Development Hough transform to detect straight lines using pre-processing filter

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ABSTRACT

Image recognition is one of the most important field in image processing that in recent decades had much attention .Due to expansion of related fields with image processing and various application of this science in machine vision, military science, geography, aerospace and artificial intelligence and lots of other aspects, outstand the importance of this subject. One of the most important aspects of this case is line detection that it could be base of image recognition. Because all pictures, shapes and even whole written in picture has been formed from lines. Accordingly, exact line detection can be basis to exact image recognition. One of the methods that are used in this aspect is to identifying lines, using Hough transform .This technique is one of useful and practical method in the field of line detection. Problem with this approach is the errors in detected lines that with improving this method can lead thy way for improvement of lots of other application in this topic. In this thesis a method for improving the Hough transform in line detection is proposed. According to the achieved results it can be seen the effect of proposed method in improving Hough transform.

Keywords

Hough transform, Images text, Morphological operators, Wiener filter, Edge detection canny

1. Introduction

The Hough transform is a technique in which features are extracted that is used in image analysis and digital image processing. Previously the classical Hough Transform worked on the identification of lines in the image but later it has been extended to identifying positions of shapes like circles and ellipses [1,15,18]. In automated analysis of digital images, there was a problem of detecting simple geometric shapes such as straight lines, circle, etc. So in the pre-processing stage edge detector has been used to obtain points on the image that lie on the desired curve in image space. But due to some imperfections in image data or in the edge detector, some pixels were missing on the desired curve as well as special deviation between the geometric shape used and the noisy edge pixels obtained by the edge detector. So to refine this problem Hough transform is used. In this the grouping of edge pixels into an object class is performed by choosing appropriate pixels from the set of parametric image objects.(Fig1)

The simplest case of Hough transform is finding straight lines that are hidden in large amounts of image data. For detecting lines in images, the image is first converted into binary image using some form of thresholding and then the positive or suitable instances are added into the dataset. The main part of Hough transform is the Hough space. Each point (d, T) in Hough space matches to a line at angle T and distance d from the origin in the data space. The value of a function in Hough space gives the point density along a line in the data space. (Fig2)

For each point in the Hough space, consider all the lines which go through that point at an actual discrete set of angles that are chosen on the priority basis. For each angle T, calculate the distance to the line through the point at that angle and discretize that distance using a priori chosen discretization, giving value d. Now make a corresponding discretization of the Hough space. This will result in a set of boxes in Hough space. These boxes are called the Hough accumulators. For each line it considers above, we increment a count (initialized at zero) in the Hough accumulator at point (d, T). After considering all the lines through all the points, a Hough accumulator with a high value will probably correspond to a line of points.[2,3]

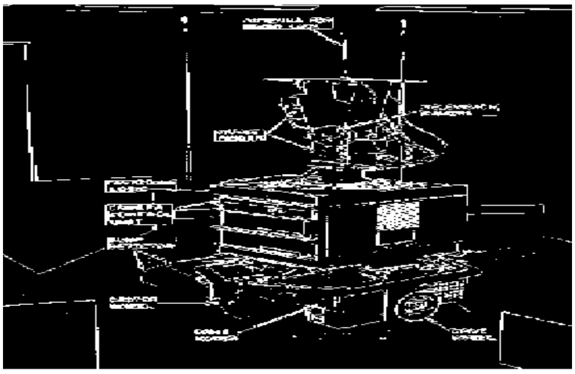


Figure 1: Input Image

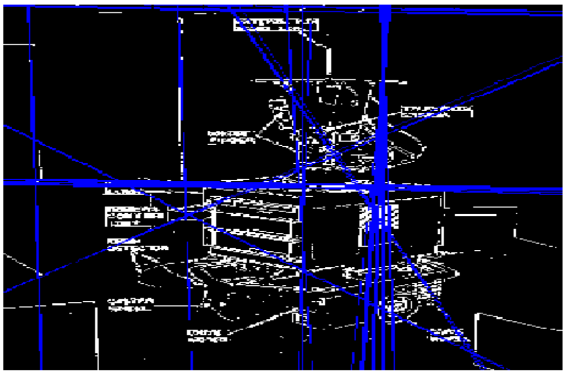


Figure 2: Finding lines using Hough Transform[1]

