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Determining and Ranking Efficiency of Various Product Grades Produced in Jam Polyethylene Plants using Fuzzy Data Envelopment Analysis

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Abstract

With the growing trend in globalization and market competitiveness, process and resource optimization and the production of highly efficient products have become among the concerns of corporate managers. This research aims to evaluate and rank different polyethylene product grades produced at the Jam Petrochemical Complex using the Fuzzy Data Envelopment Analysis (FDEA) based upon fuzzy arithmetic [1]. The input-oriented fuzzy BCC model was suitable and applied to obtain the fuzzy efficiencies of different grades of polyethylene produced at the Jam Petrochemical Complex (13 DMUs) based on identified input and output indicators (Two inputs and three outputs). Then, a preference-degree approach is applied to compare and rank fuzzy DMU efficiencies. Based on the results, products HD52518, HD52505UV, and HM9450F were ranked first to third, respectively. The results highlight significant disparities in efficiency among the grades, providing a basis for targeted improvements.

Keywords: Fuzzy Data Envelopment Analysis; Efficiency; Ranking; Polyethylene.

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1. Introduction

The industrialization of societies and the advancement of technology have intensified global business competition, necessitating the use of management science and process optimization to gain a competitive edge. In the petrochemical industry, specifically polyethylene production, assessing the efficiency of various product grades is crucial for optimizing production processes and maintaining market competitiveness.

This research focuses on the polyethylene production units at the Jam Petrochemical Complex, evaluating the efficiency of various product grades. The Fuzzy Data Envelopment Analysis (FDEA) [1] is employed to handle the inherent uncertainty and fuzziness in the data, offering a comprehensive efficiency assessment. Also, this research demonstrates the applicability of FDEA in the petrochemical industry, offering a framework for efficiency evaluation and decision-making. This research aims to rank the different grades of polyethylene produced at the Jam Petrochemical Complex using the FDEA. By identifying and analyzing suitable input and output criteria and determining the appropriate FDEA model, this research seeks to evaluate the efficiency of each product grade produced in Jam Polyethylene Plants and provide actionable insights for process improvement. We could not find the use of FDEA to rank the efficiency of polyethylene grades in previously published research. Using FDEA for this purpose is the main novelty of current research.

2. Literature Review

2.1. DEA and Fuzzy DEA

DEA is a non-parametric method in operations research and economics for estimating production frontiers. Frontiers are applied to evaluate the efficiency of decision-making units (DMUs). It was introduced by Farrell (1957) [2] and further developed by Charnes, Cooper, and Rhodes (1978) [3]. Various theoretical extensions have been developed for DEA [4-8]. DEA allows for assessing relative efficiency among similar units, considering multiple inputs and outputs without requiring explicit functional forms.

Fuzzy logic first appeared in the scene of new computing in 1965, following the introduction of the theory of fuzzy sets by Lotfizadeh. A well-known approach for determining an efficient frontier under uncertainty is Stochastic Frontier Analysis, introduced in 1977, which can identify efficient units by taking several inputs and one output. In 1990, Fare and Grosskopf changed the above model with more complexity to a model with multiple inputs and outputs. The complexities in this model made it unattractive. Cooper et al. (1999) introduced Imprecise Data Envelopment Analysis. This model can evaluate the performance of DMUs in the presence of ambiguous data. In chance-constrained DEA, inputs are definite, and outputs are random. The Monte Carlo simulation method, introduced by some researchers such as Banker in 1987, was used in data coverage analysis. In 1998, Simar and Wilson used the bootstrapping method to introduce uncertainty in DEA by constructing confidence intervals regarding performance scores. Other methods in this field include regression analysis, statistical tolerance, and super-

efficiency techniques such as Anderson and Peterson [5], Fuzzy DEA, and Gray DEA [9-11].
FDEA extends DEA by incorporating fuzzy set theory to handle imprecise data. It can be particularly beneficial in real-world scenarios where data uncertainty and ambiguity are prevalent. FDEA has been applied in various industries, including healthcare, manufacturing, and energy, to provide a more robust efficiency evaluation framework.

