Numerical Investigation of Birefringene Effect on the Light Reflection

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Abstract: In the present paper, the problem of light reflection from a birefringent medium and thin film is considered. First, the analytical equations governing the propagation of a plane and harmonic electromagnetic wave in an infinite, birefringent, linear, non-dispersive, non-absorbing, and non-magnetic medium is derived from Maxwell equations. Then, using phase matching condition and boundary conditions, the governing equations of reflection and transmission from a birefringent medium is obtained. Next, the reflection of *s* and *p* polarizations in incidence of *s*-polarized, *p*-polarized, and circularly polarized light on a plane surface is calculated using a massive computer code developed by the authors. Calculations show that the polarizations are mixed and converted to each other. On the other hand, dependence of reflection on azimuthal incidence angle is revealed. Then, the problem of interfering reflection from a birefringent thin film is regarded. The computer code calculates reflection of light from the film by considering the successive reflections and transmissions from the upper and lower surfaces of the film through two-reflection approach. Calculations show that, in reflection of white light from the film, a kind of banding is developed which is absent in isotropic films. Observation of reflection increase by increasing birefringent properties is another finding of the paper.

Key words: Birefringent medium, Birefringent thin film, Reflection.

1. INTRODUCTION

Refractive index in biaxially anisotropic media is dependent on light propagation direction. Such media which are existent both naturally (e.g. crystals) and synthetically (e.g. photonic crystals, sculptured thin films, magnetized plasmas, electro-optical, magneto-optical, elasto-optical, acousto-optical and other materials) are of great importance in optics [1]. They are used in many optical components such as modulators, switches, tunable filters, phase plates, and more recently in high-capacity optical memories, omnidirectional reflectors, antireflection coatings, enhanced polarization converters, anisotropic diffraction

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gratings, and many other applications [2-7].

The problem of light propagation in birefringent media and its reflection and transmission in interfaces has a long history in optics [8-10]. Due to their growing variety and complicated physics, the theoretical, numerical, and experimental study of such media optics is still in the focus of both geometric [11-13] and wave [14-17] optics community. The feasibility of well-known phenomena such as total internal reflection and Brewster angle in biaxial media and their comparison with isotropic cases has attracted great attention [18-21]. The appearance of strange optical phenomena in the biaxial media like negative and amphoteric refraction has also amused researchers [22-24]. Even, light interaction with inhomogeneous or finite-sized biaxial media which leads to scattering and diffraction has not been neglected in the literature [25-28].

The problem of successive reflections and transmissions at biaxial interfaces and resulting interference pattern is also very important. Especially, much time has been expended to study the light interference associated with a biaxial monolayer or multilayer [29-31]. Analytical investigation of the interference patterns by infinite-beam approaches which include an infinite number of reflected and transmitted beams is greatly simplified by incorporating 4×4 or 2×2 matrix methods [29,32,33]. Simple approximation of the 4×4 matrix method for ultra-thin films is presented in [34-36]. However, such infinite-beam approaches are appropriate only when the incident light has infinite longitudinal coherence (e.g. laser light) to keep its phase relation after each reflection and transmission. In other words, it is the coherence length of the incident light that determines the necessary number of reflections and transmissions contributing in the interference. Hence, using more reflections and transmissions not only does not increase the accuracy but also can lead to inaccurate results. In situations where the applied light has a short coherence length (e.g. natural light) the inclusion of only two beams, i.e. one reflected from the upper interface and the other reflected from the lower one of the film, suffices to give correct interference pattern. Such a double-reflection approach can correctly explains many natural interference patterns such as colorful soap bubbles, the colors of butterfly wings, the rainbow color of oil stain on the water, and etc. However, despite many studies regarding biaxial thin film interference by infinite-beam approach, one can hardly find research works in references by double-reflection approach [7,37].

