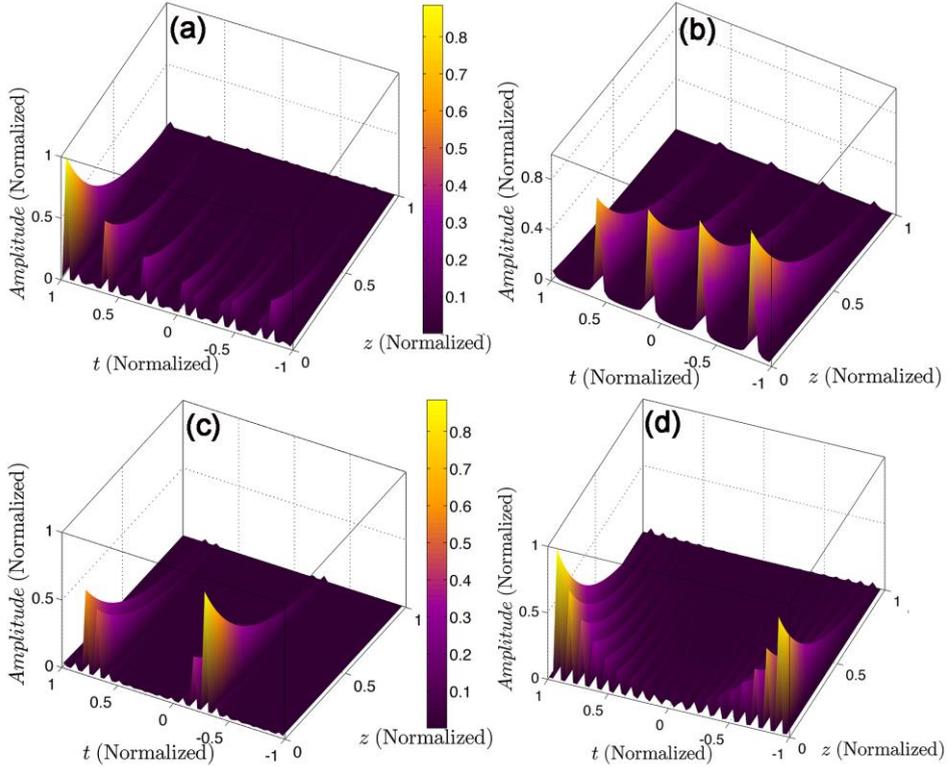


Fig 6. Spatiotemporal evolution of plasmon-soliton modes for (a) $\gamma'1 / \eta'1 = 0.8 \times 10^{-4}$; (b) $\gamma'2 / \eta'2 = 0.9 \times 10^{-3}$, $\gamma'2 = 11.25 \gamma'1$, $\eta'2 = \eta'1$; (c) $\gamma'3 / \eta'3 = 1.4 \times 10^{-4}$, $\gamma'3 = 11 \gamma'2$, $\eta'3 = 7.6 \times 10^3 \eta'2$; (d) $\gamma'4 / \eta'4 = 8 \times 10^{-7}$, $\gamma'4 = 8 \times 10^2 \gamma'3$, $\eta'4 = 1.3 \times 10^5 \eta'3$.

C. Considering actual parameters

In real systems (like the plasmonic waveguides), the values of the nonlinear and absorption coefficients read the relations $\gamma \propto \chi^{(3)}$ and $\eta \propto \chi^{(1)}$ where $\chi^{(3)}$ and $\chi^{(1)}$ are the third order and linear susceptibility of the system structure respectively [45]. If $\chi^{(1)}$ is reduced to unity, one can write $\chi^{(3)} \simeq (4\pi\epsilon_0)^6 \hbar^8 / m^4 e^{10}$ [44]. Even so, one may notice that the ratio of γ' / η'

can be varied depending on the characteristics of the waveguide such as the working wavelength, number of 2D layers, Fermi energy, driving voltage/light intensity, etc. A variety of studies has shown the Fano resonance and Lorentz-type solitons as prevalent dynamical regime of the plasmonic waveguide in the time domain [41-43]. In corroboration, the results obtained for Ansatz II can be truly assigned to the temporal nonlinear dynamics of the plasmon-solitons. Energy exchange between the dark and bright modes causes a Fano resonance. This in turn, reveals that the plasmon-solitons entity is intrinsically nonlinear. The resonant interaction of SPs as the spatial localized modes and solitary waves as the temporal wave envelopes can be controlled by tuning the nonlinearity and absorption. While the large absorption implies the more localization, the strong nonlinearity can convey an amplification, beneficial for generating the long range SPs.

