# Mathematical Calculation of $^{14}_{6}C$ Radioactivity lifetime in Archeology

Amir Hashempoormafi[1*], Mohammad Hosein Salmani Yengejeh[2]
[1]Department of Archeology, Chalous Branch, Islamic Azad University, Chalous, Iran
[2] Department of Mathematics, Chalous Branch, Islamic Azad University, Chalous, Iran
Email: hashempoormafi.amir@gmail.com(Corresponding Author ), hoseinsallmani@gmail.com[2]

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## Abstract

The radiocarbon dating method of $^{14}_{6}C$ is one of the newest methods in archeology. Knowing and applying this method is possible by using the methods of natural sciences in general and mathematical sciences in particular. The use of radiocarbon dating method has been expanded in recent years in Iran in archaeological excavations and often, citing this method, findings and works related to them are dated absolutely (Mofidi Nasrabadi, 2018: 23-41). Therefore, this study has been done by describing and determining the radioactive equations used in calculating the age of $^{14}_{6}C$. $^{14}_{6}C$ is one of the isotopes of carbon, a radioactive substance and the result of interactions with $^{14}_{6}N$. Living organisms absorb some $^{14}_{6}C$ in their food cycle, and when they die, the absorption of radioisotopes stops, and the absorbed $^{14}_{6}C$ gradually decreases according to the half-life of radioactive carbon with the physical characteristic, i.e., the rate of transformation (the numerical interpretation of the derivative). This physical law is expressed in the form of a mathematical equations, and two basic and dynamic mathematical tools are used, i.e. differential and integral, and With the help of $^{14}_{6}C$ properties, the activity equation (exponential function) is obtained, which is used in the calculation of $^{14}_{6}C$ dating.

**Keywords:** $^{14}_{6}C$ dating, radiocarbon, mathematics in archaeology, radioactivity equations, radioisotope.

## 1. Introduction:

Mathematics emerged in ancient times depending on the needs of life and gradually became a system of various knowledge. Mathematics, like other sciences, is a reflection of the laws of nature and is used as a powerful tool to understand nature. But since mathematics is too abstract and subjective, its new fields are not accessible to those who are not specialized in work. Since ancient times, this abstract feature of mathematics has given rise to subjectivist ideas about its lack of connection with nature. Today, despite its abstract and subjective nature, the immeasurable extent of its application has been shown to everyone. In this regard, in archaeology, the successful crystallization of the idea of applying other sciences, especially mathematics, can be seen from the developments of the beginning of the 20th century in the collection and processing of data in archaeological research. By accepting the rate of use of mathematics by other sciences, in the ranking of this use, it can be seen that archeology is at the bottom of the table. A look at the history of archeology and possibly the traditional thinking in the application of modern archeology is indicative of this fact.

The first real application of mathematics in archaeological research should be found in the revolutionary discovery of radiocarbon annuity by Professor Libby and his colleagues in 1947. This article is a plan of how to apply and explain the mathematical model and function used in the calculations of the annuity of (6^14)C. By examining the theoretical principles of the exponential function, (6^14)C radioactivity, along with the examination of its cycle and transformation or activity equations, the necessary model and pattern used in annuity of (6^14)C are determined..

## 2. Radioactivity

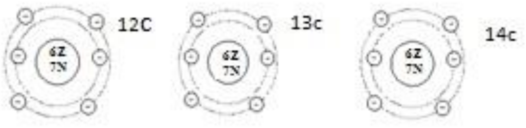
Radiocarbon dating is the most famous and well-known absolute dating method for determining the age of organic materials and materials containing carbon. This method was presented by Libby and his colleagues in 1947 AD (Libby, W.F. et al., 1974, pp931-936). Libby was able to measure this substance in methane gas in the mines of the Baltimore Islands in the United States with complex enrichment methods. slow. Two years later, Libby and Anderson used this method to date ancient Egyptian objects whose ages were known, and the results were in very good agreement with known dates (Arnold, J.R and W.F. Libby, 1949, pp678-680). ) Currently, there are various methods for dating carbon 14. The method invented by Libby has gradually evolved over the years and is now used in many laboratories around the world. This method requires large amounts of samples (from grams to kilograms) and the testing process may take several months, but at the same time, this method is very accurate. Mass Spectrometry) have been invented which is widely used in various research fields. Analysis using this method requires a small amount of sample. Of course, each of the methods and equipment used in carbon 14 analysis have their own advantages and complications. Carbon 14 dating has a special place in archeology researches in Iran. Iranian and foreign archaeological research teams used to send many samples to carbon 14 dating laboratories in Iran and abroad for dating. The first carbon 14 dating of Iranian samples belongs to Hasanlu region, which was done at the University of Pennsylvania (Ralph, E.K, 1959). The results of the dating of different samples from the ancient areas of Iran are published in different magazines such as (Bovington, R.H. et al, 1979, pp, 195-195), and other specialized magazines of archeology and archeology (Amirlu, 1366: 51-74).