In the present paper, using propagating waves in biaxial media and applying boundary conditions at the interfaces, the interference pattern of a biaxial thin film is numerically calculated by double-reflection method. The results are compared with that of isotropic thin films. The paper is organized as follows: In the second and third sections a brief introduction concerning light propagation, reflection, and transmission in biaxial media and interfaces is presented. The

fourth section is devoted to numerical calculation of interference pattern associated with biaxial thin films by double-reflection approach. Summary and conclusion are drawn in the last section.

2. LIGHT PROPAGATION IN A BIAXIAL MEDIUM

In this section the propagation of a harmonic plane wave in an infinite, homogeneous, linear, and non-magnetic biaxial medium with no absorption and dispersion is considered based on the formulation presented in [38-40]. The dielectric tensor of a biaxial medium is a diagonal matrix in its principal coordinate system:

$$\ddot{\varepsilon} = \varepsilon_0 \begin{pmatrix} n_x^2 & 0 & 0 \\ 0 & n_y^2 & 0 \\ 0 & 0 & n_z^2 \end{pmatrix} \tag{1}$$

Where n_x, n_y, and n_z are principal refractive indices along principal axes x, y, and z respectively and ε_0 is vacuum permittivity. The electric field of a harmonic plane wave has the form:

$$\vec{E}(\vec{r},t) = \vec{E}_0\, e^{i(\vec{K}\cdot\vec{r}-\omega t)} \tag{2}$$

in which $\vec{E}_0$ is the vector amplitude and ω and $\vec{K}$ are frequency and wave vector, respectively, with relation:

$$\vec{K} = \frac{N}{c}\omega\, \hat{e}_K \tag{3}$$

N in (3) is the effective refractive index, c is light speed in free space, and $\vec{e}_K$ is a unit vector in propagation direction. $\vec{E}_0$ and N in (2) and (3) are unknowns and must be determined in terms of ω and $\vec{e}_K$. Defining $\vec{D}$ and $\vec{H}$ fields of the wave as:

$$\vec{D}(\vec{r},t) = \vec{D}_0\, e^{i(\vec{K}\cdot\vec{r}-\omega t)} = \ddot{\varepsilon}\,.\,\vec{E}(\vec{r},t)$$
$$\vec{H}(\vec{r},t) = \vec{H}_0\, e^{i(\vec{K}\cdot\vec{r}-\omega t)} = \frac{1}{\mu_0}\vec{B}_0\, e^{i(\vec{K}\cdot\vec{r}-\omega t)} \tag{4}$$

and using curl equations of Maxwell yields:

$$\vec{K} \times \vec{B}_0 = -\frac{\omega}{c^2} \bar{\bar{\varepsilon}} \cdot \vec{E}_0$$
$$\vec{K} \times \vec{E}_0 = \omega \vec{B}_0 \tag{5}$$

Their combination leads to:

$$N^2 \left(\hat{e}_K \left(\hat{e}_K . \vec{E}_0 \right) - \vec{E}_0 \right) + \bar{\bar{\varepsilon}} \cdot \vec{E}_0 = \vec{0} \tag{6}$$

with the matrix form of:

$$\begin{pmatrix} n_x^2 - N^2(e_{K,y}^2 + e_{K,z}^2) & N^2 e_{K,x} e_{K,y} & N^2 e_{K,x} e_{K,z} \\ N^2 e_{K,x} e_{K,y} & n_y^2 - N^2(e_{K,x}^2 + e_{K,z}^2) & N^2 e_{K,y} e_{K,z} \\ N^2 e_{K,x} e_{K,z} & N^2 e_{K,y} e_{K,z} & n_z^2 - N^2(e_{K,x}^2 + e_{K,y}^2) \end{pmatrix} \begin{pmatrix} E_{0x} \\ E_{0y} \\ E_{0z} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{7}$$

Equating the determinant of the coefficient matrix with zero in order to find nontrivial amplitude $\vec{E}_0$ results in:

$$\frac{e_{K,x}^2}{N^2 - n_x^2} + \frac{e_{K,y}^2}{N^2 - n_y^2} + \frac{e_{K,z}^2}{N^2 - n_z^2} - \frac{1}{N^2} = 0 \tag{8}$$

