Machine Learning-driven Group Ranking in Data Envelopment Analysis: Applications in the Banking Sector

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Abstract:

This paper explores the intersection of Group Ranking in Data Envelopment Analysis (DEA) and the potent capabilities of Machine Learning (ML) within the insurance sector, aiming to redefine group efficiency assessment. While DEA has been a cornerstone for evaluating Decision-Making Units (DMUs), the traditional models fall short in the nuanced insurance sector. To address these limitations, ML is integrated into DEA, enabling more effective DMU ranking. The study includes an empirical application within the banking industry, emphasizing the methodology's relevance and potential to transform the landscape of this industry. Specifically, we evaluate the efficiency of 525 branches of Mellat Bank in Iran, divided into 21 groups, each comprising 25 members. We use the BPNN neural network algorithm to predict the group efficiency score of the 21st group and compare the results with those obtained from the CCR model.

Keywords: Group Efficiency, Banking Groups, Machine Learning, Neural Network, Data Envelopment Analysis, Ranking

1. Introduction

Efficiency measurement of Decision-Making Units (DMUs) has been a focal point in organizational performance assessment since the pioneering work of Farrell in 1957, who laid the groundwork for Data Envelopment Analysis (DEA) [1]. DEA, introduced by Charnes et al. in 1978 [2], employs linear programming techniques to evaluate the efficiency of DMUs with multiple inputs and outputs, thus offering a comprehensive framework for performance evaluation. Over the years, DEA has evolved, with new methodologies and applications emerging to address the complexities of diverse organizational settings across sectors such as

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government, healthcare, finance, and manufacturing. Despite its widespread adoption, traditional DEA approaches encounter challenges, particularly when assessing the efficiency of new DMUs. The proliferation of DMU datasets, fueled by the advent of big data, compounds this challenge. For instance, mainland China has witnessed a rapid increase in the number of small and micro-sized companies, surpassing 73 million [3]. Various methods have been developed to enhance the discrimination power of basic Data Envelopment Analysis (DEA) models and address challenges in evaluating organizational performance. Over the past three decades, numerous ranking methods have emerged within the DEA context. Adler et al. [4] and Chen [5] have extensively reviewed these ranking methods. Among them, the cross-efficiency evaluation, initially proposed by Sexton et al. [6], stands out as the most popular. This method utilizes peer evaluation instead of self-evaluation information, overcoming some limitations of basic DEA models like CCR and BCC. However, its utility may diminish due to the non-uniqueness of optimal weights [7]. To mitigate this issue, Doyle and Green [8] introduced aggressive and benevolent formulations as secondary goals to select a solution from multiple optimal weights.

Another method proposed for ranking efficient units is super-efficiency, introduced by Anderson and Peterson [9]. This method involves eliminating the DMU under assessment from the DMU set and computing its distance from the new efficient border. However, this elimination can render associated DEA models infeasible [10]. To address this challenge, Memariani et al. [11] modified the non-radial model presented by Mehrabian et al. [10], ensuring the feasibility of the linear programming model.

While the mentioned methods primarily focus on individual evaluation of DMUs, many real-world scenarios involve DMUs as members of distinct groups. For instance, sales units of a brand operate independently but under a single management for achieving common targets. In such cases, group efficiency evaluation becomes crucial. Ang et al. [12] examined this evaluation from two perspectives: group efficiency evaluation based on average performance and weakest performance.

To develop this method, Shahbazifar et al. [13] proposed a novel two-stage network group efficiency evaluation method. This method, based on network DEA, offers a broader insight and yields more accurate results compared to conventional approaches. Their goal is to determine the true ranking of two-stage network production groups, presenting new network DEA models for efficiency evaluation based on both average and weakest performance criteria.