The results also suggest that the analytical solutions of Duffing equation (Eq.(3)) can describe the stationary states in a plasmonic waveguide with a simple but comprehensive manner.

4. CONCLUSION

Solitary solutions of the temporal nonlinear dynamics in a plasmonic waveguide has been derived using the analytical solution of the Duffing equation as the temporal part of the spatiotemporal nonlinear amplitude equation. It has been proposed that the Duffing oscillator waveforms obtained for the complex values of the absorption and nonlinear coefficients stand for the temporal nonlinear dynamics of plasmon-solitons. It has been shown that the ratio of the nonlinear to absorption coefficient is an effective factor for determining the plasmon-soliton modes. Accordingly, the nonlinear modulation of plasmon-solitons can be achieved by tuning the nonlinearity and absorption. The results are thus useful for effective design process of the plasmonic waveguides. In particular, this paper suggests an all-plasmonic modulation method to attain highly localized SPs with long propagation range. The simple quasi-classical model presented in this study is simple yet mathematically rigorous. The results are in a good agreement with those of previously proposed models based on the quantum theory.

REFERENCES

- [1] Senthilnathan K, Porsezian K. *Bright and dark spatial solitons in coupled photorefractive media*. J.Mod. Opt. [Online] 51(3). (2004 Feb. 14) 15-21. Available: <https://www.tandfonline.com/doi/abs/10.1080/09500340408235533>
- [2] Babourina-Brooks E, Doherty A, Milburn GJ. *Quantum noise in a nanomechanical duffing resonator*. New. J. Phys. [Online] 10(10). (2008 Oct 31)105020. Available: <https://iopscience.iop.org/article/10.1088/1367-2630/10/10/105020/meta>
- [3] Moon G, Jhe W, Noh HR. *Duffing Oscillation in an Intensity-modulated Magneto-optical Trap: An Analytical Study for the (1+ 3) Atomic Energy Structure*. J. Korean Phys. Soc. [Online] 58(5). (2011 May 13) 1105-1109. Available: <https://www.jkps.or.kr/journal/view.html?doi=10.3938/jkps.58.1105>
- [4] Barnes WL, Dereux A, Ebbesen TW. *Surface plasmon subwavelength optics*. Nature. [Online] 424(6950). (2003 Aug) 824-830. Available: <https://www.nature.com/articles/nature01937>
- [5] Joly AG, Gong Y, El-Khoury PZ, Hess WP. *Surface plasmon-based pulse splitter and polarization multiplexer*. J. Phys. Chem. Lett. [Online] 9(21). (2018 Oct 11) 6164-61698. Available: <https://pubs.acs.org/doi/abs/10.1021/acs.jpcclett.8b02643>
- [6] Singh SP, Tiwari NK, Akhtar MJ. *Spoof Surface Plasmon-Based Coplanar Waveguide Sensor for Dielectric Sensing Applications*. IEEE Sens. J. [Online] 20(1). (2019 Sep 23) 193-201. Available: <https://ieeexplore.ieee.org/abstract/document/8846053>
- [7] Selvendran S, Susheel A, Tarun PV, Muthu KE, Raja AS. *A novel surface plasmon based photonic crystal fiber sensor*. Opt Quantum Electron. [Online] 52. (2020 Jun) 1-2. Available: <https://link.springer.com/article/10.1007%2Fs11082-020-02403-8>
- [8] Zhang Y, Han Z. *Experimental demonstration of spoof surface plasmon based THz antennas for huge electric field enhancement*. Plasmonics. [Online] (2) (.2018 Apr;13) 531-535. Available: <https://link.springer.com/article/10.1007/s11468-017-0540-2>
- [9] Xu L, Li F, Wei L, Zhou J, Liu S. *Design of surface plasmon nanolaser based on MoS2*. Appl. Sc. [Online] 8(11) (2018 Nov) 2110. Available: <https://www.mdpi.com/2076-3417/8/11/2110>