This is a sixth degree equation in terms of N and has six solutions: $\pm N_1$, $\pm N_2$, and $\pm \infty$. The only physically meaningful solutions are $+N_1$ and $+N_2$. If these indices are found for all propagation directions and their loci are plotted, a three-dimensional double-shell is formed in which one of the shells is within the other. The double-shell is known as iso-frequency double-shell. Inserting N_1 and N_2 in (7), one can obtain their respective eigen-polarization:

$$\vec{E}_{0,1} = A_1 \begin{pmatrix} \dfrac{e_{K,x}}{N_1^2 - n_x^2} \\ \dfrac{e_{K,y}}{N_1^2 - n_y^2} \\ \dfrac{e_{K,z}}{N_1^2 - n_z^2} \end{pmatrix} \quad , \quad \vec{E}_{0,2} = A_2 \begin{pmatrix} \dfrac{e_{K,x}}{N_2^2 - n_x^2} \\ \dfrac{e_{K,y}}{N_2^2 - n_y^2} \\ \dfrac{e_{K,z}}{N_2^2 - n_z^2} \end{pmatrix} \tag{9}$$

where A_1 and A_2 are arbitrary constants. So, two aforementioned unknowns (namely effective index, N, and amplitude, $\vec{E}_0$) are obtained. This discussion shows that a harmonic plane wave can propagate in any direction of a biaxial

medium but cannot have any polarization except two eigen-polarizations of (9). Because of different refractive indices, these two eigen-polarizations cannot combine to form various elliptic polarizations as in isotropic media. These eigen-polarizations are known as ordinary and extra-ordinary waves in uniaxial media. They have different nomenclature in biaxial media such as slow-wave and fast-wave (depending of their phase velocity) or outer-shell and inner-shell waves (depending on their effective index position in the outer or inner shell of the double-shell iso-frequency).

3. REFLECTION AND TRANSMISSION OF A PLANE HARMONIC WAVE AT A FLAT INTERFACE BETWEEN ISOTROPIC AND ANISOTROPIC MEDIA

In this section the reflection and transmission of a harmonic plane wave incident from a semi-infinite isotropic medium on a flat surface of a semi-infinite biaxial medium with arbitrary orientation of principal axes is considered, Fig. 1. In addition to the principal frame, a laboratory frame is also needed, because initial information of incident wave is more easily given in lab frame rather than principal one. Lab axes x_{Lab} and y_{Lab} lie in the interface so that the z_{Lab} axis is directed into the biaxial medium. Using Euler rotation matrix which includes three successive rotations, first by α about z_{Lab}, second by β about new x_{Lab}, and third by γ about new z_{Lab}, the lab frame coincides with the principal frame.
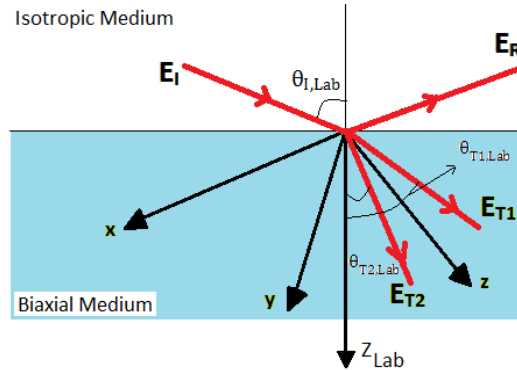

Fig. 1. Schematic of the reflection and transmission at the plane interface between isotropic and biaxial media with principal axes of x, y, and z.

