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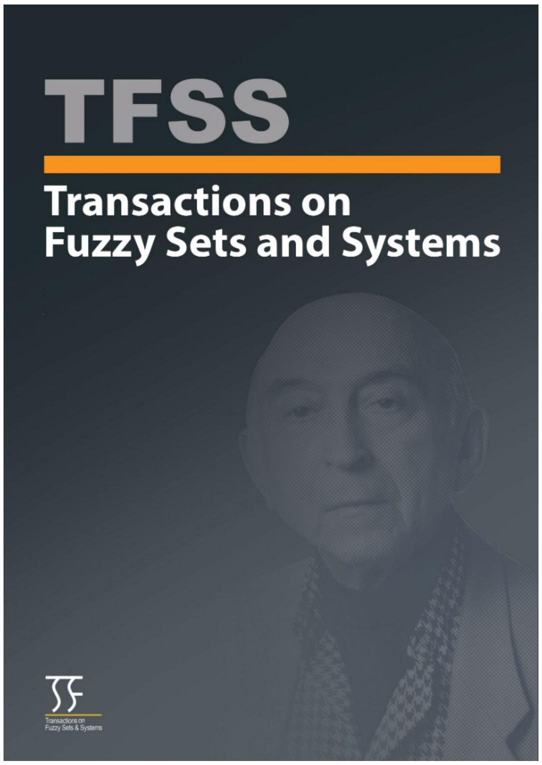

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Fermatean Fuzzy CRADIS Approach Based on Triangular Divergence for Selecting Online Shopping Platform

Mijanur Rahaman Seikh*, **Arnab Mukherjee**

Abstract. In the evolving landscape of e-commerce, selecting an optimal online shopping platform is crucial for businesses aiming to enhance customer experience and operational efficiency. This paper introduces a novel approach that combines the Fermatean fuzzy set theory with the triangular divergence distance measure in Compromise Ranking of Alternatives from Distance to Ideal Solution (CRADIS) method to streamline the decision-making process in online platform selection. By applying the CRADIS method, businesses can systematically evaluate and select an online shopping platform that best meets their operational needs and strategic goals, thereby enhancing their e-commerce effectiveness and customer satisfaction. Through a comprehensive example, we illustrate the application of this approach in evaluating and ranking four distinct online shopping platforms based on multiple criteria. This result shows that Myntra ($\widehat{\chi}_4$) is the best choice. Through this integrated approach, decision-makers can gain valuable insights into the relative merits of each online shopping platform, allowing them to make informed choices aligned with their preferences and requirements. Furthermore, by accommodating uncertainty and imprecision, the Fermatean fuzzy set theory enhances the robustness of the decision-making process, minimizing the risk of making sub-optimal decisions. Overall, this paper demonstrates the practical applicability of Fermatean fuzzy set theory in decision support systems for online platform selection. To demonstrate the proposed method's applicability, we have compared the results with existing Multi-attribute decision making (MADM) methods. To establish its stability, we conducted a sensitivity analysis. By leveraging the CRADIS method alongside Fermatean fuzzy set theory, decision-makers can navigate the complex landscape of online shopping platforms with greater confidence and efficiency, ultimately leading to more satisfactory outcomes for both consumers and businesses alike.

AMS Subject Classification 2020: 90B50; 03E72; 62A86
Keywords and Phrases: Multi-attribute decision-making, Fermatean fuzzy set, Distance measure, CRADIS, Triangular divergence, Online shopping platform selection.

1 Introduction

Distance measures play an important role in handling fuzzy information. Several distance measures have been developed for different fuzzy environments over the years. Recently, Ganie et al. [1] define an innovative picture fuzzy distance measure and novel multi-attribute decision-making method. Puška et al. [2] proposed a comprehensive decision framework for selecting distribution center locations: a hybrid improved fuzzy SWARA and fuzzy compromise ranking of alternatives from distance to ideal solution (CRADIS) approach. Deng et al. [3] proposed a new distance measure in Fermatean fuzzy sets (FFSs). Palanikumar et al. [4] present the novelty of different distance approaches for multi-criteria decision-making challenges using q-rung vague sets. Robot sensors process based on generalized Fermatean normal different aggregation operators

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framework is proposed by Palanikumar et al. [5]. Some recent decision-making problems can be found in various fuzzy environments [6, 7, 8, 9].

There are a few distance measures for Fermatean fuzzy sets in the literature. Senapati et al. [10] proposed the general-Euclidean distance measure (GEDM) for FFSs and used Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) to solve some illustrative Multi-attribute decision making (MADM) problems. Onyeke and Ejegwa [11] defined a modified distance measure for FFSs to fulfill the axiomatic description of the distance function. These distances are not completely competent for calculating accurate distances between FFSs. FFSs, being more advanced and efficient in depicting fuzzy information, automatically call for the development of modified distance measures for better decision-making methods. Recently, the triangular divergence measure, a method generally used in probability distributions, has been researched to develop distance measures based on it. We find instances where the existing distance measures fail to evaluate the distances between FFSs accurately. The triangular divergence measure, proposed by Yehudayoff [12], has been extended to form distance measures by some researchers. Liu [13] defines a distance measure of Fermatean fuzzy sets based on triangular divergence and its application in medical diagnosis. Sahoo [14] uses similarity measures for Fermatean fuzzy sets and its applications in group decision-making. Mandal and Seikh [15] explain the interval-valued Fermatean fuzzy (TOPSIS) method and its application to sustainable development programs. In recent times, FFSs have been utilized in various decision-making problems [16, 17, 18, 19, 20]. Seikh and Chatterjee [21] establish the determination of the best renewable energy sources in India using SWARA-ARAS in a confidence level-based interval-valued Fermatean fuzzy environment. Seikh and Mandal [22] mentioned interval-valued Fermatean fuzzy Dombi aggregation operators and SWARA-based PROMETHEE II method for bio-medical waste management.

Several MADM methods use the distance measure to identify the best alternative. One such method is CRADIS. CRADIS is a relatively new method proposed by Puška et al. [23] in 2022. Hence, it has relatively fewer applications in decision-making problems and fewer extensions in other fuzzy environments. It identifies the best alternatives more comprehensively and simply by using the merits of MARCOS, ARAS, and TOPSIS. Yuan et al. [24] proposed a novel distance measure and CRADIS method in picture fuzzy environment. Further, Puška et al. [25] clarify fuzzy multi-criteria analysis on green supplier selection in an agri-food company. Krishankumar et al. [26] select the IoT service provider for sustainable transport using q-rung orthopair fuzzy CRADIS and unknown weights. Most of these studies utilized the GEDM or the Hamming distance measure (HDM) in the CRADIS method. From the thorough review of the literature, it is observed that Fermatean fuzzy numbers (FFNs) are efficient in expressing fuzzy information and are a popular research area. Also, triangular divergence distance measure forming effective distance measures has few studies on them. Moreover, the CRADIS method is a recently developed and strong method combining the merits of various decision-making methods that have been applied to solve a variety of decision-making problems. Hence, modification of the CRADIS method would eventually make it better and stronger. In this study, the triangular divergence-based distance measure (TDDM) for FFSs is proposed. To improve the existing distance measure, the hesitancy degree of FFSs is included in the distance formula. We further employ it in the CRADIS method to improve the existing CRADIS method.

There are several motivations for this study. They are as follows:

- FFSs have two popular distance measures, but they are not entirely competent in calculating distances between all FFNs. It leads to the requirement of a new and better distance measure for achieving more accurate results in decision-making problems.

- Triangular divergence measure is a popular classical method, mostly used in probability distributions. It has been utilized for distance measures for FFSs and Interval-valued intuitionistic fuzzy sets (IVIFSs). FFNs are better at expressing fuzzy information than FFSs and IVIFSs. Hence, extending the triangular divergence-based distance measure to FFNs will be more beneficial and realistic.

- The previous application of triangular divergence-based measure does not include the hesitancy degree. It leaves out certain fuzzy information from the calculated data and thus may cause discrepancies in results. Hence, including the hesitancy degree in the distance measure will make it more precise.

The following are some significant contributions of this study.

- A new triangular divergence-based distance measure for FFSs is proposed and its properties are discussed.
- The hesitancy degree is also included in the triangular divergence-based distance measure for FFSs to overcome the loss of information.
- The proposed distance measure is utilized in the CRADIS method, eventually modifying the method to give better results to decision-making problems.
- The proposed method is used to solve a real-life decision-making problem of selection of the best online shopping platform.

The study has been organized in the following way: Section 2, consists of the preliminaries. Section 3 has the newly proposed distance measure with its properties and we establish the superiority of the proposed triangular divergence-based distance measure. In Section 4, we iterate the modified-CRADIS method used to solve an illustrative MADM problem. In Section 5, we use an example to apply our proposed method and solve it followed by a comparative and sensitivity analysis. Lastly, Section 6 has the conclusion, research implications, limitations, and future research scopes. Table 1 presents the list of abbreviations used in the manuscript.

