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Implication of Quantum Effects on Non-Linear Propagation of Electron Plasma Solitons

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Abstract: We have studied the electron exchange-correlation effect on the characteristics of the two-component unmagnetized dense quantum plasma with streaming motion. For this purpose, we have used the quantum hydrodynamic model (including the effects of a quantum statistical Fermi electron temperature) for studying the propagation of an electrostatic electron plasma waves in such that plasma consisting of quantum electrons and immobile ions. It is found that by regarding the latter effect, it possible the excitation of two distinct modes. Some different cases such as: unmagnetized, collisionless, classical cases and some formulas presented and discussed. By using the reduced quantum hydrodynamic (QHD) model, the Korteweg de Vries (KdV) equation incorporating the electron exchange-correlation effect is derived. It was shown that the electron exchange-correlation phenomenon on the main quantities for both rarefactive and compressive types of solitary-wave propagation can be important. In particular, the arbitrary amplitude of electron solitary-wave experiences a spreading as the effect of exchange-correlation becomes effective. Variations of the width of the electron solitary wave for different plasma values were depicted. It was shown that by increasing the exchange values, the width of soliton decrease.

Keywords: Electron exchange-correlation, Quantum plasma, Electron plasma waves, Quantum hydrodynamic model, Korteweg de Vries equation.

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1. Introduction

Over the past decade many researchers have been interested in the studying of the linear and nonlinear properties of waves in classical and quantum plasma [1, 2]. Since the electron thermal energy is much smaller than their Fermi energy it is expected that the statistical of plasma particles describe by the antisymmetric wave function i.e. Fermi-Dirac statistics. Study of spin effects in degenerate plasmas lead to an interesting topic i.e. the exchange and the correlation potentials, these effects embody a short-range electric potential, which depends only on the number density of the Fermi particles [3]. The electron exchange-correlation potential is a function of electron density and is given by

$$V_{xc} = -0.985 \frac{e^2}{\epsilon_{eff}} n_e^{1/3} \left[1 + \frac{0.034}{n_e^{1/3} a_B} \ln(1 + 18.37 n_e^{1/3} a_B) \right] \quad (1)$$

Which can be obtained via the adiabatic local-density approximation [4], where $a_B = \epsilon_{eff} \hbar^2 / m_e e^2$ is the Bohr radius and ϵ_{eff} is the effective dielectric permeability of system. The last equation shows that as density increases, the exchange effects increase, while correlation effects decrease. Because $18.376 n_e^{1/3} a_B \ll 1$, for the sake of simplicity we keep (4) to the second order [5, 6]

$$V_{xc} = -1.6(e^2 / \epsilon_{eff}) n_e^{1/3} + 5.65(\hbar^2 / m_e) n_e^{2/3} \quad (2)$$

Quantum plasma effects are ubiquitous from astrophysical system [7] to biophotonics [8], laser produced plasmas [9], miniaturized semiconductor devices and quantum wells and quantum diodes [10, 11]. By adding the Bohm potential, one may generalize the QHD model for plasma with the inclusion of a quantum correction term. The Bohm potential first appeared to describe negative differential resistance in resonant tunneling diodes [12]. Since the QHD model is a simple model and is a straight forward approach to answer, it has been used in magnetized and unmagnetized quantum plasmas for studying in different aspects of linear and non-linear wave propagation in the past decade mainly due to the realization of their occurrence in both the laboratory and space environments [12-14]. For instance, the electron-hole dynamics in semiconductors are studied too [15]. Furthermore, the dynamics and formation of dark soliton and vortices in quantum plasma have studied by Shukla and Eliasson [13]. Using the same model, the KdV solitary wave structure for electron solitary wave [16] and ion acoustic waves in planar and non-planar geometries [17-20], electro-acoustic waves [21], the modulational instability of electron plasma waves in a quantum plasma [2, 22] have been studied by many researchers.

