International Journal of analytical and Numerical Methods in Mechanical Design
Winter & Spring 2023 Vol 2 , Issue 1, 49-55

Investigation of Stress in the Perforated Plate with the Presence of Edge Crack

Nooredin Sabzian Moradabadi [a], Hamed Bazvandi [b], ✉, Mojtaba Nazari [a]

[a] Department of Mathematics, Khorramabad Branch, Islamic azad university, Khorramabad, Iran
[b] Department of Mechanical Engineering, Khorramabad Branch, Islamic azad university, Khorramabad, Iran
Received 20 March 2023; Revised 25 April 2023; Accepted 28 May 2023
✉ bazvandi@khoiau.ac.ir

Abstract

The phenomenon of failure in objects is one of the major issues that human beings have been facing for a long time, and because of advances in technology in the present age, this issue is more important than in the past.All engineering materials, on the other hand, have tiny cracks from which failure begins.Therefore, estimating the residual life of thin plates made from these materials and used in space and offshore structures requires knowledge of the stress distribution due to cracking in these components. Because of the singularity of the crack tip due to large stresses, the presence of a relatively small crack can lead to a hazardous situation. Therefore, this area should be given more attention.In this research, using the Inglis formula and considering the correction coefficient of the compensatory free surface, the value of the stress coefficient for edge crack is obtained. Then, by replacing the new stress coefficient in Westergaard formula, we calculate the stress field of Mode I (Opening mode) and Mode II (sliding mode) in the perforated plane containing the edge crack. Finally, we examine the effects of various parameters such as loading angle, crack length and hole radius on the values obtained for stress in both modes by plotting.

Keywords: failure; edge crack; Inglis formula; stress field

1. Introduction

Thin sheets are widely used in offshore and space structures. Usually, defects in engineering structures or structures are involuntarily or specifically designed for structures.One of these geometrical defects is to create holes in the plates. Different types of holes are created in the plates for functional reasons such as weight loss of the structure and access to other system equipment.Plates containing holes when exposed to tensile or shear loading have caused severe local stress and reduced structural resistance, resulting in unexpected hazardous events such as the collapse of planes, spacecraft and ships, etc. over the past decades.This necessitates a more accurate understanding of the mechanics of failure in objects, and the problem of failure mechanics since the beginning of the present century has always been of interest to researchers and designers of mechanical components. Application of Complex Variable Functions to Plate Problems Based on Klosov's Elasticity Theory was studied[1] . Investigation of inherent defects in materials, and stress fields in elliptical perforated plates by Inglis was showed[2]. Application of the complex mapping method in the problem of circular hole with edge crack and also circular hole with a pair of symmetric edge crack in unlimited plate under uniform stress was studied by Bowie[3]. The expression of the mixed-mode method in two-dimensional elastic theory by Muskelyshvili and his use of coherent mapping to analytically solve the stress problem around the hole in two-dimensional elastic bodies [4, 5]. the stress concentration around holes of different shapes (triangular-rectangular, etc.) for isotropic and non-isotropic planes using the Savin Measured to calculate [6]. the definition of special function expansions, the application of the improved cumulative boundary method, and the Muskiewicli mixing variable method, and computation for Newman's elliptical and circular bifurcations was used[7]. Numerical calculation of stress intensity coefficients using integral equations for circular perforations having an edge elasticity in infinite elastic material under uniform loading and also analysis of stress intensity factors under conditions where there is loading on the hole and crack by Tweed and Ruck was studied[8, 9]. tensile problems in a plate containing an elliptic hole through which two symmetric edge cracks are branched, and numerical results for mid-length cracks by Neyestani and Isida[10, 11] was Analyzed. the Semi-Unlimited and Isotropic Plate Problem of a Triangular Slit, from which a crack originated from a distal tensile load, and the use of the Muskelyshvili method and the congruent mapping as a sum of fractional expressions to provide an exact solution for the stress distribution by Hasseb[12] And Ida was Investigated. Use of Green's Function and Muskelyshvili's Method to Calculate Stress Coefficients of First and Second Mode Stress Intensity Coefficients for an Unlimited Isotropic Plate Containing a Circular Hole with an Edge Crack by Shivakomar and Foreman was studied[13]. The stress distribution and intensity around triangular holes by Theocariz and Petro was Investigated [14]. Unlimited Isotropic Plate Problem Containing Cracked Circular Cracks and Crack Displacement Rate Using Doddel Plastic Area Model for Small Cracks by Bostrom was showed[15]. the Buckner principle to compute the stress intensity factor of the first and second mode infinite isotropic planes containing the cracked circular hole, also investigate the problem of having one crack and two cracks of different lengths and two cracks of arbitrary angle originating from the hole by Lin et al was Applyed [16]. the fractional numerical method, in calculating the stress intensity coefficients in cracks originating from circular and square holes in an isotropic boundless plate by Yan[17, 18] was Used. a convex mapping to investigate the problem of a perforated edge crack in a linear elastic plane under intra-plane elongation and using this mapping to move the outside of the cracked semicircle into a single circle by Abdulmola et al[19] was Introduced. the stress distribution adjacent to a hole and a crack for plane problems with crack originating from an elliptic hole Also use the mixed method to calculate the stress intensity coefficients of single and double edge cracks originating from an elliptic hole by Guo et al[20-24] was Obtained. the stress distribution problem around a polygonal hole in an infinite plane under uniform load over long distances using the Batista-modified Muskhelishvili variable method[25] was Solved. The stress distribution around polygon hole in infinite plane under dual axis loading and investigation of the effect of hole geometry and loading by Sharma[26] was Calculated. the mixed variable method, in the study of the problem of elliptical holes with two edge cracks with unequal length in the isotropic plate and under tensile load in the distance and obtaining analytical solutions for stress functions and stress intensity coefficient of the first mode by Liu and Duan[27-32] was Used. the fractional method, to calculate stress intensity coefficients on a rectangular plate with two equal cracks by Miao et al[33, 34] was studied. The First Mode Stress Intensity Coefficients in Single and Double Circular Plate Finite Circular Plates Using New Weight Function by Kim and Hill[35] was Presented. The fracture behavior due to an elliptical hole with two asymmetric cracks in a one-dimensional quasi-crystal hexagon with piezoelectric effect by Yang et al[36] was Investigated. In this research, by combining the Inglis formula with the Westergaard method and considering the correction factor of the compensatory free surface, we introduce a new method for calculating the stress field of a perforated plate containing edge cracks for modes 1 and 2, Then the effects of crack length, loading angle and hole radius on the stress level for both modes are compared.