2.2. Applications in Industry

The application of DEA and FDEA in industrial contexts, especially in the petrochemical and polymer industries, has shown promising results. Studies have demonstrated the utility of DEA in evaluating operational efficiencies, environmental impacts, and productivity improvements. For example, Han et al. (2015) used FDEA to assess energy efficiency in ethylene production systems, highlighting significant efficiency variations and potential areas for optimization [12]. Similarly, Mardani et al. (2017) provided a comprehensive review of DEA applications in energy efficiency, underscoring its relevance and adaptability across different industrial sectors [13]. Du et al. have used this method to plan the production of various units in a central management system in such a way that the overall efficiency of the system is maximized [14]. Ghiyasi and Mokhberian (2024) rummage through the environmental and technical performance of Iranian gas refineries through the DEA [15]. D'Angelo et al. (2023) used the DEA to compare the assessed technologies in terms of economic and environmental life cycle assessment (LCA) indicators, ranking the different technologies and estimating their required improvement targets [16]. Tabatabaei et al. (2022) present a common-weights approach for a relational network DEA model in a fuzzy environment to measure the efficiencies of the system and the component processes in gas refineries [17]. Peng et al. (2021) present an attempt to combine the methods of generalized DEA and TOPSIS. By establishing a green material index system, the G-CCR model of generalized DEA was first used to select effective materials from the candidate samples, and TOPSIS was then used to sort the effective suppliers [18]. Ahmadi and Ahmadi (2012) evaluated the technical efficiency of 23 industries in Iran and the overall efficiency of the provinces with two methods of output-oriented DEA [19]. Lotfi et al. (2013) researched measuring the productivity and ranking of chemical companies using the combination of DEA models and selective hierarchical analysis [20]. Xin and Sun (2014) have used the DEA method to evaluate the environmental level of the petrochemical industry park [21]. In general, the use of this method in ranking industries and measuring productivity can see in many researches [9, 12-13, 22-30].

3. Methodology

3.1. Research Design

This research utilizes the FDEA to evaluate the efficiency of polyethylene product grades at the Jam Petrochemical Complex (**Figure 1**). The FDEA is applied for its ability to handle data uncertainty, providing a more accurate efficiency assessment.

3.2. Expert team formation

An expert team was formed consisting of professionals based on the following characteristics:

- Familiarity with the production, sale, purchase, repair, and maintenance processes.

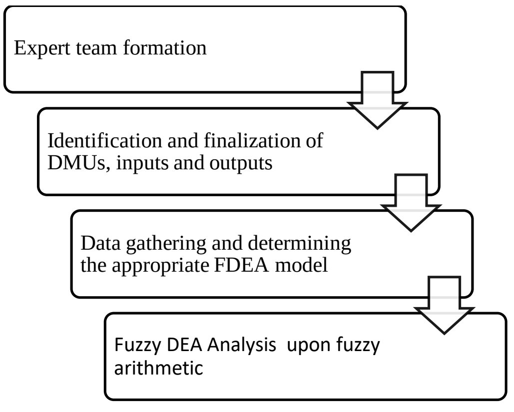

Figure 1. Methodology Diagram

- Familiarity with production grades in polyethylene plants.
- Knowledge of financial indicators in calculating the cost of products.
- Work experience.
- The degree of influence.

3.3. Identification and finalization of DMUs, inputs and outputs

By considering production history, processed data, and experts' opinions, DMUs can identified.

The initial list of inputs and outputs can identify through literature review. Then, a questionnaire can create and given to the experts to collect their opinions. Experts' views can collect through descriptive variables in the consensus meeting. Then, the final list of inputs and outputs can identify considering experts' views.

3.4. Data gathering and determining the appropriate FDEA model

The value of inputs and outputs should be in triangular fuzzy number (TFN) format. At this stage, first, the TFNs of the inputs and outputs of each DMU should determine. The TFN DMUs' data gathering for inputs and outputs can done by considering production history, processed data, and experts' opinions considering the nature of inputs and outputs.

Then, the appropriate FDEA model should determine. Some features of DEA models are mentioned in [36].

There are three types of DEA models. In the input-oriented DEA models, the DMUs reach the efficient frontier by decreasing input without increasing output values. In the output-oriented DEA models, the DMUs reach the efficient frontier by increasing output without decreasing input values. In input-output-oriented DEA models, efficient DMUs drive the efficient frontier by reducing input and increasing output values. So, to determine suitable DEA models, defining the type of return to scale (such as fixed or variable) and the nature of the model (such as input-oriented or output-oriented) are necessary.