Evaluating the efficiency of each new DMU using conventional DEA methods necessitates substantial computational resources in terms of memory and CPU time .In response, there is a growing interest in leveraging Machine Learning (ML) algorithms to predict efficiency scores without the need for extensive re-analysis. ML algorithms, which learn from data to make predictions or decisions, present a promising approach to enhance efficiency assessment. Previous studies have explored various ML-DEA methodologies, including neural network back-propagation DEA algorithms (NNDEA), genetic algorithms, support vector machines (SVM), and integrated support vector machines (ISVM) [14]. While existing literature highlights the potential of hybrid ML-DEA methodologies, several limitations persist. Many studies predominantly focus on neural networks or back-propagation neural networks for prediction tasks, with limited exploration of other ML algorithms. Moreover, there is a scarcity of research comparing the performance of integrated ML-DEA models with individual ML models[3]. To address these gaps, Emrouznejad et al. [3] aim to bridge traditional DEA methods with ML algorithms by proposing a comprehensive ML-DEA framework. Specifically, they introduce ML-DEA algorithm: DEA-CCR model combined with back-propagation neural network (BPNN-DEA). Subsequent to the seminal work by Charnes et al. [2], numerous sophisticated applications have emerged, incorporating additional variables and complex models to assess the efficiency and productivity changes of Decision-Making Units (DMUs). These endeavors aim to enhance organizational performance across various sectors, both public and private. Also, Mahamoudi et al. [15] proposed an incremental weighted cross-entropy loss function for convolutional neural networks to tackle class imbalance. Their method enhances performance by gradually increasing the weight of minority classes during training, showing superior results compared to other techniques on various datasets. Additionally, due to the intricate nature of DEA calculations, specialized software tools have been developed to facilitate analysis. However, in the process of assessing organizational performance, the addition of a new Decision-Making Unit (DMU) necessitates the rerunning of the DEA model. To circumvent the need for recalculating the efficiency of all DMUs, some studies have proposed predicting the DEA efficiency of new DMUs by integrating the DEA model with various Machine Learning (ML) algorithms. For instance, Liu et al. [16] utilized DEA, a three-stage DEA, and artificial neural network (ANN) to evaluate the technical efficiency of 29 semiconductor firms in Taiwan. They observed that employing different approaches (DEA vs. NN) within a similar methodological framework yielded divergent results.

In today's era of rapid big data expansion, the growth of datasets has surged exponentially. Consequently, conducting Data Envelopment Analysis (DEA) on large datasets containing numerous inputs and outputs poses significant challenges due to the immense computational resources required, including memory and CPU time. Emrouznejad et al. [17] introduced a novel solution to this issue with their proposal of a neural network back-propagation DEA algorithm (NNDEA). This innovative algorithm aims to streamline the efficiency assessment process by randomly selecting a subset of Decision-Making Units (DMUs) for neural network training. Subsequently, the trained model can be leveraged to estimate efficiency scores without the need to solve linear programming problems for each individual DMU. Given that NNDEA significantly reduces the computational requirements in terms of computer memory and CPU time compared to traditional DEA-CCR models, it emerges as a valuable tool for efficiency measurement in large datasets. Moreover, researchers have explored various methodologies to integrate DEA with other techniques for efficiency assessment in different domains. For example, Misiunas et al. [18] combined DEA with Artificial Neural Networks (ANN) to develop a healthcare analytics methodology aimed at predicting the functional status of organ recipients. Their study focused on thoracic datasets comprising extensive records of lung and heart transplants performed in the US, totaling 16,771 records and 442 variables. By addressing the challenge of predicting patient outcomes in organ transplant operations, their proposed methodology yielded highly promising results, thus validating its efficacy.

Furthermore, in the realm of financial services, researchers have investigated the integration of DEA with machine learning techniques to enhance efficiency assessment in banking operations. For instance, Thaker et al. [19] developed a hybrid DEA-Random Forest model to evaluate the operational efficiency of commercial banks. By leveraging the capabilities of both DEA and Random Forest algorithms, their approach enabled accurate efficiency prediction while efficiently handling large-scale banking datasets.

Additionally, in the context of manufacturing industries, Lee et al. [20] proposed a novel approach that combines DEA with Convolutional Neural Networks (CNN) for assessing the operational efficiency of production processes. Their methodology, applied to a dataset encompassing diverse manufacturing parameters, demonstrated significant improvements in efficiency prediction accuracy compared to traditional DEA methods. Barros et al. [21] introduced a DEA-BPNN approach for evaluating and forecasting the efficiency ratings of insurance firms in Mozambique, while Kwon et al. [22] developed a similar method for assessing efficiency in major US banking institutions. Yang et al. proposed [23] a teleoperation method for robots based on human-robot interaction using visual information. This method, enhanced by an extreme learning machine algorithm, allows a robot to autonomously complete tasks by learning from a single human demonstration, achieving satisfactory performance.