2. Hough transform

The HT is a global operation that maps a line to a point. Consider object points in image space (the xy-plane) as

shown in Fig. 3(a). Each such object point, say object pointA, in image space has an infinite number of straight linesthat could pass through it according to its slope-interceptrepresentation, $y_i = \alpha x_i + \beta$Rewriting this representationas $\beta = -\alpha x_i + y_i$and upon realizingthat α and β can takemany possible values; one can construct a $\alpha\beta$-space, whichis called Hough space. In Hough space, the equation for allpossible lines going through point A(xi, yi) is representedas $\beta = -\alpha x_i + y_j$. Thus, a single point in an image spacecorresponds to a straight line in the Hough space shown inFig. 3(b).The Hough space is usually searched for local maximawhere the parameters of lines that connect the points in theinput image are determined. Once a Hough space is developed, the line detection problem in an image space becomesa considerably simpler local maxima searching problem inHough space.With a line slope of infinity, the y-intercept of a line becomes negative infinity. The most popular way to overcomethis problem is by using an alternate analytic representationof lines [15,18]. A straight line can be expressed in polar coordinates as $\rho = x\cos\theta + y\sin\theta$ where the pair (ρ, θ)definesa vector from the origin to the nearest point on the line.The main utility of the $\rho\theta$-Hough space is that it can easilyhandle verticallinesarisinginimagespace.Regardlessofwhichrepresentation is used, the main disadvantage oftheHT is that the extent ofthelineissometimesdifficulttodetermine. Further processing must be usedto determine theextent of a line.

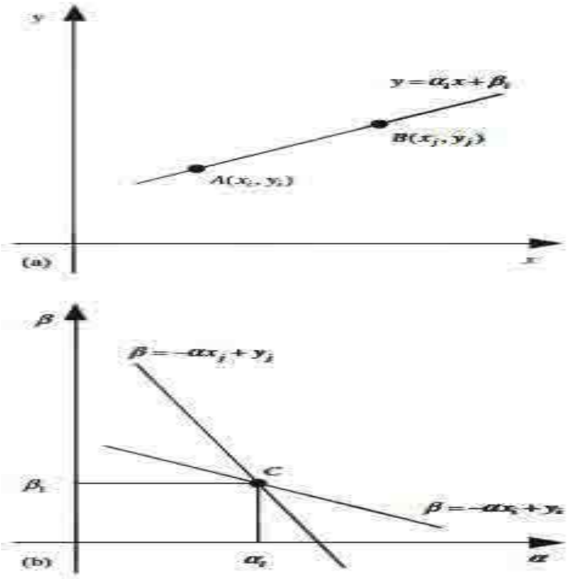


Fig3:Slope-intercept representation of the Hough trans form.

3. Edge Detection

Edge detection works on the idea of the identification of points in the digital image at which the image brightness changes sharply. The points at which image brightness changes sharply are organized into a set of curved line segments termed as edges[4,5,6]. Edge detection is afundamental tool in image processing particularly in the areas of feature detection and feature extraction. Applying an edge detection algorithm to an image may significantly reduce the amount of data to be processed

and may therefore filter out information that may be regarded as less relevant, while preserving the important structural properties of an image. If the edge detection step is successful, the subsequent task of interpreting the information contents in the original image may therefore be substantially simplified. However, it is not always possible to obtain such ideal edges from real life images of moderate complexity.[7,15,16]

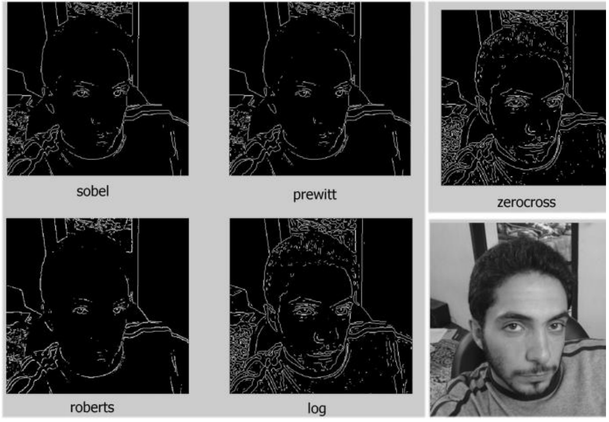
Is usually the first step in the analysis of image segmentation. The action part of the image is divided into parts of individual constituents. The segmentation of the issue depends on the intended use is when objects were separated must be stopped segmentation. The boundary between an object and an image of the border between overlapping objects, defined edges. Ideally, if we assume that the intensity of the same image with different levels of intensity surrounding objects, then any significant change in the amount of edge intensity can be taken into account.By this definition, if the edges of an image to be determined, place all the objects in the specified image and its basic properties such as the environment, etc. can easily be measured.[8,9,15].

