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Efficiency analysis in semi-additive production technology in DEA in the presence of production trade-offs

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Abstract

In this paper, we evaluate the performance of decision-making units (DMUs) in semi-additive production technology in the presence of production trade-offs. We introduce the semi-additive production technology. The semi-additive technology is based on all the DMUs observed and the set of aggregated DMUs corresponding to these DMUs in data envelopment analysis (DEA). We obtain production trade-offs on input and output components in the production process in semi-additive technology. We present a single-stage model to measure efficiency in the presence of production trade-offs in semi-additive production technology. This model also identifies inefficiencies in all input and output components. We show an application of the presented model in the banking industry and at the end we bring the results of the paper.

Keywords: Data envelopment analysis; Efficiency; semi-additive production technology; production trade-offs.

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1. Introduction

DEA is a planning methodology for measuring the technical efficiency of a set of DMUs. This method was introduced by Charnes et al. [1] and was widely used as a powerful technique in various fields such as banking, healthcare, and education (Cook and Seiford [2], Liu et al. [3], Emrouznejad and Yang [4]). Production technology is a significant part of DEA. Different technologies are presented in DEA literatures. Charnes et al. [1] presented the first technology called production technology with constant returns to scales (CRS). Banker et al. [5] presented another technology with the property of variable returns to scales (VRS) under the name of BCC model. Next, other technologies were presented under the title of non-increasing efficiency technology by Koopmans [6]. Deprins et al. [7] introduced free disposal hull technology by removing the assumption of convexity. Another type of removal of the convexity principle from the principles of the subject of building the production possibility set (PPS) and providing another technology was presented by Petersen [8] which included two linear programming models. Green and Cook [9] presented another technology called additive technology based on observed DMUs and aggregation units that are formed by combining observed units. As stated, we can provide production technology based on observed DMUs and aggregation DMUs that create by observation DMUs.

The additive assumption states that the two observed units DMU1 and DMU2 can aggregate their activities and create a new unit called cumulative unit DMU1+DMU2. In this technology, by accepting the additive assumption, we assume that if observed units DMU1 and DMU2 are able to product, then the new unit created as an aggregated unit in the form of DMU1+DMU2 is also able to product and can exist. This principle is used in the relevant articles as additive and semi additive axioms. If we want to distinguish between these two assumptions, we can say that by accepting the as additive axiom, if an observed unit such as DMU1 belongs to the production technology and has the possibility of activity, then new aggregated units such as DMU1+DMU1 and DMU1+2DMU2 also belong to the set of production possibilities. But the principle of semi additive states that the new aggregation activities of units DMU1 and DMU2 in the form of DMU1+DMU2 belong to the set of production possibility and have the possibility of production and that the units constituting the new aggregation unit are distinct, that is, DMU1 and DMU2 are two different units namely $DMU1 \neq DMU2$. But based on the additive assumption, these units can be the same and have the same inputs and outputs (Ghiyasi [10], Ghiyasi and Cook [11]).

A way of applying value judgement of the decision-maker (DM) in the process of evaluating the performance of the DMUs in DEA is the use of weight restrictions on the input or output components. At first, Allen et al. [12] and Rolle et al. [13] presented the application of weight restrictions in multiple DEA models. Also, they showed that applying weight restrictions on input or output components, the efficiency score of DMU change. The efficiency score of the DMUs may decrease compared to before applying weight restrictions. Allen et al. [12] showed that applying weight restrictions in DEA models may lead to the model becoming infeasible. Podinovski [14] investigated the relationship between weight restrictions in multiplier DEA models and production trade-offs in envelopment DEA models. He presented production technology in the presence of production trade-offs in CRS technologies. Podinovski [12] calculated efficient targets in DEA models with production trade-offs. Podinovski and Bouzdine-Chameeva [15] raised the issue of unlimited and free production of output vector in production technology. They showed that we have unlimited and free production of outputs then DEA model in presence of production trade-offs will have unbounded optimal solution. The envelopment DEA model become infeasible. Podinovski and Bouzdine-Chameeva [16] investigated the relationship between weight restrictions in multiplicative DEA models and production trade-offs in envelopment DEA models. They presented production technology in the presence of production trade-offs in CRS and VRS technologies. They presented the necessary and sufficient conditions for the compatibility of weights in production trade-offs and stated how to avoid the inconsistency of weights by choosing a suitable production trade-offs matrix. Podinovski and Bouzdine-Chameeva [17] investigated consistent weight restrictions in DEA. Papaioannou and Podinovski [18] introduced production technologies with ratio inputs and outputs.

