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Research article

The influence of various boundary conditions on dynamic stability of a beam-moving mass system

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Abstract
In this paper, the effect of different boundary conditions on dynamic stability of a beam located on a viscoelastic medium stimulated by moving masses and periodic axial force is studied. Partial differential equations governing the system are derived using Hamilton's method and based on Euler-Bernoulli beam theory. Then, equations are converted into a set of ordinary differential equation with time-varying coefficients using Galerkin method along with trigonometric shape functions. The time-varying position of moving loads causes these time-varying coefficients in the governing equations. By applying Floquet's theory to the obtained equations, the conditions of parametric resonance are analyzed for different values of mass and velocity of passing loads. The results obtained from this research show that the stiffness and viscosity of the elastic medium have positive effects on the stability of the beam under moving and fluctuating axial loads. So, with a suitable choice for these values in practical applications, it is possible to prevent unexpected vibrations of the structure. In addition, the use of fixed supports for the two ends of the beam exposed to the mentioned loadings has high reliability in the discussion of dynamic stability.

Keywords: Dynamic stability, Boundary condition, Viscoelastic medium, Moving mass

1- Introduction

The problem of vibration of structures under dynamic loading is one of the important fields of engineering with many industrial applications, such as pipes carrying fluids, cranes carrying moving loads, gun barrels, movement of vehicles on bridges, and the passage of trains on rails located on elastic medium and so on. Therefore, attention to solving such problems analytically and experimentally has been the focus of many researchers for many years [1-7]. The research can be divided into two main groups. The first group of these researches examines the time or frequency response of the beam under moving mass. The second group mainly focuses on system stability analysis. This category includes the identification of those parameters of the system, according to which instability occurs in the system and as a result, the beam experiences vibration with increasing amplitude. In this category, the results usually lead to analytical or numerical calculations that determine the stable and unstable states of the problem. In the following, a review of the papers analyzing the dynamic stability of the beam under moving mass loading in different boundary and medium conditions is given. Mackrtich [8] investigated the dynamic stability of Euler-Bernoulli and Timoshenko beams located on elastic medium under the passage of moving masses with constant velocities and equal distances, using the Floquet theory, and showed that the Euler-Bernoulli has a wider stable region in the plane of the mass-velocity. Senalp et al [9] investigated the dynamic response of Euler-Bernoulli beam with finite length and located on linear and non-linear viscoelastic substrates, stimulated under moving masses at different speeds. Mirzabigi and Bakhtiari-Nejad [10] investigated the stability of the beam-moving mass system with elastic boundary conditions. Pirmoradian et al. [11] investigated the instability and resonance of a Timoshenko beam excited by the successive passage of moving masses. In their study, they used the incremental harmonic balance method to investigate parametric resonance conditions. In another study Pirmoradian et al. [12] studied instability of plates lying on elastic foundations traversed by inertial loads. The stability of the plate vibrations was investigated via incremental harmonic balance method.

By reviewing the previous papers, it can be seen that the investigation of the effect of different boundary conditions and the effect of the axial force on the plane of instability of the beam- moving mass system has not been done so far. Therefore, in this article, besides considering these cases, how the viscoelastic medium affects the stability plane of the beam- moving mass problem is investigated.

2- Problem formulation

In this section, the equations of motion governing the beam on the viscoelastic medium with different boundary conditions under moving mass and alternating axial loadings are obtained using Hamilton's principle and based on Euler-Bernoulli theory. In deriving the equations, small amplitude vibration (linear model) is considered. In addition, all acceleration terms of the moving mass including Criollis and centripetal accelerations are applied in the dynamic equations.

The beam has a length of ℓ, thickness of h, cross-sectional area of A, density of ρ and Young's modulus of E, while beam is located on a viscoelastic medium with a stiffness of k and damping coefficient of c. The schematic of the considered model is shown in Fig. 1. In the following, the governing equations are derived using Hamilton's principle.