2. Problem Statement

Suppose an indefinite plate containing crack length 2a under uniform tension σ locates in the far distance (Fig. 1). For this problem, the simplest function of Westergaard is as follows[37].

$$\boldsymbol{z_I} = \frac{\boldsymbol{\sigma z}}{(\boldsymbol{z^2} - \boldsymbol{a^2})^{1/2}} \qquad (\mathbf{1})$$

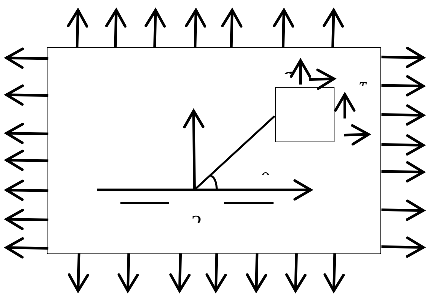


Fig.1. Infinite plate containing cracks of length 2a under uniform stresses σ at infinity

If in Fig. 1, instead of normal tention, σ applies uniform shear tension, the simplest Westergaard function is The following is the case[37]:

$$z_{II} = -\frac{itz}{(z^2-a^2)^{1/2}} \qquad (2)$$

2.1. A) Mode I (Opening mode)

Consider Equation (1). If we transfer the source of the coordinates to the crack tip, then ξ= z - a Then the equation for points near the crack tip (ξ≪ a) can be calculated as follows:

$$\mathrm{z_I} = \frac{\sigma(\xi+a)}{[\xi(\xi+2a)]^{1/2}} = \frac{\sigma(\xi+a)}{\xi^{1/2}.(\xi+2a)^{1/2}}$$

$$= \frac{\sigma(\xi+a)}{\xi^{1/2}\left(2a\left(1+\frac{\xi}{2a}\right)\right)^{1/2}} =$$

$$\frac{\sigma(\xi+a)}{(2a\xi)^{1/2}\left(1+\frac{\xi}{2a}\right)^{1/2}}$$

$$= \frac{\sigma(\xi+a)}{(2a\xi)^{1/2}}\left[1-\frac{1}{2}\left(\frac{\xi}{2a}\right)+\frac{3}{8}\left(\frac{\xi}{2a}\right)^2 - \frac{15}{24}\left(\frac{\xi}{2a}\right)^3 + \cdots\right]$$