It can be said that the main contribution of this paper is as follows. We first present the semi-additive production technology by introducing the production axioms in DEA. In the following, we present a single-stage model for the simultaneous measurement of radial efficiency and mix inefficiency in the presence of production trade-offs in this technology.

The following of this paper is organized as follows. The second section presents the semi-additive production technology. The third section present a single model for measuring efficiency of DMUs with production trade-offs in the semi-additive production technology. The fourth section illustrate models with a numerical example. The fifth section proposed an application of single-stage model in banking and at the end we present the results of the research.

2. Semi-additive production technology in DEA

Suppose we have n DMUs with m inputs and s outputs. For $DMU_j = (X_j, Y_j)$, $j = 1, \dots, n$, we consider the vectors output as $Y_j = (y_{1j}, \dots, y_{sj})^T \in R_+^s$ and input vector $X_j = (x_{1j}, \dots, x_{mj})^T \in R_+^m$. Ghiyasi [11] proposed semi-additive production technology. To introduce the above technology, we first introduce the following axioms for introducing production technology.

P1. Feasibility of observations.
This axiom states that all observed DMUs belong to the production technology, i.e. $(X_j, Y_j) \in T$, $j = 1, \dots, n$.

P2. Free disposability.
This axiom implies that if $(X_1, Y_1) \in T$ and if a point (X_2, Y_2) is such that, $X_2 \geq X_1$, $Y_1 \geq Y_2$, then $(X_2, Y_2) \in T$.

P3. Convexity.
This axiom states that if $(X_1, Y_1) \in T$, $(X_2, Y_2) \in T$, then $\mu(X_1, Y_1) + (1-\mu)(X_2, Y_2)$ for all $\mu \in (0,1)$.

P4. Radial rescaling.
This assumption states that if $(X, Y) \in T$, then $\mu(X, Y) \in T$, for all $\mu \geq 0$.

P5. Semi-additive.
This axiom implies that $(X_i, Y_i) \in T$, $(X_j, Y_j) \in T$, then $\left((X_i, Y_i) + (X_j, Y_j)\right) \in T$, that $i \neq j$.

P6. Minimum extrapolation.
This assumption states that the set T is the smallest set that holds in the above assumptions. In other words, the T set is the subscription of all sets of production

technologies that have the above properties.

Ghiyasi and Cook [12] proved that the PPS under semi-additive condition by accepting axioms P1-P6 is as follows.

$$T^{SA} = \tag{1}$$

$$\left\{ (X,Y) \middle| \begin{array}{c} \sum_{j=1}^{n} \lambda_j X_j \leq X, \sum_{j=1}^{n} \lambda_j Y_j \geq Y, \\ \sum_{j=1}^{n} \lambda_j \geq 1,\ 0 \leq \lambda_j \leq 1, j = 1, \dots, n \end{array} \right\}$$

The efficiency evaluation model of the unit under evaluation, i.e. $DMU_o = (X_o, Y_o)$, in the input oriented based on semi-additive technology based on the the new PPs namely T^{SA} will be as follows.

$$\begin{aligned} &\theta_{SA}^{*} = \min\ \theta_{SA} \\ &S.t.\ \sum_{j=1}^{n} \lambda_j\, x_{ij} \leq \theta_{SA} x_{io}\ , i = 1, \dots, m \\ &\sum_{j=1}^{n} \lambda_j\, y_{rj} \geq y_{ro}, r = 1, \dots, s, \\ &\sum_{j=1}^{n} \lambda_j \geq 1,\ 0 \leq \lambda_j \leq 1,, j = 1, .., n. \end{aligned} \tag{2}$$

Definition 1. DMU_o is called semi-additive efficient in evaluation with model (2), if $\theta_{SA}^{*} = 1$, otherwise it is called a semi-additive inefficient DMU.