Overall, the integration of DEA with machine learning algorithms offers a promising avenue for addressing efficiency assessment challenges across various domains, ranging from healthcare and finance to manufacturing and beyond. These innovative methodologies enable more accurate and efficient evaluation of organizational performance in the era of big data.

In this paper, we have evaluated 21 banking groups, each consisting of 25 branches, using the Back-Propagation Neural Network (BPNN) method. We predict the group efficiency scores using this approach.

The rest of this paper is organized as follows. Section 2 provides a brief introduction of data envelopment analysis and group efficiency evaluation method and BPNN machine learning algorithm. The research structure is explained in Section 3. In Section 4, some empirical application is presented, and the efficiency scores obtained from the various methods are reported and analyzed. The last section includes conclusions and possible future research.

2. Methodologies

2.1 DEA and group efficiency evaluation

DEA, a method for evaluating DMU efficiency, employs linear programming to envelop observed input/output vectors tightly. The DEA-CCR model, introduced by Charnes et al. [2], focuses on the ratio of multi-outputs to multi-inputs, representing how effectively a DMU utilizes its resources to produce valuable outputs. This model sets a condition where these ratios must be less than or equal to one for all other DMUs, without the need for predefined weights on inputs and outputs.

Consider a scenario where n Decision Making Units (DMUs) are assessed based on m inputs and s outputs. Let $x_{ij} (i = 1, \dots, m)$ and $y_{rj} (r = 1, \dots, s)$ represent the input and output values of DMU_j $(j = 1, \dots, n)$ respectively. The efficiency score of DMU_k can be determined using the formula:

$$E_{kk} = \frac{\sum_{r=1}^{s} u_{rk} y_{rk}}{\sum_{i=1}^{m} v_{ik} x_{ik}} \tag{1}$$

In this context, v_{ik} and u_{rk} represent the weights attributed to the i-th input and r-th output of DMU_k respectively. The CCR model used to assess DMU_k can be articulated as follows:

$$
\begin{aligned}
&E_{kk} = max \sum_{r=1}^{s} u_{rk} y_{rk} \\
&s.t. \ \sum_{i=1}^{m} v_{ik} x_{ik} = 1, \\
&\quad \sum_{r=1}^{s} u_{rj} y_{rj} - \sum_{i=1}^{m} v_{ij} x_{ij} \leq 0, \ j = 1, \dots, n, \\
&\quad u_{rj}, v_{ij} \geq 0, \ r = 1, \dots, s, \ i = 1, \dots, m, \ j = 1, \dots, n,
\end{aligned} \tag{2}
$$

In this model, v_{ij} and u_{rj} represent virtual multipliers for the i-th input and r-th output respectively. The optimal solution to model (1) for DMU_k is denoted as u_{rk}^* $(r = 1, \dots, s)$ and v_{ik}^* $(i = 1, \dots, m)$. DMU_k is considered efficient if and only if $E_{kk}^* = 1$, while if the value is below one, the DMU is considered inefficient.

Let's consider a scenario where n DMUs are organized into K groups, with each group k $(k = 1, \dots, K)$ comprising D_k members. Each member DMU_{d_k} $(d_k = 1, \dots, D_k)$ within a group has m inputs $\boldsymbol{x}_{d_k} = (x_{id_k})$ and s outputs $\boldsymbol{y}_{d_k} = (y_{rd_k})$.. For each group t $(t = 1, \dots, K)$ under evaluation, the group efficiency score based on average performance is obtained by solving the optimization model (3).