In the present study, we focus on the influence of exchange-correlation effects on the linear and nonlinear properties of electron plasma waves using the QHD model in unmagnetized, collisionless, ultracold electron-ion quantum plasma with streaming motion. It is shown that this effect can significantly influence the dynamic, formation and properties of solitary wave structure. The paper is organized as follow. In the sections II to IV, we bring our main equations. In section V our results are discussed. Section VI is supplied by our conclusion.

2. Theoretical Model

Let us consider a system of an unmagnetized collisionless dense quantum plasma consisting of two component charge particles (electrons and ions of density n_e , n_i , respectively). It is assumed the electrons move with certain non-relativistic streaming velocity (U_0) while the ions fill a uniform neutralizing background and are taken to be immobile. The nonlinear dynamic equations of the IAWs are governed by the continuity, the momentum-balance, and the Poisson equations [23, 24]

$$\frac{\partial N_e}{\partial T} + \frac{\partial}{\partial X}(N_e U_e) = 0 \quad (3)$$

$$\left(\frac{\partial}{\partial T} + U_e \frac{\partial}{\partial X}\right)U_e = \frac{\partial \psi}{\partial X} - N_e \frac{\partial N_e}{\partial X} + \frac{H^2}{2} \frac{\partial}{\partial X} \left(\frac{1}{\sqrt{N_e}} \frac{\partial^2}{\partial X^2} \sqrt{N_e} \right) - \lambda \frac{\partial N_e^{2/3}}{\partial x} + \gamma \frac{\partial N_e^{1/3}}{\partial x} \quad (4)$$

$$\frac{\partial^2 \psi}{\partial X^2} = (N_e - N_i) \quad (5)$$

Where N_e , U_e and ψ are the number of electron density.

The electron fluid velocity and electrostatic potential respectively, and $H = \hbar \omega_{pe} / (2 K_B T_{Fe})$, $\lambda = 5.65 \hbar^2 n_0^{2/3} / (2 K_B T_{Fe} m_e)$ and $\gamma = 1.6 e^2 n_0^{1/3} / (2 K_B T_{Fe} m_e \epsilon_{eff})$ are non-dimensional quantum parameter proportional to the quantum diffraction, the exchange and the correlation effects, respectively. For a Fermi gas, the fluid quantum pressure is defined as $P_e = m_e V_{Fe}^2 n_e^3 / (3 n_0^2)$ [25] in which $V_{Fe} = \sqrt{2 k_B T_{Fe} / m_e}$ is the Fermi thermal speed, T_{Fe} is the electron Fermi temperature, k_B is Boltzmann's constant. The parameters appearing in basic Eqs. (3)- (5) have been appropriately normalized

$$\begin{aligned} X &\rightarrow x \omega_{pe} / V_{Fe} & T &\rightarrow t \omega_{pe} & N_e &\rightarrow n_e / n_0 \\ U_e &\rightarrow u_e / V_{Fe} & \omega_{pe} &\rightarrow \sqrt{n_0 e^2 / \epsilon_0 m_e} & \psi &\rightarrow e \varphi / 2 K_B T_{Fe} \end{aligned} \quad (6)$$

By using the set of Eqs. (3)- (5), one study the non-linear electron plasma wave propagation in a quantum plasma.

3. Linear Dispersion Characteristics

Now in the linear limit and assuming that all the field variables are varying as $\exp[i(kx - \omega t)]$ (where frequency ω and wavenumber k are normalized), Eqs. (3)- (5) can be linearized to give the following wave dispersion relations for the 'fast mode' and the 'slow mode', respectively

$$\omega_{f,s} = kU_0 \pm \sqrt{1 + k^2 + \frac{H^2}{4}k^4 + \frac{2\lambda}{3}k^2 - \frac{\gamma}{3}k^2}. \quad (7)$$

Note that in the absence of the exchange and correlation effects ($\lambda \rightarrow 0$, $\gamma \rightarrow 0$), Eqs. (7) reduce to Eqs. (10) and (11) of Ref. [26]. In this paper, we shall interest to study the behavior of the fast mode only. In Fig. 1 impact of the exchange correlation on the normalized dispersion relation of fast mode is plotted versus of the normalized wavenumber k for given values of quantum diffraction parameter $H = 1.3$ and different values of streaming velocity λ and γ . As we see, the phase velocity of the fast mode increases by increasing the exchange correlation effects. It means that the electron exchange correlation effects enhance the wave frequency.