Table 1: List of abbreviation.

Abbreviation	Full-form
CRADIS	Compromise ranking of alternatives from distance to ideal solution
TOPSIS	Technique for order performance by similarity to ideal solution
GEDM	General-Euclidean distance measure
HDM	Hamming distance measure
VIKOR	VlseKriterijumska Optimizacija I Kompromisno Resenje
MADM	Multi-attribute decision making
FFSs	Fermatean fuzzy sets
FFNs	Fermatean fuzzy numbers
IoT	Internet of things
SWARA	Stepwise weight assessment ratio analysis
ARAS	Additive ratio assessment method
MARCOS	Measurement of alternatives and ranking according to the compromise solution
IVIFSs	Interval valued intuitionistic fuzzy sets
TDDM	Triangular divergence-based distance measure
LVS	Linguistic variables
PDM	Positive distance matrix
NDM	Negative distance matrix

2 Preliminaries

In this section, some basic definitions and preliminaries are recalled. Throughout the manuscript, the universal set is consistently denoted as Υ.

Definition 2.1. [10] *Let $\Re$ be an FFS over Υ and is defined as follows:*

$$\Re = \{\langle\ \hbar, \alpha_{\Re}(\hbar), \beta_{\Re}(\hbar)\rangle | \hbar \in \Upsilon\}$$

introducing this condition

$$0 \leq (\alpha_{\Re}(\hbar))^3 + (\beta_{\Re}(\hbar))^3 \leq 1.$$

For all $\hbar \in \Upsilon$, the numbers $\alpha_{\Re}(\hbar)$ and $\beta_{\Re}(\hbar)$ denote the degree of membership and the degree of non membership.

Where, $\alpha_{\Re} : \Upsilon \longrightarrow [0,1]$ and $\beta_{\Re} : \Upsilon \longrightarrow [0,1]$.

For any Fermatean fuzzy set $\Re$ and $\hbar \in \Upsilon$.

$$\gamma_{\Re}(\hbar) = \sqrt[3]{1 - (\alpha_{\Re}(\hbar))^3 - (\beta_{\Re}(\hbar))^3}$$

is define as the degree of indeterminacy of $\hbar$ to $\Re$.

Definition 2.2. [10] *Let $\Re = (\alpha_{\Re}, \beta_{\Re})$, $\Re_1 = (\alpha_{\Re_1}, \beta_{\Re_1})$ and $\Re_2 = (\alpha_{\Re_2}, \beta_{\Re_2})$ be three FFNs, then some operation are defined as below:*

1. $\Re_1 \bigcap \Re_2 = (min\{\alpha_{\Re_1}, \beta_{\Re_2}\}, max\{\beta_{\Re_1}, \beta_{\Re_2}\})$.
2. $\Re_1 \bigcup \Re_2 = (max\{\alpha_{\Re_1}, \beta_{\Re_2}\}, min\{\beta_{\Re_1}, \beta_{\Re_2}\})$.
3. $\Re^c = (\beta_{\Re}, \alpha_{\Re})$.

Definition 2.3. [10] *Let $\Re = (\alpha_{\Re}, \beta_{\Re})$, $\Re_1 = (\alpha_{\Re_1}, \beta_{\Re_1})$ and $\Re_2 = (\alpha_{\Re_2}, \beta_{\Re_2})$ be three FFNs and λ >0, then some mathematical operations are formulated as below:*

1. $\Re_1 \boxplus \Re_2 = (\sqrt[3]{\alpha^3_{\Re_1} + \alpha^3_{\Re_2} - \alpha^3_{\Re_1}\alpha^3_{\Re_2}}, \beta_{\Re_1}\beta_{\Re_2})$.
2. $\Re_1 \boxtimes \Re_2 = (\alpha_{\Re_1}\alpha_{\Re_2}, \sqrt[3]{\beta^3_{\Re_1} + \beta^3_{\Re_2} - \beta^3_{\Re_1}\beta^3_{\Re_2}})$.
3. $\lambda\Re = (\sqrt[3]{1 - (1 - \alpha^3_{\Re})^{\lambda}}, \beta^{\lambda}_{\Re})$.
4. $\Re^{\lambda} = (\alpha^{\lambda}_{\Re}, \sqrt[3]{1 - (1 - \beta^3_{\Re})^{\lambda}})$.

Definition 2.4. *Let $\Re_1$=$(\alpha_{\Re_1}, \beta_{\Re_1})$ and $\Re_2$=$(\alpha_{\Re_2}, \beta_{\Re_2})$ be two FFNs. Then the Euclidean distance measure [10] $d_e(\Re_1, \Re_2)$ and the Hamming distance measure [3] $d_h(\Re_1, \Re_2)$ between $\Re_1$ and $\Re_2$ are defined as follow:*

$$d_e(\Re_1, \Re_2) = \sqrt{\frac{1}{2}[(\alpha^3_{\Re_1} - \alpha^3_{\Re_2})^2 + (\beta^3_{\Re_1} - \beta^3_{\Re_2})^2 + (\gamma^3_{\Re_1} - \gamma^3_{\Re_2})^2]} \tag{1}$$

$$d_h(\Re_1, \Re_2) = \frac{1}{2}|[(\alpha^3_{\Re_1} - \alpha^3_{\Re_2})^2 + (\beta^3_{\Re_1} - \beta^3_{\Re_2})^2 + (\gamma^3_{\Re_1} - \gamma^3_{\Re_2})^2]|. \tag{2}$$

Definition 2.5. [12] *The set $\Xi_n = \{M = (m_1, m_2, ..., m_n) | m_i > 0, \sum\limits_{i=1}^{n} m_i = 1\}$, with $n \geq 2$, represents a collection of finite discrete probability distributions. For $\forall\ M, P \in \Xi_n$, the classical triangular divergence measure between M and P is defined as follows:*

$$\Delta(M, P) = \sum_{i=1}^{n} \frac{(m_i - p_i)^2}{m_i + p_i}.$$

Greater triangular divergence indicates greater difference between the probability distributions M and P. Using the above-mentioned equation, the square root of the triangular divergence is presented in the following manner:

$$d(M,P)=\sqrt{\sum_{i=1}^{n}\frac{(m_i-p_i)^2}{m_i+p_i}}$$

where, by convention, $0/0=0$.

3 Distance Measure Based on Triangular Divergence for Fermatean Fuzzy Sets

In this section, we proposed the new distance measure for Fermatean fuzzy sets based on triangular divergance measure.

Definition 3.1. *Let $\Re_i=\langle\ \hbar,\alpha_{\Re_i}(\hbar),\beta_{\Re_i}(\hbar)\rangle$ for $i=1,2$ be two FFSs in $\Upsilon=\{\hbar_1,\hbar_2\}$, then the triangular divergence-based modified distance measure(TDDM) between FFSs $\Re_1$ and $\Re_2$ denoted by d_T is given by*

$$d_T(\Re_1,\Re_2)=\sqrt{\frac{1}{2n}\sum_{j=1}^{n}\left[\frac{\left(\alpha^3_{\Re_1}(\hbar_j)-\alpha^3_{\Re_2}(\hbar_j)\right)^2}{\alpha^3_{\Re_1}(\hbar_j)+\alpha^3_{\Re_2}(\hbar_j)}+\frac{\left(\beta^3_{\Re_1}(\hbar_j)-\beta^3_{\Re_2}(\hbar_j)\right)^2}{\beta^3_{\Re_1}(\hbar_j)+\beta^3_{\Re_2}(\hbar_j)}+\frac{\left(\gamma^3_{\Re_1}(\hbar_j)-\gamma^3_{\Re_2}(\hbar_j)\right)^2}{\gamma^3_{\Re_1}(\hbar_j)+\gamma^3_{\Re_2}(\hbar_j)}\right]}. \quad (3)$$

Theorem 3.2. *The distance measure $d_T(\Re_1,\Re_2)$, between the two FFSs $\Re_1$ and $\Re_2$, follows the following properties. Here $\Re_1$, $\Re_2$ and $\Re_3$ are FFSs.*

I. $d_T(\Re_1,\Re_2)=0\Leftrightarrow\Re_1=\Re_2$;

II. $d_T(\Re_1,\Re_2)=d_T(\Re_2,\Re_1)$;

III. $0\le d_T(\Re_1,\Re_2)\le 1$;

IV. If $\Re_1\le\Re_2\le\Re_3$, then $d_T(\Re_1,\Re_2)\le d_T(\Re_1,\Re_3)$ and $d_T(\Re_2,\Re_3)\le d_T(\Re_1,\Re_3)$.

Proof. I. Let $d_T(\Re_1,\Re_2)=0$ for any $\hbar\in\Upsilon$. Then we can say that

$$d_T(\Re_1,\Re_2)=\sqrt{\frac{1}{2n}\sum_{j=1}^{n}\left[\frac{\left(\alpha^3_{\Re_1}(\hbar_j)-\alpha^3_{\Re_2}(\hbar_j)\right)^2}{\alpha^3_{\Re_1}(\hbar_j)+\alpha^3_{\Re_2}(\hbar_j)}+\frac{\left(\beta^3_{\Re_1}(\hbar_j)-\beta^3_{\Re_2}(\hbar_j)\right)^2}{\beta^3_{\Re_1}(\hbar_j)+\beta^3_{\Re_2}(\hbar_j)}+\frac{\left(\gamma^3_{\Re_1}(\hbar_j)-\gamma^3_{\Re_2}(\hbar_j)\right)^2}{\gamma^3_{\Re_1}(\hbar_j)+\gamma^3_{\Re_2}(\hbar_j)}\right]}=0.$$