3. The efficiency analysis with production trade-offs in the Semi-additive production technology

Let k judgements judgement production trade-offs in the following form.

$$(\mathrm{P}_t, \mathrm{Q}_t), t = 1, \dots, k. \tag{3}$$

We consider the vectors P_t, Q_t for modifying the inputs and outputs of production unit respectively. Assume $V \in R_{+}^{m}$ and $U \in R_{+}^{s}$ are the weight vectors correspond to the components of input and output respectively. Podinovski (2004) introduces a form of weight restrictions on the vectors $V \in R_{+}^{m}$ and $U \in R_{+}^{s}$ of input and output weights, as follows.

$$U^T \mathrm{P}_t - V^T \mathrm{Q}_t \leq 0,\ t = 1, \dots, L, \tag{4}$$

We consider the vectors P_t, Q_t non-zero, then are linked. We presented a single-stage model to obtain strongly efficient targets for inefficient DMUs in the presence of production trade-offs in the input oriented as follows.

$$\begin{aligned} &\min\ \theta_{SA}^{TO} - \epsilon\left(\sum_{i=1}^{m} \gamma_i + \sum_{r=1}^{s} \delta_r\right) \\ &s.t. \sum_{j=1}^{n} \lambda_j\, x_{ij} + + \sum_{t=1}^{k} \sigma_t\, \mathrm{P}_{it} + h_i = \\ &\theta_o^{PT-T} x_{io} - \gamma_i\ ,\ i = 1, \dots, m, \\ &\sum_{j=1}^{n} \lambda_j\, y_{rj} + \sum_{t=1}^{k} \sigma_t\, \mathrm{Q}_{rt} = y_{ro} + \delta_r\ , \\ &r = 1, \dots, s, \\ &\sum_{j=1}^{n} \lambda_j \geq 1,\ 0 \leq \lambda_j \leq 1,, j = 1, .., n \\ &\theta_{SA}^{TO} x_{io} - \gamma_i \geq 0\ ,\ i = 1, \dots, m, \\ &\gamma_i \geq 0,\ h_i \geq \mathrm{o}, i = 1, \dots, m, \theta_{SA}^{TO} \text{ sign free}, \\ &\delta_r \geq \mathrm{o}, r = 1, \dots, s, \sigma_t \geq 0, t = 1, \dots, k. \end{aligned} \tag{5}$$

Let $\left(\theta_{SA}^{TO^*}, \lambda_j^*, h_i^*, \sigma_t^*, \gamma_i^*, \delta_r^* : j = 1, \dots, n, i = 1, \dots, m,\ r = 1, \dots, s,\ t = 1, \dots k\right)$ is an optimal solution of model (5). The efficiency score of $DMU_o = (X_o, Y_o)$ in the presence of production trade-offs and input oriented is equal to $\theta_{SA}^{TO^*}$.

Definition 2. We call DMU_o efficient in the presence of production trade-offs and input oriented, if $\theta_{SA}^{TO^*} = 1$, otherwise it is inefficient.

The target corresponding to $DMU_o = (X_o, Y_o)$ resulting of model (5) is obtained as follows.

$$(X^*, Y^*) = \left(\theta_{SA}^{TO^*} X_o - \gamma^*, Y_o + \delta^*\right). \tag{6}$$

4. Case study

To demonstrate the applications of proposed approach in this paper, we analyze the performance of banking industry in Turkey. Examining banking efficiency is important both from the point of view of macroeconomics and microeconomics. From a macroeconomic point of view, the efficiency of the banking sector affects the cost of financial intermediation and the stability of the entire financial system. From the point of view of microeconomics, banking efficiency is particularly important for improving regulations and institutional supervision, and especially for improving the competitiveness of banks. Increasing the efficiency of banks leads to a better distribution of financial resources and, as a result, better support for investment and economic growth. Analyzing bank performance discovers problems in the system by considering a set of clearly defined inputs and outputs. These banks have the same activity in the global market. To evaluate the banks, we consider three input and two output. The input and output variables in the evaluation of banks are as follows.

Inputs: Personnel Expenses, Total interest expenses., non-interest expenses.

Outputs: Net interest income, non-interest income.

The data sets and characteristics of commercial banks are given in Tables 1 and 2.

At first, we obtain the efficiency of banks based on the BCC model. As can be seen, banks B01, B03, B04, B06, B07, B08, B14, B15, B16, B18 and B20 are efficient and other banks are inefficient. In the following, we calculate the efficiency of banks in semi-additive technology based on model (2). Due to the importance of inputs, we use models in the input oriented in this evaluation. As can be seen, banks B03, B04, B06, B14 and B15 are efficient and other banks are inefficient. The results are different in VRS and semi-additive technologies. For example, banks B01, B07, B08, B16, B18 and B20 are inefficient in semi-additive technology, while they are efficient in VRS technology. The results are in the last two columns of Table 2.