But around the crack tip which is ξ=z-a (values | ξ | are very small, so assuming k_1=σ√πa can be written as:

$$z_1 = \frac{\sigma a}{(2a\xi)^{\frac{1}{2}}} = \frac{\sigma\sqrt{a}}{(2\xi)^{\frac{1}{2}}} = \frac{\sigma\sqrt{\pi a}}{\sqrt{2\pi\xi}} = \frac{k_1}{\sqrt{2\pi\xi}} \qquad (3)$$

Now by considering the polar coordinates r, θ, and and ξ=re^iθ apply the law of motion.

$$\sigma_x = \frac{k_1}{\sqrt{2\pi r}} cos\frac{\theta}{2}\left(1 - sin\frac{\theta}{2}sin\frac{3\theta}{2}\right) \qquad (4)$$

$$\sigma_y = \frac{k_1}{\sqrt{2\pi r}} cos\frac{\theta}{2}\left(1 + sin\frac{\theta}{2}sin\frac{3\theta}{2}\right) \qquad (5)$$

$$T_{xy} = \frac{k_1}{\sqrt{2\pi r}} cos\frac{\theta}{2}\left(sin\frac{\theta}{2}cos\frac{3\theta}{2}\right) \qquad (6)$$

B) II (sliding mode)

By applying the relation (2)and the process used for the first mode, different tension states can be calculate for the crack with the length of 2a in an unlimited sheet under uniform shear tension in the plate (τ) as follow:

$$\sigma_x = \frac{k_{II}}{\sqrt{2\pi r}} sin\frac{\theta}{2}\left(2 + cos\frac{\theta}{2} cos\frac{3\theta}{2}\right) \quad (7)$$

$$\sigma_y = \frac{k_{II}}{\sqrt{2\pi r}} sin\frac{\theta}{2}\left(cos\frac{\theta}{2} cos\frac{3\theta}{2}\right) \quad (8)$$

$$T_{xy} = \frac{k_{II}}{\sqrt{2\pi r}} cos\frac{\theta}{2}\left(1 - sin\frac{\theta}{2} sin\frac{3\theta}{2}\right) \quad (9)$$

Where (k_II) the tention coefficient of mode II is for the following figure.

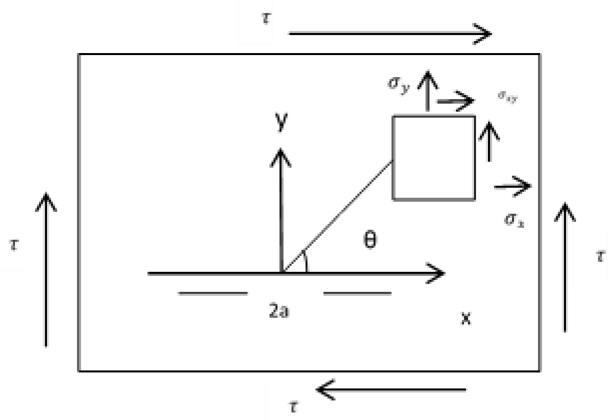


Fig.2. Infinity plate containing cracks of length 2a under uniform shear stresses τ at infinity

$$\left(k_{II} = \tau\sqrt{\pi a}\right) \quad (10)$$

On the other hand,taking into account the British equation [2],the tension distribution in the perforated plate containing the elliptic hole For Fig. 3, can be obtain according to the following formula.

$$\sigma_A = \sigma(1 + \tfrac{2a}{b}) \quad (11)$$

This relationship gives us the amount of tension on a plate with elliptic holes 2a and 2b. Also according to [38].The stress intensity coefficient for a fracture crack is obtained from relation(11) that Its σ is the applied stress and a0 is the length of the fracture crack.

$$k = 1.12\sigma_A\sqrt{\pi a_0} \quad (12)$$

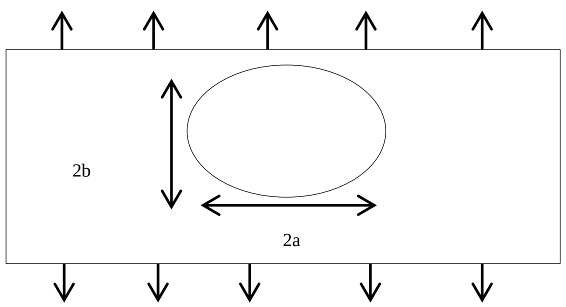


Fig.3. The plate has an elliptical hole of diameter 2 a, 2b under uniform stresses σ at infinity

Now, if we consider the simultaneous effects of holes and cracks, we can integrate the relationships(11) and (12) for obtaning the following relation:

$$k^* = 1.12\sigma(1 + \tfrac{2a}{b})\sqrt{\pi a_0} \quad (13)$$

Of course, the relation (13)for the time (a0 << R) means that the dimensions of the crack are very low compared to the cavity radius.