Then

$$\frac{\left(\alpha^3_{\Re_1}(\hbar_j)-\alpha^3_{\Re_2}(\hbar_j)\right)^2}{\alpha^3_{\Re_1}(\hbar_j)+\alpha^3_{\Re_2}(\hbar_j)}=\frac{\left(\beta^3_{\Re_1}(\hbar_j)-\beta^3_{\Re_2}(\hbar_j)\right)^2}{\beta^3_{\Re_1}(\hbar_j)+\beta^3_{\Re_2}(\hbar_j)}=\frac{\left(\gamma^3_{\Re_1}(\hbar_j)-\gamma^3_{\Re_2}(\hbar_j)\right)^2}{\gamma^3_{\Re_1}(\hbar_j)+\gamma^3_{\Re_2}(\hbar_j)}=0.$$

That is,

$$\left(\alpha^3_{\Re_1}(\hbar_j)-\alpha^3_{\Re_2}(\hbar_j)\right)^2=\left(\beta^3_{\Re_1}(\hbar_j)-\beta^3_{\Re_2}(\hbar_j)\right)^2=\left(\gamma^3_{\Re_1}(\hbar_j)-\gamma^3_{\Re_2}(\hbar_j)\right)^2.$$

Again we know that

$$0\le\alpha_{\Re_1},\alpha_{\Re_2},\beta_{\Re_1},\beta_{\Re_2},\gamma_{\Re_1},\gamma_{\Re_2}\le 1.$$

Hence, we have $\alpha_{\Re_1}(\hbar_j)=\alpha_{\Re_2}(\hbar_j),\beta_{\Re_1}(\hbar_j)=\beta_{\Re_2}(\hbar_j),\gamma_{\Re_1}(\hbar_j)=\gamma_{\Re_2}(\hbar_j)$.
Therefore,

$$\Re_1=\Re_2.$$

Conversely, when $\Re_1 = \Re_2$, one has

$$\alpha_{\Re_1}(\hbar_j) = \alpha_{\Re_2}(\hbar_j), \beta_{\Re_1}(\hbar_j) = \beta_{\Re_2}(\hbar_j), \gamma_{\Re_1}(\hbar_j) = \gamma_{\Re_2}(\hbar_j).$$

Then, we can obtain

$$d_T(\Re_1,\Re_2) = \sqrt{\frac{1}{2n}\sum_{j=1}^{n}\left[\frac{\left(\alpha^3_{\Re_1}(\hbar_j)-\alpha^3_{\Re_2}(\hbar_j)\right)^2}{\alpha^3_{\Re_1}(\hbar_j)+\alpha^3_{\Re_2}(\hbar_j)} + \frac{\left(\beta^3_{\Re_1}(\hbar_j)-\beta^3_{\Re_2}(\hbar_j)\right)^2}{\beta^3_{\Re_1}(\hbar_j)+\beta^3_{\Re_2}(\hbar_j)} + \frac{\left(\gamma^3_{\Re_1}(\hbar_j)-\gamma^3_{\Re_2}(\hbar_j)\right)^2}{\gamma^3_{\Re_1}(\hbar_j)+\gamma^3_{\Re_2}(\hbar_j)}\right]} = 0.$$

Hence, property I holds.

II. Next we prove that $d_T(\Re_1,\Re_2) = d_T(\Re_2,\Re_1)$. We know that

$$\begin{aligned} d_T(\Re_1,\Re_2) &= \sqrt{\frac{1}{2n}\sum_{j=1}^{n}\left[\frac{\left(\alpha^3_{\Re_1}(\hbar_j)-\alpha^3_{\Re_2}(\hbar_j)\right)^2}{\alpha^3_{\Re_1}(\hbar_j)+\alpha^3_{\Re_2}(\hbar_j)} + \frac{\left(\beta^3_{\Re_1}(\hbar_j)-\beta^3_{\Re_2}(\hbar_j)\right)^2}{\beta^3_{\Re_1}(\hbar_j)+\beta^3_{\Re_2}(\hbar_j)} + \frac{\left(\gamma^3_{\Re_1}(\hbar_j)-\gamma^3_{\Re_2}(\hbar_j)\right)^2}{\gamma^3_{\Re_1}(\hbar_j)+\gamma^3_{\Re_2}(\hbar_j)}\right]} \\ &= \sqrt{\frac{1}{2n}\sum_{j=1}^{n}\left[\frac{\left(\alpha^3_{\Re_2}(\hbar_j)-\alpha^3_{\Re_1}(\hbar_j)\right)^2}{\alpha^3_{\Re_2}(\hbar_j)+\alpha^3_{\Re_1}(\hbar_j)} + \frac{\left(\beta^3_{\Re_2}(\hbar_j)-\beta^3_{\Re_1}(\hbar_j)\right)^2}{\beta^3_{\Re_2}(\hbar_j)+\beta^3_{\Re_1}(\hbar_j)} + \frac{\left(\gamma^3_{\Re_2}(\hbar_j)-\gamma^3_{\Re_1}(\hbar_j)\right)^2}{\gamma^3_{\Re_2}(\hbar_j)+\gamma^3_{\Re_1}(\hbar_j)}\right]} \\ &= d_T(\Re_2,\Re_1). \end{aligned}$$

Hence, property II holds.

III. We then prove that $0 \le d_T(\Re_1,\Re_2) \le 1$. From Definition 2.1, it is obvious that $0 \le d_T(\Re_1,\Re_2)$ and we observe that, $0 \le \alpha^3_{\Re_1}(\hbar) + \beta^3_{\Re_1}(\hbar) \le 1$, $0 \le \alpha^3_{\Re_2}(\hbar) + \beta^3_{\Re_2}(\hbar) \le 1$. So, the following inequality holds

$$\left(\alpha^3_{\Re_1}(\hbar)-\alpha^3_{\Re_2}(\hbar)\right)^2 \le \left(\alpha^3_{\Re_1}(\hbar)+\alpha^3_{\Re_2}(\hbar)\right)^2 \text{ and } \left(\beta^3_{\Re_1}(\hbar)-\beta^3_{\Re_2}(\hbar)\right)^2 \le \left(\beta^3_{\Re_1}(\hbar)+\beta^3_{\Re_2}(\hbar)\right)^2.$$

Then,

$$\begin{aligned} d_T(\Re_1,\Re_2) &= \sqrt{\frac{1}{2n}\sum_{j=1}^{n}\left[\frac{\left(\alpha^3_{\Re_1}(\hbar_j)-\alpha^3_{\Re_2}(\hbar_j)\right)^2}{\alpha^3_{\Re_1}(\hbar_j)+\alpha^3_{\Re_2}(\hbar_j)} + \frac{\left(\beta^3_{\Re_1}(\hbar_j)-\beta^3_{\Re_2}(\hbar_j)\right)^2}{\beta^3_{\Re_1}(\hbar_j)+\beta^3_{\Re_2}(\hbar_j)} + \frac{\left(\gamma^3_{\Re_1}(\hbar_j)-\gamma^3_{\Re_2}(\hbar_j)\right)^2}{\gamma^3_{\Re_1}(\hbar_j)+\gamma^3_{\Re_2}(\hbar_j)}\right]} \\ &\le \sqrt{\frac{1}{2n}\sum_{j=1}^{n}\left[\frac{\left(\alpha^3_{\Re_1}(\hbar_j)+\alpha^3_{\Re_2}(\hbar_j)\right)^2}{\alpha^3_{\Re_1}(\hbar_j)+\alpha^3_{\Re_2}(\hbar_j)} + \frac{\left(\beta^3_{\Re_1}(\hbar_j)+\beta^3_{\Re_2}(\hbar_j)\right)^2}{\beta^3_{\Re_1}(\hbar_j)+\beta^3_{\Re_2}(\hbar_j)} + \frac{\left(\gamma^3_{\Re_1}(\hbar_j)+\gamma^3_{\Re_2}(\hbar_j)\right)^2}{\gamma^3_{\Re_1}(\hbar_j)+\gamma^3_{\Re_2}(\hbar_j)}\right]} \\ &= \sqrt{\frac{1}{2n}\sum_{j=1}^{n}\left[\alpha^3_{\Re_1}(\hbar_j)+\alpha^3_{\Re_2}(\hbar_j)+\beta^3_{\Re_1}(\hbar_j)+\beta^3_{\Re_2}(\hbar_j)+\gamma^3_{\Re_1}(\hbar_j)+\gamma^3_{\Re_2}(\hbar_j)\right]} \\ &= \sqrt{\frac{1}{2n}\sum_{j=1}^{n}2} \\ &= 1. \end{aligned}$$

Hence, property III holds.