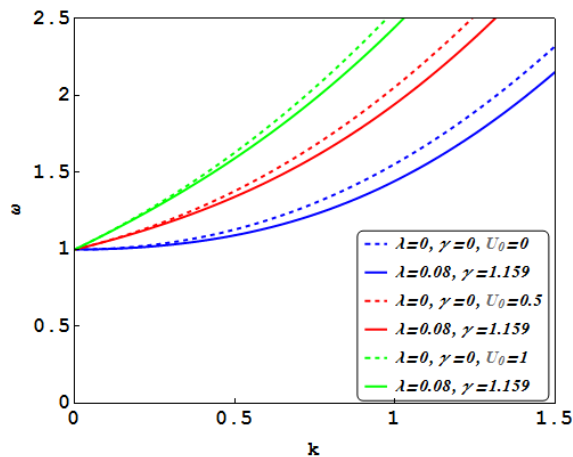


Fig. 1. Dispersion relation of fast mode versus of normalized k for different values of H and U_0

4. Nonlinear Analysis

In order to study the non-linear behavior of electron plasma waves, we use the standard reductive perturbation technique [27] as

$$\xi = \varepsilon^{1/2}(X - VT) \quad \tau = \varepsilon^{3/2}T \quad (8)$$

Where V is the linear phase velocity and ε is a small ($0 < \varepsilon \ll 1$) parameter measuring the nonlinearity. By assuming the following perturbation expansions around their equilibrium values

$$N_e = 1 + \varepsilon N_e^{(1)} + \varepsilon^2 N_e^{(2)} + \dots \quad (9)$$

$$\psi_e = \varepsilon \psi_e^{(1)} + \varepsilon^2 \psi_e^{(2)} + \dots \quad (10)$$

$$U_e = U_0 + \varepsilon U_e^{(1)} + \varepsilon^2 U_e^{(2)} + \dots \quad (11)$$

After substituting Eqs. (8)- (11) into Eqs. (3)- (5), one may derive the perturbation expansions. After a few algebraic steps and keeping terms up to order $\varepsilon^{5/2}$, one may obtain the KdV equation as

$$\frac{\partial \psi^{(1)}}{\partial \tau} + A \psi^{(1)} \frac{\partial \psi^{(1)}}{\partial \xi} + B \frac{\partial^3 \psi^{(1)}}{\partial \xi^3} = 0, \quad (12)$$

In which

$$A = \frac{\alpha^2 + 2(V - U_0)\alpha\beta + \beta^2 + (2\gamma - 2\lambda)/9}{\alpha + (V - U_0)\beta}, \quad (13)$$

$$B = \frac{1 - (V - U_0)^2 + H^2\beta/4 + (2\lambda - \gamma)/3}{\alpha + (V - U_0)\beta} \quad (14)$$

$$\alpha = \frac{3(V - U_0)}{3 - 3(V - U_0)^2 + 2\lambda - \gamma} \quad \beta = \frac{3}{3 - 3(V - U_0)^2 + 2\lambda - \gamma} \quad (15)$$

