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Teleportation via an Entangled Coherent Channel and Decoherence Effect on This Channel

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Abstract: We study an entangled two-mode coherent state within the framework of 2×2-dimensional Hilbert space. We investigate the problem of quantum teleportation of a superposition coherent state via an entangled coherent channel. By three different measures with the titles ``minimum assured fidelity (MASF)", ``average teleportation fidelity" and ``optimal fidelity (f)" we study the quality of this kind of teleportation. Decoherence properties of the entangled coherent state due to channel losses are analysed. For a symmetric noise channel, the degradation of optimal fidelity and degree of entanglement are calculated. Also by two different measures with the titles ``concurrence" and ``entanglement of formation" we study the amount of entanglement of a decohered quantum channel and discuss its details. We demonstrate that entanglement of the decohered entangled coherent state is reduced but not throughly lost. Finally we find that the optimal fidelity of the decohered entangled coherent state is more than the classical limit and the decohered entangled coherent state may be useful for quantum teleportation.

Keywords: Coherent States, Teleportation, Fidelity, Concurrence

1. INTRODUCTION

The motivation for this work has been to investigate the utility of an entangled coherent state as resource for performing the teleportation of an unknown coherent superposition state.

A state is named entangled if it is unfactorizable [1]. Entanglement is essential for many applications of quantum information processing [2-10]. Quantum teleportation is an important and vital quantum information processing task where an arbitrary unknown quantum state can be replicated at a distant location using previously shared entanglement and classical communication between the sender and the receiver. The sender and receiver are called Alice and Bob. A remarkable application of entangled states having many ramifications in information technology, quantum teleportation can also be combined with other

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operations to construct advanced quantum circuits useful for information processing [6]. The original idea of teleportation introduced by Bennett et al. [2] is implemented through a channel involving a pair of particles in a Bell State shared by the sender and the receiver and at the end of the protocol an unknown input state is reconstructed with perfect fidelity at another location while destroying the original copy. Quantum teleportation, which uses entangled quantum states as quantum channels, plays a crucial role in optical quantum computation and communication [11,12].

In this paper we study quantum teleportation with the resource of entangled coherent states. Coherent states are eigenstates of the annihilation operator $\hat{a}$, i.e., $\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$, $|\alpha\rangle = e^{-|\alpha|^2/2}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle$ [13], where α is a complex amplitude. Without losing generality, α is assumed to be real for simplicity throughout the paper. The entangled coherent state is defined as

$$|\phi_3\rangle = \frac{1}{\sqrt{N_1}}(|-\alpha\rangle|-\alpha\rangle + |\alpha\rangle|\alpha\rangle), \tag{1}$$

which were studied as quasi-Bell states [14], where $N_1 = 2(1+\mathrm{e}^{-4\alpha^2})$.

The remaining part of the paper is organized as follows. In Sec. 2 , we study the teleportation of a coherent superposition state via the state $|\phi_3\rangle$, where we provide expressions for the minimum assured fidelity (MASF), the average teleportation fidelity F_{ave} [15], and the optimal fidelity (f) [16] for the state $|\phi_3\rangle$, and deterministic perfect teleportation is possible via the state $|\phi_3\rangle$. In Sec. 3, we analyse decoherence properties of the state due to channel losses. When the quantum system is open to the outside world, the initially prepared system decoheres and becomes mixed [16]. The degradation of optimal fidelity and degree of entanglement are calculated. For this purpose we profit the concurrence [17-19] and the entanglement of formation [18]. Also we study the optimal fidelity [18] of the mixed entangled coherent state. Finally the paper is concluded in Sec. 4.

2. QUANTUM TELEPORTATION

Let us formulate the teleportation protocol between two parties Alice and Bob, with the input coherent superposition state prepared by a third party Charlie. He sends the prepared coherent superposition state to Alice. In this transmission we assume that there is no distortion of the input coherent superposition state. Since the input coherent superposition state is given by the third party, Alice have no knowledge about the received coherent superposition state which she wants to teleport, and this arbitrary coherent superposition state is given by:

$$|\psi\rangle_a = A|\alpha\rangle_a + B|-\alpha\rangle_a, \tag{2}$$

where the amplitudes A and B are unknown. Let us assume that two distant partners Alice and Bob share the quantum channel $|\phi_3\rangle$. The particles *b* and *c* are with Alice and Bob, respectively.