In the particular case where a = b = R. With regard to the relation (12) we will have:

$$k_1^* = 3.36\sigma\sqrt{\pi a_0} \quad (14)$$

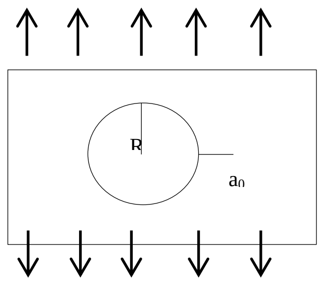
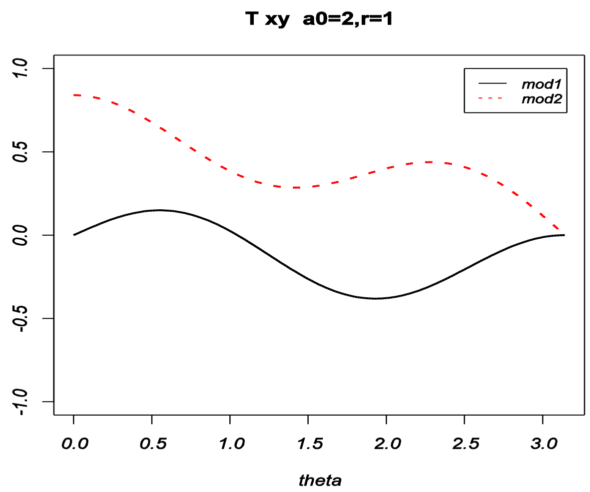


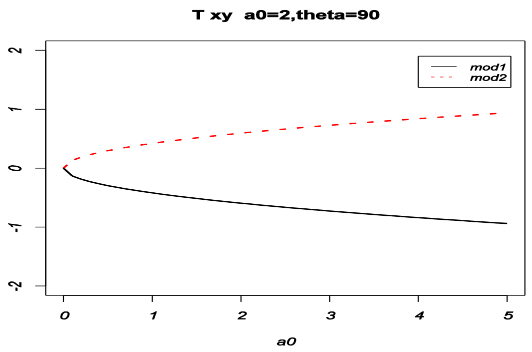
Fig.4. Short cracks of length a0, caused by an elliptical hole in an infinite sheet subjected to stress σ perpendicular to the principal axis of the hole

By substituting k_1* for k_1 in relation (4), (5)and (6) the stress field of mode II can be calculated on perforated plates containing the edge crack for Fig.4. Also, if instead of the normal stress σ, set the value of τ in the relation k_1*. then put the obtained value in the equations (7), (8)and (9). The Mode II stress value for the perforated plate containing the crack is obtained. Now to investigate the effects of various parameters such as loading angle, crack length and hole radius on mode stress values 1 and 2 using R coding software version 3.6.3 Draw and analyze the results.

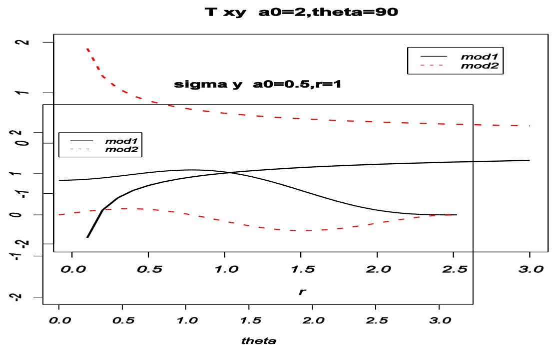
3. Results and discussion



(a)



(b)

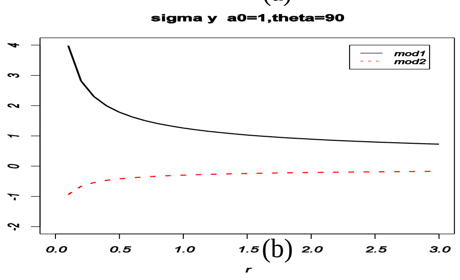


(c)