Edge detection based on the first derivative and second derivative operators Sobel operator [3,15,17], operator Prewitt

[4,15,17], Roberts operator [5,15,17] Laplacian operators. In these methods, stereotypes around the image gradient and Laplacian convolution operator adapt.The amount of the brightness level changes in several directions and reveals the range by applying a threshold value on the resulting image, edge extraction are associated with stereotypes. Although the classical methods of smoothing methods did not use them this problem by calculating average picture was solved. The reason for this is that derived estimates were calculated using a relatively stronger than the noise of neighboring pixels of two pixels, so the idea operators have been proposed.[9]. Some of them is an extended version of the previously mentioned detector. For example, a mask made by prewitt operator 7×7 Based on the assessment of horizontal columns the first derivative of the image is presented as a matrix (1) is provided:

$$H_c=\frac{1}{21}\begin{bmatrix} -1 & -1 & -1 & 0 & 1 & 1 & 1 \\ -1 & -1 & -1 & 0 & 1 & 1 & 1 \\ -1 & -1 & -1 & 0 & 1 & 1 & 1 \\ -1 & -1 & -1 & \mathbf{0} & 1 & 1 & 1 \\ -1 & -1 & -1 & 0 & 1 & 1 & 1 \\ -1 & -1 & -1 & 0 & 1 & 1 & 1 \\ -1 & -1 & -1 & 0 & 1 & 1 & 1 \end{bmatrix}$$

(a) (b) (c)



orginal image

(d) (e) (f)

Fig3: masks made of (a)sobel, (b)prewitt, (c)zerocross (d) roberts, (e) log , (f) orginal Image

Ingeneral, thesemethods are simple calculations tofind the edgesofthe image,but duetolackof system software the image ishighly sensitive to noiseandifthereis noise generated answers have a lot of errors.

We proposed in this paper you will use algorithms canny for the best algorithm to determine the edges in images.

3.1. Canyy edge detector

the Canny edge detector [9,18,19] is an edge detection algorithm that uses a multiple stage algorithm so as to detect edges in images. Its aim is to discover the optimal edge detection.

The algorithm canny runs in 5 separate steps:

1-Smoothing: Blurring of the image to remove noise.

2- Finding gradients: The edges should be marked where the gradients of the image has large magnitudes.

3- Non-maximum suppression: Only local maxima should be marked as edges.

4- Double thresholding: Potential edges are determined by thresholding. edge pixels stronger than the high threshold are marked as strong; edge pixels weaker than the low threshold are suppressed and edge pixels between the two thresholds are marked.

5- Edge tracking by hysteresis: Final edges are determined by suppressing all edges that are not connected to a very certain (strong) edge[17,19].

In this definition, an optimal edge detector includes the following things.

1.**Good detection** – low probability of not marking real edge points,and falsely markingnon-edge points

$$SNR=\frac{\left|\int_{-w}^{w} G(-x)f(x)dx\right|}{n_o\sqrt{\int_{-w}^{w} f^2(x)dx}}$$

f is the filter, G is the signal edge,denominator is the root-mean-squared response to noise $n(x)$ only.

2.Good localization – edges marked through this algorithm should approach as close as possible to the edge in the real image.

$$Localization=\frac{1}{\sqrt{E[x_0^2]}}=\frac{\left|\int_{-w}^{w} G'(-x)f'(x)dx\right|}{n_o\sqrt{\int_{-w}^{w} f'^2(x)dx}}$$

-A measure that increases as localization improves.

-Use reciprocal of the rms distance of the marked edge from the center of the true edge.

3.**Minimal response** – a given edge in the image should only be marked once so as to reduce false edges. [8,11,14,17,19]

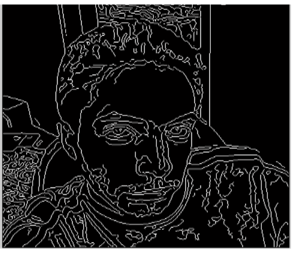


Fig4: Input Image **Fig 5: Edge detection using** Cannyalgorithm

4. Operators Morphology

Morphological operators often take a binary image and a structuring element as input and combine them using a set operator(intersection, union, inclusion, complement). They process objects in the input image based on characteristics of its shape, which are encoded in the structuring element. The mathematical details are explained in Mathematical Morphology.[11,17]

Morphological operators Continuing to improve the identification of lines using the Hough transform(HT) can be improved by using morphology operators offered. The morphology of a wide range of image processing operations that process the images based on the shapes them. The value of each pixel in the output image morphology operation by the corresponding pixel in the input image and its neighbors.- The most basic morphological operators Dilationexpansion and Erosion wear Dilation adds pixels to the image borders. While Erosion removes pixels from the image borders. Open areas usually smooth operator environment, break down and remove the protrusions of the light beam. Depending on the size and shape of the structure element is used for functional morphology.[11]

In grayscale morphology, images are functions mapping a Euclidean space or grid E into $\mathbb{R}\cup\{\infty,-\infty\}$, where $\mathbb{R}$ is the set of reals, ∞ is an element larger than any real number, and $-\infty$ is an element smaller than any real number.