In radial DEA models (CCR models), inputs and outputs have to decrease or increase with the same ratio, but in nonradial models (such as BCC models), the change of ratios can be different.

3.5. Fuzzy DEA Analysis upon fuzzy arithmetic

We applied FDEA according to [31]. The FDEA provides methods to build models and rank the DMUs based on the obtained efficiencies. This method has been used in many academic studies by many researchers. For example, we can refer to Abbasi and Kaviani (2016)[9], Arana-Jiménez et al. (2022)[32], Bagheri et al. (2022)[33], and Akram et al. (2023)[34-35].

3.5.1. Fuzzy DEA model based upon fuzzy arithmetic

A fuzzy number is a convex fuzzy set defined by a specific range of real numbers, where each number has a membership grade between 0 and 1. TFNs are the most widely used type, with their membership functions defined accordingly.

$$\mu_{\tilde{A}}(x) = \begin{cases} (x-a)/(b-a), & a \le x \le b \\ (d-x)/(d-b), & b \le x \le d \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

TFNs are often represented as (a, b, d) to keep things brief.

Consider a scenario with n DMUs, m inputs, and s outputs. Let x_{ij} *(i=1, …, m)* and y_{rj} *(r=1, …, s)* represent the input and output data for DMU_j (j=1, …, n). Without loss of generality, we assume all input and output data x_{ij} and y_{rj} are uncertain and characterized by TFNs $\tilde{x}_{ij} = \left(x_{ij}^L, x_{ij}^M, x_{ij}^U\right)$ and $\tilde{y}_{rj} = \left(y_{rj}^L, y_{rj}^M, y_{rj}^U\right)$, where $x_{ij}^L > 0$ and $y_{rj}^L > 0$ for $i = 1, \ldots, m$; $r = 1, \ldots, s$ and $j = 1, \ldots, n$. It is mentionable that crisp input and output data are treated as a special case of TFNs input ($\tilde{x}_{ij}$), and TFNs output ($\tilde{y}_{rj}$), where $x_{ij}^L = x_{ij}^M = x_{ij}^U$ and $y_{rj}^L = y_{rj}^M = y_{rj}^U$. The efficiency of DMU_j is defined as follows:

$$\tilde{\theta}_j = \sum_{r=1}^{s} u_r \tilde{y}_{rj} / \sum_{i=1}^{m} v_i \tilde{x}_{ij} \quad (2)$$

This fuzzy number is referred to as fuzzy efficiency, where u_r ($r = 1, \ldots, s$) and v_i ($i = 1, \ldots, m$) represent the weights assigned to the outputs and inputs, respectively. Based on fuzzy arithmetic, the fuzzy efficiency defined in Equation (2) can expressed as follows:

$$\begin{aligned} \tilde{\theta}_j &= \frac{\sum_{r=1}^{s} u_r \left[y_{rj}^L, y_{rj}^M, y_{rj}^U\right]}{\sum_{i=1}^{m} v_i \left[x_{ij}^L, x_{ij}^M, x_{ij}^U\right]} \\ &= \frac{\sum_{r=1}^{s} u_r y_{rj}^L, \sum_{r=1}^{s} u_r y_{rj}^M, \sum_{r=1}^{s} u_r y_{rj}^U}{\sum_{i=1}^{m} v_i x_{ij}^L, \sum_{i=1}^{m} v_i x_{ij}^M, \sum_{i=1}^{m} v_i x_{ij}^U} \\ &\approx \left[\frac{\sum_{r=1}^{s} u_r y_{rj}^L}{\sum_{i=1}^{m} v_i x_{ij}^U}, \frac{\sum_{r=1}^{s} u_r y_{rj}^M}{\sum_{i=1}^{m} v_i x_{ij}^M}, \frac{\sum_{r=1}^{s} u_r y_{rj}^U}{\sum_{i=1}^{m} v_i x_{ij}^L}\right] \end{aligned} \quad (3)$$