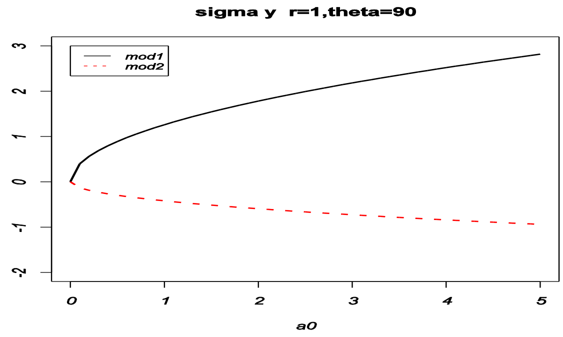
Fig.5.a,b,c respectively The effect of θ, a_0, and r on Txy in modes I and II

Fig.5 compare the rate of change of Txy in modes I and II and in cases where a_0 , r, and θ change. As we can see in the figures above, with increasing angle θ , however, the value of Txy in mode II is greater than that in mode I But these changes are oscillating, and increasing or decreasing the angle does not increase or decrease the amount of Txy at all. On the other hand, increasing a_0 in mode I reduces Txy but in mode II increases Txy. In general, as the value of a_0 increases, the value of Txy diverges in two modes. Also, increasing r in mode I increases Txy but in mode II decreases Txy. And in general, as r increases, the difference in Txy value between the two modes decreases.

(a)



(b)

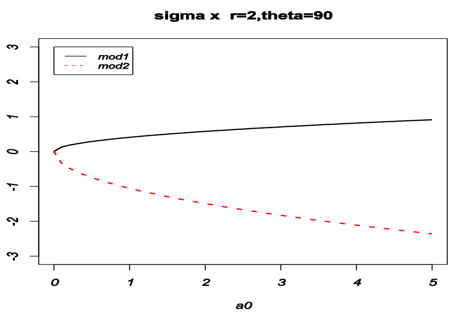


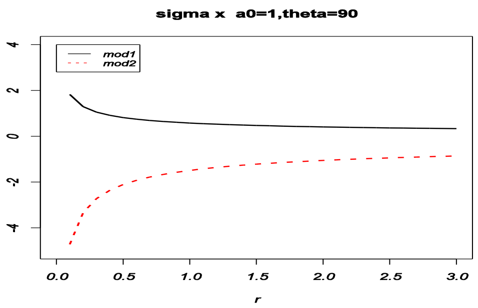
(c)

Fig.6. The effect of r, a_0 and θ on σy in modes I and II

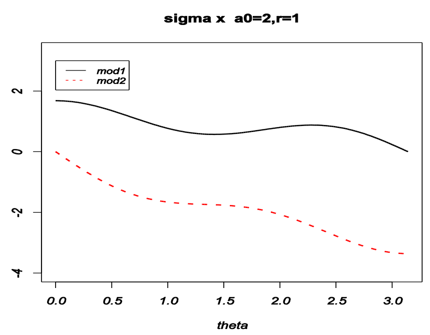
Fig.6.compare the rate of change of σy in modes I and II and in cases where a_0 , r, and θ change. As we can see in the above figures, by increasing r, the value of σy decreases in mode I but increases in mode II. On the other hand, the higher the value of r, the closer the value of σy becomes in two modes. But with increasing a_0, the value of σy increases in case I and decreases in mode II but, with increasing angle θ , however, the value of σy in mode I is greater than that in mode II. And then, with a proper increase in angle θ , the two values converge.

(a)





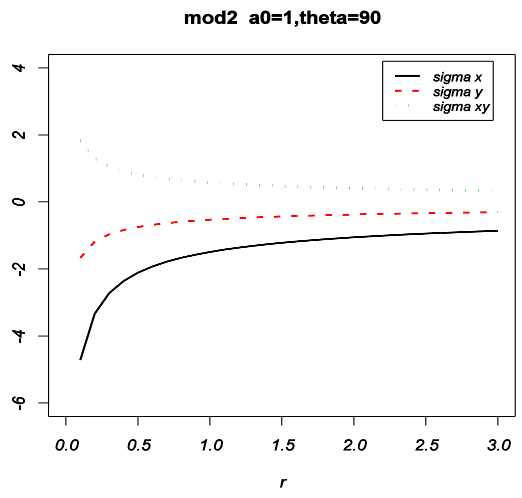
(b)