Table 1: Turkey banks data

Bank Name	Bank	Inputs		
		Personnel Expenses	Total Interest Expenses	Non-Interest Expenses
AKBANK T.A.S.	B01	414.16	1943.06	1444.78
Alternatifbank A.S.	B02	29.8	226.1	62.22
Anadolubank A.S.	B03	44.36	206.74	74.83
Birlesik Fon Bankasi Anonim Sirketi	B04	4.88	60.65	1.52
Burgan Bank A.S.	B05	32.8	161.31	96.26
Citibank A.S.	B06	21.17	24.45	55.48
Denizbank A.S.	B07	293.1	1042.17	733.04
HSBC Bank Anonim Sirketi	B08	69.71	153.84	158.58
ICBC Turkey Bank A.S.	B09	36.32	92.7	58.33
ING Bank A.S.	B10	100.94	292.27	215.88
QNB Finansbank A.S.	B11	273.25	1001.94	618.34
Sekerbank T.A.S.	B12	77.44	238.72	173.97
Turk Ekonomi Bankasi A.S.	B13	224.13	700.13	477.47
Turkish Bank A.S.	B14	4.39	4.79	10.64
Turkiye Cumhuriyeti Ziraat Bankasi A.S.	B15	691.81	4978.76	1543.7
Turkiye Garanti Bankasi A.S.	B16	609.89	1885.35	2006.85
Turkiye Halk Bankasi A.S.	B17	528.57	4739.71	1008.21
Turkiye Is Bankasi A.S.	B18	871.17	2544.86	2010.64
Turkland Bank A.S.	B19	11.76	21.82	27.61
Yapi ve Kredi Bankasi A.S.	B20	557.66	2285.09	1265.91

For the purpose of the bank's management opinion in the evaluation process, we use the production trade-offs method. For this purpose, we use the single-stage model in this paper to evaluate the performance of banks.

First, we use model (5) to evaluate the performance of branches. Table 3 shows the results of evaluating the efficiency scores of branches by selecting production trade-off matrixes as: Production trade-offs 1: $P_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$, $Q_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, in this situation, we have $i = 3$, $r = 2$, $t = 1$.

With this choice, we do not have a weight restriction on the input and output components.

The technical efficiency scores and efficient targets of the DMUs corresponding to these weight restrictions are listed in Table 3. As can be seen, banks B03, B04, B06, B14 and B15 are efficient and other banks are inefficient.

Next, in order to show the sensitivity analysis of the model results, we solve the model (5) for different choices of production trade-off matrixes. First, we use model (5) to evaluate the performance of branches. Tables 4, 5 and 6 shows the results of evaluating the efficiency of branches by selecting the production trade-off matrix as follows.

Table 3: The results of model (5) for production trade-offs 1.

Production trade-offs 1: $P_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$, $Q_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$						
DMUs	The technical efficiency scores and efficient targets					
	Efficiency	x_1^*	x_2^*	x_3^*	y_1^*	y_2^*
B01	0.9985	413.559	1901.042	1406.413	2787.97	709.58
B02	0.5565	16.1198	71.937	34.6253	91.86	27.63
B03	1	44.36	206.74	74.83	98.86	78.54
B04	1	4.88	60.65	1.52	1.39	2.16
B05	0.4751	15.5834	17.9046	40.5515	94.57	37.4994
B06	1	21.17	24.45	55.48	138.51	53.5
B07	0.9796	287.107	981.8614	718.0515	1477.93	551.42
B08	0.9335	64.3636	143.6036	148.0282	243.399	136.03
B09	0.7755	21.6813	71.8895	45.2353	94.0247	45.13
B10	0.8008	72.8502	234.0392	172.8689	411.45	85.76
B11	0.9593	259.0367	961.1629	593.1747	1384.41	88.1292
B12	0.4785	34.0017	114.2374	83.2519	223.95	62.99
B13	0.7695	159.0404	538.7165	367.3903	842.54	69.7888
B14	1	4.39	4.79	10.64	6.53	5.44
B15	1	691.81	4978.76	1543.7	5047.84	398.23
B16	0.9907	604.2346	1844.047	1976.185	3422.38	1167.37
B17	0.8127	363.4854	2482.303	819.3912	2630.84	252.98
B18	0.9851	852.9459	2506.992	1980.721	3917.8	1300.21
B19	0.4624	5.4377	9.5857	12.7666	14.19	8.1385
B20	0.9904	551.2383	2201.306	1253.755	2430.37	1059.02