To evaluate the fuzzy efficiency of DMU_j in comparison to the other DMUs, we can impose the condition $\tilde{\theta}_j \le 1$ for all $j = 1, \ldots, n$. Using these constraints, the following fuzzy DEA model is developed to assess the performance of each DMU:

$$\begin{aligned} &Max\ \tilde{\theta}_0 \approx [\theta_0^L, \theta_0^M, \theta_0^U] \\ &= \left[\frac{\sum_{r=1}^{s} u_r y_{r0}^L}{\sum_{i=1}^{m} v_i x_{i0}^U}, \frac{\sum_{r=1}^{s} u_r y_{r0}^M}{\sum_{i=1}^{m} v_i x_{i0}^M}, \frac{\sum_{r=1}^{s} u_r y_{r0}^U}{\sum_{i=1}^{m} v_i x_{i0}^L}\right], \quad (4) \\ &\text{Subjected to} \\ &\tilde{\theta}_j \approx \left[\theta_j^L, \theta_j^M, \theta_j^U\right] \\ &= \left[\frac{\sum_{r=1}^{s} u_r y_{rj}^L}{\sum_{i=1}^{m} v_i x_{ij}^U}, \frac{\sum_{r=1}^{s} u_r y_{rj}^M}{\sum_{i=1}^{m} v_i x_{ij}^M}, \frac{\sum_{r=1}^{s} u_r y_{rj}^U}{\sum_{i=1}^{m} v_i x_{ij}^L}\right] \le 1, \\ &j = 1, \ldots, n, \\ &u_r, v_i \ge 0, \quad i = 1, \ldots, m; r = 1, \ldots, s, \end{aligned}$$

where the subscript zero denotes the DMU being evaluated, i.e. DMU_0.

The fuzzy model (4) can simplified into three fractional programming models (models (5)-(7)).

$$Maximize\ \theta_0^L = \frac{\sum_{r=1}^{s} u_r y_{r0}^L}{\sum_{i=1}^{m} v_i x_{i0}^U}, \quad (5)$$

Subjected to

$$\theta_j^M = \frac{\sum_{r=1}^{s} u_r y_{rj}^M}{\sum_{i=1}^{m} v_i x_{ij}^M} \le 1, \quad j = 1, \ldots, n,$$

$$u_r, v_i \ge 0, \quad i = 1, \ldots, m; r = 1, \ldots, s,$$

$$Maximize\ \theta_0^M = \frac{\sum_{r=1}^{s} u_r y_{r0}^M}{\sum_{i=1}^{m} v_i x_{i0}^M}, \quad (6)$$

Subjected to

$$\theta_j^M = \frac{\sum_{r=1}^{s} u_r y_{rj}^M}{\sum_{i=1}^{m} v_i x_{ij}^M} \leq 1, \quad j = 1, \dots, n,$$

$$u_r, v_i \geq 0, \qquad i = 1, \dots, m; r = 1, \dots, s,$$

$$Maximize\ \theta_0^U = \frac{\sum_{r=1}^{s} u_r y_{r0}^U}{\sum_{i=1}^{m} v_i x_{i0}^L}, \tag{7}$$

Subjected to

$$\theta_j^M = \frac{\sum_{r=1}^{s} u_r y_{rj}^M}{\sum_{i=1}^{m} v_i x_{ij}^M} \leq 1, \quad j = 1, \dots, n,$$

$$u_r, v_i \geq 0, \qquad i = 1, \dots, m; r = 1, \dots, s,$$

Then, fractional models (5)-(7) can be converted to three linear programming models considering features of suitable DEA models. By solving the three linear programming models for each DMU, we obtain the TFN of DMUs.

3.5.2. A preference degree approach to compare and rank fuzzy efficiencies

Obtained TFNs can be applied to compare and rank the DMUs. Let $\tilde{a} = (a_L, a_M, a_U)$ and $\tilde{b} = (b_L, b_M, b_U)$ represent two triangular fuzzy efficiencies (TFNs). $P(\tilde{a} > \tilde{b})$ can calculated by Eq. (8).

