

Distribution Nature of Return from Nepalese Stock Market

Rashesh Vaidya^{1*}

Dilli Raj Sharma²

Jeetendra Dangol³

Recive 2023,10,02

Accept 2023,11,18

Abstract

The fitting of the return on normal distribution could lead to predicting the market trend with the help of only two parameters, i.e., mean and standard deviation for the stock or market return. The return from the major indices and sub-indices of different sectors listed in Nepalese secondary market, Nepal Stock Exchange Limited (NEPSE) were fitted on the probability distribution function (pdf) to determine the best-fitted distribution for the returns. The paper used the return data from the Nepalese stock market indices to fit the distribution nature of the return. The Kolmogorov-Smirnov Test (K-S Test) has been used in a work to objectively confirm the statistical fit between an observed sample and a theoretical probability distribution. Concurrently, the study employed the Anderson-Darling Test (A2 Test) to determine if a particular data sample is representative of a certain probability distribution. The paper found only the daily return from the Nepalese stock market fit for normal distribution while most of the sector-wise returns best fit the distribution related to extreme value theory.

Keywords: distribution nature of return, stock market, efficient market, Nepalese

JEL Classification: G10, G14

¹. PhD Scholar, Faculty of Management, Tribhuvan University, Kathmandu, Nepal. Email: rasheshvaidya@hotmail.com
(Corresponding author)*

². Dean, Faculty of Management, Tribhuvan University, Kathmandu, Nepal. Email: dr7sharma@yahoo.com

³. Associate Professor, Public Youth Campus, Faculty of Management, Tribhuvan University, Kathmandu, Nepal. Email: jdangol@gmail.com

Introduction

Eugene Francis Fama (1965) discovered extreme movements in stock market returns with a fat tail distribution that defied the assumption of normality. E.F. Fama (1965) argued that in an active market, there are many well-informed and intelligent investors where securities are priced correctly and reflect all the available information about the stock. If the market is efficient, the stock price reflects all relevant information, and the investors might adopt a passive investment strategy instead of showing eagerness to get more and more information from the market. The weak form of the efficient market hypothesis states that the stock's historical information helps to give reliable information. Standing on this hypothesis, the investor tries to exploit the signal for trading.

It is a rule of statistics that the sum or the distribution of random occurrences will conform to a normal distribution. Thus, if proportionate price changes are randomly generated events, their distribution should be approximately normal. A test on events conducted by E.F. Fama (1965) noted only slight deviations from normality. The short-term stock return can be approximately described by a symmetric normal distribution, but the normality cannot be maintained in context with the long-term performance. The shape of the distribution changes noticeably as the investment horizon extends (Bodie et al., 2018, 147).

C.V. Eberlein et al. (1998) fitted the returns' distributions using normal-inverse Gaussian and hyperbolic distributions. A.A. Drăgulescu and V.M. Yakovenko (2002) showed that the distributions of DJIA returns are approximated by exponential distributions.

S.G. Badrinath and S. Chatterjee (1988), A. Peiró (1994) rejected an assumption of

normality for the market return and A.R. Chowdhary (1994) found the scaled-t distribution empirically fitted for the four Scandinavian stock markets' returns.

A. Odabasi et al. (2004) stated that the daily return from the Istanbul stock exchange was closer to the Gaussian (1809) distribution, providing better information about the market return. The research on evaluating the distribution nature of the stock market return during the seventies and eighties put forward an argument for the transformation of market return into log-normal to produce a better result for the return analysis.

The papers try to determine the distribution nature of the return from the stock market with an assumption that it will follow a closer normality. Nevertheless, in reality, various events influence the market return. Hence, Y. Malevergne et al. (2006) stated that to capture the outliers for return caused by extreme events, exponential and stretched exponential distributions were seen as better to capture the outliers. J. Odhiambo et al. (2020) also found the return from the Nairobi Securities Exchange does not follow a Gaussian but a log-normal distribution best fitted for the market return.

Hence, the connection between the normal distribution and investment management has become a concern to statisticians, portfolio managers, and econophysicists. The normality nature of the stock's or market's return led to a simple scenario analysis, where only two parameters, i.e., the mean and standard deviation, will be sufficient to determine the probability of a future scenario. But the big question is whether the return from the world's stock markets follows the properties of normality. Hence, the paper tries to find the distribution nature of the returns from the Nepalese stock market.