A harmonic plane wave is incident on the surface by polar angle of $\theta_{I,Lab}$ and azimuthal angle of $\varphi_{I,Lab}$ with electric field:

$$\vec{E}_I(\vec{r},t) = \vec{E}_{I,0}\, e^{i(\vec{k}_I \cdot \vec{r} - \omega t)} \tag{10}$$

Where $\vec{E}_{I,0}$ is incident amplitude which can have any elliptic polarization due to isotropy of the incidence medium, and $\vec{k}_I$ is its wave vector. The reflected and transmitted waves can be written in the form:

$$\vec{E}_R(\vec{r},t) = \vec{E}_{R,0} \, e^{i(\vec{k}_R.\vec{r}-\omega t)}$$
$$\vec{E}_T(\vec{r},t) = \vec{E}_{T,0} \, e^{i(\vec{k}_T.\vec{r}-\omega t)} \tag{11}$$

Where $\vec{E}_{R,0}$ and $\vec{E}_{T,0}$ are amplitudes of the reflected and transmitted waves, and $\vec{k}_R$ and $\vec{k}_T$ are respective wave vectors. The phase matching condition in the interface implies that:

$$\vec{E}_R(\vec{r},t) = \vec{E}_{R,0} \, e^{i(\vec{k}_R.\vec{r}-\omega t)}$$
$$\vec{E}_T(\vec{r},t) = \vec{E}_{T,0} \, e^{i(\vec{k}_T.\vec{r}-\omega t)} \tag{12}$$

Applying this condition retrieves the common optics laws of isotropic media: 1) the reflected and transmitted waves lie in the incidence plane, 2) the angle of reflection wave (both polar and azimuthal) is equal to that of incident wave, and 3) the Snell's law is true in the form:

$$n \, Sin(\theta_{I,Lab}) = N_T \, Sin(\theta_{T,Lab}) \tag{13}$$

where n is refractive index of incident medium, N_T is effective refractive index of the biaxial medium in the transmission direction, and $\theta_{T,Lab}$ is the polar angle of the transmitted (or refracted) wave. N_T and $\theta_{T,Lab}$ are both unknown in (13) and must be found. In order to find a second complementary equation, the unit vector of the transmitted wave is used:

$$\hat{e}_{T,Lab} = \begin{bmatrix} Sin\,(\theta_{T,Lab})\,Cos\,(\varphi_{I,Lab}) \\ Sin\,(\theta_{T,Lab})\,Sin\,(\varphi_{I,Lab}) \\ Cos\,(\theta_{T,Lab}) \end{bmatrix} \tag{14}$$

This unit vector is transformed to principal frame by Euler matrix:

$$\hat{e}_{T,\mathrm{Princ}}(\theta_{T,Lab}) = [Euler(\alpha,\beta,\gamma)]\,\hat{e}_{T,Lab} \tag{15}$$

Inserting this unit vector in (8) the complementary equation is found:

$$\frac{e_{T,\mathrm{Princ},x}^2\left(\theta_{T,Lab}\right)}{N_T^2-n_x^2}+\frac{e_{T,\mathrm{Princ},y}^2\left(\theta_{T,Lab}\right)}{N_T^2-n_y^2}+\frac{e_{T,\mathrm{Princ},z}^2\left(\theta_{T,Lab}\right)}{N_T^2-n_z^2}-\frac{1}{N_T^2}=0 \tag{16}$$

Simultaneous solution of (13) and (16) yields two transmission angles and one refractive index for each:

$$\begin{cases}\left(\theta_{T,Lab,1},N_{T,1}\right)\\ \left(\theta_{T,Lab,2},N_{T,2}\right)\end{cases} \tag{17}$$

This equation shows that there are two transmitted waves that fulfill the Snell's law simultaneously. So, an incident wave on the biaxial medium is refracted to two transmitted waves. Hence, the transmitted wave in (11) should be modified as:

$$\vec{E}_T\left(\vec{r},t\right)=\vec{E}_{T1,0}\ e^{i\left(\vec{k}_{T1}\cdot\vec{r}-\omega t\right)}+\vec{E}_{T2,0}\ e^{i\left(\vec{k}_{T2}\cdot\vec{r}-\omega t\right)} \tag{18}$$