$$P(\tilde{a} > \tilde{b}) = \begin{cases} 1, & if\ a_L \geq b_U \\ 0, & if\ a_U \leq b_L \\ \dfrac{(a_U - b_L)^2}{(a_U - b_L + a_M - b_M)(a_U - a_L + b_U - b_L)}, & if\ (a_U > b_L) \cap (a_M \leq b_M) \\ 1 - \dfrac{(b_U - a_L)^2}{(b_U - a_L + a_M - b_M)(a_U - a_L + b_U - b_L)}, & if\ (a_M > b_M) \cap (a_L < b_U) \end{cases} \tag{8}$$

To establish a complete ranking order for these fuzzy efficiencies, follow these steps:

1. **Calculate the Degree of Preference Matrix:**

Eq. (9) shows the format of the Degree of Preference Matrix (M_p).

$$M_p = \begin{array}{c} \\ \tilde{\theta}_1 \\ \tilde{\theta}_2 \\ \vdots \\ \tilde{\theta}_n \end{array} \begin{array}{c} \begin{array}{cccc} \tilde{\theta}_1 & \tilde{\theta}_2 & \dots & \tilde{\theta}_n \end{array} \\ \begin{bmatrix} - & p_{12} & \cdots & p_{1n} \\ p_{21} & - & \cdots & p_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ p_{n1} & p_{n2} & \dots & - \end{bmatrix} \end{array} \tag{9}$$

while $p_{ij} = p(\tilde{\theta}_i > \tilde{\theta}_j)\ (i, j = 1, \dots, n)$ calculated by Eq. (8).

2. **Identify the Leading Fuzzy Efficiency:** Find a row in the degree of preference matrix where all elements, except for the diagonal element, are greater than or equal to 0.5. If this row corresponds to $\tilde{\theta}_i$, $\tilde{\theta}_i$ has the highest fuzzy efficiency.

3. **Update the Matrix:** Remove the iii-th row and iii-th column (thus removing $\tilde{\theta}_i$) from the matrix. In the reduced matrix, if $\tilde{\theta}_j$ appears as the highest fuzzy efficiency among the remaining ones, rank $\tilde{\theta}_j$ in the second place.

$\tilde{\theta}_i \overset{p_{ij}}{\gtrsim} \tilde{\theta}_j$ if $p_{ij} > 0.5$ or $\tilde{\theta}_i \sim \tilde{\theta}_j$ if $p_{ij} = p_{ji} = 0.5$.

4. **Iterate for Remaining Efficiencies:** Remove the j-th row and j-th column from the reduced matrix and repeat the process until all fuzzy efficiencies are ranked appropriately.

4. Results

4.1. Expert team

Fourteen people working in the petrochemical field (considering Section 3.1 characteristics) were selected as an expert team (**Table 1**).

Table 1. Expert team specifications

Exp. No.	Responsibilities
1	Factory manager
2	Technical assistant of factory manager
3	Chief of integrated planning
4	Head of sales and marketing
5	Head of operation
6	Head of strategic planning
7	Head of management information
8	Head of Technical Services
9	Head of process engineering
10	Head of production planning
11	Head of technical, economic, and financial evaluation of projects and plans
12	Head of HDPE plant
13	Head of LLDPE plant
14	Deputy of operation

4.2. DMUs, inputs and outputs

According to production history, processed data, and experts' opinions, 13 DMUs are identified **(Table 2)**.

Table 2. DMUs

DMU	Plant	Grade name
1	HDPE	CRP100 B
2	HDPE	CRP100 N
3	LLDPE	HD5000S
4	LLDPE	HD52505UV
5	LLDPE	HD52518
6	LLDPE	HD60507 UV
7	HDPE	HF4760 (BL3)
8	HDPE	HM5010T2N (EX3)
9	HDPE	HM9450F (EX5)
10	HDPE	HM8355 (BL4)
11	LLDPE	LL22501AA
12	LLDPE	LL22501KJ
13	LLDPE	MD38504UV

The selection criteria for input and output indicators included relevance to production efficiency, data availability, and the ability to capture accurately the performance of different polyethylene grades. In these regards, data was collected from the Jam Petrochemical Complex, focusing on various grades of polyethylene products to distinguish inputs and outputs. We finally identified the input and output indicators through a review of production processes and consultations with experts (**Table 3**).