By the Gram-Schmidt theorem, it is always possible to make orthonormal bases in *N*-dimensional vector space from any *N* linear independent vectors. Suppose orthonormal bases by superposing nonorthogonal and linear independent two coherent states $|\alpha\rangle$ and $|-\alpha\rangle$ [16]:

$$|\psi_+\rangle = \frac{1}{\sqrt{N_\theta}}(\cos\theta|\alpha\rangle - \sin\theta|\alpha\rangle), \tag{3}$$

$$|\psi_-\rangle = \frac{1}{\sqrt{N_\theta}}(-\sin\theta|\alpha\rangle + \cos\theta|-\alpha\rangle), \tag{4}$$

where

$$N_\theta = \cos^2 2\theta, \qquad \sin 2\theta = \langle -\alpha|\alpha\rangle = \exp(-2\alpha^2), \tag{5}$$

Now using (3) and (4), the state (2) may be represented as:

$$|\psi\rangle_a = A'|\psi_+\rangle_a + B'|\psi_-\rangle_a, \tag{6}$$

with $A' = A\cos\theta + B\sin\theta$ and $B' = A\sin\theta + B\cos\theta$. After sharing the quantum channel $|\phi_3\rangle$, the initial state `` $|\phi_{abc}\rangle$ " then is:

$$\begin{aligned}
|\phi_{abc}\rangle &= (A'|\psi_+\rangle_a + B'|\psi_-\rangle_a)\frac{1}{\sqrt{N_1}}(|-\alpha\rangle_b|-\alpha\rangle_c + |\alpha\rangle_b|\alpha\rangle_c) \\
&= \frac{1}{\sqrt{2N_1}}(|B_1\rangle_{ab}\left((A' + B'\sin 2\theta)|\psi_+\rangle_c + (A'\sin 2\theta + B')|\psi_-\rangle_c\right) \\
&+ |B_2\rangle_{ab}\left((A' - B'\sin 2\theta)|\psi_+\rangle_c + (A'\sin 2\theta - B')|\psi_-\rangle_c\right) \\
&+ |B_3\rangle_{ab}\left((A'\sin 2\theta + B')|\psi_+\rangle_c + (A' + B'\sin 2\theta)|\psi_-\rangle_c\right) \\
&+ |B_4\rangle_{ab}\left((A'\sin 2\theta - B')|\psi_+\rangle_c + (A' - B'\sin 2\theta)|\psi_-\rangle_c\right)),
\end{aligned} \tag{7}$$

Where $|B_{1,2}\rangle = \frac{1}{\sqrt{2}}\left(|\psi_+\rangle|\psi_+\rangle \pm |\psi_-\rangle|\psi_-\rangle\right)$, $|B_{3,4}\rangle = \frac{1}{\sqrt{2}}\left(|\psi_+\rangle|\psi_-\rangle \pm |\psi_-\rangle|\psi_+\rangle\right)$. Since the states *a* and *b* are with Alice, she performs a Bell state measurement on her states and then sends the measurement result to Bob expending two classical bits. Bob accordingly chooses one of the unitary transformations $\{I, \sigma_x, i\sigma_y, \sigma_z\}$ to perform on his part *c* of the quantum channel. Here, σ's are Pauli operators and I is the identity operator and the correspondence between the measurement outcomes and the unitary operations are $|B_1\rangle_{ab} \Rightarrow I$, $|B_2\rangle_{ab} \Rightarrow \sigma_z$, $|B_3\rangle_{ab} \Rightarrow \sigma_x$, $|B_4\rangle_{ab} \Rightarrow i\sigma_y$.