Where $\vec{E}_{T1,0}$ and $\vec{E}_{T2,0}$ are amplitudes of the two transmitted waves and $\vec{k}_{T1}$ and $\vec{k}_{T2}$ are respective wave vectors. $\vec{E}_{T1,0}$ and $\vec{E}_{T2,0}$ are obtained using (9) as

$$\vec{E}_{T1,0}=A_{T1}\begin{pmatrix}\dfrac{e_{T1,\mathrm{Princ},x}\left(\theta_{T,Lab,1}\right)}{N_{T,1}^2-n_x^2}\\[2mm]\dfrac{e_{T1,\mathrm{Princ},y}\left(\theta_{T,Lab,1}\right)}{N_{T,1}^2-n_y^2}\\[2mm]\dfrac{e_{T1,\mathrm{Princ},z}\left(\theta_{T,Lab,1}\right)}{N_{T,1}^2-n_z^2}\end{pmatrix}\ ,\quad \vec{E}_{T2,0}=A_{T2}\begin{pmatrix}\dfrac{e_{T2,\mathrm{Princ},x}\left(\theta_{T,Lab,2}\right)}{N_{T,2}^2-n_x^2}\\[2mm]\dfrac{e_{T2,\mathrm{Princ},y}\left(\theta_{T,Lab,2}\right)}{N_{T,2}^2-n_y^2}\\[2mm]\dfrac{e_{T2,\mathrm{Princ},z}\left(\theta_{T,Lab,2}\right)}{N_{T,2}^2-n_z^2}\end{pmatrix} \tag{19}$$

So far, all of the unknowns including effective refractive index and amplitude of the transmitted light were found in terms of initial values such as incidence angle, refractive indices of media, and Euler matrix. However, $\vec{E}_{R,0}$, A_{T1}, and A_{T2} are still unknown. As the reflected wave is in the isotropic medium, its amplitude $\vec{E}_{R,0}$ can be expanded in terms of p (parallel to incidence plane) and s (perpendicular to incidence plane) polarizations:

$$\vec{E}_{R,0}=E_{R,0,p}\ \hat{p}+E_{R,0,s}\ \hat{s} \tag{20}$$

With this definition, the unknown vector $\vec{E}_{R,0}$ is decomposed into two unknown

scalars $E_{R,0,p}$ and $E_{R,0,s}$. So, if the four unknowns A_{T1}, A_{T2}, $E_{R,0,p}$, and $E_{R,0,s}$ are found somehow, then the reflection and transmission problem is completely solved. To find them, boundary conditions must be applied. Continuation of tangential components of $\vec{E}$ and $\vec{H}$ fields at interface yields four scalar equations for these four unknowns and uniquely gives the amplitudes of reflected and transmitted waves. Of course, application of boundary conditions is easier in lab frame than principal one, so, the transmission amplitude in (19) must be transformed to lab frame by inverse Euler matrix. A computer code has been developed by the authors in Mathematica software and calculates the reflection and transmission of any incidence light with arbitrary polarizations.

As a numerical example assume:

$$n_1 = 1 \, , \, n_x = 1.2 \, , \, n_y = 2.2 \, , \, n_z = 3.2$$
$$\alpha = \frac{\pi}{5} \, , \, \beta = \frac{\pi}{6} \, , \, \gamma = \frac{\pi}{7} \tag{21}$$