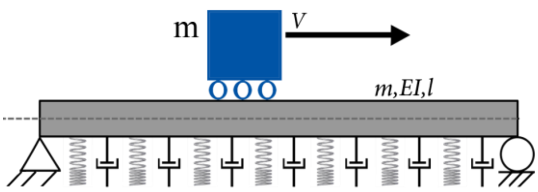


Fig. 1 Schematic of Beam-moving mass system

The displacement field equations based on the Euler-Bernoulli beam theory are defined as the following:

$$\bar{u}(x,z,t) = -z\frac{\partial w(x,t)}{\partial x} \quad (1)$$

$$\bar{v}(x,z,t) = 0 \quad (2)$$

$$\bar{w}(x,z,t) = w(x,t) \quad (3)$$

where $\bar{u}$, $\bar{v}$ and $\bar{w}$ are the general displacements of the beam in the direction of the x, y, and z axes,respectively. Also, w represents the transverse displacement of the neutral axis of the beam. Based on Euler-Bernoulli beam theory, strains can be defined as follows:

$$\varepsilon_{xx} = \frac{\partial \bar{u}}{\partial x} = -z\frac{\partial^2 w}{\partial x^2} \quad (4)$$

$$\varepsilon_{yy} = \varepsilon_{zz} = \varepsilon_{xy} = \varepsilon_{xz} = \varepsilon_{yz} = 0 \quad (5)$$

Using the above relations, the energy functions of the system can be extracted as follows.

The beam's strain energy function is defined as

$$PE_b = \frac{1}{2}\int_{\forall} \sigma_{xx}\varepsilon_{xx} d\forall \quad (6)$$

where $\forall$ states the beam volume. Applying Hook's law to Eq. (6) yields to

$$PE_b = \frac{1}{2}\int_{\forall} E\,\varepsilon_{xx}^2 d\forall \quad (7)$$

Using Eq. (4) and remembering the relationship of the second moment of inertia of the beam (I), the strain energy of the beam is obtained as

$$PE_b = \frac{1}{2}\int_0^{\ell} EI\left(\frac{\partial^2 w}{\partial x^2}\right)^2 dx \quad (8)$$

The beam kinetic energy can be stated as

$$KE_b = \frac{1}{2}\int_0^{\ell} \rho A\left(\frac{\partial w}{\partial t}\right)^2 dx \quad (9)$$

The work done by the moving mass is introduced by the following equation

$$PE_m = -\int_0^{\ell} F(x,y,t)\,w(x,t)\,dx \quad (10)$$

where the loading function $F(x,y,t)$ is defined by

$$F(x,y,t) = m\left(g - \frac{d^2 w}{dt^2}\right)\bar{\delta}(x - vt) \quad (11)$$

In Eq. (11), m is the mass of moving load, g introduces the gravitational acceleration, and $\bar{\delta}$ is the delta function used to introduce the position of the moving mass on the beam. Using some algebraic calculation will lead to

$$F(x,y,t) = m\left(g - V^2\frac{\partial^2 w}{\partial x^2} - 2V\frac{\partial^2 w}{\partial x \partial t} - \frac{\partial^2 w}{\partial t^2}\right)\bar{\delta}(x - Vt) \quad (12)$$

where $V^2\frac{\partial^2 w}{\partial x^2}$, $2V\frac{\partial^2 w}{\partial x \partial t}$, $\frac{\partial^2 w}{\partial t^2}$ introduce centripetal, Coriolis and normal accelerations, respectively.

The work done by the viscoelastic medium and the oscillating axial force are expressed by the following equations:

$$V_f = \frac{1}{2}\int_0^{\ell}\left(kw + c\frac{\partial w}{\partial t}\right)w\,dx \quad (13)$$

$$V_P = \frac{1}{2}\int_0^{\ell}\left(p_0 + p\cos(\Omega t)\right)\left(\frac{\partial w}{\partial x}\right)^2 dx \quad (14)$$

where p_0 is the static part of axial force, while p and Ω represent amplitude and frequency of the oscillatory axial force.

Equations of motion are obtained using Hamilton's principle using the following equation

$$\int_0^{t} \delta\left(KE_b - \left(PE_b + PE_m + V_f + V_p\right)\right)dt = 0 \quad (15)$$

Substituting Eqs. (8-10, 13, 14) into Eq. (15) and performing some variation calculation, the partial equation describing the motion of beam under excitation of moving mass and oscillatory axial force is derived as follows

$$EI\frac{\partial^4 w}{\partial x^4}+\rho A\frac{\partial^2 w}{\partial t^2}+kw+c\frac{\partial w}{\partial t}+ \left(p_0+p\cos(\Omega t)\right)\frac{\partial^2 w}{\partial x^2} = m\left(g-V^2\frac{\partial^2 w}{\partial x^2}- 2V\frac{\partial^2 w}{\partial x\partial t}-\frac{\partial^2 w}{\partial t^2}\right)\bar{\delta}(x-Vt) \tag{16}$$