IV. Lastly, we prove that if $\Re_1 \le \Re_2 \le \Re_3$, then $d_T(\Re_1,\Re_2) \le d_T(\Re_1,\Re_3)$ and $d_T(\Re_2,\Re_3) \le d_T(\Re_1,\Re_3))$.

When $\Re_1 \leq \Re_2 \leq \Re_3$, we have

$$\alpha^3_{\Re_1} \leq \alpha^3_{\Re_2} \leq \alpha^3_{\Re_3} \text{ and } \beta^3_{\Re_3} \leq \beta^3_{\Re_2} \leq \beta^3_{\Re_1}$$

for $0 \leq \eta_k \leq 1 (k = 1, 2)$ and $0 \leq \eta_1 + \eta_2 \leq 1$, a function $h(\hbar_1, \hbar_2)$ could be establish below

$$h(\hbar_1, \hbar_2) = \sum_{k=1}^{2} \frac{(\hbar_k - \eta_k)^2}{\hbar_K + \eta_K}, \hbar_k \in [0, 1]$$

then the partial derivative of the function $h(\hbar_1, \hbar_2)$ in term of $\hbar_i$ will be calculated as follow

$$\frac{\delta h}{\delta \hbar_K} = \frac{(\hbar_k - \eta_k)(\hbar_k + 3\eta_k)}{(\hbar_k + \eta_k)^2}. \tag{4}$$

From the partial derivation function of Equation (4) one has

$$\begin{cases} \frac{\delta h}{\delta \hbar_k} \geq 0, & 0 \leq \eta_k \leq \hbar_k \leq 1, \\ \frac{\delta h}{\delta \hbar_k} < 0, & 0 \leq \hbar_k \leq \eta_k \leq 1. \end{cases}$$

Therefore, when $\hbar_k \geq \eta_k$, $h(\hbar_1, \hbar_2)$ is monotonically increasing function for $\hbar_k$ and when $\hbar_k \geq \eta_k$, $h(\hbar_1, \hbar_2)$ is a monotonically decreasing function for $\hbar_k$.

Let, $\eta_1 = \alpha^3_{\Re_1}, \eta_2 = \beta^3_{\Re_1}$

when $\Re_1 \leq \Re_2 \leq \Re_3$

$$\eta_1 = \alpha^3_{\Re_1} \leq \alpha^3_{\Re_2} \leq \alpha^3_{\Re_3},$$
$$\beta^3_{\Re_3} \leq \beta^3_{\Re_2} \leq \beta^3_{\Re_1} = \eta_2.$$

Because, $h(\hbar_1, \hbar_2)$ is monotonically increasing when $\hbar_1 \geq \eta_1$ if $\alpha^3_{\Re_3} \geq \alpha^3_{\Re_2}$ one has

$$h(\alpha^3_{\Re_3}, \beta^3_{\Re_3}) \geq h(\alpha^3_{\Re_2}, \beta^3_{\Re_3}). \tag{5}$$

Meanwhile, because $h(\hbar_1, \hbar_2)$ is monotonically decreasing when $\hbar_3 \leq \eta_3$ if $\beta^3_{\Re_3} \leq \beta^3_{\Re_2}$ one has

$$h(\alpha^3_{\Re_2}, \beta^3_{\Re_3}) \geq h(\alpha^3_{\Re_2}, \beta^3_{\Re_2}). \tag{6}$$

Combining (5) and (6) one has

$$h(\alpha^3_{\Re_3}, \beta^3_{\Re_3}) \geq h(\alpha^3_{\Re_2}, \beta^3_{\Re_2})$$

that is,

$$\frac{(\alpha^3_{\Re_2} - \alpha^3_{\Re_1})^2}{\alpha^3_{\Re_2} + \alpha^3_{\Re_1}} + \frac{(\beta^3_{\Re_2} - \beta^3_{\Re_1})^2}{\beta^3_{\Re_2} + \beta^3_{\Re_1}} \leq \frac{(\alpha^3_{\Re_3} - \alpha^3_{\Re_1})^2}{\alpha^3_{\Re_3} + \alpha^3_{\Re_1}} + \frac{(\beta^3_{\Re_3} - \beta^3_{\Re_1})^2}{\beta^3_{\Re_3} + \beta^3_{\Re_1}}.$$

Consequently, we have

$$\begin{aligned} d_T(\Re_1, \Re_2) &= \sqrt{\frac{1}{2n} \sum_{j=1}^{n} \left[\frac{\left(\alpha^3_{\Re_2}(\hbar_j) - \alpha^3_{\Re_1}(\hbar_j)\right)^2}{\alpha^3_{\Re_2}(\hbar_j) + \alpha^3_{\Re_1}(\hbar_j)} + \frac{\left(\beta^3_{\Re_2}(\hbar_j) - \beta^3_{\Re_2}(\hbar_j)\right)^2}{\beta^3_{\Re_2}(\hbar_j) + \beta^3_{\Re_1}(\hbar_j)} \right]} \\ &\leq \sqrt{\frac{1}{2n} \sum_{j=1}^{n} \left[\frac{\left(\alpha^3_{\Re_3}(\hbar_j) - \alpha^3_{\Re_1}(\hbar_j)\right)^2}{\alpha^3_{\Re_3}(\hbar_j) + \alpha^3_{\Re_1}(\hbar_j)} + \frac{\left(\beta^3_{\Re_3}(\hbar_j) - \beta^3_{\Re_2}(\hbar_j)\right)^2}{\beta^3_{\Re_3}(\hbar_j) + \beta^3_{\Re_1}(\hbar_j)} \right]} \\ &= d_T(\Re_1, \Re_3). \end{aligned}$$

Since, the hesitancy degree is dependent on the membership and non-membership degree, we can say that $d_T(\Re_1, \Re_2) \leq d_T(\Re_1, \Re_3)$. Similarly, we can proved that $d_T(\Re_2, \Re_3) \leq d_T(\Re_1, \Re_3)$. □

Next, we utilize the following example to establish the superiority of the proposed triangular divergence-based distance measure for FFNs.

Example 3.3. Let there be three FFNs, $\bar{F}_1 = (0.65, 0.8321)$, $\bar{F}_2 = (0.85, 0.6831)$ and $\bar{F}_3 = (0.85, 0.6849)$. Clearly $\bar{F}_2 \neq \bar{F}_3$, so distance between $(\bar{F}_1, \bar{F}_2)$ and $(\bar{F}_1, \bar{F}_3)$ should not be equal. We now calculate the GEDM (proposed by Senapati [10]), HDM (proposed by Deng. [3]) and TDDM (proposed) between $(\bar{F}_1, \bar{F}_2)$ and $(\bar{F}_1, \bar{F}_3)$ using Equations (1), (2) and (3) respectively, to establish superiority of the proposed distance measure. Table 2 gives the values of the calculated distances.

Table 2: Comparison of distance measure for FFNs.

FFN Pair	GEDM [10]	HDM [3]	TDDM (proposed)
$(\bar{F}_1, \bar{F}_2)$	0.306	0.339	**0.342**
$(\bar{F}_1, \bar{F}_3)$	0.306	0.339	**0.343**

Thus we see that even though the GEDM and HDM give equal distances for the pairs, the proposed TDDM gives different distances for the given pair, thus establishing the superiority of the proposed distance measure.

From Figure 1, we notice that there is a significant difference in the distance measures between the given pairs in the case of TDDM, whereas the existing distance measures fail to distinguish between them.

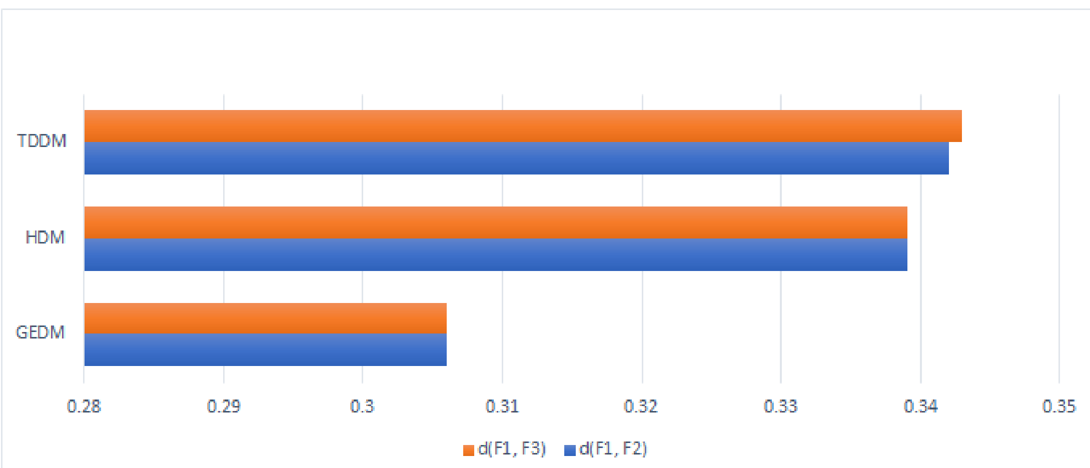

Figure 1: Superiority of the proposed distance measure.