- [10] Mansuri M, Mir A, Farmani A. *Numerical modeling of a nanostructure gas sensor based on plasmonic effect*. Journal of Optoelectrical Nanostructures. 4(2) [Online] (2019 May 1) 29-44. Available: http://jopn.miau.ac.ir/article_3476_479.
- [11] Lee DH, Lee MH. *Efficient experimental design of a long-range gapped surface plasmon polariton waveguide for plasmonic modulation applications*. IEEE Photonics J. [Online] 11(1) (2019 Jan) 241-250. Available: <https://ieeexplore.ieee.org/abstract/document/8625489>
- [12] Abdolazhadeh Ziabari A, Royanian S, Yousefi R, Ghoreishi S. *Performance Improvement of Ultrathin CIGS Solar Cells Using Al Plasmonic Nanoparticles: The Effect of the Position of Nanoparticles*. Journal of Optoelectrical Nanostructures. [Online] 5(4) (2020 Nov 1) 17-32. Available: http://jopn.miau.ac.ir/article_4506.html
- [13] Amin R, Maiti R, Gui Y, Suer C, Miscuglio M, Heidari E, Khurgin JB, Chen RT, Dalir H, Sorger VJ. *Heterogeneously integrated ITO plasmonic Mach-Zehnder interferometric modulator on SOI*. Sci. Rep. [Online] 11(1). (2021 Jan 14) 1-2. Available: <https://www.nature.com/articles/s41598-020-80381-3>
- [14] Surface plasmon amplification by stimulated emission of radiation (spaser). Stockman MI, Bergman DJ, inventors; Ramot. (2009 Aug 4.) United States patent US 7,569,188. [Online] Available: <https://patents.google.com/patent/US7569188B2/en>
- [15] Klein M, Badada BH, Binder R, Alfrey A, McKie M, Koehler MR, Mandrus DG, Taniguchi T, Watanabe K, LeRoy BJ, Schaibley JR. *2D semiconductor nonlinear plasmonic modulators*. Nat. Commun. [Online] 10(1) (2019 Jul 22) 1-7. Available: <https://www.nature.com/articles/s41467-019-11186-w>
- [16] Yuan D, Wang J, Li Y, Ding P. *A multi-wavelength SPASER based on plasmonic tetramer cavity*. J. Opt. [Online] 21(11) (2019 Oct 7) 115001. Available: <https://iopscience.iop.org/article/10.1088/2040-8986/ab46d5/meta>
- [17] Berini P. *Highlighting recent progress in long-range surface plasmon polaritons: guest editorial*. Adv. Opt. Photonics. [Online] 11(2) (2019 Jun 30) ED19-23. Available: <https://www.osapublishing.org/aop/abstract.cfm?uri=aop-11-2-ed19>
- [18] Jing JY, Wang Q, Zhao WM, Wang BT. *Long-range surface plasmon resonance and its sensing applications: A review*. Opt. Lasers Eng. [Online] 112. (2019 Jan 1) 103-118. Available:

<https://www.sciencedirect.com/science/article/abs/pii/S0143816618306365>

- [19] Abdulhalim I. *Coupling configurations between extended surface electromagnetic waves and localized surface plasmons for ultrahigh field enhancement*. Nanophotonics. [Online] 7(12). (2018 Nov 26) 1891-1916. Available: <https://www.degruyter.com/document/doi/10.1515/nanoph-2018-0129/html>
- [20] Mikhailov SA. *Influence of optical nonlinearities on plasma waves in graphene*. ACS Photonics. [Online] 4(12). (2017 Dec 20) 3018-3022. Available: <https://pubs.acs.org/doi/abs/10.1021/acsp Photonics.7b00468>
- [21] Walasik W, Rensversez G. *Plasmon-soliton waves in planar slot waveguides. I. Modeling*. Phys. Rev. A. [Online] 93(1). (2016 Jan 14) 013825. Available: <https://journals.aps.org/pr/abstract/10.1103/PhysRevA.93.013825>
- [22] Salas AH. *Exact solution to duffing equation and the pendulum equation*. Appl. Math. Sci. 8(176). (2014) 8781-8789. Available: <https://hal.archives-ouvertes.fr/hal-01356787/>
- [23] Scalora M, Vincenti MA, De Ceglia D, Cojocararu CM, Grande M, Haus JW. *Nonlinear Duffing oscillator model for third harmonic generation*. Opt Soc Am B. [Online] 32(10). (2015 Oct 1) 2129-2138. Available: <https://www.osapublishing.org/josab/abstract.cfm?uri=josab-32-10-2129>
- [24] Sharif MA, Ashabi K. *A Quasi-classical Model for Delineation of Dynamical States and Chaotic Maps in a Spaser*. Plasmonics. [Online] 16. (2021) 97-105. Available: <https://link.springer.com/article/10.1007/s11468-020-01269-6>
- [25] Sharif MA. *Spatio-temporal modulation instability of surface plasmon polaritons in graphene-dielectric heterostructure*. Physica E Low Dimens. Syst. Nanostruct. [Online] 105. (2019 Jan 1) 174-181. Available: <https://www.sciencedirect.com/science/article/abs/pii/S1386947718304818>
- [26] Doi M, Sugiyama M, Tanaka K, Kawai M. *Advanced LiNbO₃/optical modulators for broadband optical communications*. IEEE J. Sel. Top. Quantum Electron. [Online] 12(4). (2006 Aug 7) 745-750. Available: <https://ieeexplore.ieee.org/abstract/document/1668118/>
- [27] Interferometric modulator for optical signal processing. Bulow JA. (1994 May 24.) United States patent US 5,315,370. [Online]. Available: <https://patents.google.com/patent/US5315370A/en>

- [28] Moftakharzadeh A, Afkhami Aghda B, Hosseini M. *Noise Equivalent Power Optimization of Graphene-Superconductor Optical Sensors in the Current Bias Mode*. Journal of Optoelectrical Nanostructures. [Online] 3(3) (2018 Sep 1) 1-2. Available: http://jopn.miau.ac.ir/article_3040.html
- [29] Zhang Y, Yu H, Zhang R, Zhao G, Zhang H, Chen Y, Mei L, Tonelli M, Wang J. *Broadband atomic-layer MoS₂ optical modulators for ultrafast pulse generations in the visible range*. Opt.Lett. [Online] 42(3). (2017 Feb 1) 547-550.
Available: <https://www.osapublishing.org/ol/abstract.cfm?uri=ol-42-3-547>
- [30] Merolla JM, Mazurenko Y, Goedgebuer JP, Porte H, Rhodes WT. *Phase-modulation transmission system for quantum cryptography*. Opt. Lett. [Online] 24(2). (1999 Jan 15) 104-6. Available: <https://www.osapublishing.org/ol/abstract.cfm?uri=ol-24-2-104>
- [31] Urino Y, Noguchi Y, Noguchi M, Imai M, Yamagishi M, Saitou S, Hirayama N, Takahashi M, Takahashi H, Saito E, Okano M. *Demonstration of 12.5-Gbps optical interconnects integrated with lasers, optical splitters, optical modulators and photodetectors on a single silicon substrate*. Opt. Express [Online] 20(26). (2012 Dec 10) B256-B263. Available: <https://www.osapublishing.org/oe/fulltext.cfm?uri=oe-20-26-B256&id=246439>
- [32] Jabbari M, Dehghan M, Darvish G, Ghaffari-miab M. *Ultra-Compact Bidirectional Terahertz Switch Based on Resonance in Graphene Ring and Plate*. Journal of Optoelectrical Nanostructures. [Online] 4(4) (2019 Dec 1) 99-112. Available: http://jopn.miau.ac.ir/article_3761.html
- [33] Grinblat G, Zhang H, Nielsen MP, Krivitsky L, Berté R, Li Y, Tilmann B, Cortés E, Oulton RF, Kuznetsov AI, Maier SA. *Efficient ultrafast all-optical modulation in a nonlinear crystalline gallium phosphide nanodisk at the anapole excitation*. Sci. Adv.[Online] 6(34). (2020 Aug 1) eabb3123. Available: <https://advances.sciencemag.org/content/6/34/eabb3123.abstract>
- [34] Firby CJ, Elezzabi AY. *Nonlinear optical modulation in a plasmonic Bi:YIG Mach-Zehnder interferometer*, in Integrated Optics: Devices, Materials, and Technologies XXI International Society for Optics and Photonics. (2017 Feb 16) 1010617. Available: <https://www.spiedigitallibrary.org/conference-proceedings-of-spie/10106/1010617/Nonlinear-optical-modulation-in-a-plasmonic-Bi-YIG-Mach-Zehnder/10.1117/12.2252840.short>