$$
\begin{aligned}
&E_t^A = max \frac{\sum_{r=1}^{s} \sum_{d_t=1}^{D_t} u_{rt} \, y_{rd_t}}{\sum_{i=1}^{m} \sum_{d_t=1}^{D_t} v_{it} x_{id_t}} \\
&\quad \text{s.t.} \frac{\sum_{r=1}^{s} u_{rt} y_{rd_k}}{\sum_{i=1}^{m} v_{it} x_{id_k}} \leq 1, k = 1, \dots, K, \ d_k = 1, \dots, D_k, \\
&\quad \frac{\sum_{r=1}^{s} \sum_{d_t=1}^{D_t} u_{rt} y_{rd_t}}{\sum_{i=1}^{m} \sum_{d_t=1}^{D_t} v_{it} x_{id_t}} \leq 1 \\
&\quad v_{it}, u_{rt} \geq 0, \ i = 1, \dots, m, \ r = 1, \dots, s.
\end{aligned} \tag{3}
$$

Model (3) in linear form is as follows:

$$
\begin{aligned}
&E_t^A = max \sum_{r=1}^{s} \sum_{d_t=1}^{D_t} u_{rt} \, y_{rd_t} \\
&\quad s.t \sum_{r=1}^{s} u_{rt} y_{rd_k} - \sum_{i=1}^{m} v_{it} x_{id_k} \leq 0, \quad k = 1, \dots, K, \ d_k = 1, \dots, D_k, \\
&\quad \sum_{i=1}^{m} \sum_{d_t=1}^{D_t} v_{it} x_{id_t} = 1, \\
&\quad v_{it}, u_{rt} \geq 0, \ i = 1, \dots, m, \ r = 1, \dots, s.
\end{aligned} \tag{4}
$$

Suppose u_{rt}^{*}, v_{it}^{*} are the optimal solutions for model (4). The optimal solution for model (4) provides the average group efficiency score for group t as follows:

$$E_t^{A*} = \sum_{r=1}^{s} \sum_{d_t=1}^{D_t} u_{rt}^{*}\, y_{rd_t} \tag{5}$$

When the efficiency of the group reaches the optimal level, the efficiency values of each DMU in group t can be calculated as follows:

$$e_{d_t}^{A*} = \frac{\sum_{r=1}^{s} u_{rt}^{*} y_{rd_t}}{\sum_{i=1}^{m} v_{it}^{*} x_{id_t}}, \ \ d_t = 1, \dots, D_t. \tag{6}$$

In this paper, we utilize a combination of machine learning and group performance evaluation models to predict group efficiency scores. Prior to that, in the following section, we introduce our selected machine learning algorithm and present its overview.

2.2 BPNN ML algorithm

The progression of machine learning (ML) can be delineated into three distinct epochs. Initially, Hebb [24] laid the foundation for ML by pioneering neuropsychological learning mechanisms, initiating a brief period of development. However, from the mid-1960s to the late 1970s, progress stagnated due to constraints in computer memory and processing speed, impeding the realization of practical AI solutions. Since the late 1970s, ML has experienced a resurgence, expanding beyond single-concept learning to encompassing multiple concepts and exploring diverse learning strategies. This revival has attracted considerable scholarly attention, particularly amidst rapid advancements in AI and data mining, resulting in numerous breakthroughs.

ML is inherently multidisciplinary, drawing insights from diverse domains such as probability theory, statistics, approximation theory, convex analysis, and algorithm complexity theory. At its core, ML aims to emulate human learning behaviors, facilitating the acquisition of new knowledge and skills to continually enhance performance. Over decades of evolution, ML has garnered widespread recognition, featuring algorithms that scrutinize data, glean insights, and make informed decisions or predictions regarding unknown phenomena. Its applications span a plethora of domains, including data mining, computer vision, biometric recognition, stock market analysis, and robotics. In essence, ML relies on algorithms to scrutinize data, extract patterns, and derive actionable insights. This paradigm shift obviates the need for explicitly programmed tasks, instead fostering autonomous algorithmic development. ML encompasses a gamut of methodologies, including supervised learning, unsupervised learning, semi-supervised learning, and reinforcement learning, each exhibiting distinct strengths and weaknesses. Supervised learning, for instance, is widely employed for classification and regression tasks, as elucidated in spam filtering and weather forecasting applications, respectively. For further elucidation on ML, one may refer to Stuart et al. [25], Mehryar et al. [26] along with a myriad of other pertinent literature.