As be seen in KdV Eq. (12), the non-linearity coefficient A is independent of the quantum diffraction parameter (H) whereas the dispersive coefficient B depends on it and both of them depend on the exchange correlation effects. If transforming the independent variables ξ and τ into the one variable $\eta = \xi - M\tau$ where M is the normalized constant speed, by imposing the appropriate boundary conditions (namely as $\eta \rightarrow \pm\infty$ then $\partial\psi^{(1)}/\partial\eta \rightarrow 0$, $\partial^2\psi^{(1)}/\partial\eta^2 \rightarrow 0$ and $\partial^3\psi^{(1)}/\partial\eta^3 \rightarrow 0$), one may find the steady state equation

$$\psi^{(1)} = \psi_m \operatorname{sech}^2(\eta/\Delta), \quad (16)$$

$$\psi_m = 3M / A \text{ and } \Delta = \sqrt{4B / M} , \quad (17)$$

Represent the amplitude and the width of the solitary wave, respectively. Because of balance between the non-linear effect (A) and the dispersive effect (B), the solitary wave structure is formed. Thus, these two coefficients have crucial role in foundation of solitary wave structure.

5. Numerical Results and Discussion

The influence of the exchange correlation effect on the amplitude and the width (Δ) of the solitary wave structure are studied numerically (in Figures 2 and 3) for different values of the exchange-correlation parameters λ and γ . The numerical example is for typical parameters of the gold metallic plasma at room temperature [18, 19, 28]. Here the typical physical parameters are $n_0 = 5.9 \times 10^{22} \text{ cm}^{-3}$, $\omega_p = 1.37 \times 10^{16} \text{ s}^{-1}$, $V_{Fe} = 1.4 \times 10^8 \text{ cm/s}$ and $H = 0.4$. The spatial patterns of the electron solitary wave show that the amplitude and the width of compressive solitary wave will be shrinking by increasing the value of the exchange-correlation effect. It should be noted despite the fact that the amplitude of the solitary wave does not depend on the quantum effects, the soliton exhibits compression as the values of λ , γ increase or decrease respectively (see Fig. 2). Although by increasing the value of γ for $\lambda = 0$ the soliton amplitude increased, by increasing the value of λ for $\gamma = 0$ the soliton amplitude decreased.

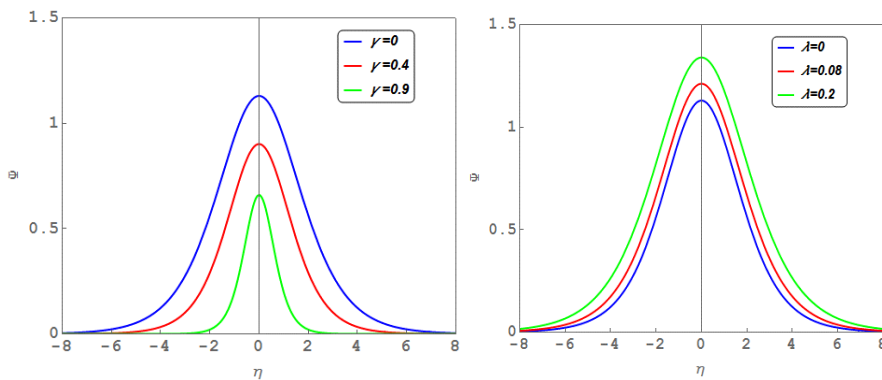


Fig. 2. Compressive solitary wave for three different values of γ when $\lambda = 0$ (left) and λ when $\gamma = 0$ (right).

Likewise (see Fig. 3), the domain of the allowable quantum diffraction parameters H becomes smaller as the values of (λ, γ) increase. It means that the energy carried of the electron exchange-correlation effect, given by

$$E = \int_{-\infty}^{+\infty} (\psi^{(1)})^2 d\eta = \frac{4}{3} \frac{\sqrt{4B/M}}{1 + 2\lambda/3 - \gamma/3} \left(\frac{3M}{A}\right)^2 = \frac{24\sqrt{B} M^{3/2}}{(1 + 2\lambda/3 - \gamma/3)A^2} \quad (18)$$

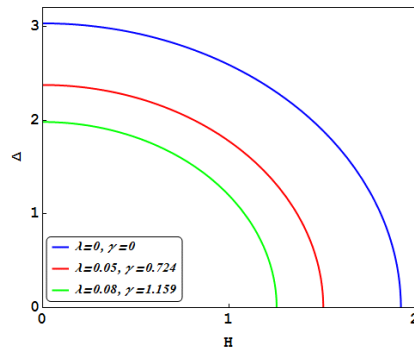


Fig. 3. Variation of the width of the electron solitary wave for different values of λ and γ .