In the conversation about the universe, two things are mentioned, matter and energy. These two quantities can be transformed into each other and exist in various forms to form all the visible and invisible objects of the universe. Matter is characterized by one of its characteristics, mass, which is made of tiny particles or atoms. The smallest component of a simple element is an atom, which is not found in free form, and the smallest component of a compound substance is a molecule, which is found in free form. The molecules of the substances that make up the compounds around us are made of the combination of atoms. Atom is the smallest constituent particle of matter that has the chemical properties of matter. All atoms are made up of two distinguishable regions. The central area of the atom, called the nucleus, with a radius of about $10^{-15}$ meters, contains protons with a positive electric charge and neutrons without an electric charge. The outer region of the atom and around the nucleus contains electrons with a negative electrical charge with a radius of about $10^{-10}$ meters. In a neutral atom, the number of electrons orbiting the nucleus is equal to the number of protons in that atom. These electrons exist in orbits around the nucleus and in separate groups. The number of protons is represented by Z and it is called atomic number. The atomic number is different for different elements and is used to indicate a special type of atom. The number of neutrons is represented by N and it is called neutron number. The total number of protons and neutrons in the nucleus of an atom is called the mass number of that atom and is denoted by A; that's mean:

$$A = N + Z$$

The number of electrons of each element and their arrangement around the nucleus indicate the chemical properties of each element. The number of neutrons in an atomic nucleus does not play a role in chemical processes, but it plays a decisive role in nuclear reactions. The number of neutrons of an element is not always the same. Nuclei that have the same number of protons but different numbers of neutrons are called isotopes. Isotopes of an element are atoms that have the same number of positive charges in the nucleus and the same number of electrons, but the number of neutrons in their nucleus is different. To distinguish between isotopes of an element, the mass number is written in the upper part of its name. For example, an atom with chemical symbol X is represented by An atom like ${}^{14}_{6}C$ represents the carbon isotope. Z may be removed because all atoms of a chemical element have the same Z. For example, carbon isotopes can be written as${}^{12}C$, ${}^{14}C$, ${}^{13}C$, etc. Most of the elements have several isotopes and because the electronic structure of the isotopes is the same, their chemical reactions are also similar. Isotopes differ from each other in terms of atomic mass.

Carbon element with atomic number 6 has 3 isotopes, (${}^{14}_{6}C$ with 6 protons and 8 neutrons), (${}^{12}_{6}C$ with 6 protons and 6 neutrons) and (${}^{13}_{6}C$ with 6 protons and 7 neutrons) are isotopes of carbon element. ${}^{14}C$ is the heaviest isotope of carbon and has an unstable nucleus.



Isotopes of carbon with the same number of protons and different nuotrons

French scientist Henri Bequrel first used the concept of radioactivity. Radioactivity is the spontaneous emission of particles or electromagnetic waves from the nucleus of an atom. In fact, decay is a phenomenon during which an unstable atom emits its excess energy. Among the carbon isotopes, ${}^{14}C$ is a radioactive substance due to the instability of its nucleus and has a specific half-life. Half-life is the time required to reduce any radioactive isotope to half of its initial value, and it is a measure of the rate of conversion of that radioactive isotope into another isotope, and it is an invariable property for every isotope. The basis of ${}^{14}C$ annuity is the radioactivity of ${}^{14}C$.