The discretization of the equations of motion is achieved using the Galerkin method. For the Euler-Bernoulli beam theory, the variable w which is a function of x and t is expressed by

$$w(x,t)=\sum_{i=1}^{\infty}\eta_i(x)\,\bar{w}_i(t) \tag{17}$$

where $\bar{w}_i(t)$ is the generalized coordinate (time domain), and $\eta_i(x)$ is the unknown shape function (space domain). The shape functions should be selected in such a way that satisfy the beam boundary conditions. Three types of boundary conditions are considered, which will be discussed in the following. For beam with simple-simple boundary condition, we have

$$\eta_i(0)=0,\eta_i(\ell)=0, \quad \frac{\partial^2\eta_i(0)}{\partial x^2}=0,\frac{\partial^2\eta_i(\ell)}{\partial x^2}=0, \tag{18}$$

while for the beam with fixed-fixed boundary condition, we have

$$\eta_i(0)=0,\eta_i(\ell)=0, \quad \frac{\partial\eta_i(0)}{\partial x}=0,\frac{\partial\eta_i(\ell)}{\partial x}=0, \tag{19}$$

and these relations for a beam with simple-fixed boundary conditions are

$$\eta_i(0)=0,\eta_i(\ell)=0, \quad \frac{\partial\eta_i(0)}{\partial x}=0,\frac{\partial^2\eta_i(\ell)}{\partial x^2}=0 \tag{20}$$

Accordingly, shape functions and natural frequencies of beam with simple-simple, fixed-fixed and simple-fixed boundary conditions can be expressed as Eqs. (21) to (23), respectively.

$$\eta_i(x)=\sqrt{\frac{2}{\ell}}\sin\left(\frac{i\pi x}{\ell}\right), \quad \omega_i=\left(\frac{i\pi}{\ell}\right)^2\sqrt{\frac{EI}{\rho A}} \tag{21}$$

$$\eta_i(x)=\sqrt{\frac{2}{3\ell}}\left(\cos\left(\frac{2i\pi x}{\ell}\right)-1\right), \quad \omega_i=\sqrt{\frac{16}{3}}\left(\frac{i\pi}{\ell}\right)^2\sqrt{\frac{EI}{\rho A}} \tag{22}$$

$$\eta_i(x)=\sqrt{\frac{1}{\ell}}\left(\cos\left(\frac{3i\pi x}{2\ell}\right)-\cos\left(\frac{i\pi x}{2\ell}\right)\right), \quad \omega_i=\frac{8}{5}\left(\frac{i\pi}{\ell}\right)^2\sqrt{\frac{EI}{\rho A}} \tag{23}$$

In order to apply the Galerkin method, Eq. (17) is inserted into Eq. (16). Then, by multiplying the resulted equation by $\eta_j(x)$ and integrating along the length of the beam and considering the condition of orthogonality between the vibration modes and considering the integral property of the Dirac delta function, the ordinary differential equations governing the beam-moving mass system in the vector-matrix form is extracted, in the form of (24).

$$\mathbf{M}(t)\ddot{\bar{W}}+\mathbf{B}(t)\dot{\bar{W}}+\mathbf{K}\bar{W}=\mathbf{F}(t) \tag{24}$$

where the components of the matrices and vectors are as follows

$$M_{ij}=\delta_{ij}+\frac{m}{\rho A}\eta_i(x_m)\eta_j(x_m)$$

$$B_{ij}=\frac{c}{\rho A}\delta_{ij}+\frac{2mv}{\rho A}\cdot\frac{\partial\eta_i\left(x_m\right)}{\partial x}\eta_j\left(x_m\right)$$
$$K_{ij}=\omega_i^2\delta_{ij}+\frac{k}{\rho A}\delta_{ij}+$$
$$\frac{mV^2}{\rho A}\cdot\frac{\partial^2\eta_i\left(x_m\right)}{\partial x^2}\eta_j\left(x_m\right) \tag{25}$$
$$F_j=\frac{mg}{\rho A}\eta_j\left(x_m\right)$$
$$\bar{w}_j=\begin{bmatrix}\bar{w}_1\\ \bar{w}_2\\ \vdots\\ \bar{w}_j\end{bmatrix}_{j*1}$$