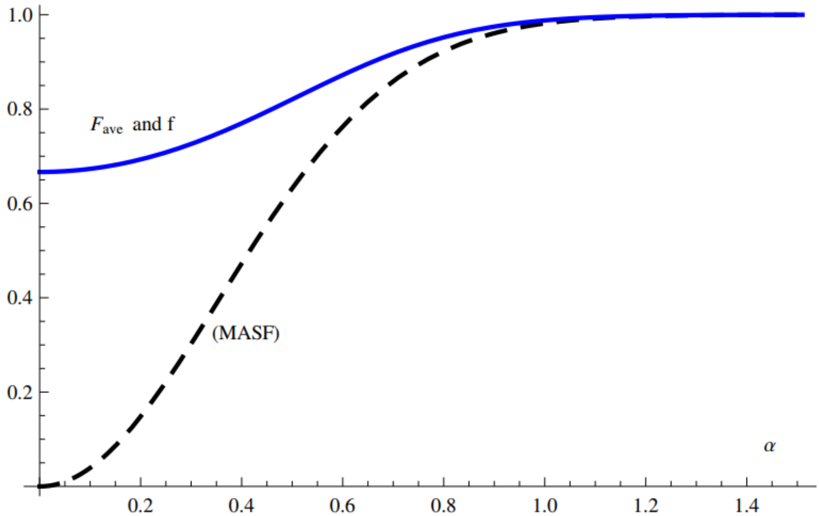

Fig. 1. Variation of the minimum assured fidelity (MASF), the average teleportation fidelity (F_{ave}) and the optimal fidelity (f) with the amplitude α for teleportation of the state (2) via the channel $|\phi_3\rangle$.

Thereafter Bob transmits this state to the third party Charlie whose task would be to measure the efficiency of the teleportation protocol.

The average teleportation fidelity (F_{ave}) and the minimum assured fidelity (MASF) can be used as measures of quality of teleportation. We may now compute MASF for teleportation of the state (2) via the channel $|\phi_3\rangle$, by using the procedure adopted in [15] and obtain

$$\text{(MASF)} = 1 - |\langle -\alpha|\alpha\rangle|^2 = 1 - \exp(-4\alpha^2), \tag{8}$$

that the diagram of this equation has been shown in Fig. 1. From this figure one can see that for $\alpha \geq 0.95$, (MASF) =1, and hence for $\alpha \geq 0.95$, the state $|\phi_3\rangle$ leads to the deterministic perfect teleportation of the state (2). Note that if $\alpha \geq 0.95$ then $\sin 2\theta (= \langle -\alpha|\alpha\rangle) \simeq 0$, $N_1 \simeq 2$, and Eq. (7) can be rewritten as:

$$\begin{aligned} |\phi_{abc}\rangle = \frac{1}{2}(& |B_1\rangle_{ab}\left(A'|\psi_+\rangle_c + B'|\psi_-\rangle_c\right) \\ & + |B_2\rangle_{ab}\left(A'|\psi_+\rangle_c - B'|\psi_-\rangle_c\right) \\ & + |B_3\rangle_{ab}\left(B'|\psi_+\rangle_c + A'|\psi_-\rangle_c\right) \\ & + |B_4\rangle_{ab}\left(-B'|\psi_+\rangle_c + A'|\psi_-\rangle_c\right)). \end{aligned} \tag{9}$$

We may now compute the average teleportation fidelity F_{ave} for teleportation of the state (2) via the channel $\left|\phi_3\right\rangle$, by using the procedure adopted in [15] and obtain

$$F_{ave}=\frac{3+\left|\langle -\alpha|\alpha\rangle\right|^2}{3(1+\left|\langle -\alpha|\alpha\rangle\right|^2)}=\frac{3+\exp(-4\alpha^2)}{3(1+\exp(-4\alpha^2))}, \quad (10)$$

that the diagram of this equation has been shown in Fig. 1. From this figure one can see that for $\alpha \geq 0.95$, $F_{ave}=1$. The figure 1 shows the average teleportation fidelity F_{ave} is more than 2/3 for $\alpha \geq 0$.