Literature Review

R.L. Hagerman (1978) investigated the nature of the distribution for the return on the NYSE and AMEX stock exchanges and discovered that the return did not follow the stable normal distribution. The paper concluded that a theoretical and empirical assumption of the stable normal distribution of returns is not reasonable.

P. Praetz and E.J.G. Wilson (1978) examined the empirical frequency distributions of continuously compounded monthly returns on the Melbourne Stock Exchange for the period of 1958–73 and found that the monthly return was seen to be fitted by the stable Paretian and Student's *t* distributions.

S.G. Badrinath and S. Chatterjee (1988) investigated the distribution of the daily and monthly data of CRSP from 1962 to 1985 and found that the returns for both the daily and monthly returns departed from normality.

B. Gray and D. French (1990) found the return for S&P 500 was best fitted to the logistic distribution with fatter tails resembling the Paretian distribution.

P. Peiró (1994) found that the return from the stock market was far away from the normal distribution. Similarly, E. Eberlein and U. Keller (1995) also found that a hyperbolic distribution was best fitted for the daily return from DAX.

F.M. Aparicio and J. Estrada (2001) tested the assumption of normality for the return from thirteen European securities markets. The paper found that the scaled *t*-distribution was likely suitable for the daily returns, but the normal distribution was seen as plausible for the monthly return.

G.D. Gettinby et al. (2001) found that the extreme returns from the market could be determined by the distribution accommodating the extreme value theory (EVT) of the market return. Hence, the generalized logistic and generalized extreme value distributions were seen as

best fitted for the return from the shares of the stock markets of the US, UK, and Japan.

According to P. Mondal et al. (2010), the assumption of normality for fitting the market return is widely accepted. Hence, the paper tested the distribution pattern of the return from the banking stocks listed on the Bombay Stock Exchange. The paper concluded that Burr, Dagum, log-logistic, and Cauchy distribution were seen as better fitted for the stocks' return. R. Kumar and R.S. Dhankar (2011) tested the normality of the daily, weekly, monthly, and annual return distribution of BSE-listed indices, BSE SENSEX, BSE 100, and BSE 500, and found that the daily and weekly returns were not normally distributed. The monthly and annual return distribution was symmetrical.

E.L. Naiman and V.Y. Khokhlov (2012) found that Student's *t* distribution and Laplace distribution were found to be superior to the normal distribution for the Ukrainian stock market returns.

F. Pizzutilo (2013) found that all the listed shares on the Nikkei 225 described the Pearson Type IV distribution. The paper also found that the short period data better described the nature of the return.

C.G. Corlu et al. (2016) show a concern with the assumption of normality for the stock market return. Hence, the researchers used the data of ten developed stock market daily returns and ten of the emerging stock market indexes and found that the generalized lambda distribution is a prominent distribution for modeling the behavior of the daily return from the stock market.

K. Borowski (2018) tested the normality of the overnight, daily, weekly, monthly, quarterly, and yearly closing to closing, opening to closing, opening to opening, and overnight return for 65 stock markets around the world. The paper found that daily and weekly returns for all periods

were not distributed normally. But for the monthly and yearly, returns from nine markets were accepted and the remaining eight were not fitted with a normal distribution. The paper concluded that the higher the data compression in the time of an index, the fewer chances of rejection from the normality of the return distribution.

D. Toth and B. Jones (2019) investigated the major stock market indexes' daily return distributions for normality and discovered the normal distribution is not the best for returns, even after testing for big data. Similarly, a paper proposed Laplace distribution as a suitable model for a daily market return.

S-Y. Choi and J-H. Yoon (2020) investigated the distribution nature of four stock market returns: the HSCEI, KOSPI 200, S&P 500, and EURO STOXX 50, and discovered that generalized hyperbolic fit best for the HSCEI and S&P 500, variance-gamma for the KOSPI 200, and hyperbolic for the EURO STOXX 50. Similarly, E. Afuecheta et al. (2020) found that the Generalized Gamma and Burr III distributions were best fitted for the return from the S&P 500 and DJIA.

According to C. Liu and C. Chang (2021), who investigated the S&P 500 return probability distribution, market returns are not best-fitted on Gaussian but have power-law tails.