Grayscale structuring elements are also functions of the same format, called "structuring functions".

Denoting an image by f(x) and the structuring function by b(x), the grayscale dilation off by b is given by

$$(f\oplus b)(x)=\sup_{y\in E}[f(y)+b(x-y)],$$

Where "sup" denotes the supremum.

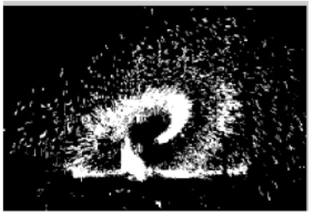
Similarly, the erosion of f by b is given by

$$(f\ominus b)(x)=\inf_{y\in E}[f(y)-b(y-x)],$$

Where "inf" denotes the infimum.

Just like in binary morphology, the opening and closing are given respectively by

$f\circ b=(f\ominus b)\oplus b$, and $f\bullet b=(f\oplus b)\ominus b$

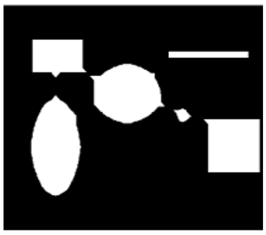


Binary image

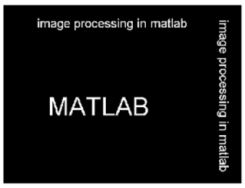
imclose

imopen

Binary image



imerode



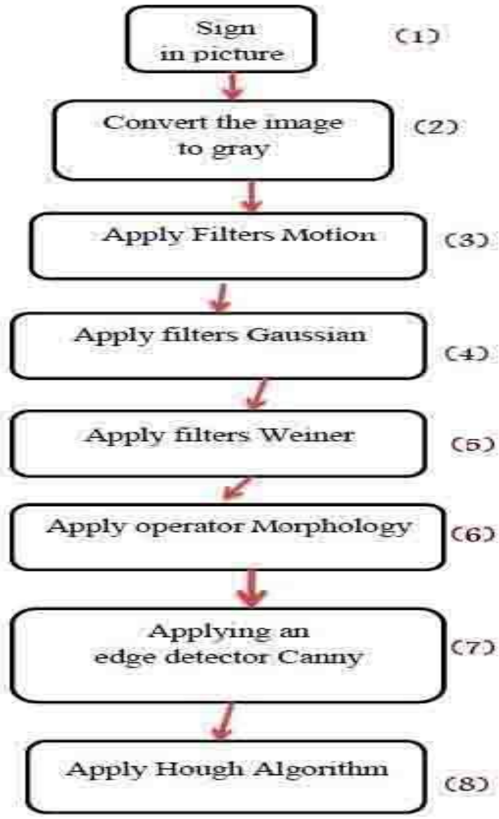
Binary imge

im dilation

5. The proposed Improved Hough transform algorithm

Eight step algorithm is proposed that the first step is entering the image pre-processing step and seven-step. Finally, the Houghtransform toidentify lines usedin the first stage Wienerfilter andoperatorsMorphology and the Houghtransform apply the proposedmethodolog to improve theidentificationof lines by transformingHoughseven asa preprocessing step usedin the process, is proposed steps tofullyprocess theproposedalgorithm canbe seen later.[15,19]

Step One: Sign in picture
Second step: convert the image to gray
Step Three: Apply Filters move
Step Four: Apply filters Gaussian
Step Five: Apply filters Weiner
Step Six: Apply operator morphology
Seventh step: Applying an edge detector canny
Eighth step: Apply Hough algorithm



Step One: Sign in picture

At this point, the image canbe brought in for processing and identification of lines. The best image to image recognition accuracy with the lowest noise in the picture.

Second step: convert the image to gray

The model image, gray image is important because in most applications there is no need for a color image having only a gray image will be enough. In this step of the method the input image is converted into a gray image and then processing the image of gray will be used also if the image is a color entry can convert the image to gray.