Now we may check the optimal fidelity of the teleportation scheme by using the criterion introduced by Horodecki et al. in [20]. The optimal fidelity of teleportation in any general scheme by means of trace-preserving local quantum operations and classical communication via a single channel may be obtained from the maximal singlet fraction of the channel. The relation is

$$f(\rho)=\frac{2F(\rho)+1}{3}, \quad (11)$$

Where $f(\rho)$ is the optimal fidelity for the given quantum channel ρ, and $F(\rho)$ is the maximal singlet fraction of the channel. $F(\rho)$ is defined as $\max\langle\varphi|\rho|\varphi\rangle$ where the maximum is taken over all the 2×2 maximally entangled states. Any 2×2 channel becomes useless for quantum teleportation when the optimal fidelity $f(\rho)$ is less than the classical limit 2/3. Here, we may express the optimal fidelity for teleportation of the state (2) via the channel $\left|\phi_3\right\rangle$ as

$$f(\rho)=\frac{3+\left|\langle -\alpha|\alpha\rangle\right|^2}{3(1+\left|\langle -\alpha|\alpha\rangle\right|^2)}=\frac{3+\exp(-4\alpha^2)}{3(1+\exp(-4\alpha^2))}, \quad (12)$$

that the diagram of this equation has been shown in Fig. 1. From this figure one can see that the optimal fidelity is more than the classical limit 2/3 for $\alpha \geq 0$. Using (10) and (12), we can easily verify that $f=F_{ave}$ for the state $\left|\phi_3\right\rangle$. Thus, the teleportation scheme realized using the state $\left|\phi_3\right\rangle$ as the quantum channel is optimal.

3. DECOHERENCE PROPERTIES

In this section, we will discuss decoherence properties of the state $\left|\phi_3\right\rangle$. We can model such photon losses by interacting the signal with a vacuum mode $\left|0\right\rangle_E$ in a beam splitter with the properly chosen transmissivity parameter η. The effect of decoherence can be represented as [18]:

$$\left|\alpha\right\rangle_{1(2)}\left|0\right\rangle_{E} \rightarrow \left|\sqrt{\eta}\alpha\right\rangle_{1(2)}\left|\sqrt{1-\eta}\alpha\right\rangle_{E}, \tag{13}$$

where $\left|0\right\rangle_{E}$ refers to the environment mode and η is the noise parameter which means the fraction of photons that survive the noise. For simplicity we then assume that both modes are equally lossy. We will introduce two auxiliary environment modes E1 and E2, which coupled with mode 1 and 2 respectively and the initial state reads $\left|\phi_3\right\rangle_{1,2}\otimes\left|0\right\rangle_{E1}\left|0\right\rangle_{E2}$. After passing the noisy channel, we can get the final state as:

$$\begin{aligned}\left|\phi_3\right\rangle_{1,2}\otimes\left|0\right\rangle_{E1}\left|0\right\rangle_{E2} \rightarrow \frac{1}{\sqrt{N_1}}(&\left|-\sqrt{\eta}\alpha\right\rangle_1\left|-\sqrt{\eta}\alpha\right\rangle_2\left|-\sqrt{1-\eta}\alpha\right\rangle_{E1}\left|-\sqrt{1-\eta}\alpha\right\rangle_{E2} \\ &+\left|\sqrt{\eta}\alpha\right\rangle_1\left|\sqrt{\eta}\alpha\right\rangle_2\left|\sqrt{1-\eta}\alpha\right\rangle_{E1}\left|\sqrt{1-\eta}\alpha\right\rangle_{E2}),\end{aligned} \tag{14}$$