Given that the technical efficiency derived from the DEA-CCR model appears as continuous data, this study investigates a machine learning algorithm tailored specifically for regression tasks known as the back-propagation neural network (BPNN).

To introduce BPNN, we must first delve into Artificial Neural Networks (ANNs) (Mcculloch et al. [27]. ANNs, prominent in machine learning and cognitive science, are inspired by biological neural networks found in the central nervous systems of animals. They serve to estimate or approximate functions that may depend on numerous inputs, typically of unknown nature. Let's provide a succinct overview of the original concept of ANNs: The foundational neuron model can be depicted as shown in Figure 1, representing the simplest form of a neuron. This model serves as an exemplar to elucidate the fundamental concept of ANNs. Imagine there are m Decision-Making Units (DMUs), each possessing n features (i.e., n inputs denoted as $x_1, x_2, \dots, x_n$ in Figure 1. Additionally, each DMU has a target variable y, also known as the output variable, unique to each DMU (labeled as y_i for DMU_i). Given the varying importance of each feature for DMU_i, distinct weights are assigned to them (represented as $w_{i1}, w_{i2}, \dots, w_{in}$ in Figure 1). Subsequently, the weighted sum of inputs and y_i establishes a mapping relationship through an activation function.

$$y_i = \varphi\left(\sum_{j=0}^{n} w_{ij} x_j\right) - \theta \qquad (7)$$

In equation (7), $\sum_{j=0}^{n} w_{ij} x_j$ represents the weighted sum of inputs, θ denotes the intercept term, and φ symbolizes the activation function. Common activation functions include the sigmoid function, tanh function, rectified linear unit function (ReLU), softmax function, etc. By collecting data from n DMUs with known inputs and outputs, the weights w_{ij} and θ can be estimated based on equation (7), a process known as model training. Once the trained model is obtained, new DMUs with known inputs but unknown outputs can be evaluated using the model. Any necessary adjustments to the weights w_{ij} and θ can be made accordingly. The principles of the multilayer neural network model closely resemble this process; for further details, refer to Cheng [28].

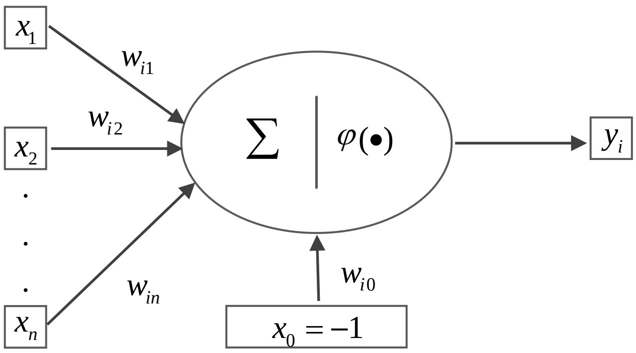

Figure. 1 Diagram of single neuron model

Following extensive development, BPNN has emerged as the predominant method for training ANNs (Rumelhart et al. [29]). It boasts two primary features: (a) being a supervised learning method, extending the delta rule, and (b) requiring the use of activation functions that are differentiable everywhere.

The BPNN algorithm evolved from ANN, with its mathematical intricacies extensively discussed in various literature. This paper provides a concise introduction to its core principles: propagation and weight adjustment (namely, the computation of actual output proceeds from input to output, while weight and threshold modifications occur in the opposite direction).

Phase 1: Propagation involves two main steps:

a. Forward propagation entails passing a training pattern's input through the neural network to produce output activations.

b. Back-propagation involves propagating the output activations generated in step (a) back through the neural network using the target associated with the training pattern. This step calculates the deltas for all output and hidden neurons.

During this phase, the output value of each node is computed based on various factors, including the output values of nodes in the preceding layer, the weights connecting the current node to all nodes in the previous layer, the current node's threshold, and the activation function. Commonly, the sigmoid function is employed as an activation function in this context.

Phase 2: Weight Update comprises the following steps:

a. Compute the gradient of the weight by multiplying its output delta with the input activation.

b. Adjust the weight in the direction opposite to the gradient by subtracting a portion of it from the weight.