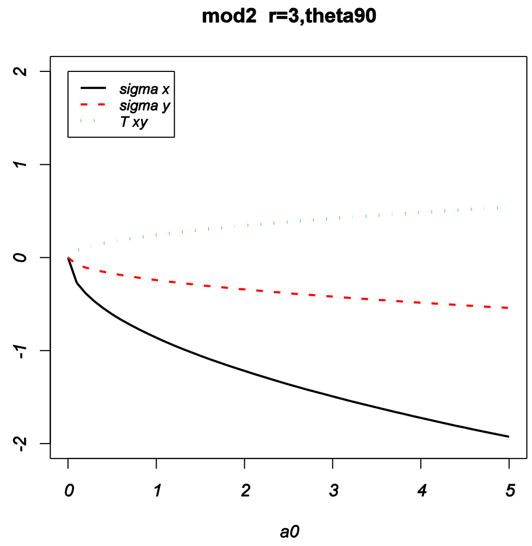
(c)

Fig.7. The effect of r, a_0 and θ on σx in modes I and II

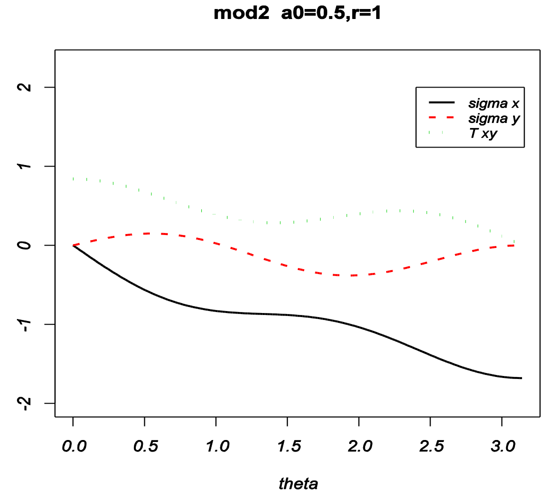
Fig.7 show the rate of change of σx in states where a_0, r, and ɵ change. As shown in the diagrams. Increasing r in mode I decreases σx but in mode II increases σx . But in the case of increasing a_0, the opposite is true, ie in mode I the value of σx increases but in mode II it decreases. Also, as the angle. Increases, the difference in the value of σx between the modes increases.



(a)

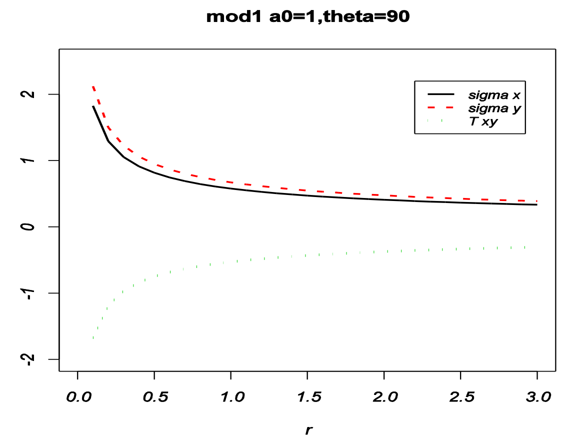


(b)

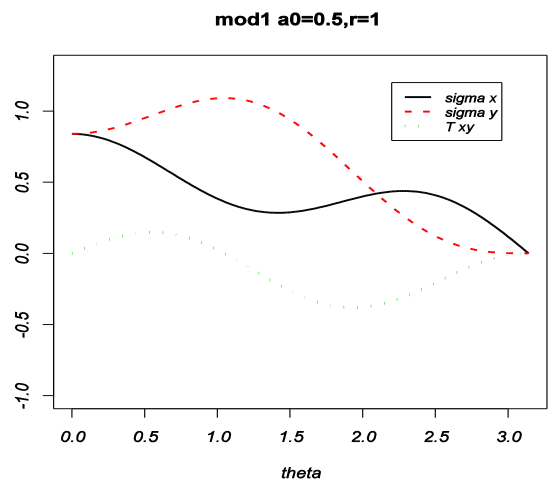


(c)

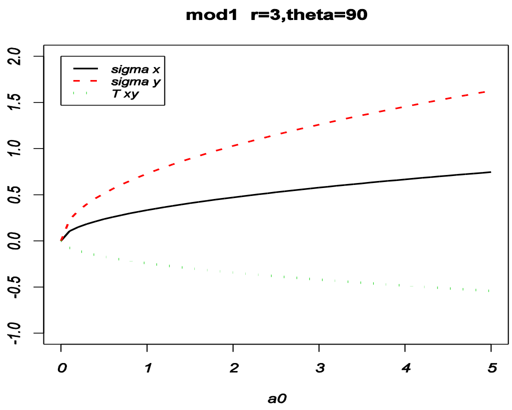
Fig.8. The effect of r,a_0 and θ on σ_x, σ_y andTxy in mode II