Step Three: Apply the Motion filter Motion blur

is the apparent streaking of rapidly moving objects in a still image or a sequence of images such as a movie or animation. It results when the image being recorded changes during therecordingofasingleexposure, either due to rapid movement or longexposure.Suppose you're taking pictures with a camera in your hands, if in the time taken tomovethe camerahave the image will be drawn. Filter algorithm of the same operation on Image. Len is the determinant of motion. A (theta) theta also determines the anglehedoes. For example 90 to thet a as shock and the image up and down or a value of 0 to theta as you move the image to the left and right. This filter is one-of the inputs for theWienerfilter.[13,18]

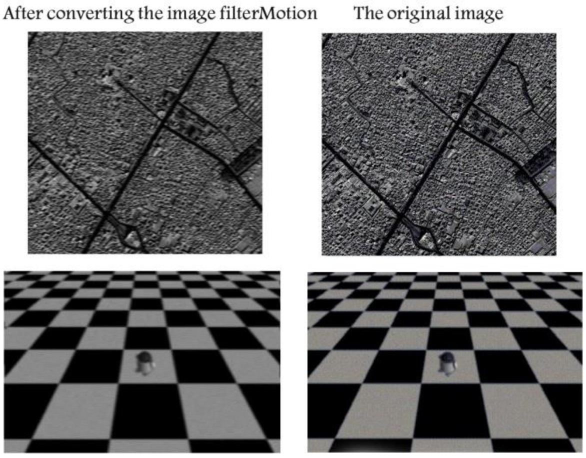


Fig : (b) : output image (a) : input image

Step Four: Apply Gaussian

Low frequency pixel intensity difference with neighboring pixels is small Relaxed the image . In contrast, high frequency pixel pixel brightness differences of neighboring pixels with pixels high. Edges and noise (Gaussian filter, a low-pass filter is applied to the calming. Filter to image animage inwhichthe edgesare smooth and relat ively quiet noise are achieved and apply this filter makes a series of additional detail in theimage isdeleted, this step is one of the inputs Wiener filter.

1.Gaussian smoothing is very effective for removing Gaussian noise.[12,15]
2.The weights give higher significance to pixels near the edge (reduces edge blurring).
3.They are low linear pass filters
4.Computationally efficient (large filters areimplemented using small 1D filters) .
5.Rotationally symmetric (perform the same in all directions).
6.The degree of smoothing is controlled by σ (larger σ for more intensive smoothing.

First step: involves smoothing the image [6,7,8,17] by convolving with the Gaussian filter. Smoothing is to reduce the noise within an image. Gaussian smoothing function is given by

$$G_\sigma(x,y)=\frac{1}{2\pi\sigma^2}\exp(-\frac{x^2+y^2}{2\sigma^2})$$

Where σ is standard deviation

Example:

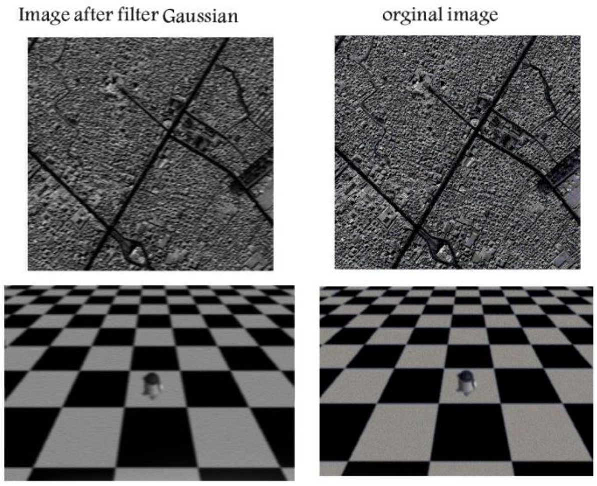
m = n = 7
σ² = 2

$$\begin{bmatrix} 1 & 4 & 7 & 10 & 7 & 4 & 1 \\ 4 & 12 & 26 & 33 & 26 & 12 & 4 \\ 7 & 26 & 55 & 71 & 55 & 26 & 7 \\ 10 & 33 & 71 & 91 & 71 & 33 & 10 \\ 7 & 26 & 55 & 71 & 55 & 26 & 7 \\ 4 & 12 & 26 & 33 & 26 & 12 & 4 \\ 1 & 4 & 7 & 10 & 7 & 4 & 1 \end{bmatrix}$$

2-D convolution operation is

$$I'(x,y)=\sum_{k=-N}^{N}\sum_{l=-N}^{N} g(k,l)I(x-k,y-l)$$

Where g (k, l) = Convolution Kernel
I(x,y) = Original image The larger the width of Gaussian mask, the lower is the Detector's sensitivity to noise.