This phase constitutes the error back-propagation process. The fundamental concept behind BPNN is to refine network parameters by minimizing the error between the output layer and the expected value, thereby reducing the overall error.

3. Research structure

The research framework investigates the amalgamation of the DEA method with the BPNN ML algorithm. The DEA-CCR model is initially utilized to evaluate the efficiency of each Decision Making Unit (DMU) within the training datasets. This assessment categorizes DMUs based on their technical efficiency, with the DEA efficiency serving as the target variable and the input/output indicators of the DEA model acting as feature variables. Subsequently, the BPNN algorithm is employed to analyze these categorized DMUs and derive patterns: What input/output combinations correspond to specific DEA efficiencies? Following training with the datasets, the BPNN model is refined until it meets evaluation criteria, and then it's applied to unclassified DMUs with unknown DEA efficiencies. This process facilitates the prediction of efficiency for these DMUs using the trained BPNN model.

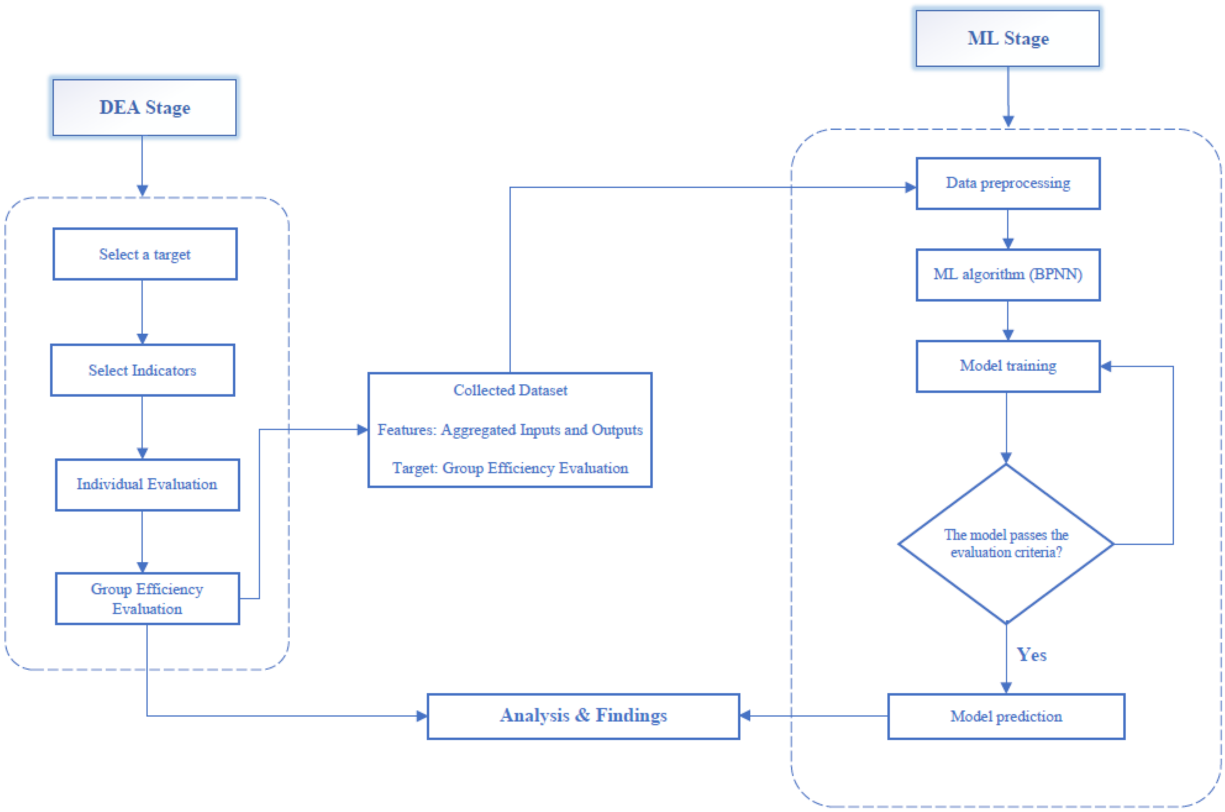

Figure.2 Research structure exploring the connection between the DEA and ML

The research framework involves two main stages: the DEA stage and the ML stage, illustrated in Figure 4. In the first stage, known as the DEA stage, DMUs with suitable input and output indicators are selected based on real-world considerations. Their DEA efficiencies are then assessed using a DEA model. Moving to the second stage, known as the ML stage, the DEA outcomes are utilized to predict the DEA efficiency of unclassified DMUs through a frontier formed by the BPNN algorithm. Within this ML stage, four steps are undertaken:

Step1: Data Preprocessing. Primarily involves standardizing the data.

Step2: Model Training. Utilizing the training datasets containing DMUs marked by their DEA efficiency to extract rules, specifically determining the input/output combinations corresponding to specific DEA efficiencies.

Step3: Evaluation Criteria. If the model meets predefined standards regarding accuracy and stability, it is considered trained. Otherwise, further training is conducted.

Step4: Model Prediction. Utilizing the trained BPNN model to predict the DEA efficiency of new DMUs. This involves adding the new DMUs to the testing datasets and executing a Python code, which automatically calculates their predicted efficiency.

During the comparison and conclusion phase, the DEA efficiency is meticulously analyzed and juxtaposed with the ML-DEA efficiency (i.e., prediction efficiency). This analysis encompasses assessing the accuracy and stability of the model, conducting statistical tests, and making inferences.

4. Empirical analysis

In this section, we present the empirical application of our research, aimed at evaluating the efficiency of 525 branches of Mellat Bank in Iran. These branches are divided into 21 groups, each comprising 25 members include the following financial information:

Inputs:

I_1**: Regulatory Compliance Costs:** Expenses related to ensuring compliance with banking regulations and laws, including hiring compliance officers, conducting audits, and implementing compliance software.

I_2**: Marketing and Advertising Expenses:** Costs related to advertising campaigns, promotional materials, and marketing strategies aimed at attracting customers.

Outputs:

O_1: Investment Gains: Revenue earned from returns on the bank's investments in stocks, bonds, and other financial instruments.

O_2: Loan Interest Income: Revenue generated from interest payments on loans provided to customers, including personal loans, business loans, and mortgages.

These inputs and outputs were selected to comprehensively capture the factors influencing the bank's operational efficiency and financial outcomes, ensuring that the model provides a meaningful analysis of the bank's performance. (See Table 1)

Table. 1 Inputs and Outputs with Related Articles in Banking Industry Studies.

Inputs/Outputs	Description	References
I_1: Regulatory Compliance Costs	Expenses related to ensuring compliance with banking regulations and laws.	[30],[31]
I_2: Marketing and Advertising Expense	Costs related to advertising campaigns, promotional materials, and marketing strategies.	[30],[31]
O_1: Investment Gains	Revenue earned from returns on the bank's investments in stocks, bonds, and other financial instruments	[30],[31]
O_2: Loan Interest Income	Revenue generated from interest payments on loans provided to customers	[30],[31]

Our analysis focuses on assessing the individual efficiency scores of members within Group 21 and subsequently determining the group efficiency score (GE) of Group 21. Table 2 presents the summary statistics of the data collected from the Iranian Mellat Bank branches. The data encompass two inputs (I_1 & I_2) and two outputs (O_1 & O_2).

We employ the Data Envelopment Analysis (DEA) approach, specifically the DEA-CCR model, to assess the efficiency of the Mellat Bank branches. Additionally, we utilize the DEA-BPNN model to enhance the accuracy of our predictions.

Table. 2 Description of Data for 525 Mellat Bank Branches.

	I_1	I_2	O_1	O_2
Count	525	525	525	525
Mean	4.8E+09	6.84E+09	3.59E+08	7.17E+09
Std	1.59E+10	4.31E+10	2.06E+09	4.51E+10
Min	22595676	15879188	-9.1E+09	12697732
25%	7.63E+08	4.55E+08	45309186	5.06E+08
50%	1.69E+09	1.24E+09	97692782	1.32E+09

75%	3.75E+09	3.4E+09	2.28E+08	3.43E+09
Max	2.91E+11	9.15E+11	4.12E+10	9.6E+11

Table 3 presents the results of our analysis, including individual efficiency scores of members within Group 21 and the corresponding group efficiency scores.