4 MADM Process Using Modified Distance Based FFNs-CRADIS Method

In this section, we propose a MADM process utilizing a modified version of the CRADIS method by implementing the proposed distance measure for FFNs.

Let the set of alternatives be $\widehat{\chi} = \{\widehat{\chi}_1, \widehat{\chi}_2, ..., \widehat{\chi}_m\}$ such that there are "m" alternatives and "n" criteria such that $SC = \{SC_1, SC_2, ..., SC_n\}$ be the set of criteria where their weights are $\varpi_1, \varpi_2, ..., \varpi_n$ respectively. Here $0 \leq \varpi_j \leq 1$ and $\sum_{j=1}^{n} \varpi_j = 1$.

General the initial decision matrices is $B = (b_{ij})_{m\times n}$. A panel of specialists has been invited to offer their assessments in order to achieve the desired ranking of the "m" alternatives regarding "n" attributes. Then construct the Fermatean fuzzy evaluation matrix as $B = (b_{ij})_{m\times n}$, where $i = 1, 2..., m$ and $j = 1, 2, ..., n$ also $b_{ij} = (\alpha_{ij}, \beta_{ij})$. The algorithm shown in Figure 2.

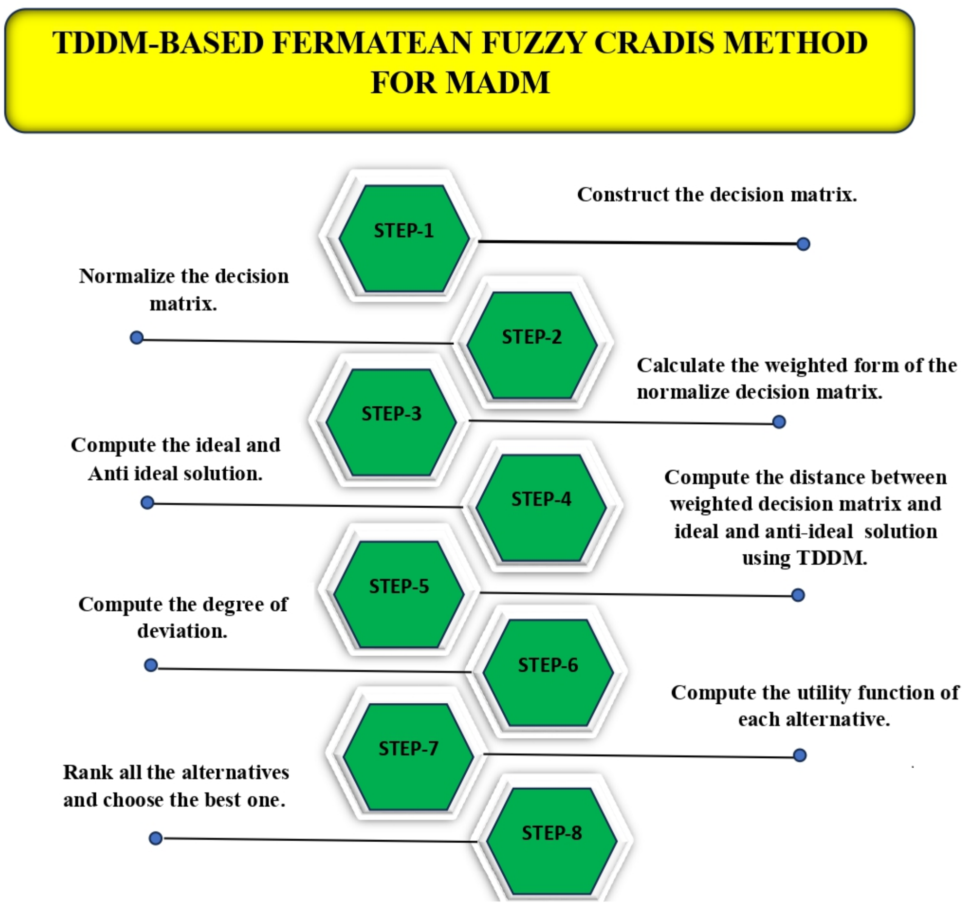

Figure 2: CRADIS-based MADM process using the triangular divergence distance measure.

The triangular distance-based CRADIS method is used for ranking the alternatives. The algorithm of the method is as follows:

Step 1: Construct the decision matrix for the FFNs.

Step 2: Now normalized the decision matrix. The two group are cost type and benefit type. The normalized

Fermatean fuzzy decision matrix, $D = (d_{ij})_{m\times n}$ is as follows:

$$D = (d_{ij})_{m\times n} = \begin{cases} b_{ij} = (\alpha_{ij}, \beta_{ij}), & SC_j \text{ is a benefit attribute,} \\ b^c_{ij} = (\alpha_{ij}, \beta_{ij}), & SC_j \text{ is a cost attribute.} \end{cases}$$

Step 3: Set up the Fermatean fuzzy weighted decision matrix.

$T = (t_{ij})_{m\times n}$ based on the subsequent formula

$$\begin{aligned} T &= d_{ij}\varpi_j \\ &= (\sqrt[3]{1-(1-\alpha^3_{d_{ij}})^{\varpi_j}}, \beta^{\varpi_j}_{d_{ij}}) \end{aligned} \tag{7}$$

where, d_{ij} is the component of normalized decision matrix D and ϖ_j is the attribute weight SC_j.

Step 4: Find out the ideal and anti-ideal solution.

Ideal alternatives ,

$$\begin{aligned} \widehat{\chi}_0 &= \{t_{01}, t_{02}, ..., t_{0n}\} \\ t_{0j} &= (\alpha_{t_{oj}}, \beta_{t_{oj}}) \\ &= \{\max_{1\leq i\leq m} \alpha_{t_{ij}}, \min_{1\leq i\leq m} \beta_{t_{ij}}\}, j = 1, 2, ..., n \end{aligned}$$

and

anti-ideal alternatives ,

$$\begin{aligned} \widehat{\chi}_{m+1} &= (t_{m+11}, ..., t_{m+1n}) \\ t_{m+1j} &= (\alpha_{tm+ij}, \beta_{tm+ij}) \\ &= \{\max_{1\leq i\leq m} \alpha_{t_{ij}}, \min_{1\leq i\leq m} \beta_{t_{ij}}\}, j = 1, 2, ..., n. \end{aligned} \tag{8}$$

Step 5: Deviation are obtain by TDDM

$$d^+_T = d(t_{ij}, t_{0j}) \text{ for } i = 1, 2..., m \text{ } and \text{ } j = 1, 2, ..., n. \tag{9}$$

$$d^-_T = d(t_{ij}, t_{m+ij} \text{ for } i = 1, 2..., m \text{ } and \text{ } j = 1, 2, ..., n. \tag{10}$$

Using the formula (3).

Step 6: Determine the degree of deviation of every option from the ideal and undesirable solution

$$s^+_i = \sum_{j=1}^{n} d^+_j, s^-_i = \sum_{j=1}^{n} d^-_j.$$

Step 7: Analysis of every alternative utility function concerning its deviation from ideal option

$$k^+_i = \frac{s^+_0}{s^+_i}, k^-_i = \frac{s^-_i}{s^-_0}$$

where, s^-_0 is the optimal choice that is situated at the greatest distance from anti-ideal solution and s^+_0 is the best option that is the closet to the ideal solution.

Step 8: Ranking possible option. Finding the average departure of the option from the degree of value yield the final ranking

$$Q_i = \frac{(k^+_i + k^-_i)}{2}. \tag{11}$$

The selection possessing the greatest numerical magnitude is the ideal choice Q_i.

5 Illustrative Example

This section illustrates the proposed MADM technique by solving a numerical problem. Suppose there are four online platforms for shopping. There are four alternatives Amazon($\widehat{\chi}_1$); Flipkart($\widehat{\chi}_2$); Meesho($\widehat{\chi}_3$); Myntra($\widehat{\chi}_4$). The decision expert will assess the alternative online shopping platform based on the following eight criteria: SC_1 is the price competitiveness; SC_2 is the delivery charge; SC_3 is the subscription; SC_4 is the customer reviews and rating; SC_5 is the fast delivery; SC_6 is the return policy; SC_7 is the product quality; SC_8 is the customer support. Among the criteria SC_1, SC_2 and SC_3 are the "cost criteria" and SC_4, SC_5, SC_6, SC_7 and SC_8 are "benefit criteria".