Table 3- Indicator data sources

Indicator	Unit	Required data	Data provider
Utility consumption	Gj/day	Utility consumption	Process engineering, production control, management information, and energy.
Operational difficulty	-	Questionnaire, non-conforming product amount, maintenance, and process problems.	Maintenance, operation, process engineering, and management information.
Amount of product	Ton/day	The amount of product	Process engineering, production control, management information
Gross profit	$/day	Price and quantity of raw materials, chemicals, utilities, and products	Sales, process engineering, finance, management information, NPC.
Market demand	-	Questionnaire	Marketing and sales data, management information.

Then, the identification of inputs and outputs was necessary. So, we created a suitable questionnaire based on a literature review. Then, according to the results obtained from the completed questionnaires, as shown in **Tables 4 and**

5, "utility consumption" and "operational difficulty" were identified as inputs, and "product quantity", "gross profit" and "market demand" as outputs.

Table 4. Inputs

I	Name	Description	Unit
1	Utility consumption	They aren't used directly in the production process, such as water, electricity, steam, air, nitrogen, cooling water, etc.	Gj/day
2	Operational difficulty	Factors that stop/reduce production, create process problems, produce inconsistent products, clog lines and equipment, and increase the number of maintenance operations.	-

Table 5. Outputs

O	Name	Description	Unit
1	Product quantity	The amount of each grade.	Ton/day
2	Gross profit	The product sales price subtraction from the cost of feed, chemicals and utility services.	$/day
3	Market demand	The amount of product demand in the market.	-

4.3. Inputs' TFNs, outputs' TFNs, and appropriate DEA models

We identified the TFN of inputs and outputs according to processed data and experts' opinions.

Through data processing based on DMUs, the data was validated, sorted, summarized, aggregated, and classified. We analyzed I1, O1, and O2 values presented on different days (quantitative data) using SPSS software. In this regard, the minimum, average, and maximum values obtained from the output of SPSS software were considered as the lower, middle, and upper limit values of TFNs, respectively.

As seen in **Tables 4 and 5**, the value of inputs and outputs in the last column is "-" (I2 and O3) because the variable is descriptive and should gathered through experts' views. We determined the TFN of I2 and O3 (qualitative data) by considering experts' views using descriptive variables fuzzification (**Table 6**).

Table 6. Descriptive variables fuzzification method

Linguistic scale	5-point qualitative scale	TFN of I2	TFN of O3
Very Poor	1	(5, 6, 7)	(1, 2, 3)
Poor	2	(4, 5, 6)	(2, 3, 4)
Average	3	(3, 4, 5)	(3, 4, 5)
Good	4	(2, 3, 4)	(4, 5, 6)
Very Good	5	(1, 2, 3)	(5, 6, 7)

Tables 7 and 8 show the TFN of DMUs for inputs and outputs, respectively.

Table 7. TFN of inputs

DMU	I1	I2
1	(1760, 2224, 2543)	(5, 6, 7)
2	(1569, 2179, 2560)	(4, 5, 6)
3	(1972, 2179, 2297)	(3, 4, 5)
4	(1732, 2050, 2213)	(1, 2, 3)
5	(1470, 1952, 2133)	(1, 2, 3)
6	(1746, 2007, 2110)	(3, 4, 5)
7	(1497, 2157, 2614)	(1, 2, 3)
8	(1745, 2321, 2544)	(3, 4, 5)
9	(1002, 2201, 2590)	(1, 2, 3)
10	(2024, 2240, 2600)	(1, 2, 3)
11	(1875, 2128, 2260)	(5, 6, 7)
12	(1645, 2103, 2257)	(5, 6, 7)
13	(2054, 2078, 2097)	(3, 4, 5)

Table 8. TNF of outputs

DMU	O1	O2	O3
1	(563,792,948)	(182881,310861, 613071)	(5,6,7)
2	(552,830,973)	(52808,303440, 584204)	(5,6,7)
3	(570,873,984)	(-260,200664, 352438)	(3,4,5)
4	(603,957,1074)	(32325,141023, 254236)	(4,5,6)
5	(551,969,1124)	(-12545,139779, 326130)	(5,6,7)
6	(616,926,1019)	(92058,156842, 230828)	(2,3,4)
7	(560,901,1066)	(26409,182818, 460306)	(5,6,7)
8	(615,767,841)	(65030,183200, 279536)	(3,4,5)
9	(553,901,1019)	(83097,236039, 492700)	(4,5,6)