Tracing over all environment modes, we obtain the density operator:

$$\begin{aligned}\rho_{1,2} = \frac{1}{N_1}(&\left|\sqrt{\eta}\alpha\right\rangle_1\left\langle\sqrt{\eta}\alpha\right|\otimes\left|\sqrt{\eta}\alpha\right\rangle_2\left\langle\sqrt{\eta}\alpha\right| \\ &+\left|-\sqrt{\eta}\alpha\right\rangle_1\left\langle-\sqrt{\eta}\alpha\right|\otimes\left|-\sqrt{\eta}\alpha\right\rangle_2\left\langle-\sqrt{\eta}\alpha\right| \\ &+\exp\left(-4(1-\eta)\alpha^2\right)(\left|\sqrt{\eta}\alpha\right\rangle_1\left\langle-\sqrt{\eta}\alpha\right|\otimes\left|\sqrt{\eta}\alpha\right\rangle_2\left\langle-\sqrt{\eta}\alpha\right| \\ &+\left|-\sqrt{\eta}\alpha\right\rangle_1\left\langle\sqrt{\eta}\alpha\right|\otimes\left|-\sqrt{\eta}\alpha\right\rangle_2\left\langle\sqrt{\eta}\alpha\right|)),\end{aligned} \tag{15}$$

The orthonormal basis vectors are now η-dependent

$$\left|\psi_+(\eta)\right\rangle = \frac{1}{\sqrt{N_\Theta}}(\cos\Theta\left|\sqrt{\eta}\alpha\right\rangle - \sin\Theta\left|-\sqrt{\eta}\alpha\right\rangle), \tag{16}$$

$$\left|\psi_-(\eta)\right\rangle = \frac{1}{\sqrt{N_\Theta}}(-\sin\Theta\left|\sqrt{\eta}\alpha\right\rangle + \cos\Theta\left|-\sqrt{\eta}\alpha\right\rangle), \tag{17}$$

where

$$N_\Theta = \cos^2 2\Theta, \qquad \sin 2\Theta = \left\langle-\sqrt{\eta}\alpha\middle|\sqrt{\eta}\alpha\right\rangle = \exp(-2\eta\alpha^2), \tag{18}$$

The density matrix $\rho_{1,2}$ is representing in the orthonormal basis $\left|\Psi_-(\eta)\right\rangle\left|\Psi_-(\eta)\right\rangle\}$, $\left|\Psi_-(\eta)\right\rangle\left|\Psi_+(\eta)\right\rangle$, $\left|\Psi_+(\eta)\right\rangle\left|\Psi_-(\eta)\right\rangle$, $\{\left|\Psi_+(\eta)\right\rangle\left|\Psi_+(\eta)\right\rangle$ as:

$$\rho_{1,2} = \begin{pmatrix} A & B & B & C \\ B & D & D & B \\ B & D & D & B \\ C & B & B & A \end{pmatrix}, \tag{19}$$

where

$$\sin\Theta = \sqrt{\frac{1-(1-\exp(-4\eta\alpha^2))^{1/2}}{2}}, \qquad \cos\Theta = \sqrt{\frac{1+(1-\exp(-4\eta\alpha^2))^{1/2}}{2}},$$

$$A = \frac{1}{N_1}(\sin^4\Theta + \cos^4\Theta + 0.5\ \sin^2 2\Theta\ \exp(-4(1-\eta)\alpha^2)),$$

$$B = \frac{\sin 2\Theta}{2N_1}(1+\exp(-4(1-\eta)\alpha^2)),$$

$$C = \frac{1}{N_1}(0.5\ \sin^2 2\Theta + (\sin^4\Theta + \cos^4\Theta)\exp(-4(1-\eta)\alpha^2)),$$

$$D = \frac{\sin^2(2\Theta)}{2N_1}(1+\exp(-4(1-\eta)\alpha^2)). \tag{20}$$

In the following, we analyse decoherence properties of the channel $|\phi_3\rangle$ due to channel losses. For this purpose we profit two measures with the title of the concurrence, and the entanglement of formation. Also we study the optimal fidelity of the mixed entangled coherent state.