A plane wave with *p*, *s*, and circular polarizations is incident on the biaxial medium in arbitrary polar and azimuthal angles. The reflection coefficient for *s* and *p* polarization was calculated by the computer code and plotted in left and right column of Fig. 2, respectively. The upper row of the figure is for incident *s* polarization, the middle row for *p* and the lower row for circular ones. The upper row shows that, in reflection of incident *s* polarization, both *s* and *p* polarizations are created. This is contradicting with isotropic media that only *s* polarization is created upon reflection of incident *s* polarized light. Another important thing is that again in contrast to isotropic media, the reflection is dependent on azimuthal angle in addition to polar angle of incidence.
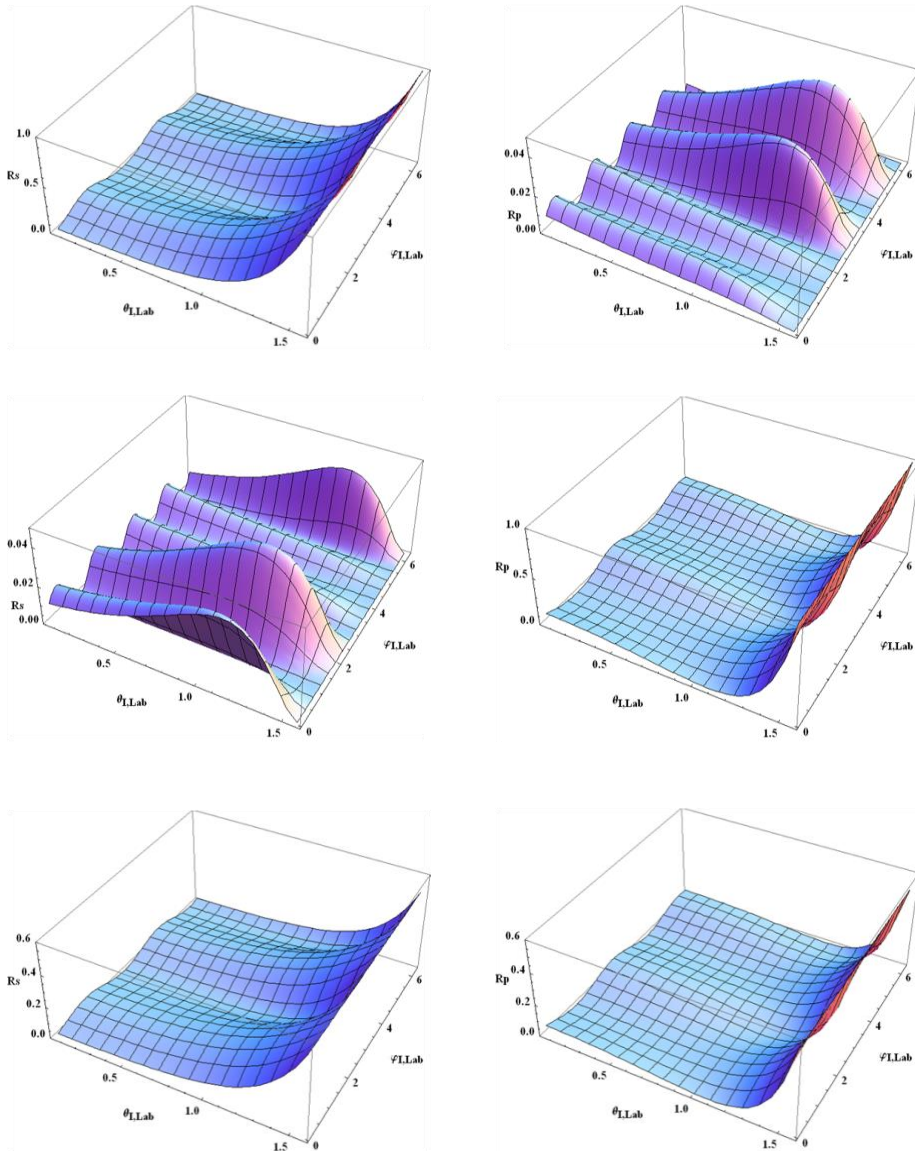
Fig. 2. Reflection coefficient of *s* (left) and *p* (right) polarizations as functions of incidence polar, $\theta_{I,Lab}$, and azimuthal, $\varphi_{I,Lab}$, angles. Upper row is for incident *s* polarization, and middle and lower rows for *p* and circular ones, respectively.