Production trade-offs 2:

$$P_1 = \begin{pmatrix} 3 \\ 2 \\ -2 \end{pmatrix}, Q_1 = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$$

Production trade-offs 3:

$$P_1 = \begin{pmatrix} -1 \\ 0 \\ -2 \end{pmatrix}, Q_1 = \begin{pmatrix} -4 \\ 1 \end{pmatrix}$$

Production trade-offs 4:

$$P_1 = \begin{pmatrix} 1 \\ 1 \\ -6 \end{pmatrix}, Q_1 = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$$

The weight restriction corresponding to these matrixes will be as follows respectively.

Weight restriction 2:

$$-3u_2 + 2u_1 - 3v_1 - 2v_2 + 2v_3 \leq 0.$$

Weight restriction 3:

$$u_2 - 4u_1 + v_1 + 2v_3 \leq 0.$$

Weight restriction 4:

$$5u_2 - 2u_1 - v_1 - v_2 + 6v_3 \leq 0.$$

The efficiency score and efficient targets of the DMUs corresponding to these weight restrictions are listed in Tables 4, 5 and 6. By changing the production exchange matrix, the efficiency values and the corresponding objectives of the units are changed.

Table 4: The results of model (5) for production trade-offs 2.

Production trade-offs 2: $P_1 = \begin{pmatrix} 3 \\ 2 \\ -2 \end{pmatrix}, Q_1 = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$						
DMUs	The technical efficiency scores and efficient targets					
	Efficiency	x_1^*	x_2^*	x_3^*	y_1^*	y_2^*
B01	0.9985	413.559	1901.042	1406.413	2787.97	709.58
B02	0.5528	16.4725	70.1114	34.3932	91.86	27.63
B03	1	44.36	206.74	74.83	98.86	78.54
B04	1	4.88	60.65	1.52	1.39	2.16
B05	0.4751	15.5834	17.9046	40.5515	94.57	37.4994
B06	1	21.17	24.45	55.48	138.51	53.5
B07	0.9796	287.107	981.8614	718.0515	1477.93	551.42
B08	0.9335	64.3636	143.6036	148.0282	243.399	136.03
B09	0.7755	21.6813	71.8895	45.2353	94.0247	45.13
B10	0.7793	78.6616	227.7634	168.2334	411.45	98.7578
B11	0.9538	260.6168	955.6171	589.7521	1384.41	284.3555
B12	0.475	36.7806	108.0232	82.6281	223.95	62.99
B13	0.7558	169.3965	529.1554	360.8699	842.54	184.3454
B14	1	4.39	4.79	10.64	6.53	5.44
B15	1	691.81	4978.76	1543.7	5047.84	398.23
B16	0.9907	604.2346	1844.047	1976.185	3422.38	1167.37
B17	0.8009	423.3073	2475.88	807.4289	2630.84	252.98
B18	0.9792	841.6185	2492.026	1968.897	3917.8	1300.21
B19	0.4624	5.4377	9.5857	12.7666	14.19	8.1385
B20	0.9903	552.2602	2203.917	1253.652	2430.37	1059.02

Table 5: The results of model (5) for production trade-offs 3.