3. 1. CONCURRENCE

The entanglement can be measured by the concurrence [17-19]. The concurrence is written as:

$$C = \max(0,\ \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}), \tag{21}$$

Where the parameters λ_i (i=1,2,3,4) are the eigenvalues of the non-Hermitian matrix $\rho\tilde{\rho}$, and

$$\tilde{\rho} = (\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y), \tag{22}$$

Here ρ^* is the complex conjugate of ρ in the same basis (19). Performing the preceding calculations, one can get:

$$\tilde{\rho}_{1,2} = \begin{pmatrix} A & -B & -B & C \\ -B & D & D & -B \\ -B & D & D & -B \\ C & -B & -B & A \end{pmatrix}, \qquad \rho_{1,2}\ \tilde{\rho}_{1,2} = \begin{pmatrix} a & b & b & c \\ -b & d & d & -b \\ -b & d & d & -b \\ c & b & b & a \end{pmatrix}, \tag{23}$$

where

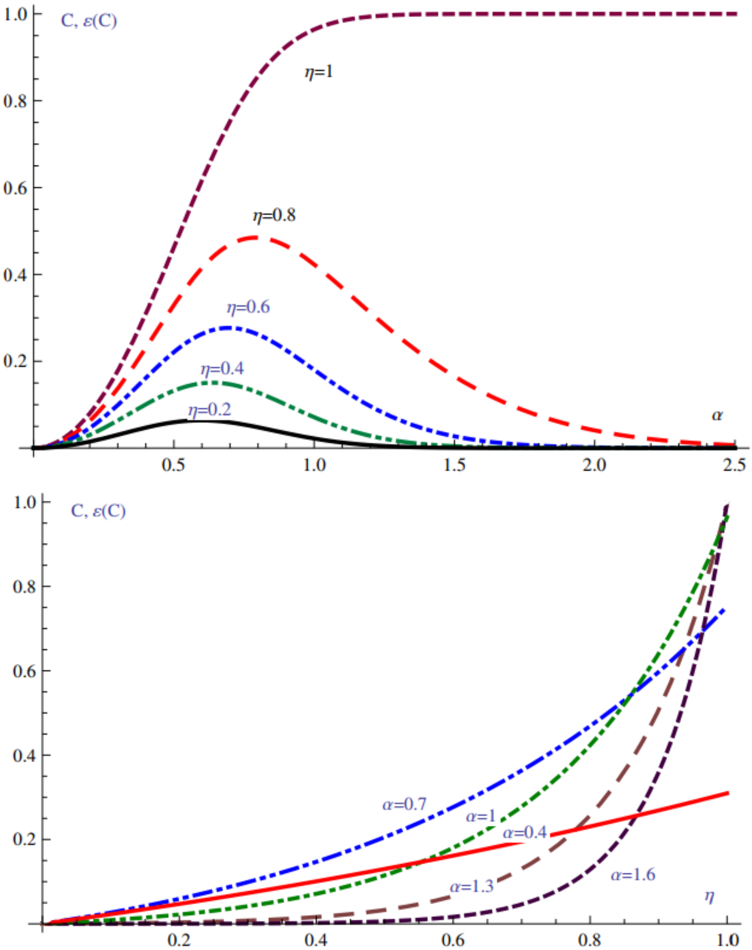

Fig. 2. Variation of the concurrence and the entanglement of formation of $\rho_{1,2}$ with the amplitude α for $\eta = 0.2, 0.4, 0.6, 0.8, 1$. Variation of the concurrence and the entanglement of formation of $\rho_{1,2}$ with the channel noise parameter η for $\alpha = 0.4, 0.7, 1, 1.3, 1.6$.

$$\begin{aligned} a &= A^2 - 2B^2 + C^2, \\ b &= -AB - BC + 2BD, \\ c &= 2AC - 2B^2, \\ d &= -2B^2 + 2D^2. \end{aligned} \tag{24}$$