4. REFLECTION INTERFERENCE FROM A BIREFRINGENT THIN FILM

A flat biaxial thin film of thickness *h* placed between two isotropic media is considered. A plane wave with given polarization is incident on the film at polar

angle $\theta_{I,Lab}$ and azimuthal angle $\varphi_{I,Lab}$. A part of the wave is reflected from the upper surface and the other part is transmitted to film in the form of two refracted waves. These two waves reach the lower surface of the film. There, a part of them is reflected back to the biaxial medium and the other part is transmitted to the lower medium. Phase matching condition in the lower surface implies that each of the two waves is converted to two reflected and one transmitted waves. The four reflected waves move toward the upper surface and generate four transmitted waves to upper medium and eight reflected waves to biaxial medium, Fig. 3. This procedure continues until an infinite number of reflected and transmitted waves in upper, biaxial, and lower media are produced. As mentioned in references [for example 29 and 32] the superposition of these infinite waves results in a single up-going wave in the upper medium, a single down-going wave in the lower medium, and four (two up-going and two down-going) waves in the biaxial film. This is, indeed, an infinite-reflection approach. On the other hand, the method that includes only one reflection from the upper surface and one reflection from the lower surface and calculates the superposition of the five waves sent to the upper medium is called double-reflection approach. This approach, as mentioned in the introduction, is true only when the incident light has not sufficient coherence length to involve higher order reflections in the calculations. An important point evident in Fig is that each incident wave on internal surface of the film produces two reflected waves with different angles. This is the other optical properties of the biaxial media.
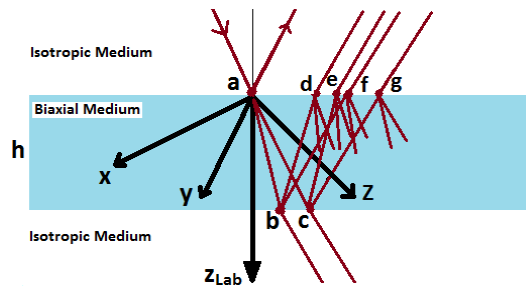

Fig. 3. Schematic of a biaxial thin film placed between two isotropic media with *x*, *y*, and *z* as its principal axes. Each incident wave is converted to two reflected waves with different angles upon internal reflections.

   The developed computer code can numerically calculate interference patterns of biaxial thin films with arbitrary number of reflections. In the double-reflection mode, the code finds the amplitudes of the five waves sent to the upper medium by applying boundary conditions in points a, b, c, d, e, f, and g denoted in Fig and sums over them to form the interference pattern. As an example, the interference pattern of a biaxial thin film with thickness $h=1\mu m$ placed in vacuum is

considered. The Euler angles are assumed as:

$$\alpha = \frac{\pi}{5} , \beta = \frac{\pi}{6} , \gamma = \frac{\pi}{7} \qquad (22)$$

A circularly polarized white light in the wavelength range of $400 \, nm \le \lambda \le 800 \, nm$ is incident on the biaxial film under constant incident angles. The interference patterns of $\theta_{I,Lab} = \pi/3$ and $\varphi_{I,Lab} = \pi/3$ for films with increasing birefringence (but with equal values of average refractive index of n_{ave}=1.8) are plotted in Fig. 4 from left to right.
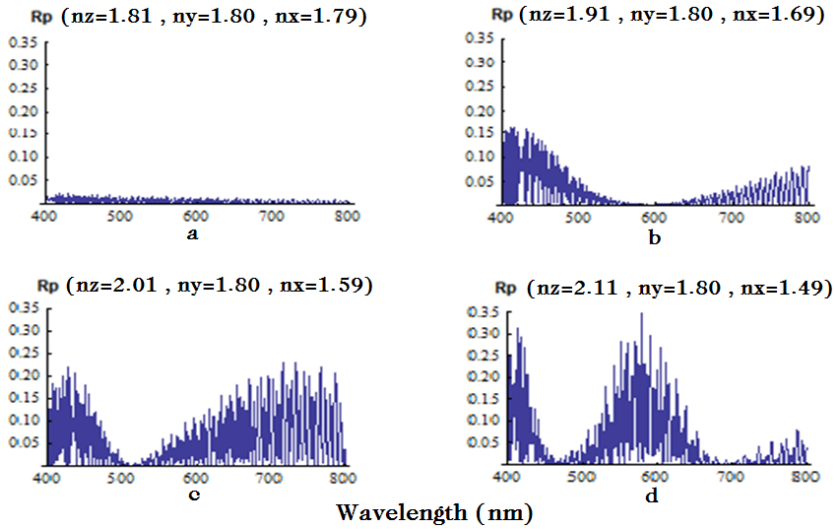