Step Five: Apply filters Weiner

The weiner filtering is optimal in terms of the mean square error. In other words, it minimizes the overall mean square error in the process of inverse filtering and noise smoothing. The weiner filtering is a linear estimation of the original image. The approach is based on a stochastic framework. The orthogonality principle implies that the weiner filter in Fourier domain can be expressed as follows:

$$W(f_1,f_2)=\frac{H^*(f_1,f_2)S_{xx}(f_1,f_2)}{|H(f_1,f_2)|^2S_{xx}(f_1,f_2)+S_{\eta\eta}(f_1,f_2)},$$

Where are $S_{xx}(f_1,f_2),\ S_{\eta\eta}(f_1,f_2)$ respectively power spectra of the original image and the additive noise, and

$H(f_1, f_2)$ is the blurring filter. It is easy to see that the weiner filter has two separate parts, an inverse filtering part and a noise smoothing part. It not only performs the deconvolution by inverse filtering (high pass filtering) but also removes the noise with a compression operation (low pass filtering). To implement the weiner filter in practice we have to estimate the power spectra of the original image and the additive noise. For white additive noise the power spectrum is equal to the variance of the noise. To estimate the power spectrum of the original image many methods can be used. A direct estimate is the period gram estimate of the power spectrum computed from the observation.

$$S_{yy}^{per} = \frac{1}{N^2}[Y(k,l)Y(k,l)^*]$$

Where y(k,l) is the dft of the observation.Theadvantage of the estimate is that it can be implemented very easily without worrying about the singularity of the inverse filtering[5]. Another estimate which leads to a cascade implementation of the inverse filtering and the noise smoothing is

$$S_{xx} = \frac{S_{yy} - S_{\eta\eta}}{|H|^2}$$

Which is a straightforward result of the fact: $S_{yy} = S_{\eta\eta} + S_{xx}|H|^2$ the power spectrum S_{yy} can be estimated directly from the observation using the period gram estimate. This estimate results in a cascade implementation of inverse filtering and noise smoothing[6]

$$W = \frac{1}{H}\frac{S_{yy}^{per} - S_{\eta\eta}}{S_{yy}^{per}}.$$

The disadvantage of this implementation is that when the inverse filter is singular, wehaveto use the generalized inverse filtering. People also suggest the power spectrum of the original image can be estimated based on a model

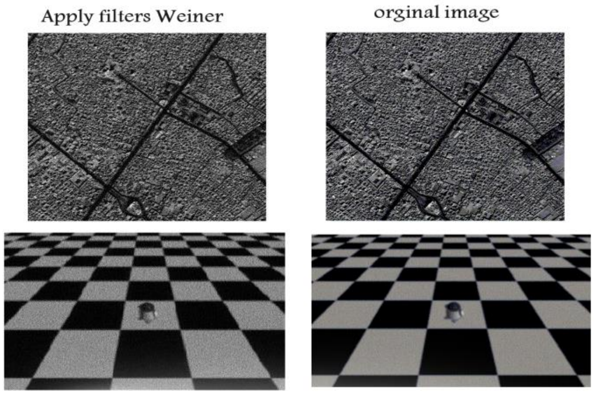
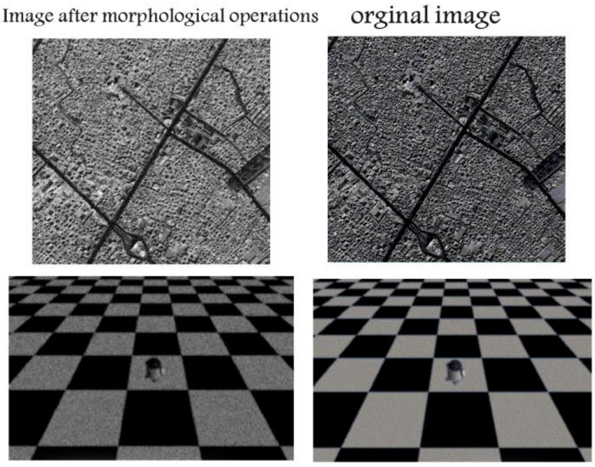


Fig:

Sixth step: Applying morphological operators

In the sixth step after applying the Wiener filter should be applied functional morphology, given that each of the operators Morphology applied on the image can have specific results in the application of these operators from four opening,closeing, Dilation and Erosion which is used can have the most optimal solutionof course on any one of these four operator with optimum output image compared to other methods. In this step. In this step Recovery is done on-line, which means that the lines are thick with low thickness and lines Noise mode that probably will be deleted.