DMU	O1	O2	O3
10	(715,925,1028)	(138019,191532, 292962)	(2,3,4)
11	(557,851,988)	(-6954,96209, 210772)	(3,4,5)
12	(579,835,977)	(-10869,188687, 277715)	(3,4,5)
13	(922,1013,1058)	(356674,408392, 430647)	(1,2,3)

According to the properties of the case study and DEA models (described in section 3.4), a fuzzy input-oriented BCC model was suitable. So, fractional models (5)-(7) can convert to three input-oriented BCC models (models (10)-(12)).

$$Maximize\ \theta_0^L = \sum_{r=1}^{S} u_r y_{r0}^L - u_0^L, \qquad (10)$$

$$\text{S.t. } \sum_{i=1}^{m} v_i x_{i0}^U = 1,$$

$$\sum_{r=1}^{S} u_r y_{rj}^U - \sum_{i=1}^{m} v_i x_{ij}^U - u_0^L \le 0, \quad j = 1, \dots, n,$$

$$u_r, v_i \ \ge 0, \qquad i = 1, \dots, m; r = 1, \dots, s.$$

$$Maximize\ \theta_0^M = \sum_{r=1}^{S} u_r y_{r0}^M - u_0^M, \qquad (11)$$

$$\text{S.t } \sum_{i=1}^{m} v_i x_{i0}^M = 1,$$

$$\sum_{r=1}^{S} u_r y_{rj}^U - \sum_{i=1}^{m} v_i x_{ij}^U - u_0^M \le 0, \ j = 1, \dots, n,$$

$$u_r, v_i \ \ge 0, \qquad i = 1, \dots, m; r = 1, \dots, s.$$

$$Maximize\ \ \theta_0^U = \sum_{r=1}^{S} u_r y_{r0}^U - u_0^U, \qquad (12)$$

$$\text{S.t } \sum_{i=1}^{m} v_i x_{i0}^L = 1,$$

$$\sum_{r=1}^{S} u_r y_{rj}^U - \sum_{i=1}^{m} v_i x_{ij}^U - u_0^U \le 0, \quad j = 1, \dots, n,$$

$$u_r, v_i \ \ge 0, \qquad i = 1, \dots, m; r = 1, \dots, s.$$

4.1. Fuzzy efficiency of DMUs

By solving models (10)-(12) for each DMU (with GAMS software), the TFN of DMUs was obtained (Error! Reference source not found.).

Table 9. TFN of DMUs

DMU	Lower bound (Model 10)	Middle value (Model 11)	Upper bound (Model 12)
1	0.394023	0.45054	1
2	0.391406	0.459844	1
3	0.436221	0.459844	0.508114
4	0.452779	0.5	1
5	0.469761	0.51332	1
6	0.474882	0.499253	0.573883
7	0.383321	0.5	1
8	0.393868	0.431711	0.574212
9	0.386873	0.5	1
10	0.385385	0.5	1
11	0.443363	0.470865	0.5344
12	0.443952	0.476462	0.609119
13	0.477826	0.482194	0.572458

4.1. Ranking and interpretation of results

A preference-degree approach (section 3.5.2) was applied to rank the TFN of DMUs. In step 1, the matrix of the degree of preference is calculated by Eq. (8) for the fuzzy efficiencies to compare and rank DMUs **(Table 10)**.