J. Pekár and M. Pčolár (2022) analyzed the daily returns for the 30 stock markets around the world, where the paper concluded that the generalized skewed t distribution and generalized lambda distribution were seen as suitable for the quality risk estimation for the market.

Methodology and Data

The paper used the concept of a probability density function (pdf). A simple comparative graphical device to study the shape of the probability density

function (pdf) of a random variable is the normal probability plot (Gujarati et al., 2014, 137).

A paper has adopted the Kolmogorov-Smirnov Test (K-S Test) to statistically verify the goodness of fit between an observed sample and a theoretical probability distribution. At the same time, the paper used the Anderson-Darling Test (A2 Test) to find whether a given sample of data is drawn from a given probability distribution. It is one of the strongest statistical tools for detecting most departures from normality and whether the parameters determined by the distribution are significant or not.

The paper has used all the indices published by the only secondary market in Nepal, the Nepal Stock Exchange Limited (NEPSE). The NEPSE yearly return data from the fiscal year 1994 was taken for the paper.

The return for the NEPSE Sensitive Index, an index calculated for listed stocks having a minimum paid-up capital of NRS 20 million, having more than 1000 shareholders, a company that has booked a net profit for three consecutive years, having a higher book value than a par value, and having published an annual report within six months of the completion of the fiscal year, was taken from the day of its calculation, i.e., January 1, 2007, to the end of the fiscal year 2022. The Nepalese fiscal year ends in mid-July of the Gregorian calendar.

The NEPSE Float Index and NEPSE Sensitive Float Index were introduced on September 15, 2008, covering the values of the ordinary shares of all listed companies and the ordinary shares listed at NEPSE that are eligible for the NEPSE Sensitive Index.

The NEPSE monthly return and NEPSE daily return data are covered from mid-July, 1998. Similarly, the sub-indices used in the paper are used from mid-July, 1998, while the return from the

segregated sub-indices, namely micro-finance, life, and non-life insurance, is used from July 17, 2018. After the segregation of the insurance index, the old insurance sub-index has not been in use on the NEPSE floor. Furthermore, an investment sub-index was introduced on the NEPSE floor on February 28, 2021, by segregating the listed companies involved in making investment portfolios in big projects and infrastructure development. Similarly, on the same date, the hotel sub-index was renamed as the hotel and tourism sub-index. For all the indices, the closing index was till the fiscal year 2022, which ended in the month of July.

Empirical Findings

Best Fitted Distribution as per Probability Distribution Function (pdf)

In probability theory, for a continuous random variable, probability is not considered at a particular point (which is always zero) but a probability is considered in a small interval of magnitude. Hence, $p(x)$ is called the pdf of the random variable, X .

Best-Fitted Distribution for Major Indices as per Probability Distribution Function (pdf)

The Table 1 elaborates the best-fitted distribution for the major indices as per pdf:

Table 1

Best-Fitted Distribution for Major Indices as per Probability Distribution Function (pdf)

Indices	Distribution	K-S Test	A ² Test	Parameters
NEPSE Yearly	Log-normal(3P)	0.08454	0.2738	$\sigma=0.62548$, $\mu=4.0025$, $\gamma=-51.606$
NEPSE Monthly	Logistic	0.0434	0.70619	$\sigma=4.0527$, $\mu=1.1493$
NEPSE Daily	Normal	0.1090	140.04	$\sigma=1.2830$, $\mu=0.0497$
NEPSE Sensitive	Student's t	0.0551	28.394	$\nu=2$
NEPSE Float	GEV	0.08773	58.022	$k=0.15994$, $\sigma=1.1933$, $\mu=-0.50267$
NEPSE Sensitive Float	Student's t	0.03525	3.8397	$\nu=3$
Banking	Log-normal(3P)	0.07035	0.28981	$\sigma=0.3878$, $\mu=4.6341$, $\gamma=-92.961$
Mfg. & P.	Rayleigh (2P)	0.13971	0.6636	$\sigma=40.796$, $\gamma=29.762$
Hotel and Toursim	Log-normal(3P)	0.15223	0.66139	$\sigma=0.54248$, $\mu=4.3863$, $\gamma=-70.502$
Trading	Gumbel Max.	0.28279	3.2281	$\sigma=65.557$, $\mu=7.6103$
Insurance	Laplace	0.24201	1.64999	$\sigma=0.01272$, $\mu=58.146$
Finance	Log-normal(3P)	0.13096	0.41417	$\sigma=0.81938$, $\mu=4.0297$, $\gamma=-52.29$
Dev. Bank	GEV	0.12382	0.37737	$k=0.35585$, $\sigma=36.425$, $\mu=2.9924$
Hydro	Gumbel Max.	0.25994	1.4961	$\sigma=61.447$, $\mu=13.602$
'Other'	Rayleigh (2P)	0.32684	1.9409	$\sigma=86.936$, $\gamma=81.045$
Life*	Gumbel Max.	0.05207	0.20715	$\sigma=9.206$, $\mu=-1.6274$
Non-life*	Gamma (3P)	0.11595	0.42789	$\alpha=4.8382$, $\beta=4.8844$, $\gamma=-20.647$
Micro-finance*	GEV	0.10224	0.57149	$k=0.10434$, $\sigma=6.9834$, $\mu=-1.8243$
Investment*	GEV	0.17111	0.2882	$k=0.14583$, $\sigma=4.9953$, $\mu=-2.3241$