The linguistic variables (LVs) used to rate the importance of decision experts, and criteria and evaluate the alternatives are given in Table 3. The framework is shown in Figure 3.

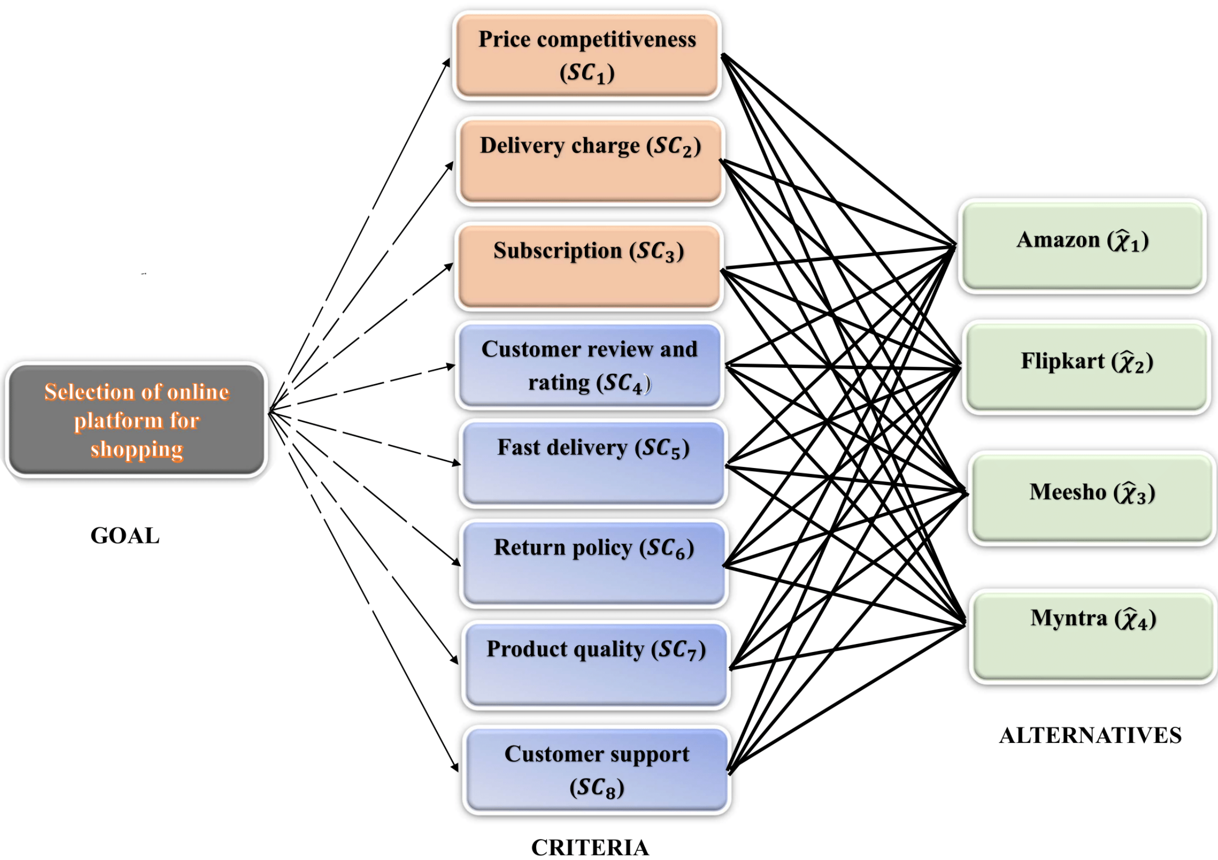

Figure 3: Framework for selecting online shopping platform alternatives.

Table 3: LVs for assessing importance of DEs, criteria and alternatives.

LVs	FFNs
Extremely good (EG)	(0.98,0.3)
Good (G)	(0.9,0.6)
Medium (M)	(0.85,0.7)
Bad (B)	(0.78,0.8)
Very bad (VB)	(0.3,0.98)

The weight of the criteria's are $\varpi_1 = 0.15, \varpi_2 = 0.22, \varpi_3 = 0.18, \varpi_4 = 0.22, \varpi_5 = 0.12, \varpi_6 = 0.08, \varpi_7 = 0.15, \varpi_8 = 0.08$ for SC_1 to SC_8 respectively.

The next and last step is the application of the proposed triangular divergence distance measure-based CRADIS method for ranking of the alternatives. The initial decision matrix made by the decision maker is given in Table 4.

Table 4: Decision matrix for in terms of LVs.

Alternative	SC_1	SC_2	SC_3	SC_4	SC_5	SC_6	SC_7	SC_8
$\widehat{\chi}_1$	(G)	(M)	(G)	(B)	(VB)	(M)	(G)	(EG)
$\widehat{\chi}_2$	(EG)	(M)	(G)	(VB)	(B)	(M)	(EG)	(B)
$\widehat{\chi}_3$	(G)	(B)	(M)	(VB)	(EG)	(B)	(M)	(G)
$\widehat{\chi}_4$	(B)	(M)	(VB)	(EG)	(B)	(M)	(G)	(G)

Step 1 The decision matrix made by the decision experts is given below in Table 5.

Table 5: Decision matrix.

Alternative	SC_1	SC_2	SC_3	SC_4	SC_5	SC_6	SC_7	SC_8
$\widehat{\chi}_1$	(0.9,0.6)	(0.85,0.7)	(0.9,0.6)	(0.78,0.8)	(0.3,0.98)	(0.85,0.7)	(0.9,0.6)	(0.98,0.3)
$\widehat{\chi}_2$	(0.98,0.3)	(0.85,0.7)	(0.9,0.6)	(0.3,0.98)	(0.78,0.8)	(0.85,0.7)	(0.98,0.3)	(0.78,0.8)
$\widehat{\chi}_3$	(0.9,0.6)	(0.78,0.8)	(0.85,0.7)	(0.3,0.98)	(0.98,0.3)	(0.78,0.8)	(0.85,0.7)	(0.9,0.6)
$\widehat{\chi}_4$	(0.78,0.8)	(0.85,0.7)	(0.3,0.98)	(0.98,0.3)	(0.78,0.8)	(0.85,0.7)	(0.9,0.6)	(0.9,0.6)

Step 2 Construct the normalized decision matrix for the FFNs which is given in Table 6.

Table 6: Normalized decision matrices.

Alternative	SC_1	SC_2	SC_3	SC_4	SC_5	SC_6	SC_7	SC_8
$\widehat{\chi}_1$	(0.6,0.9)	(0.7,0.85)	(0.6,0.9)	(0.78,0.8)	(0.3,0.98)	(0.85,0.7)	(0.9,0.6)	(0.98, 0.3)
$\widehat{\chi}_2$	(0.3,0.98)	(0.7,0.85)	(0.6,0.9)	(0.3,0.98)	(0.78,0.8)	(0.85,0.7)	(0.98,0.3)	(0.78,0.8)
$\widehat{\chi}_3$	(0.6,0.9)	(0.8,0.78)	(0.7,0.85)	(0.3,0.98)	(0.98,0.3)	(0.78,0.8)	(0.85,0.7)	(0.9,0.6)
$\widehat{\chi}_4$	(0.8,0.78)	(0.7,0.85)	(0.98,0.3)	(0.98,0.3)	(0.78,0.8)	(0.85,0.7)	(0.9,0.6)	(0.9,0.6)

Step 3 We calculate the weighted decision matrix using Equation (7). The weighted decision matrix is given in Table 7.

Table 7: Weighted decision matrix.

Alternative	SC_1	SC_2	SC_3	SC_4	SC_5	SC_6	SC_7	SC_8
$\widehat{\chi}_1$	(0.330,0.984)	(0.445,0.965)	(0.349,0.981)	(0.234,0.995)	(0.148,0.997)	(0.419,0.972)	(0.562,0.926)	(0.587,0.908)
$\widehat{\chi}_2$	(0.160,0.997)	(0.445,0.965)	(0.349,0.981)	(0.081,0.999)	(0.420,0.974)	(0.419,0.972)	(0.702,0.835)	(0.369,0.982)
$\widehat{\chi}_3$	(0.330,0.984)	(0.526,0.947)	(0.418,0.971)	(0.082,0.999)	(0.660,0.865)	(0.369,0.982)	(0.511,0.948)	(0.463,0.960)
$\widehat{\chi}_4$	(0.467,0.963)	(0.445,0.965)	(0.736,0.805)	(0.380,0.976)	(0.420,0.973)	(0.419,0.972)	(0.562,0.926)	(0.462,0.959)

Step 4 Establish the ideal and anti-ideal solution which is given in Table 8 using Equation (8).