The eigenvalues of $\rho_{1,2}\tilde{\rho}_{1,2}$ are $\{\lambda_1 = a + c + 2d, \lambda_2 = a - c, \lambda_{3,4} = 0\}$. The concurrence of $\rho_{1,2}$ is given by:

$$C=\max(0,\sqrt{a+c+2d}-\sqrt{a-c})=\sqrt{a+c+2d}-\sqrt{a-c}. \quad (25)$$

The relation between concurrence and the entanglement of formation of a pure state is expressed as [18]:

$$E(\psi)=\varepsilon(C(\psi)), \quad (26)$$

where the function ε is defined as

$$\varepsilon(C)=H(\frac{1+\sqrt{1-C^2}}{2}), \quad (27)$$

$$H(x)=-x\log_2 x-(1-x)\log_2(1-x). \quad (28)$$

The above formula still holds for mixed states of bipartite systems [19]. Therefore the entanglement of formation of $\rho_{1,2}$ is just $E(\rho_{1,2})=\varepsilon(C)=H((1+(1-(\sqrt{a+c+2d}-\sqrt{a-c})^2)^{1/2})/2)$. In Fig. 2, the diagrams of the concurrence and the entanglement of formation of $\rho_{1,2}$ have been shown. From this figure one can see that in the limit of $\alpha\to 0$, the concurrence approaches 0. For $\eta=1$ (the pure entangled coherent state) and $\alpha\geq 0.95$, the concurrence approaches 1. Approximately for $0.2\leq\alpha\leq 1.1$, the concurrence of $\rho_{1,2}$ is larger than the other amplitudes (except for $\eta=1$). It is seen in Fig.2 that the mixed state $\rho_{1,2}$ is entangled regardless of the channel noise parameter. The entangled coherent states with large coherent amplitudes decohere faster than those with small amplitudes. This is in agreement with the fact that macroscopic quantum effects are not easily seen because it is more fragile [16]. The concurrence and the entanglement of formation have completely similar results for entanglement of the decohered entangled coherent state as shown in Fig. 2.

3. 2. OPTIMAL FIDELITY

Now we may check the optimal fidelity of the teleportation scheme by using the criterion introduced in [18]. According to this criterion, $F(\rho_{1,2})$ is written as:

$$F(\rho_{1,2})=\max_{\beta}\langle B_1(\beta)|\rho_{1,2}|B_1(\beta)\rangle, \quad (29)$$

where $|B_1(\beta)\rangle$ is of the similar form of $|B_1\rangle$ so reads:

$$F(\rho_{1,2})=\max_{\beta}\frac{(1+\exp(-4(1-\eta)\alpha^2))}{2(1+\exp(-4\alpha^2))(1-\exp(-4\beta^2))^2}\times \quad (30)$$
$$(\exp(-(\beta-\sqrt{\eta}\alpha)^2)+\exp(-(\beta+\sqrt{\eta}\alpha)^2)-2\exp(-3\beta^2-\eta\alpha^2))^2.$$

The problem is to find the maximal value of $F(\rho_{1,2})$. We investigate the derivative of $F(\rho_{1,2})$ and find that $\frac{d}{d\beta}F(\rho_{1,2})=0$ when:

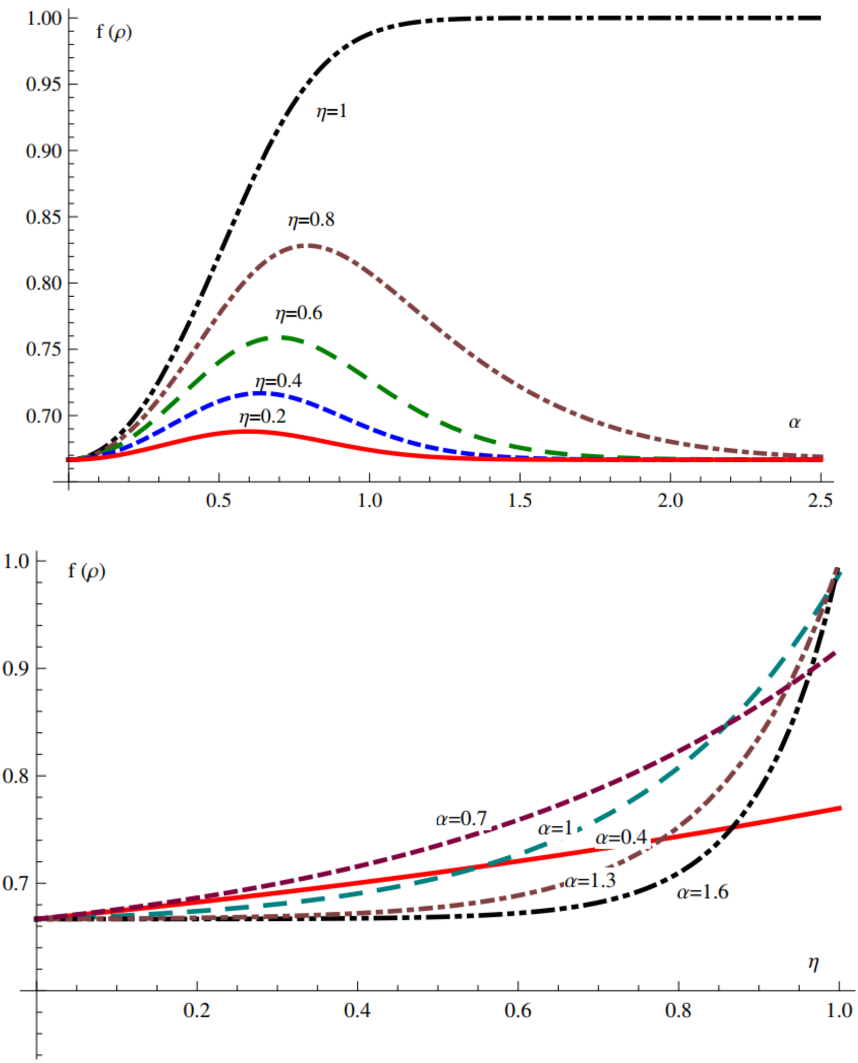

Fig. 3. Variation of the optimal fidelity of $\rho_{1,2}$ with the amplitude α for different parameters $\eta = 0.2, 0.4, 0.6, 0.8, 1$. Variation of the otptimal fidelity of $\rho_{1,2}$ with the channel noise parameter η for $\alpha = 0.4, 0.7, 1, 1.3, 1.6$.

$$\beta = \sqrt{\eta}\alpha, \tag{31}$$

Now using (11), (30) and (31), the optimal fidelity of the channel is equal to

$$f(\rho_{1,2}) = \left(\frac{(1+\exp(-4(1-\eta)\alpha^2))}{(1+\exp(-4\alpha^2))} + 1\right)/3, \tag{32}$$

The diagrams of this equation have been shown in Fig. 3. This figure has similar results to Fig. 2. The figure 3 shows the optimal fidelity $\rho_{1,2}$ is more than the classical limit 2/3 and we can see that the mixed channel may be useful for quantum teleportation.

4. CONCLUSION

To summarize, in this paper we have studied the teleportation of an unknown coherent superposition state via an entangled coherent channel. We have computed minimum assured fidelity, average fidelity, and optimal fidelity of the entangled coherent channel. For $\alpha \geq 0.95$, this channel leads to the deterministic perfect teleportation.

We have investigated decoherence properties of the entangled coherent state. For a symmetric noise channel, we have studied the concurrence and the entanglement of formation where these criteria have completely similar results. The mixed entangled coherent state is entangled regardless of the channel noise parameter. In the limit of $\alpha \to 0$, the concurrence approaches 0. For $\eta = 1$ and $\alpha \geq 0.95$, the concurrence approaches 1. Approximately for $0.2 \leq \alpha \leq 1.1$, the concurrence of $\rho_{1,2}$ is larger than the other amplitudes. The entangled coherent state with large coherent amplitudes decohere faster than those with small amplitudes. This is in agreement with the fact that macroscopic quantum effects are not easily seen because it is more fragile. Also we have studied the optimal fidelity of the mixed entangled coherent state. The optimal fidelity of the decohered entangled coherent state is more than the classical limit 2/3 and the mixed channel may be useful for quantum teleportation for $\alpha \geq 0.95$.

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