Figure 4 shows that the amplitude and the width of the rarefactive soliton will change for different values of λ and γ . According to the definitions of λ and γ , these two parameters change with values of number of electron density as $n_0^{2/3}$ and $n_0^{1/3}$, respectively. These parameters show how the electron exchange-correlation may evaluate by increasing (or decreasing) with the value of n_0 . It is clear that the amplitude (width) of rarefactive soliton is independent (dependent) on H but unlike compressive soliton its amplitude (width) increases (expands) with increasing of λ and γ . It is clear from Figure 4 (left) that the width of soliton may expand by increasing the value of γ . On the other hand, it is obviously from Figure 4 (right) that the width of soliton may sharp by increasing the value of λ .

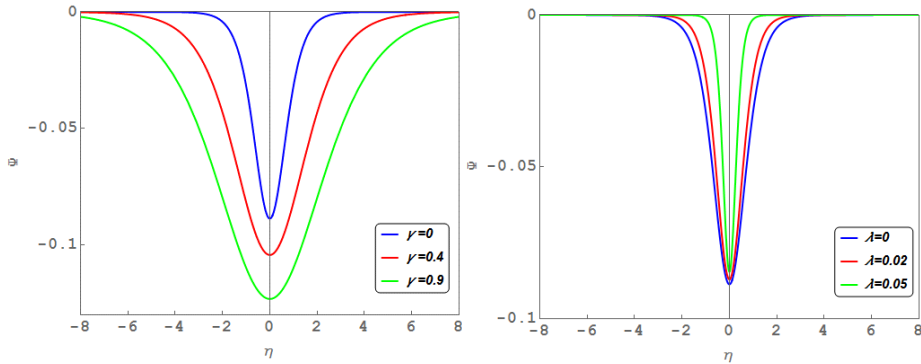


Fig. 4. Rarefactive solitary wave for three different values of γ when $\lambda = 0$ (left) and λ when $\gamma = 0$ (right).

6. Conclusion

To sum up, in this paper, it has been attempted to use the quantum hydrodynamic model (including the effects of a quantum statistical Fermi electron temperature) for studying the propagation of an electrostatic electron plasma waves in two-component unmagnetized quantum plasma consisting of quantum electrons and immobile ions. The electron exchange-correlation effect on the linear and non-linear properties of wave with considering streaming motion have been investigated. The general, in fact two transcendental linear dispersion relations are derived for electron quantum plasma waves, which depend on the exchange-correlation, quantum effects and streaming motion. Some different cases such as: unmagnetized, collisionless, classical cases and some formulas presented in the previous studies have been obtained. By using the standard reductive perturbation technique, we have derived the KdV equation and obtained its stationary localized solutions. It is found that the spatial patterns of both rarefactive and compressive types of solitary-wave propagation can be significantly affected by the exchange-correlation effects. In particular, although, the amplitude of compressive electron solitary-wave can decrease, the amplitude of compressive rarefactive would increase as the effect of exchange-correlation becomes effective. Variations of the width of the electron solitary wave for different plasma values were depicted. It was shown that by increasing the exchange values, the width of soliton decrease. Our results may be useful for understanding the origin of electrostatic fluctuations and associated phenomena in dense electron-ion quantum plasma such as can be found in metal nanostructure, intense laser-solid plasma experiments as well as in some astrophysical environment.

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