Table 8: Ideal and anti-ideal solution.

ideal and anti ideal	SC_1	SC_2	SC_3	SC_4	SC_5	SC_6	SC_7	SC_8
(t_{0j})	(0.467,0.963)	(0.527,0.947)	(0.736,0.805)	(0.380,0.976)	(0.660,0.865)	(0.419,0.972)	(0.702,0.835)	(0.587,0.908)
(t_{m+ij})	(0.160,0.997)	(0.445,0.965)	(0.350,0.981)	(0.0812,0.999)	(0.148,0.997)	(0.369,0.982)	(0.510,0.948)	(0.369,0.982)

Step 5 Next, we find the distance of every alternative from both the ideal and anti-ideal using the proposed TDDM given in Equations (9) and (10). The positive distance matrix (PDM) and negative distance matrix (NDM) are recorded in Table 9.

Table 9: PDM and NDM.

Alterna-tive	SC_1		SC_2		SC_3		SC_4		SC_5		SC_6		SC_7		SC_8	
	D^+	D^-	D^+	D^-	D^+	D^-	D^+	D^-	D^+	D^-	D^+	D^-	D^+	D^-	D^+	D^-
$\widehat{\chi}_1$	0.136	0.118	0.098	0	0.478	0	0.143	0.075	0.449	0	0	0.067	0.231	0.077	0	0.281
$\widehat{\chi}_2$	0.218	0	0.098	0	0.478	0	0.185	0	0.339	0.184	0	0.067	0	0.302	0.281	0
$\widehat{\chi}_3$	0.136	0.118	0	0.098	0.438	0.064	0.185	0	0	0.449	0.067	0	0.302	0	0.177	0.121
$\widehat{\chi}_4$	0	0.218	0.098	0	0	0.478	0	0.185	0.339	0.184	0	0.067	0.231	0.077	0.177	0.121

Step 6 In this step, we calculate the degree of deviation using Equation (11). The deviation values of alternatives are enlisted in Table 10.

Table 10: Deviation of alternatives.

Alternative	S_i^+	S_i^-
$\widehat{\chi}_1$	1.536	0.619
$\widehat{\chi}_2$	1.600	0.553
$\widehat{\chi}_3$	1.306	0.851
$\widehat{\chi}_4$	0.845	1.332

Step 7 We now compute utility function of each alternative using Equation (11) which are given in table 11.

Step 8 In the last step, we rank the alternatives using Equation (11) as depicted in Table 11.

Table 11: Ranking.

Alternative	K_i^+	K_i^-	Q_i	*Ranking*
$\widehat{\chi}_1$	0.550	0.465	0.507	3
$\widehat{\chi}_2$	0.528	0.416	0.471	4
$\widehat{\chi}_3$	0.647	0.639	0.643	2
$\widehat{\chi}_4$	1	1	1	1

After ranking of all alternatives we get that $\widehat{\chi}_4$ is the best option.

5.1 Comparative Analysis

In this section, the proposed modified CRADIS method is compared with the existing methods and distance measures. The comparison has been performed with the TOPSIS and VIKOR methods. The algorithm of the TOPSIS method as given by Kirisci [27], is used to solve the illustrative problem. In place of the distance measure used by Kirisci [27], we have used GEDM, HDM, and the proposed TDDM. To compare with the VIKOR method, the algorithm given by Gül [28] is used where, the GEDM, HDM, and proposed TDDM are used to calculate distances of alternatives from an ideal solution.

The TDDM-based CRADIS method gives the ranking $\widehat{\chi}_4 > \widehat{\chi}_3 > \widehat{\chi}_1 > \widehat{\chi}_2$. First, we compare the TDDM-based CRADIS method with the GEDM-based CRADIS method which is given in Equation (1) to evaluate the distance between the alternatives and the PIS and NIS and we see that the ranking order is $\widehat{\chi}_4 > \widehat{\chi}_3 > \widehat{\chi}_2 > \widehat{\chi}_1$. Next, we used HDM-based CRADIS which is given in Equation (2) in place of TDDM-based CRADIS for comparison. The outcome remains unchanged in the case of the GEDM-based CRADIS method, i.e., $\widehat{\chi}_4 > \widehat{\chi}_3 > \widehat{\chi}_2 > \widehat{\chi}_1$.

Now the comparison is done by the VlseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR) [29] method. The comparison is performed by using TDDM-based VIKOR in place of the modified TDDM-based CRADIS method and the result obtained is $\widehat{\chi}_4 > \widehat{\chi}_3 > \widehat{\chi}_1 > \widehat{\chi}_2$, which is the same as the proposed method. We also use GEDM-based VIKOR and HDM-based VIKOR in place of the modified TDDM-based CRADIS method for comparison. In both cases, we see that the ranking is equivalent to the proposed method, i.e., $\widehat{\chi}_4 > \widehat{\chi}_3 > \widehat{\chi}_1 > \widehat{\chi}_2$.

Lastly, a comparison is done with a novel TOPSIS [27] method. For comparison, we use TDDM-based TOPSIS in place of TDDM-based CRADIS and here the ranking result is $\widehat{\chi}_4 > \widehat{\chi}_3 > \widehat{\chi}_1 > \widehat{\chi}_2$ which is the same as TDDM-based CRADIS method. We also compare GEDM-based TOPSIS and we get the ranking $\widehat{\chi}_3 > \widehat{\chi}_4 > \widehat{\chi}_1 > \widehat{\chi}_2$. Comparing TDDM-based CRADIS method with HDM-based TOPSIS, it gives the ranking $\widehat{\chi}_4 > \widehat{\chi}_3 > \widehat{\chi}_1 > \widehat{\chi}_2$ which is the same as TDDM-based CRADIS method.

Given in Table 12 are the ranking orders using different distance measures. Therefore, it can be stated that the proposed TDDM method is superior as well as reliable.

Table 12: Comparison of ranking results using different distance-based MADM methods.

MADM method	Alternatives ranked based on	Distance measure	Ranking result
CRADIS (proposed)	Degree of value yield	GEDM	$\widehat{\chi}_4 > \widehat{\chi}_3 > \widehat{\chi}_2 > \widehat{\chi}_1$
		HDM	$\widehat{\chi}_4 > \widehat{\chi}_3 > \widehat{\chi}_2 > \widehat{\chi}_1$
		TDDM	$\widehat{\chi}_4 > \widehat{\chi}_3 > \widehat{\chi}_1 > \widehat{\chi}_2$
VIKOR [28]	Compromise measure	GEDM	$\widehat{\chi}_4 > \widehat{\chi}_3 > \widehat{\chi}_1 > \widehat{\chi}_2$
		HDM	$\widehat{\chi}_4 > \widehat{\chi}_3 > \widehat{\chi}_1 > \widehat{\chi}_2$
		TDDM	$\widehat{\chi}_4 > \widehat{\chi}_3 > \widehat{\chi}_1 > \widehat{\chi}_2$
TOPSIS [27]	Closeness coefficient	GEDM	$\widehat{\chi}_3 > \widehat{\chi}_4 > \widehat{\chi}_1 > \widehat{\chi}_2$
		HDM	$\widehat{\chi}_4 > \widehat{\chi}_3 > \widehat{\chi}_1 > \widehat{\chi}_2$
		TDDM	$\widehat{\chi}_4 > \widehat{\chi}_3 > \widehat{\chi}_1 > \widehat{\chi}_2$

Thus, we see for all the cases, the best alternative is $\widehat{\chi}_4$, except GEDM based TOPSIS.

Figure 4 shows the comparative analysis concerning different distance measures.

5.2 Sensitivity Analysis

In this particular subsection, a sensitivity analysis is conducted to ascertain the stability of the method put forth. Four attributes were utilized in the case study. To conduct the sensitivity analysis, six different sets of attribute weights are employed, which are derived from the rearrangement of the initially computed attribute weights. Through the examination of the model's reaction to various weighting methods, it is possible to pinpoint the attributes that carry the greatest influence on the outcomes and detect probable sources of uncertainty.

Through this iterative process, sensitivity analysis improves decision-making by elucidating the resilience of the model across various scenarios, directing stakeholders towards better informed and adaptable decisions.