Step Seven: Identify the edges of the image using the canny algorithm

At this point you're using the edges detected. Usually edge detection algorithms like you .A threshold level of the threshold by subtracting the intensity level where the intensity is low, edge detection. Where high brightness is too weak to be strong edge detection. When you need to be strong edge detection with the use strong slope of you. Edge detection algorithm, do you have three different threshold brightness level, if the difference between the intensity level of the first threshold level, she is more known as edge, even if it is smaller than the second threshold value, an edge, and if the two weak edge, the edge of the preserve continuity.

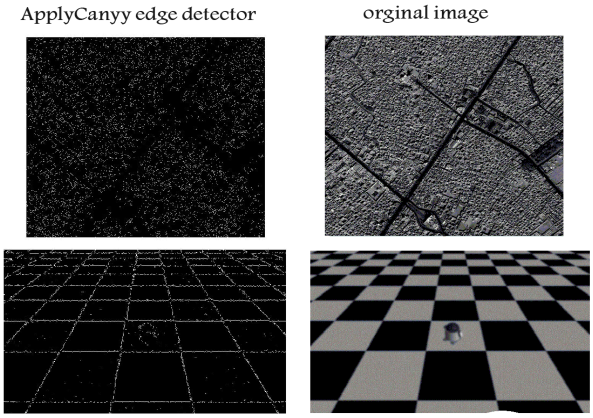
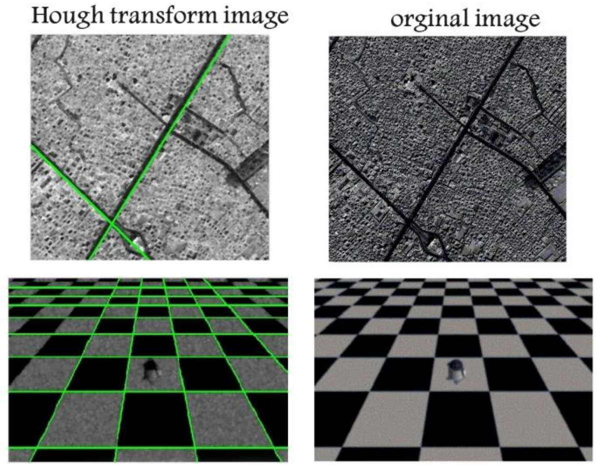


Fig Eighth step: Apply algorithm Hough

At this stage, after applying the input image pre-processing algorithms to identify the lines enter Hough willbe,willbecome. Infact, this isthe part that identifies lines. But according to the Hough transform is highly sensitive to noise,so it doesnot producesignificant results inid entifying linesthe endresult is toproduce shortcomi-ngs. The proposed method was explained that the main problem, asfarassignificantly reduced and also the chan ges that occur in the image, improve outcome in recognition of the lines.[15,18]

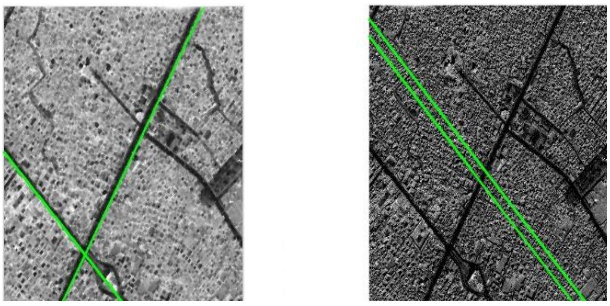


5. Comparison Results

To be able to identify the lines of the proposed algorithm Review, we have the Hough transform method compared with conventional methods. The benchmark comparison of the number of lines detected correctly. We will use as a benchmark. In this section we try to compare the use of multiple images with different characteristics, to improve the proposed algorithm can be seen in all kinds of images. Also at the end to compare the number of lines per picture will be compared properly detected and the number of lines Methods to identify the wrong line, also will be evaluated. Below are some pictures that will be used to compare the two methods, have been proposed, and the purpose of this algorithm is also marked on them that we produce the desired output for the specific purposeofAnd optimal image output is output that tries to fully reach the desired goal.