*Monthly returns are considered for calculation.

For the normal distribution, the probability distribution function (pdf) curve is pictured as exactly bell-shaped. Similarly, the area under the pdf curve for the normal distribution is equal to one. It is a statistical function for the return that describes all the possible values and likelihoods that a random variable can take within a given range. The determination of the pdf for the respective returns from the portfolio helps an investor to determine the probability of loss and helps in risk management.

The log-normal (3P) distribution, or three-parameter log-normal distribution, is the best-fitted probability density function (pdf) for the return from the NEPSE yearly return and three sectors of companies listed on the NEPSE floor. The return pattern for NEPSE-listed stocks in the banking, hotel, and finance sectors is also best-fitted to a log-normal distribution. This shows that the yearly return and the returns from three sectors of stocks listed at NEPSE are reflecting the bubbling pressure, or in other words, intense pressure, to give a higher return from the trading of the respective sector's stocks.

Logistic distribution is best suited for the NEPSE monthly returns. This distribution is stated to be the best distribution for modeling growth. The NEPSE monthly return distribution, hence, is seen as having a nature of slow growth in an initial stage, then gaining momentum, and finally slowing down when the return is saturated or some form of equilibrium.

The NEPSE daily return, Gaussian (1809) distribution is the best-fitted distribution, which reflects that the daily return at the NEPSE has an equal probability of positive deviation or negative deviation of the same magnitude. This indicates that more than 99 percent of the returns are expected to fall within three standard deviations of the mean, allowing an investor to make a statistical inference

about NEPSE's expected daily return and risk.

The Student's t distribution is the best-fitted distribution for the NEPSE Sensitive Index return as well as for the NEPSE Sensitive Float Index. The distribution is closer to the normal distribution with a bit of a lower and wider bell-shape. A. A. Peir (1994) found Student's t distribution was best fitted for the daily market return. It is more prone to extreme values than the normal distribution. Hence, the return from the stocks measured under the NEPSE Sensitive Index and the NEPSE Sensitive Float Index gives a realistic picture of the VaR from the market.

Toth and B. Jones (2019), as well as G. Harckbart (2019), proposed a Laplace distribution as a better option for modeling a volatile stock market. The pdf for the return from insurance reflected the Laplace distribution (1812), also known as a double exponential distribution. This indicates that the probability of a crash from the peak is unprecedented for the respective sectors following the Laplace distribution. Similarly, different unrelated factors other than the related one govern the fluctuation in the return for the Laplace distribution fitted return. The calculation of the insurance sub-index has been stopped and the overall insurance sub-index has been segregated into life insurance sub-index and non-life insurance sub-index. The return from the stocks of the life-insurance sector is defined by the Gumbel Max distribution, while the return from stocks of non-life insurance is defined by the Gamma (3P) distribution. This shows that the return for the stocks of the life-insurance sector represents the distribution of maxima that is closely related to extreme value theory (EVT), while the return from the stocks of non-life insurance shows a positively skewed distribution and is waiting for a specific event to occur in the future.

Similarly, the return on stocks of life-insurance companies followed the Gumbel Max distribution, as did the return on stocks of listed trading and hydropower companies.

The newly segregated sub-index of micro-finance and investment best fitted the Generalized Extreme Value (GEV) distribution. Similarly, the return from the stocks of the development bank sector as well as the NEPSE Float show that the return distribution has a combined property of Gumbel, Weibull, and Fréchet distributions. This reflects that the returns from the respective sub-sectors are highly unpredictable and risky.