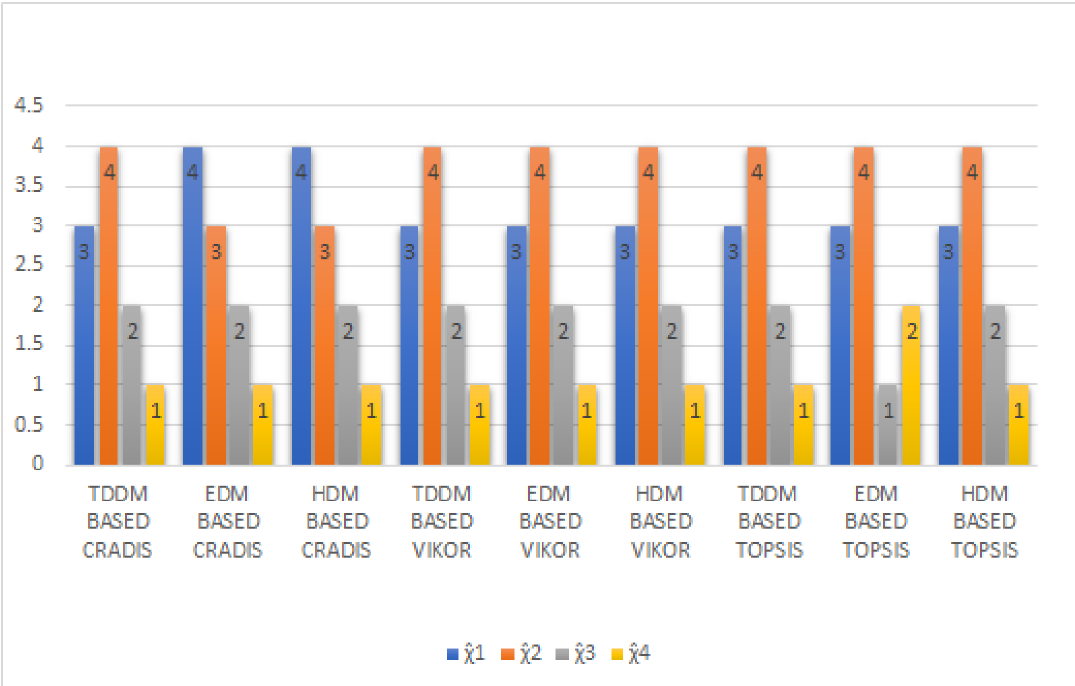

Figure 4: Comparison analysis.

The criteria weights for six sets are presented in Table 13. These particular sets of criteria weights are employed in the suggested approach, leading to alterations in the prioritization of the alternatives. Table 13 presents the selection of weight sets utilized for conducting sensitivity analysis. It is observed that in sets 2 and 4, the variable SC_1 holds the maximum weight, while for sets 4 and 6, SC_7 is assigned the minimum weight.

Table 13: Set of weight of criteria.

Sets	SC_1	SC_2	SC_3	SC_4	SC_5	SC_6	SC_7	SC_8
Set 1	0.15	0.22	0.18	0.02	0.12	0.08	0.15	0.08
Set 2	0.22	0.18	0.15	0.12	0.08	0.02	0.08	0.15
Set 3	0.18	0.02	0.12	0.08	0.15	0.08	0.15	0.22
Set 4	0.22	0.18	0.15	0.08	0.08	0.15	0.02	0.12
Set 5	0.12	0.08	0.15	0.08	0.15	0.22	0.18	0.02
Set 6	0.08	0.15	0.08	0.15	0.22	0.18	0.02	0.12

From the sensitivity analysis results presented in Table 14, it is evident that the ordering of alternatives remains consistent across all six sets of attribute weights, with the exception of set 5. The rankings consistently place Myntra ($\widehat{\chi}_4$) in the first position, followed by Meesho ($\widehat{\chi}_3$), Amazon ($\widehat{\chi}_1$) and Flipkart ($\widehat{\chi}_2$). However, in set 5, there is a deviation where Flipkart ($\widehat{\chi}_2$) is ranked third and Amazon ($\widehat{\chi}_1$) is ranked fourth. This shift in rankings can be attributed to variations in the weights assigned to the criteria, leading to a reduction in the overall utility of the alternatives. Therefore, the consistency observed in the ranking outcomes indicates that the proposed methodology exhibits significant stability and effectiveness across different configurations of criterion weights.

From Figure 5 we show the graphical demonstration of the sensitivity analysis.

Table 14: Sensitivity of proposed method.

Sets	Q_1	Q_2	Q_3	Q_4	Ranking
Set 1	0.507	0.472	0.643	1	$\widehat{\chi}_4 > \widehat{\chi}_3 > \widehat{\chi}_1 > \widehat{\chi}_2$
Set 2	0.501	0.338	0.521	1	$\widehat{\chi}_4 > \widehat{\chi}_3 > \widehat{\chi}_1 > \widehat{\chi}_2$
Set 3	0.563	0.430	0.589	1	$\widehat{\chi}_4 > \widehat{\chi}_3 > \widehat{\chi}_1 > \widehat{\chi}_2$
Set 4	0.487	0.309	0.519	1	$\widehat{\chi}_4 > \widehat{\chi}_3 > \widehat{\chi}_1 > \widehat{\chi}_2$
Set 5	0.436	0.463	0.541	1	$\widehat{\chi}_4 > \widehat{\chi}_3 > \widehat{\chi}_2 > \widehat{\chi}_1$
Set 6	0.513	0.377	0.625	1	$\widehat{\chi}_4 > \widehat{\chi}_3 > \widehat{\chi}_1 > \widehat{\chi}_2$

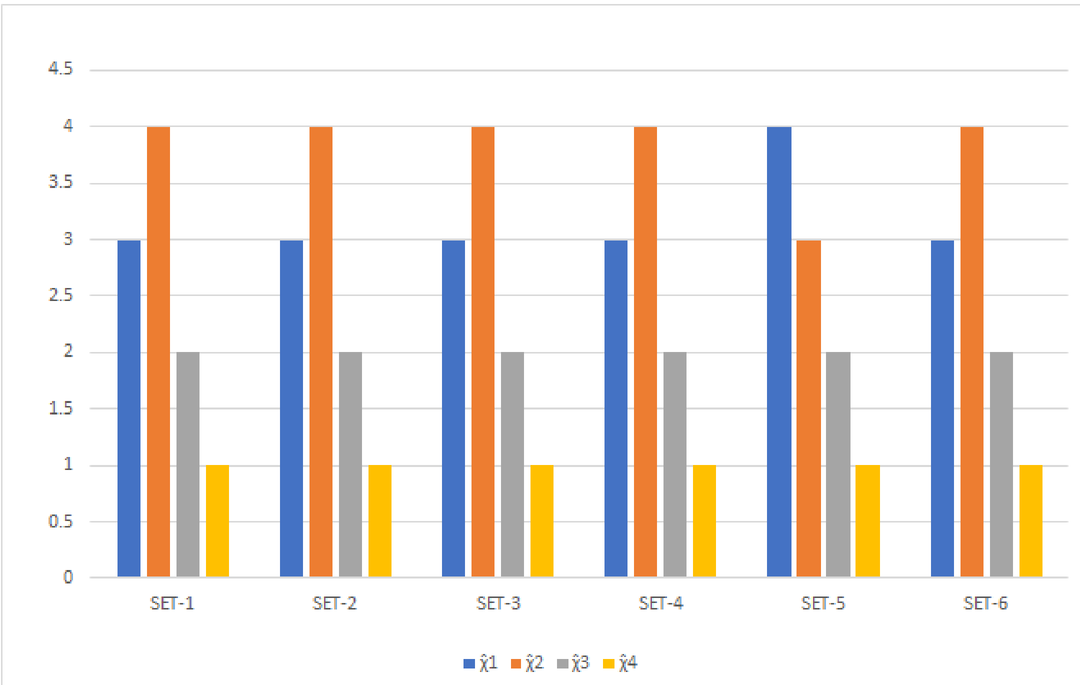

Figure 5: Sensitivity analysis.

6 Conclusion

This study proposes a modified CRADIS based on triangular divergence distance measure in the Fermatean Fuzzy sets. The FFNs are more efficient at accommodating fuzzy information compared to fuzzy extensions. Given the shortcomings of the existing distance measures for FFNs, a triangular divergence-based distance measure is proposed. To prevent any loss of information, the proposed triangular divergence-based distance measure includes the hesitancy degree of FFNs. The superiority of the proposed TDDM is established by an example where the proposed TDDM successfully distinguishes the distances between two given pairs of FFNs. To validate the proposed modified CRADIS method's practical applicability, it is used to solve a numerical problem. To check the applicability of the proposed modified CRADIS method, it has been compared with existing methods and distance measures. The comparative analysis suggests the superiority and reliability of the proposed method. We also conduct a sensitivity analysis to check its stability.

However, every research endeavor inevitably encounters certain constraints. In this paper, only the CRADIS method has been modified using the proposed TDDM. Utilizing the proposed TDDM in other distance-based MADM methods can establish the practicality of the proposed distance measure. Another limitation is that we have addressed only one numerical problem using the proposed modified CRADIS method. We have involved only one decision expert. Including more decision experts in the decision-making process can give better results. Also, we have assumed the weight of the criteria to maintain the simplicity of the method. Using different criteria weight determination methods can improve the model significantly.

There are numerous avenues for future investigation. The proposed distance measure can be extended to other fuzzy environments like 3,4-quasirung fuzzy sets [30], p,q-quasirung orthopair fuzzy sets [31], hesitant fuzzy sets [32], neutrosophic fuzzy sets [33], linear diophantine fuzzy sets [34], q-rung linear diophantine fuzzy hypersoft fuzzy set [35]. Also, the proposed TDDM can be applied to other distance-based MADM methods. The proposed TDDM-based modified CRADIS can be applied to other real-life MADM problems to establish further applicability of the method.

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