(a)



(b)



(c)

Fig.9. The effect of r,a_0 and θ on σ_x, σ_y andTxy in mode I

Fig.8 Comparison diagrams of σ_x , σ_y and Txy values in case II and Fig.9 show a comparison chart of the values mentioned in Mode I

4. Conclusion

As the value of r increases, σ_x has a decreasing trend in mode I but an increasing trend in mode II. T_{xy} also increases in mode I but decreases in mode II. And σ_y increases in mode II but decreases in mode I. Increasing the value of $_{\circ}$ a in mode I decreases T_{xy} and increases σ_x and σ_y. While in Mode II the trend is different, ie T_{xy} increases but σ_x and σ_y decrease. By increasing ɵ in both modes in the considered range, we do not see ever decreasing or ascending behavior of any of the values σ_x , σ_y and T_{xy}. Rather, the behavior of the relevant graphs in different intervals is sometimes ascending and sometimes descending. And it can not be said at all that increasing ɵ in both modes increases or decreases the values of σ_x , σ_y and T_{xy}.

Acknowledgments

The authors are greatly thankful to Khorramabad Branch of Islamic Azad University.

References

[1] Abdelmoula, R., Semani, K., & Li, J. (2007). Analysis of cracks originating at the boundary of a circular hole in an infinite plate by using a new conformal mapping approach. *Applied mathematics and computation*, 188(2), 1891-1896.

[2] Batista, M. (2011). On the stress concentration around a hole in an infinite plate subject to a uniform load at infinity. *International Journal of Mechanical Sciences*, 53(4), 254-261.

[3] Boström, L. A. (1989). The Dugdale model used for short radial cracks emanating from a circular hole in an infinite sheet. *Engineering fracture mechanics*, 34(4), 823-829.

[4] Bowie, O. L. (1956). Analysis of an infinite plate containing radial cracks originating at the boundary of an internal circular hole. *Journal of mathematics and physics*, 35(1-4), 60-71.

[5] Du, Y., Liu, S., Duan, S., & Li, Y. (2013). Electro-elastic fields of piezoelectric materials with an elliptic hole under uniform internal shearing forces. *Chinese Journal of Mechanical Engineering*, 26(3), 454-461.

[6] Gdoutos, E. E. (2020). Fracture mechanics: an introduction (Vol. 263). Springer Nature.

[7] Guo, J. H., & Liu, G. T. (2008). Analytic solutions to problem of elliptic hole with two straight cracks in one-dimensional hexagonal quasicrystals. *Applied Mathematics and Mechanics*, 29(4), 485-493.

[8] Guo, J., & Liu, G. (2007). Stress analysis for an elliptical hole with two straight cracks. 力学学报, 23(5), 699-703.

[9] Guo, J.-H., et al., The behavior of two non-symmetrical permeable cracks emanating from an elliptical hole in a piezoelectric solid. *European Journal of Mechanics-A/Solids*, 2010. 29(4): p. 654-663

[10] Guo, J.-H., et al., Exact solutions for anti-plane problem of two asymmetrical edge cracks emanating from an elliptical hole in a piezoelectric material. International *Journal of Solids and Structures*, 2009. 46(21): p. 3799-3809

[11] Huaimin, G., Guanting, L., & Jiandong, P. (2007). Stress analysis of an ellipse hole with a straight edge-crack by complex variable method. *ACTA MECHANICA SOLIDA SINICA-CHINESE EDITION-*, 28(3), 308.

[12] Inglis, C.E., Stresses in a plate due to the presence of cracks and sharp corners. *Trans Inst Naval Archit*, 1913. 55: p. 219-241.

[13] Isida, M., Chen, D. H., & Nisitani, H. (1985). Plane problems of an arbitrary array of cracks emanating from the edge of an elliptical hole. *Engineering fracture mechanics*, 21(5), 983-995.

[14] Kim, J., & Hill, M. R. (2016). Weight functions for a finite width plate with single or double radial cracks at a circular hole. *Engineering Fracture Mechanics*, 168, 112-130.

[15] Kolosov, G. V. (1909). On the application of complex function theory to a plane problem of the mathematical theory of elasticity (Orig Russian) (Doctoral dissertation, Doctoral thesis, University of Yuriew, Yuriew Publ Co).

