1 تحلیل میکروپلار پیزوترموالاستیسیته در یک سیلندر توخالی

مرجع پیزوالکتریک:

Steady-state thermal and mechanical stresses in Two-Dimensional Functionally Graded Piezoelectric Materials (2D-FGPMs) for a hollow in

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روابط کرنش-جابجایی:

$$\varepsilon_{rr} = \frac{\partial u}{\partial r}, \qquad \varepsilon_{\theta\theta} = \frac{1}{r}\frac{\partial v}{\partial \theta} + \frac{u}{r},$$

$$\varepsilon_{r\theta} = \frac{1}{2}\left(\frac{1}{r}\frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial r} - \frac{v}{r}\right),$$

$$E_r = \frac{\partial \psi}{\partial r}, \qquad E_\theta = \frac{1}{r}\frac{\partial \psi}{\partial \theta}, \tag{12}$$

where u, v, and ψ are the displacement components, electric potential, and the radial and circumferential directions, respectively.

The asymmetric stress-strain and electric dis-

$$e_{rr} = \frac{\partial u}{\partial r} \tag{1}$$

$$e_{\theta\theta} = \frac{1}{r}(u + \frac{\partial v}{\partial \theta})$$

$$e_{r\theta} = \frac{\partial v}{\partial r} - \phi$$

$$e_{\theta r} = \frac{1}{r}(\frac{\partial u}{\partial \theta} - v) + \phi$$

$$\chi_{rz} = \frac{\partial \phi}{\partial r}$$

$$\chi_{\theta z} = \frac{1}{r}\frac{\partial \phi}{\partial \theta}$$

روابط خطی تنش-کرنش میکروپلار ترموالاستیک بر اساس قانون هوک

The asymmetric stress-strain and electric displacement relations of FGPMs are as follows [11]:

$$\sigma_{rr} = C_{11}\varepsilon_{rr} + C_{12}\varepsilon_{\theta\theta} + e_{21}E_r - C_1^T T(r,\theta),$$

$$\sigma_{\theta\theta} = C_{12}\varepsilon_{rr} + C_{22}\varepsilon_{\theta\theta} + e_{22}E_r - C_2^T T(r,\theta),$$

$$\sigma_{zz} = C_{12}(\varepsilon_{rr} + \varepsilon_{\theta\theta}) + e_{23}E_r - C_3^T T(r,\theta),$$

$$\sigma_{r\theta} = 2C_{44}\varepsilon_{r\theta} + e_{24}E_\theta,$$

$$D_{rr} = e_{21}\varepsilon_{rr} + e_{22}\varepsilon_{\theta\theta} - \varepsilon_{22}E_r + g_{21}T(r,\theta),$$

$$D_{\theta\theta} = 2e_{24}\varepsilon_{r\theta} - \varepsilon_{21}E_\theta + g_{22}T(r,\theta), \quad (13)$$

$$\begin{aligned}
\sigma_{rr} &= (2\mu+\lambda+\kappa)e_{rr} + \lambda e_{\theta\theta} \\
\sigma_{\theta\theta} &= (2\mu+\lambda+\kappa)e_{\theta\theta} + \lambda e_{rr} \\
\sigma_{r\theta} &= (\mu+\kappa)e_{r\theta} + \mu e_{\theta r} \\
\sigma_{\theta r} &= (\mu+\kappa)e_{\theta r} + \mu e_{r\theta} \\
m_{rz} &= \gamma\chi_{rz} \\
m_{\theta z} &= \gamma\chi_{\theta z}
\end{aligned} \quad (2)$$

که γ, κ, μ, λ ثابت‌های میکروپلار هستند همچنین در روابط فوق $\sigma_{ij}(i,j=r,\theta)$ و $m_{ij}(i,j=r,\theta,z)$ به ترتیب تنش نیرو و تانسور تنش کوپل می‌باشد.

معادلات تعادل میکروپلار الاستیسیته در مختصات استوانه‌ای دو بعدی (r,ϴ)

The stress and electric displacement equilibrium equations are written as follows:

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r}.\frac{\partial \sigma_{r\theta}}{\partial \theta} + \frac{1}{r}(\sigma_{rr} - \sigma_{\theta\theta}) = 0,$$

$$\frac{\partial \sigma_{r\theta}}{\partial r} + \frac{1}{r}.\frac{\partial \sigma_{\theta\theta}}{\partial \theta} + \frac{2}{r}(\sigma_{r\theta}) = 0,$$

$$\frac{\partial D_{rr}}{\partial r} + \frac{1}{r}.\frac{\partial D_{\theta\theta}}{\partial \theta} + \frac{1}{r}(D_{rr}) = 0. \qquad (15)$$

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{\theta r}}{\partial \theta} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = 0$$

$$\frac{\partial \sigma_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{\theta\theta}}{\partial \theta} + \frac{\sigma_{\theta r} + \sigma_{r\theta}}{r} = 0 \qquad (3)$$

$$\frac{\partial m_{rz}}{\partial r} + \frac{1}{r}\frac{\partial m_{\theta z}}{\partial \theta} + \frac{m_{rz}}{r} + \sigma_{r\theta} - \sigma_{\theta r} = 0$$

همچنین روابط سازگاری در مختصات قطبی به شکل زیر تعریف می‌شود[152]:

$$\frac{\partial e_{\theta r}}{\partial r} - \frac{1}{r}\frac{\partial e_{r}}{\partial \theta} + \frac{e_{\theta r} + e_{r\theta}}{r} - \frac{\partial \phi_z}{\partial r} = 0$$

$$\frac{\partial e_{\theta}}{\partial r} - \frac{1}{r}\frac{\partial e_{r\theta}}{\partial \theta} + \frac{e_{\theta} - e_{r}}{r} - \frac{1}{r}\frac{\partial \phi_z}{\partial \theta} = 0 \qquad (104\text{-}3)$$

$$\frac{\partial m_{\theta z}}{\partial r} - \frac{1}{r}\frac{\partial m_{rz}}{\partial \theta} + \frac{m_{\theta z}}{r} = 0$$