Production trade-offs 3: $P_1 = \begin{pmatrix}-1\\0\\-2\end{pmatrix}, Q_1 = \begin{pmatrix}-4\\1\end{pmatrix}$						
DMUs	The technical efficiency scores and efficient targets					
	Efficiency	x_1^*	x_2^*	x_3^*	y_1^*	y_2^*
B01	0.9866	408.6265	1917.099	1425.477	2787.97	753.8436
B02	0.3464	3.3394	78.3122	21.5506	91.86	83.1231
B03	0.3503	5.7645	72.4165	26.2113	98.86	79.4072
B04	0.198	-2.8512	12.0116	0.301	1.39	36.4987
B05	0.2994	7.455	48.2898	28.8164	94.57	72.4345
B06	1	21.17	24.45	55.48	138.51	53.5
B07	0.9653	281.4958	1006	707.5991	1477.93	551.42
B08	0.6728	40.3722	103.5087	106.6979	173.32	136.03
B09	0.2835	1.67	26.2842	16.5389	61.35	73.4016
B10	0.8008	72.8502	234.0392	172.8689	411.45	85.76
B11	0.9593	259.0367	961.1629	593.1747	1384.41	88.1292
B12	0.4688	32.6392	111.9206	81.5635	223.95	62.99
B13	0.7695	159.0404	538.7165	367.3903	842.54	69.7888
B14	1	4.39	4.79	10.64	6.53	5.44
B15	0.9964	683.1768	4961.083	1538.219	5047.84	472.31
B16	0.9863	601.5079	1859.438	1972.416	3422.38	1167.37
B17	0.673	281.7874	3189.629	678.4836	2630.84	485.8606
B18	0.9851	852.9459	2506.992	1980.721	3917.8	1300.21
B19	0.3841	3.3426	8.3818	10.606	14.19	18.3335
B20	0.8939	448.4842	2042.661	1131.608	2430.37	1059.02

Table 6: The results of model (5) for production trade-offs 4.

Production trade-offs 4: $P_1 = \begin{pmatrix}1\\1\\-6\end{pmatrix}, Q_1 = \begin{pmatrix}-2\\5\end{pmatrix}$						
DMUs	The technical efficiency scores and efficient targets					
	Efficiency	x_1^*	x_2^*	x_3^*	y_1^*	y_2^*
B01	0.9985	413.559	1901.042	1406.413	2787.97	709.58
B02	0.5459	16.2691	74.2646	33.9686	91.86	27.63
B03	0.5585	24.7732	27.4887	5.341	98.86	78.54
B04	1	4.88	60.65	1.52	1.39	2.16
B05	0.4751	15.5834	17.9046	40.5515	94.57	37.4994
B06	1	21.17	24.45	55.48	138.51	53.5
B07	0.906	265.5431	944.1866	664.1206	1477.93	568.0958
B08	0.6267	43.6868	80.7233	7.1432	173.32	136.03
B09	0.4317	15.6794	17.426	10.9419	61.35	45.13
B10	0.7511	75.8122	219.513	162.1393	411.45	178.6853
B11	0.9241	252.5076	925.8826	571.4018	1384.41	556.8126
B12	0.4497	34.8282	107.363	78.242	223.95	70.5385
B13	0.7261	162.7359	508.3491	346.6805	842.54	375.0417
B14	1	4.39	4.79	10.64	6.53	5.44
B15	1	691.81	4978.76	1543.7	5047.84	398.23
B16	0.989	603.1949	1864.653	1959.785	3422.38	1167.37
B17	0.7215	381.3649	2586.387	727.4267	2630.84	308.1661
B18	0.9199	801.4179	2341.1	1849.654	3917.8	1766.336

B19	0.4624	5.4377	9.5857	12.7666	14.19	8.1385
B20	0.8267	461.0406	1805.215	862.763	2430.37	1059.02

5. Conclusion

In this paper, we presented a new model for efficiency analysis in the semi-additive technology in presence of production trade-off. By using production trade-off, we can apply the decision maker's opinion in the performance evaluation process. In semi-additive technology, we create new DMUs that have the total level of inputs and outputs of the observed units. These units can be efficient or inefficient compared to other units. The efficiency of each DMUs is calculated based on the frontier of the new PPS in the semi-additive technology in presence of production trade-off. The targets are efficient corresponding to inefficient DMUs. As seen in the numerical example and case study section, the technical efficiency scores can be different in VRS and semi-additive technologies with production trade-off. We proposed a single-stage model to measure efficiency in the presence of production trade-offs in semi-additive production technology. We have shown that applying weight restrictions in multiplier DEA model is equivalent to considering production trade-offs in single-stage DEA models. We showed that by applying weight restrictions in single-stage models, the efficiency score of DMUs may decrease and the rank of units may change in comparison. By using the models presented in this paper, we can involve the importance of the ratios of inputs and outputs in their evaluation. As future work, we can develop performance evaluation models in semi-additive technology for other data structures in DEA, such as data with a two-stage network structure. We can also develop the above models for the inverse DEA process in the presence of production trade-offs.

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