5.1. Terms of performance

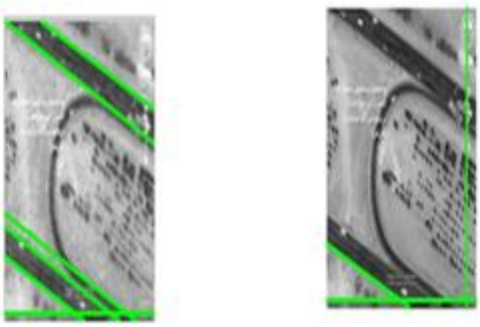
To run and compare the results for both approaches, namely conventional Hough improved application environment MATLAB is used for the comparison of images of the same size and same quality with the same suffix usedAnd to complete the comparison conditions for both methods were equal to conclude there was an error No. In the form of an aerial view of a city to find lines the streets of the city. In a specific direction the lines identified by the green line are used. As can be seen, in Haugh Typical of the left two lanes have been identified, none of the lines detected line is not correct and Error occurred one hundred percent, but the figure on the left is generated using the proposed method Two completely correct, is detected. That is to say, one hundred percent recovery.



A:Hough improved B: Hough normal

5.2. comparison of two Hough normal output and improved Hough

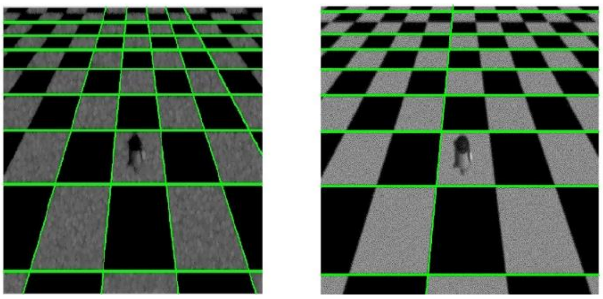
In figure (5-2) you can see an aerial view of the road is the goal in this picture Find Road ways has been given. As you can see in the image on the right to usethe conventional Houghlineshavebeenidentified, only a sp ecifically identified, but left image using Hough developed specified identify the correct line number five, which is seen a five-fold growth.



A:Hough improved B: Hough normal

5.3. comparisonof two Hough normal output and improved Hough

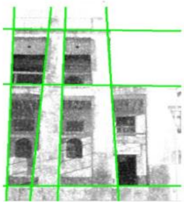
In figure(5-3)which is shaped like a chessboard, to find the horizontalandverticallinesbetween housesis chess.
As intherightfigure lines havebeenidentified using conv-entional Hough, seethe only horizntallines and
vertical lines havebeenidentified, but the proposedmethodfour vertical line more than half of the ordinary is detected.



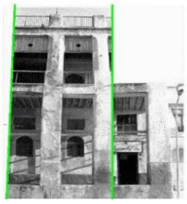
A:Hough improved B: Hough normal

5.4. comparison of two Hough normal output and improved Hough

Figure 5-4 that form the facade of a building facing is to find linesin the imageTheresultsproducedby conventional half right and left imagesproduced Using Hough algorithm is prop osed.



A:Hough improved



B: Hough normal

5.5. comparison of two Hough normal output and improved Hough

And more form:

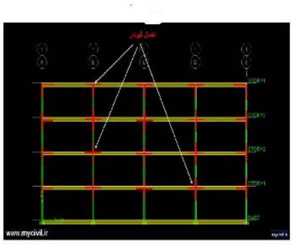
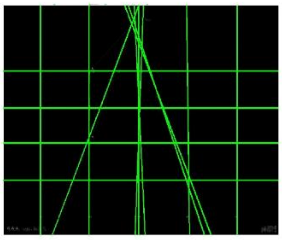
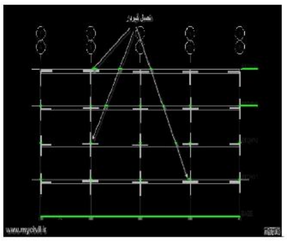


ımage orginal

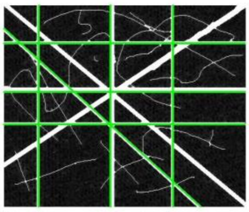
A:Hough improved



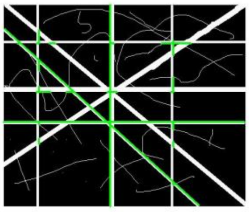
B: Hough normal



Improved Hough



General Hough



7- Results and Discussion

Given the importance of identifying lines and Hough transform, given that one of the methods in the field lines is detected, the better this method is also very important. Accordingly, in this article was to provide a way to improve the conversion and according to the results produced in this section see results resulting from the proposed method is a significant improvement in the detection lines. Also according to the accuracy of the results the proposed method produced accurate results also greatly increased and it can be said that the method proposed the improved Hough transform using Wiener filter and morphological operators a substantialimprovementinresultsandasaresulthascreateed could be related to the identification process lines used
The proposed approach is likely to find straight lines and right to a 70% increase.

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