Rayleigh (2P) [1880] was seen as best fitted for the return from the stocks of manufacturing and processing and the "other" sector. The return from the stocks of manufacturing and processing sector companies as well as from the 'Others' sector reflected a short-term distribution of amplitudes of narrow bands of positive return for investors.

Conclusion

The debate about whether the stock market's return follows normality has been going on since the early seventies. The distribution of the stock market's return has long concerned statisticians, financial analysts, and investors. The expectation is to fit the normal distribution and predict the market trend with basic statistical parameters, namely, mean, standard deviation, skewness, and kurtosis.

Nevertheless, in the real world, the assumption of normality in the stock market's return is quite complex. Thus, the paper tested the distribution nature of Nepalese stock market returns using time-based data, namely, yearly, monthly, and daily market returns. The paper also examined the distribution pattern for the return from the NEPSE's major indices and all sub-indices. The paper found that

except for the daily return from the NEPSE, none of the returns followed normality. Similarly, the return from the stocks listed under the NEPSE Sensitive Index and NEPSE Sensitive Float Index are closer to the normal distribution with a bit of a lower and wider bell shape.

The paper found that most of the sub-indices returned best fitted to the extreme value theory (EVT) related distribution. The paper concluded that measuring the risk using the usual variance rather than adopting the principle of extreme value theory was found to be more relevant in examining stock return movements. This also shows that no in-depth information on distribution function is required for applying asymptotic theory in context to analyze the returns that fit under EVT.

References

- Afuecheta, E. Semeyutin, A., Chan, S., Nadarajah, S. & Ruiz, D.A.P. (2020). Compound distribution for financial returns. *PLOS ONE*, 15(10), e0239652. <https://doi.org/10.1371/journal.pone.0239652>
- Aparicio, F.M. & Estrada, J. (2001). Empirical distributions of stock returns: European securities markets, 1990-95. *The European Journal of Finance*, 7(1). 1-21. <https://doi.org/10.1080/13518470121786>
- Badrinath, S.G., & Chatterjee, S. (1988). On measuring skewness and elongation in common stock return distribution: The case of the market index. *The Journal of Business*, 61(4), 451-472. <https://doi.org/10.1086/296443>
- Bodie, Z., Kane, A. & Marcus, A.J. (2018). *Investments*. McGraw-Hill Education.
- Borowski, K. (2018). Testing 65 equity indexes for normal distribution of returns. *Journal of Economics and Management*, 34(4), 5-38. <https://doi.org/10.22367/jem.2018.34.01>
- Choi, S-Y., & Yoon, J-H. (2020). Modeling and risk analysis using parametric distributions with an application in equity-linked securities. *Mathematical Problems in Engineering*, 2020(9863065), 1-20. <https://doi.org/10.1155/2020/9763065>
- Corlu, C.G., Meterelliyoz, M., & Tiniç, M. (2016). Empirical distributions of daily equity index returns: A comparison. *Expert Systems With Applications*, 54(15), 170-192. <https://doi.org/10.1016/j.eswa.2015.12.048>
- Drăgulescu, A.A. & Yakovenko, V.M. (2002). Probability distribution of returns in the Heston model with stochastic volatility. *Quantitative Finance*, 2(6), 443-453. <https://doi.org/10.1080/14697688.2002.0000011>
- Eberlein, E. & Keller, U. (1995). Hyperbolic distributions in finance. *Bernoulli*, 1(3), 281-299. <https://doi.org/10.0.2307/3318481>
- Eberlein, C.V., Morra, M.J., Guttieri, M.J., Brown, P.D., & Brown, J. (1998). Glucosinolate production by five field-grown *Brassica napus* cultivars used as green manures. *Weed Technology*, 12(4), 712-718. <https://doi.org/10.1017/S0890037X00044596>
- Fama, E.F. (1965). The behavior of stock-market prices. *Journal of Finance*, 38(1), 34-105. <https://doi.org/10.1086/294743>
- Gauss, J.C.F. (1809). *Theoria motus corporum coelestium in sectionibus conicis solem ambientium [Theory of the motion of the heavenly bodies moving about the sun in conic sections]*. Friedrich Perthes and I.H. Besser.
- Gettinby, G.D., Sinclair, C.D., Power, D.M., & Brown, R.A. (2006). An analysis of the distribution of extremes in indices of share returns in the US, UK and Japan from 1963 to 200. *International Journal of Finance & Economics*, 11(2), 97-113. <https://doi.org/10.1002/ijfe.280>
- Gray, B., & French, D. (1990). Empirical comparisons of distributional models for stock index returns. *Journal of Business, Finance & Accounting*, 17(3), 451-459. <https://doi.org/10.1111/j.1468-5957.1990.tb01197.x>
- Gujarati, D.N., Porter, D.C., & Gunasekar, S. (2014). *Basic econometrics*. McGraw Hill Education (India) Private Limited.
- Hagerman, R.L. (1978). More evidence on the distribution of security returns. *The Journal of Finance*, 33(4), 1213-1221. <https://doi.org/10.1111/j.1540-6261.1978.tb02058.x>
- Harckbart, G. (2019). Laplace versus the normal distribution for daily stock market returns. *SSRN Electronic Journal*, 1-7. <http://dx.doi.org/10.2139/ssrn.3479681>
- Kumar, R., & Dhankar, R.S. (2011). Distribution of risk and return: A test of normality in Indian stock market. *South Asian Journal of Management: SAJM*, 18(1), 109-118.
- Laplace, P-S. (1812). *The'orie Analytique des Probabilit'es [Analytical theory of probabilities]*. Ve. Courcier.
- Liu, C. & Chang, C. (2021). Combination of transition probability distribution and stable Lorentz distribution in stock markets. *Physica A: Statistical Mechanics and its Applications*, 565, 125554. <https://doi.org/10.1016/j.physa.2020.125554>