[16] Lin, S., & Hills, D. A. (1996). Stress intensity factors for cracks emanating from a semicircular notch in a half-plate. *The Journal of Strain Analysis for Engineering Design*, 31(6), 433-439.

[17] Liu, S. H., Li, Y. Q., & Shen, Y. M. (2012). The electro-elastic fields of piezoelectric materials

with an elliptic hole [J]. *Engineering Mechanics*, 29(12), 45-50.

[18] Liu, S., & Duan, S. (2014). Analytical solutions of cracks emanating from an elliptical hole under shear. *Chinese Journal of Aeronautics*, 27(4), 829-834.

[19] Liu, S., Qi, Y., Feng, D., Shi, X., & Wan, T. (2012). Numerical analysis of crack emanating from elliptical hole. Jixie Gongcheng Xuebao(Chinese *Journal of Mechanical Engineering*), 48(20), 83-87.

[20] Liu, S., Shen, Y., & Liu, J. (2012). Exact solutions for piezoelectric materials with an elliptic hole or a crack under uniform internal pressure. *Chinese Journal of Mechanical Engineering*, 25(4), 845-852.

[21] Liu, S., & Duan, S. (2014). Analytical solutions of cracks emanating from an elliptic hole in an infinite plate under tension. *Chinese Journal of Mechanical Engineering*, 27(5), 1057-1063.

[22] Miao, C., Wei, Y., & Yan, X. (2013). Interactions of two collinear circular hole cracks subjected to internal pressure. *Applied Mathematics and Computation*, 223, 216-224.

[23] Miao, C., Wei, Y., & Yan, X. (2015). A numerical analysis of a center circular-hole crack in a rectangular tensile sheet. *Applied Mathematics and Computation*, 250, 356-367.

[24] Muskhelishvili, N. I. (1966). Some Basic Problems of Mathematical Elasticity Theory [in Russian] Nauka.

[25] Muskhelishvili, N. I. (2013). The fundamental law of the theory of elasticity. The basic equations. In Some Basic Problems of the Mathematical Theory of Elasticity: Fundamental Equations Plane Theory of Elasticity Torsion and Bending (pp. 52-84). Dordrecht: Springer Netherlands.

[26] Newman, J. C. (1971). An improved method of collocation for the stress analysis of cracked plates with various shaped boundaries. National Aeronautics and Space Administration.

[27] Nisitani, H., & Isida, M. (1973). Stress intensity factor for the tension of an infinite plate having an elliptical hole with two cracks emanating from its apexes. *Trans. Japan Soc. Mech. Eng*, 39(317), 7-14.

[28] Norio, H., & Jiro, I. (1978). A crack originating from a triangular notch on a rim of a semi-infinite plate. *Engineering Fracture Mechanics*, 10(4), 773-782.

[29] Savin, G. N. (1961). Stress concentration around holes. (No Title).

[30] Sharma, D. S. (2012). Stress distribution around polygonal holes. *International Journal of Mechanical Sciences*, 65(1), 115-124.

[31] Shivakumar, V., & Forman, R. G. (1980). Green's function for a crack emanating from a circular hole in an infinite sheet. *International Journal of Fracture*, 16, 305-316.

[32] Theocaris, P. S., & Petrou, L. (1986). Stress distributions and intensities at corners of equilateral triangular holes. *International Journal of Fracture*, 31, 271-289.

[33] Tweed, J., & Rooke, D. P. (1979). The stress intensity factor for a crack at the edge of a loaded hole. *International Journal of solids and structures*, 15(11), 899-906.

[34] Tweed, J., & Rooke, D. P. (1973). The distribution of stress near the tip of a radial crack at the edge of a circular hole. International *Journal of Engineering Science*, 11(11), 1185-1195.

[35] Yan, X. (2004). Analysis for a crack emanating from a corner of a square hole in an infinite plate using the hybrid displacement discontinuity method. *Applied Mathematical Modelling*, 28(9), 835-847.

[36] Yan, X. (2005). Stress intensity factors for cracks emanating from a triangular or square hole in an infinite plate by boundary elements. *Engineering Failure Analysis*, 12(3), 362-375.

[37] Yang, J., Zhou, Y. T., Ma, H. L., Ding, S. H., & Li, X. (2017). The fracture behavior of two asymmetrical limited permeable cracks emanating from an elliptical hole in one-dimensional hexagonal quasicrystals with piezoelectric effect. *International Journal of Solids and Structures*, 108, 175-185.