## 3. Transformation or Activity Equations

Derivation is one of the most effective methods in mathematics that is used to solve various problems. In radioactive studies, the first studies have shown that each radioisotope (unstable nuclei) is known by the characteristics of its transformation rate (the numerical interpretation of the derivative). It is proved that the ratio of atoms being transformed is proportional to N number of atoms available for transformation. so;

$$\frac{dN}{dt} = -\lambda N \tag{1}$$

The negative sign indicates that the number of atoms decreases with time. The activity ratio or transformation activity of unstable nuclei is called radioation activity and it is displayed in terms of transformation per time unit. Actives are expressed in terms of the number of collapses that occur in a certain time interval. It represents the fraction of atoms that are transformed per unit of time. This coefficient is called transformation coefficient or decay constant. If the relationship (1) is written as;

$$\frac{dN}{N} = -\lambda dt \tag{2}$$

Then we get;

$$\int \frac{dN}{N} = \int -\lambda dt$$

$$\ln(N) = -\lambda t + c$$

where ln is the natural logarithm function and c is a constant value. If at time t = 0 we represent the number of N atoms with $N_0$ (the number of first unstable nuclei), then;

$$c = \ln(N_0)$$

After placing we get;

$$\ln N - \ln N_0 = -\lambda t$$

and finally;

$$N = N_0 e^{-\lambda t} \qquad (3)$$

It is an exponential equation and it is a general formula for the transformation of radioactive substances. In (3), N0 is the number of atoms available at the time t = 0 and N is the number of remaining atoms Fig(1).

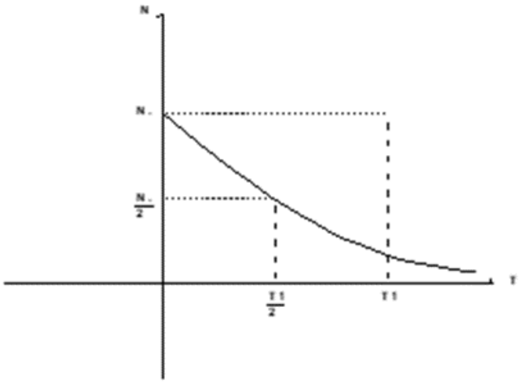


Fig1: Change in the number of radioactive nuclei with time

The number of transformed atoms after time t can be easily obtained;

$$N' = N_0 - N \qquad (4)$$

(the number of transformed nuclei)

By placing (3) in (4), we will have;

$$N' = N_0 - N_0 e^{-\lambda t} = N_0(1 - e^{-\lambda t}) \qquad (5)$$

That is, the number of transformed atoms of a radioactive substance also follows an exponential function Fig(2).

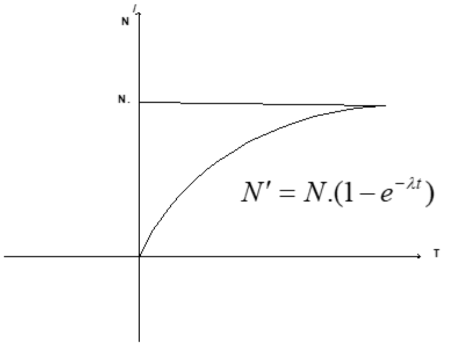


Fig2.:Variation of the number of transformed nuclei with time t

In practice, relative activity R (the number given by the counter) is usually used instead of absolute activity, so it can be written;

$$R = R_0 e^{-\lambda t} \qquad (6)$$

where $R_0$ is the relative number of atoms present at time t = 0. With the explanations given and the obtained equations, in the simplest case, the general equation of the relative radioactive transformation can be written as follows;

(future)= $e^{-\lambda t}$(present).

## 4. $^{14}C$ radioactive carbon cycle

The bombardment of Earth by cosmic rays produces a steady source of neutrons in the atmosphere. These neutrons react with nitrogen in the atmosphere and produce radioactive $^{14}C$. $^{14}C$ atoms, which are formed in the upper layers of the atmosphere, form carbon dioxide ($CO_2$) by interacting with air oxygen. Radioactive carbon dioxide remains in the atmosphere for a long time and mixes with normal carbon dioxide (consisting of $^{12}C$ and $^{13}C$). Plants absorb biomolecules containing $^{14}C$ through photosynthesis and metabolism, and animals absorb $^{14}C$ containing biomolecules through feeding, therefore radioactive activity is observed in the bodies of animals and in the trunks of plants.