- [35] Jia Y, Liao Y, Wu L, Shan Y, Dai X, Cai H, Xiang Y, Fan D. *Nonlinear optical response, all optical switching, and all optical information conversion in NbSe₂ nanosheets based on spatial self-phase modulation*. *Nanoscale*. [Online] (10). (2019;11) 4515-4522. Available: <https://pubs.rsc.org/en/content/articlelanding/2019/nr/c8nr08966c/unauth#!divAbstract>
- [36] Xie ZT, Wu J, Fu HY, Li Q. *Tunable Electro-and All-Optical Switch Based on Epsilon-Near-Zero Metasurface*. *IEEE Photonics J.* [Online] 12(4). (2020 Jul 20) 1-10. Available: <https://ieeexplore.ieee.org/abstract/document/9144401/>
- [37] Gorbach AV. *Nonlinear graphene plasmonics: amplitude equation for surface plasmons*. *Phys. Rev. A.* [Online] 87(1). (2013 Jan 25) 013830. Available: <https://journals.aps.org/pr/abstract/10.1103/PhysRevA.87.013830>
- [38] Hilgevoord J. *The uncertainty principle for energy and time*. *Am. J. Phys.* [Online] 64(12). (1996 Dec) 1451-1456. Available: <https://aapt.scitation.org/doi/abs/10.1119/1.18410>
- [39] Goyal A, Raju TS, Kumar CN. *Lorentzian-type soliton solutions of ac-driven complex Ginzburg–Landau equation*. *Appl. Math. Comput.* [Online] 218(24). (2012 Aug 15) 11931-11927. Available: <https://www.sciencedirect.com/science/article/abs/pii/S0096300312005796>
- [40] Golovinski PA, Yakovets AV, Astapenko VA. *Linear build-up of Fano resonance spectral profiles*. *Appl. Phys. B.* [Online] 124(6). (2018 Jun) 1-8. Available: <https://link.springer.com/article/10.1007/s00340-018-6983-0>
- [41] Krasavin AV, Vo TP, Dickson W, Bolger PM, Zayats AV. *All-plasmonic modulation via stimulated emission of copropagating surface plasmon polaritons on a substrate with gain*. *Nano. Lett.* [Online] 11(6). (2011 Jun 8) 2231-2235. Available: <https://pubs.acs.org/doi/abs/10.1021/nl200255t>
- [42] Golovinski PA, Yakovets AV, Khramov ES. *Application of the coupled classical oscillators model to the Fano resonance build-up in a plasmonic nanosystem*. *arXiv preprint arXiv:1711.02498* [Online] (2017 Nov 7.) Available: <https://arxiv.org/abs/1711.02498>
- [43] Jafari M. *Electronic Transmission Wave Function of Disordered Graphene by Direct Method and Green's Function Method*. *Journal of Optoelectrical Nanostructures*. [Online] 1(2) (2016 Sep 16) 57-68. Available: http://jopn.miau.ac.ir/article_2049_0.html

- [44] Ávalos-Ovando O, Besteiro LV, Wang Z, Govorov AO. *Temporal plasmonics: Fano and Rabi regimes in the time domain in metal nanostructures*. Nanophotonics. [Online] 9(11). (2020 Aug 18) 3587-3595. Available: <https://www.degruyter.com/document/doi/10.1515/nanoph-2020-0229/html>
- [45] Boyd RW. Nonlinear optics. Academic press; 2020 Mar 30.