Fig. 4. Reflection coefficient of p polarization for circularly polarized white light incident with $\varphi_{I,Lab}$=π/3 and $\theta_{I,Lab}$=π/3 angles. The birefringence is increased from 'a' to 'd' with a fixed average index of n_{ave}=1.8.

The second example is for the same film as previous example but with different incident angles of $\theta_{I,Lab} = \pi/3$ and $\varphi_{I,Lab} = \pi/6$. The interference pattern is plotted in Fig. 5.

Inspection of these figures reveals important things. The first is that, by increasing the birefringence value a kind of banding (or interference pattern) is observed in reflection. On the other hand some bands of wavelength have higher reflection and some smaller one. The second finding is that, with a fixed value of average index, increasing birefringence value leads to higher level of reflection (see the increasing level of reflection from 'a' to 'd' in both figures).
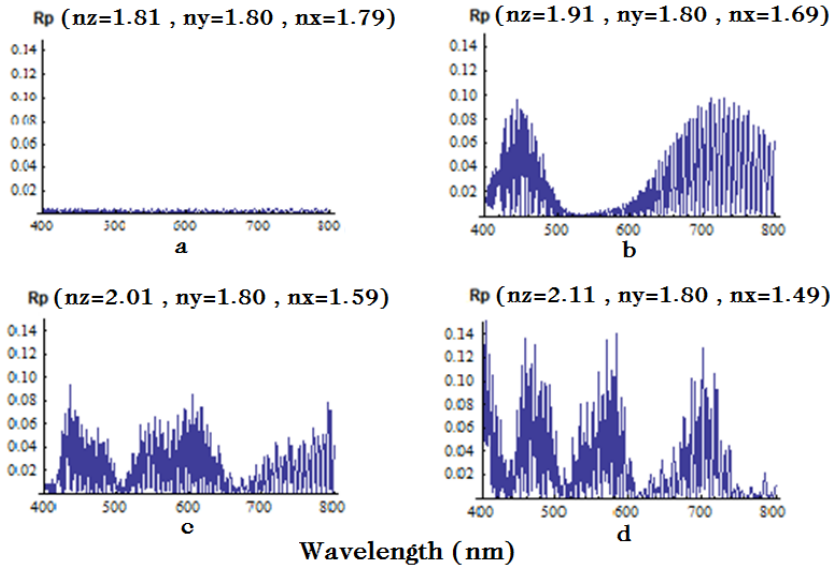
Fig. 5. Reflection coefficient of *p* polarization for circularly polarized white light incident with $\varphi_{I,Lab}=\pi/6$ and $\theta_{I,Lab}=\pi/3$ angles. The birefringence is increased from 'a' to 'd' with a fixed average index of $n_{ave}=1.8$.

5. SUMMARY AND CONCLUSION

In this paper, the interference pattern of a biaxial thin film with arbitrary orientation of principal axes is calculated numerically by a home-made computer code. Calculations show that upon reflection from birefringent films, polarizations are mixed which each other and it is related on azimuthal incident angle in addition to polar one. It was also observed that a kind of modulation (or banding) appears in the interference pattern produced by white light illumination. The number of bands is increased by increasing birefringence. On the other hand, increasing birefringence leads to increased reflection. The reported banding of reflection coefficient due to birefringence is a new finding that is not reported in the literature and the authors are indeed the first ones who are observing it. One of the applications of such a discovery is determination of birefringence properties of crystals and also study of electro-optical effects such as 'Kerr' and 'Pockels' ones which is considered as future works.

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