Various theories and experiments show that there is a balance between the decay rate of radioactive carbon and its production rate in all organisms. In living organisms, the ratio of radioactive carbon isotope ($^{14}C$) to the number of non-radioactive carbon isotopes ($^{12}C$) is approximately 1 to $10^{12}$. As a plant or animal breathes, $^{14}C$ remains at a constant level in its respiratory structure, but when living organisms die, absorption of the radioisotope stops and radioactive $^{14}C$ undergoes a decay process. As a result of this action, the activity of the radioactive substance gradually decreases, which will be proportional to the half-life of radioactive carbon. Using highly equipped devices, scientists calculated the half-life of $^{14}C$ to be $5730 \pm 40$ years. This means that for every 100 carbon atoms, only 50 atoms of $^{14}C$ remain after a half-life (i.e. 5730 years). The rest are converted to nitrogen (N) by emitting a beta ray. It has been proven that 1% of $^{14}C$ decays every 830 years, thus the natural $^{14}C$ that has been created since the Earth was formed has completely decayed.

## Conclusion

The use of different series of samples made it possible to determine the relative age tendency of the structural periods and, to this order, control and confirm the results of stratification. The interesting issue is that radiocarbon testing for all structural periods shows an age of about 50 to 200 years older than the initial dating based on pottery and written texts.

It is also interesting to note that this is the case for data enclosures.

Another ancient site such as Tel Melian in Fars province and also some other places. It has been observed in Mesopotamia as well as in Egypt, so that in these places too, the radiocarbon method has dated the findings several decades older than the era that archaeologists estimated based on written texts and matching pottery.

In general, it can be concluded that the dating of several series of samples by radiocarbon method can be used to determine relative dating, but this method is not able to date the data in an absolute way and is often associated with several decades of error. The age obtained from the examination of carbon samples is such that they can be estimated up to 50 thousand years. However, other isotopes such as potassium or uranium are also used in estimating the lifetime of objects. These isotopes have a much longer half-life and are used to determine very old geological events that should be considered millions or billions of years old.

How will the equation of relative activity that was obtained in the general state be in the $^{14}C$ annihilation? As mentioned before, in living organisms, the ratio of radioactive carbon isotope ($^{14}C$) to the number of non-radioactive carbon isotopes ($^{12}C$) is approximately 1 to $10^{12}$. Therefore, in the mentioned equation, it is determined at time t = 0 (the time when living organisms die) we have:

$$R0 = \frac{1}{10^{12}}$$

But how is the coefficient (decay coefficient) determined? We use the definition of half-life to determine. Because during the half-life, the activity reaches half of its original size, therefore, we put the expression $R0 = \frac{1}{10^{12}}$ in the equation $R = R_0 e^{-\lambda t}$ and take the natural logarithm from both sides and obtain;

$$t_{1/2} = \frac{0.693}{\lambda} \qquad (7)$$

Because the half-life time for radioactive $^{14}C$ has been determined to be 5730, by putting it in equation (7), we get;

$$\lambda = \frac{0.693}{5730} \cong 0.00012094$$

Or

$$5730 = \frac{0.693}{\lambda} \qquad (8)$$

Therefore, the transformation equation $R = R_0 e^{-\lambda t}$ for radioactive $^{14}C$ is obtained as follows;

$$R = \frac{1}{10^{12}} e^{-0.00012094t} \qquad (9)$$

This is the equation of relative activity of $^{14}C$ that is used in the calculation of $^{14}C$ annihilation.

For example; Suppose that the ratio of $^{14}C$ to $^{12}C$ in a newly discovered skull (by measuring the concentration of $^{14}C$ using an accelerating mass spectrometer (AMS)) is

$$R = \frac{1}{2} 10^{-13}$$

The life of this skull in the $^{14}C$ dating model, according to equation (9) is equal to;

$$\frac{1}{2} \times \frac{1}{10^{13}} = \frac{1}{10^{12}} e^{-0.00012094t}$$

$$\frac{1}{20} = e^{-0.00012094t}$$

$$-\ln 20 = -0.00012094t$$

$$t = \frac{\ln 20}{0.00012094} \cong 24770 \text{ years}$$

Therefore, the radiocarbon age of the skull is approximately 24,770 years. By taking into account the conditions, the chronological age is obtained.

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