Table. 3 Prediction of AGE Using the BPNN-DEA Method.

GRP	DMUs	DEA-CCR	AGE	DEA-BPNN	AGE-BPNN	P-Value
Group 21	1	0.662	0.675	0.727	0.707	P<=0.001
	2	0.757		0.721		P<=0.001
	3	0.870		0.744		P<=0.001
	4	0.542		0.706		P<=0.001
	5	0.592		0.716		P<=0.001
	6	0.769		0.746		P<=0.001
	7	0.791		0.755		P<=0.001
	8	0.742		0.672		P<=0.001
	9	0.492		0.678		P<=0.001
	10	0.564		0.627		P<=0.001
	11	0.737		0.728		P<=0.001
	12	0.781		0.770		P<=0.001
	13	0.426		0.697		P<=0.001
	14	0.915		0.740		P<=0.001
	15	0.660		0.703		P<=0.001
	16	0.847		0.794		P<=0.001
	17	0.685		0.723		P<=0.001
	18	0.674		0.711		P<=0.001
	19	0.680		0.601		P<=0.001
	20	0.644		0.709		P<=0.001
	21	0.666		0.714		P<=0.001
	22	0.740		0.720		P<=0.001
	23	0.560		0.581		P<=0.001
	24	0.600		0.702		P<=0.001
	25	0.548		0.685		P<=0.001

As depicted in Table 3, the DEA-CCR scores for individual branches within Group 21 range from 0.492 to 0.915, indicating varying levels of efficiency in resource utilization. Furthermore, the DEA-BPNN scores demonstrate slight variations compared to the DEA-CCR scores, suggesting the effectiveness of neural network-based modeling in capturing nuanced efficiency metrics.

Notably, the p-values associated with the DEA-BPNN scores are all less than or equal to 0.001, indicating statistical significance and reinforcing the reliability of our findings.

The results obtained from our empirical analysis provide valuable insights into the efficiency of Iranian Mellat Bank branches. By identifying and analyzing the efficiency scores of individual branches within Group 21, we gain a deeper understanding of the factors influencing bank performance. Moreover, the application of advanced modeling techniques such as DEA-BPNN enhances the accuracy of our predictions, enabling more robust decision-making processes within the banking sector. In conclusion, our empirical application underscores the efficacy of Data Envelopment Analysis (DEA) methodologies in assessing the efficiency of Mellat Bank branches in Iran. Through meticulous analysis, we have elucidated the nuanced factors influencing bank performance, providing valuable insights for strategic management decisions. By accurately predicting individual and group efficiency scores, our approach equips bank managers with reliable metrics to inform decision-making processes. With a clearer understanding of resource allocation efficiency, managers can make informed strategic choices aimed at optimizing operations and enhancing overall bank performance.

Moreover, our findings offer invaluable implications for risk management within the banking sector. By identifying inefficiencies and areas for improvement, banks can proactively mitigate risks and bolster their resilience in an increasingly competitive market landscape.

In essence, our research not only contributes to enhancing the operational efficiency of Mellat Bank branches but also empowers managers with the tools and insights necessary to navigate challenges and capitalize on opportunities in the dynamic banking industry.

5. Conclusion

In this paper, we have applied Data Envelopment Analysis (DEA) in conjunction with Backpropagation Neural Network (BPNN) modeling to evaluate the efficiency of Mellat Bank branches in Iran. Through rigorous analysis of input-output relationships, we have provided valuable insights into the factors shaping bank performance. By accurately predicting individual and group efficiency scores, our approach equips bank managers with reliable metrics to drive strategic decision-making processes. The findings of this study offer actionable insights for optimizing resource allocation and enhancing overall bank performance. Additionally, they underscore the significance of advanced modeling techniques in capturing nuanced efficiency metrics within the banking sector. By leveraging DEA and BPNN methodologies, we have contributed to the empirical literature on bank efficiency evaluation and provided practical implications for strategic management in the banking industry.

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