Table 10. Matrix of the degree of preference for TFN of DMUs

DMU	01	02	03	04	05	06	07	08	09	10	11	12	13
01	-	0.4935	0.8182	0.4352	0.4173	0.6816	0.4669	0.7925	0.4654	0.4660	0.7705	0.6890	0.7027
02	0.5065	-	0.8285	0.4411	0.4229	0.6903	0.4725	0.7992	0.4710	0.4716	0.7802	0.6978	0.7121
03	0.1818	0.1715	-	0.0518	0.0266	0.0890	0.1371	0.5456	0.1330	0.1347	0.3396	0.2150	0.1047
04	0.5648	0.5589	0.9482	-	0.4801	0.8137	0.5298	0.8932	0.5284	0.5290	0.9058	0.8093	0.8377
05	0.5827	0.5771	0.9734	0.5199	-	0.8542	0.5490	0.9175	0.5476	0.5482	0.9372	0.8415	0.8739
06	0.3184	0.3097	0.9110	0.1863	0.1458	-	0.2652	0.7883	0.2616	0.2631	0.7879	0.5656	0.5711
07	0.5331	0.5275	0.8629	0.4702	0.4510	0.7348	-	0.8236	0.4986	0.4992	0.8210	0.7385	0.7570
08	0.2075	0.2008	0.4544	0.1068	0.0825	0.2117	0.1764	-	0.1730	0.1744	0.3711	0.2806	0.2300
09	0.5346	0.5290	0.8670	0.4716	0.4524	0.7384	0.5014	0.8270	-	0.5006	0.8250	0.7418	0.7607
10	0.5340	0.5284	0.8653	0.4710	0.4518	0.7369	0.5008	0.8256	0.4994	-	0.8234	0.7404	0.7592
11	0.2295	0.2198	0.6604	0.0942	0.0628	0.2121	0.1790	0.6289	0.1750	0.1766	-	0.3325	0.2539
12	0.3110	0.3022	0.7850	0.1907	0.1585	0.4344	0.2615	0.7194	0.2582	0.2596	0.6675	-	0.4842

13	0.2973	0.2879	0.8953	0.1623	0.1261	0.4289	0.2430	0.7700	0.2393	0.2408	0.7461	0.5158	-

The results of applying steps 2 to 4 are:

$$5 \overset{51.99\%}{\succ} 4 \overset{52.84\%}{\succ} 9 \overset{50.06\%}{\succ} 10 \overset{50.08\%}{\succ} 7 \overset{52.75\%}{\succ} 2 \overset{50.65\%}{\succ} 1 \overset{68.16\%}{\succ} 6 \overset{57.11\%}{\succ} 13 \overset{51.58\%}{\succ} 12 \overset{66.75\%}{\succ} 11 \overset{66.04\%}{\succ} 3 \overset{54.56\%}{\succ} 8$$

where $5 \overset{51.99\%}{\succ} 4$ reads as DMU5 performs better than DMU4 to the extent of 51.99%. DMU5 has the best performance, followed by DMUs 4, 9 and 10 (**Table 11**).

Table 11. DMUs ranking

Rank	DMU	Grade	Production plant
1	5	HD-52518	LLDPE
2	4	HD-52505UV	LLDPE
3	9	HM 9450F (EX5)	HDPE
4	10	HM8355 (BL4)	HDPE
5	7	HF 4760 (BL3)	HDPE
6	2	CRP100 N	HDPE
7	1	CRP100 B	HDPE
8	6	HD-60507 UV	LLDPE
9	13	MD-38504UV	LLDPE
10	12	LL-22501KJ	LLDPE
11	11	LL-22501AA	LLDPE
12	3	HD-5000S	LLDPE
13	8	HM-5010T2N	HDPE

Now, we want to analyze the results. The first four grades of **Table 11** (DMU5, DMU4, DMU9, and DMU10) are the most consumed products in the market, which allocated about 75% of the total volume and 72% of the production days of the products, while they have only 67% of the gross profit. The last three grades (DMU11, DMU3, and DMU8) account for 8.1% of total volume, 9.6% of product production days, and 7.5% of gross profit. The above data shows that the grades with a high rating compared to grades with a low rating have a higher production capacity. Also, despite having fewer production days of high rating grades, their profit is less.

Also, we compared the efficiency results with existing benchmarks and previous studies in the Jam petrochemical complex. The comparison validated the findings and highlighted the effectiveness of the FDEA in capturing efficiency variations.

The efficiency scores and rankings provide a detailed picture of the performance of different polyethylene grades. The high-performing grades demonstrate effective utilization of resources, while the lower-performing grades suggest areas for potential improvements.

5. Conclusion

This research applied the FDEA to evaluate and rank the efficiency of different polyethylene product grades at the Jam Petrochemical Complex. The findings revealed significant efficiency variations, providing actionable insights for process optimization.

Future research could explore the application of FDEA incorporating more diverse and comprehensive data sets. Additionally, longitudinal studies could provide insights into efficiency trends and the impact of implemented improvements.

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