Considering the first mode of vibration $\left(i=j=1\right)$, and inserting equation (21) into equation (25), the ordinary differential equation governing the beam with simple supports is obtained as follows

$$\left(1+2\frac{m}{\rho A\ell}\sin^2\left(\frac{\pi Vt}{\ell}\right)\right)\frac{d^2\bar{w}}{dt^2}+$$
$$\left(\frac{4m\pi V}{\rho A\ell}\sin\left(\frac{\pi Vt}{\ell}\right)\cos\left(\frac{\pi Vt}{\ell}\right)+\frac{c}{\rho A}\right)\frac{d\bar{w}}{dt}+$$
$$\left(\omega_1^2-\frac{2m\pi^2V^2}{\rho A\ell^3}\sin^2\left(\frac{\pi Vt}{\ell}\right)+\frac{k}{\rho A}-\frac{\pi^2}{\ell^2\rho A}\left(p_0+p\cos\left(\Omega t\right)\right)\right)\bar{w}=$$
$$\sqrt{\frac{2}{\ell}}\cdot\frac{m}{\rho A}g\sin\left(\frac{\pi Vt}{\ell}\right) \tag{26}$$

Equation (26) is an ordinary differential equation with time-varying coefficients. As long as the mass is moving on the beam, the coefficients of the equation change with time. When the mass leaves the beam, equation (26) becomes the free vibration equation of the beam. Therefore, in order to analyze the stability of the system, the passage of moving masses is considered intermittently. The periodicity of the passage is $T=\ell/V$. To reflect this assumption in the governing equations, the Fourier expansion of time-varying coefficients (coefficients related to the moving mass) of Eq. (26) is written as Eq. (27)

$$\left(1+\frac{m}{\rho A\ell}\left(1-\cos\left(\frac{2\pi Vt}{\ell}\right)\right)\right)\frac{d^2\bar{w}}{dt^2}+$$
$$\left(2\frac{m\pi V}{\rho A\ell^2}\sin\left(\frac{2\pi Vt}{\ell}\right)+\frac{c}{\rho A}\right)\frac{d\bar{w}}{dt}+$$
$$+\left(\omega_1^2-\frac{m\pi^2V^2}{\rho A\ell^3}\left(1-\cos\left(\frac{2\pi Vt}{\ell}\right)\right)+\frac{k}{\rho A}-\frac{\pi^2}{\ell^2\rho A}\left(p_0+p\cos\left(\Omega t\right)\right)\right)\bar{w}=$$
$$\sqrt{\frac{2}{\ell}}\cdot\frac{m}{\rho A}g\cdot$$
$$\cdot\left(\frac{2}{\pi}+\sum_{k=1}^{\infty}\frac{4}{\pi\left(1-4i^2\right)}\cos\left(\frac{2k\pi Vt}{\ell}\right)\right) \tag{27}$$

By defining the following dimensionless parameters

$$\alpha\triangleq\frac{m}{\rho A\ell},\beta\triangleq\frac{\pi V}{\ell\omega_1},c^*\triangleq\frac{c}{\rho A\omega_1},$$
$$k^*\triangleq\frac{k}{\rho A\omega_1^2},\tau\triangleq\frac{\pi Vt}{\ell},W\triangleq\frac{\bar{w}}{\ell^{\frac{3}{2}}},$$
$$g^*\triangleq\frac{g}{\ell\omega_1^2},\lambda\triangleq\frac{p_0l^2}{\pi^2EI},\gamma\triangleq\frac{pl^2}{\pi^2EI}, \tag{28}$$

the dimensionless governing equation for a beam with simply supports under the alternating passage of moving masses is resulted

$$\beta^2\left(1+\alpha\left(1-\cos\left(2\tau\right)\right)\right)\frac{d^2W}{d\tau^2}$$
$$+\left(2\alpha\beta^2\sin\left(2\tau\right)+\beta c^*\right)\frac{dW}{d\tau}$$

$$+\begin{pmatrix} 1-\alpha\beta^2\left(1-\cos\left(2\tau\right)\right)+k^*- \\ \left(\gamma+\lambda\cos\left(2\tau\right)\right) \end{pmatrix}W$$
$$=\sqrt{2}\alpha g^*\left(\frac{2}{\pi}+\sum_{k=1}^{\infty}\frac{4}{\pi\left(1-4k^2\right)}\cos\left(2k\tau\right)\right) \tag{29}$$