- Malevergne, Y., Pisarenko, V., & Sornette, D. (2006). On the power of Generalized Extreme Value (GEV) and Generalized Pareto Distribution (GPD) estimators for empirical distributions of stock returns. *Applied Financial Economics*, 16(3), 271-289.
<https://doi.org/10.1080/09603100500391008>
- Mondal, P., Puthoo, S., Venugopal, A., Maniyara, R., Charul, A. & Parashar, N. (2010). Understanding distribution pattern of banking sector stock prices in Indian stock market. *Asia Pacific Journal of Management Research and Innovation*, 6(3), 88-98.
<https://doi.org/10.1177/097324701000600307>
- Naiman, E.L., & Khokhlov, V.Y. (2012). The distribution of daily stock returns. *Finance of Ukraine*, 2, 70-79.
<http://finukr.org.ua/docs/FU-12-02-070-uk.pdf>
- Odabasi, A., Aksu, C., & Akgiray, V. (2004). The statistical evolution of prices on the Istanbul stock exchange. *The European Journal of Finance*, 10(6), 510-525.
<https://doi.org/10.1080/1351847032000166931>
- Odhiambo, J., Weke, P., & Wendo, J. (2020). Modeling of returns of Nairobi securities exchange 20 share index using log-normal distribution. *Research Journal of Finance and Accounting*, 11(8), 77-82.
<https://doi.org/10.7176/RJFA/11-8-08>
- Peiró, A. (1994). The distribution of stock returns: International evidence. *Applied Financial Economics*, 4(6), 431-439.
<https://doi.org/10.1080/758518675>
- Pekár, J. & Pčolár, M. (2022). Empirical distribution of daily stock returns of selected developing and emerging markets with application to financial risk management. *Central European Journal of Operations Research*, 30 (2), 699–731.
<https://doi.org/10.1007/s10100-021-00771-4>
- Praetz, P. & Wilson, E.J.G. (1978). The distribution of stock market returns: 1958-1973. *Australian Journal of Management*, 3(1), 79-90.
<https://doi.org/10.1177/031289627800300106>
- Pizzutilo, F. (2013). The distribution of the returns of Japanese stocks and portfolios. *Asian Economic and Financial Review*, 3(9), 1249-1259.
- Toth, D. & Jones, B. (2019). *Against the norm: Modeling daily stock returns with the Laplace distribution*.
<https://arxiv.org/abs/1906.10325>
- Rayleigh, L. (1880). On the resultant of a large number of vibrations of the same pitch and of arbitrary phase. *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science*, 10(60), 73-78.
<https://doi.org/10.1080/14786448008626893>