It should be mentioned that the axial oscillating force is considered in such a way that the frequency of its oscillations is twice the frequency of mass passing on the beam $\left(\Omega=2\pi Vt/\ell\right)$. With a similar process and by using the corresponding shape functions, the equation governing the fixed-fixed beam and simple-fixed beam, under the passage of moving mass and alternating axial force loading, in the dimensionless form are obtained

$$\beta^2\left(1+\alpha\left(1-\frac{4}{3}\cos\left(2\tau\right)+\frac{1}{3}\cos\left(4\tau\right)\right)\right)\frac{d^2W}{d\tau^2}+\left(\beta c^*+\frac{8}{3}\alpha\beta^2\left(\sin\left(4\tau\right)\right)\right)\frac{dW}{d\tau}$$
$$+\left(1+k^*-\frac{4}{3}\alpha\beta^2\left(1-2\cos\left(2\tau\right)+\cos\left(4\tau\right)\right)-\left(\gamma+\lambda\cos\left(2\tau\right)\right)\right)W$$
$$=\sqrt{\frac{2}{3}}\alpha g^*\left(\cos\left(2\tau\right)-1\right) \tag{30}$$

$$\beta^2\left(1+\alpha\left(1+\frac{1}{2}\sum_{i=1}^{\infty}\frac{8i}{\pi\left(4i^2-9\right)}\sin\left(2i\tau\right)-\frac{1}{2}\sum_{i=1}^{\infty}\frac{8i}{\pi\left(4i^2-1\right)}\sin\left(2i\tau\right)-\cos\left(2\tau\right)\right)\right)\frac{d^2W}{d\tau^2}$$
$$+\left(\beta c^*+\alpha\beta^2\left(2\sin\left(\tau\right)-\frac{3}{2}\left(\frac{2}{3\pi}+\sum_{i=1}^{\infty}\frac{12}{\pi\left(9-4i^2\right)}\cos\left(2i\tau\right)\right)\right.\right.$$
$$\left.\left.+\frac{1}{2}\left(\frac{2}{\pi}+\sum_{i=1}^{\infty}\frac{4}{\pi\left(1-4i^2\right)}\cos\left(2i\tau\right)\right)\right)\right)\frac{dW}{dt}+\left(1+k^*-\alpha\beta^2\left(\frac{5}{4}-\frac{9}{8}\sum_{i=1}^{\infty}\frac{8i}{\pi\left(4i^2-1\right)}\right.\right. \tag{31}$$
$$\left.\left.\cdot\sin\left(2i\tau\right)+\frac{9}{8}\sum_{i=1}^{\infty}\frac{8i}{\pi\left(4i^2-9\right)}\sin\left(2i\tau\right)-\frac{5}{4}\cos\left(2\tau\right)-\left(\gamma+\lambda\cos\left(2\tau\right)\right)\right)\right)W$$
$$=\alpha g^*\left(-\frac{2}{3\pi}+\sum_{i=1}^{\infty}\frac{12\left(-1\right)^i}{\pi\left(4i^2-9\right)}\cos\left(i\tau\right)-\frac{2}{\pi}+\sum_{i=1}^{\infty}\frac{4\left(-1\right)^i}{\pi\left(4i^2-1\right)}\cos\left(i\tau\right)\right)$$

3- Stability analysis

As seen, due to the successive passage of moving masses and applying the fluctuating axial loading, the ordinary differential equations governing the problem became equations with time varying coefficients. In this section, the Floquet theory is applied to find stable and unstable regions of the mentioned system in plane of speed-mass of moving masses. In addition, the influence of beam boundary conditions, axial loading amplitude, stiffness and damping of the elastic medium on the stability of the system is studied.

Floquet theory is a standard theory for analyzing the features of a system without fully solving its governing equations [13].

Based on this method, the instability of a periodic system can be checked by determining and identifying the basic matrix in a time interval. The eigenvalues of this matrix can be considered as a criterion for determining the stability of the system. If for the selected parameters of the problem, all the eigenvalues of the basis matrix are located inside the unit circle centered $(0,0)$ in the complex plane, then the system has asymptotic stability and otherwise the system is unstable. To determine the unstable region of the problem parameters plane, a code has been written in MATLAB software, which after finding the basic matrix for the points of this plane, calculates its eigenvalues and the stability of the system for the selected parameters. The stability plane belonging to the beam with simple-simple, fixed-fixed and simple-fixed boundary conditions under the successive passage of moving masses and alternating axial loading are shown in Figs. 1 to 3, respectively. In these analyses, the dimensionless parameters are $k^*=0.5$, $c^*=0.1$, $\gamma=0.1$ and $\lambda=1$. In these figures, the horizontal and vertical axes represent the dimensionless velocity and mass of passing loads, respectively, and the hatched areas are the unstable regions and the white areas are the stable regions. As can be seen, in addition to the wide unstable areas, there are also tongues of instability in the stable area of the parameters plane. By comparing Figs. 1-3, it can be concluded that although the unstable regions belonging to the simple-fixed boundary conditions are slightly wider than the simple-simple boundary conditions, the stable region associated with the boundary conditions of the fixed-fixed is wider than other cases. Therefore, as a practical result, the use of fixed supports for two ends of the beam exposed to the passage of moving masses and axial oscillating force has high reliability in the discussion of dynamic stability.

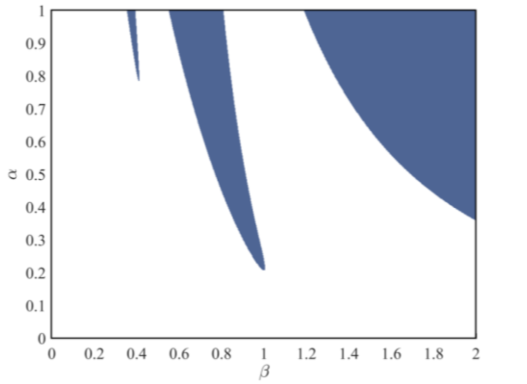


Fig. 1 Stability plane of beam with simple-simple boundary conditions

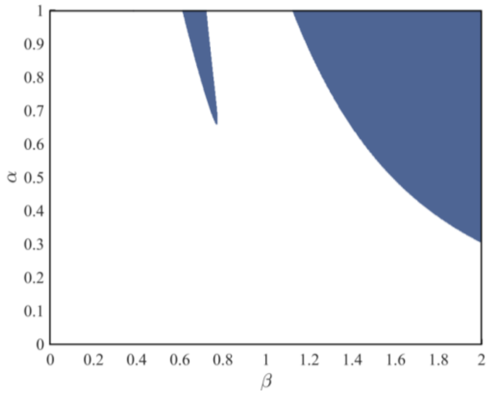


Fig. 2 Stability plane of beam with fixed-fixed boundary conditions

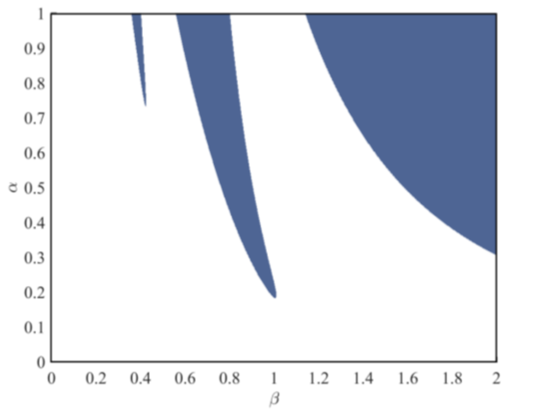


Fig. 3 Stability plane of beam with simple-fixed boundary conditions

As stated in the system modeling, a viscoelastic substrate was selected for the beam under loading. Now, in this section, the effect of the stiffness and damping parameters on the parametric plane is studied. In Fig. 4, the effect of substrate stiffness is considered. In these analyses, dimensionless parameters are set as $c^* = 0.1$, $\gamma = 0.1$ and $\lambda = 1$. From these figures, it can be concluded that the stiffness has a positive effect on the stable region of the parameter plane, so that with its increase, the extent of the stable areas increases and the beam will be stable for greater amounts of mass and velocity of loads transition.

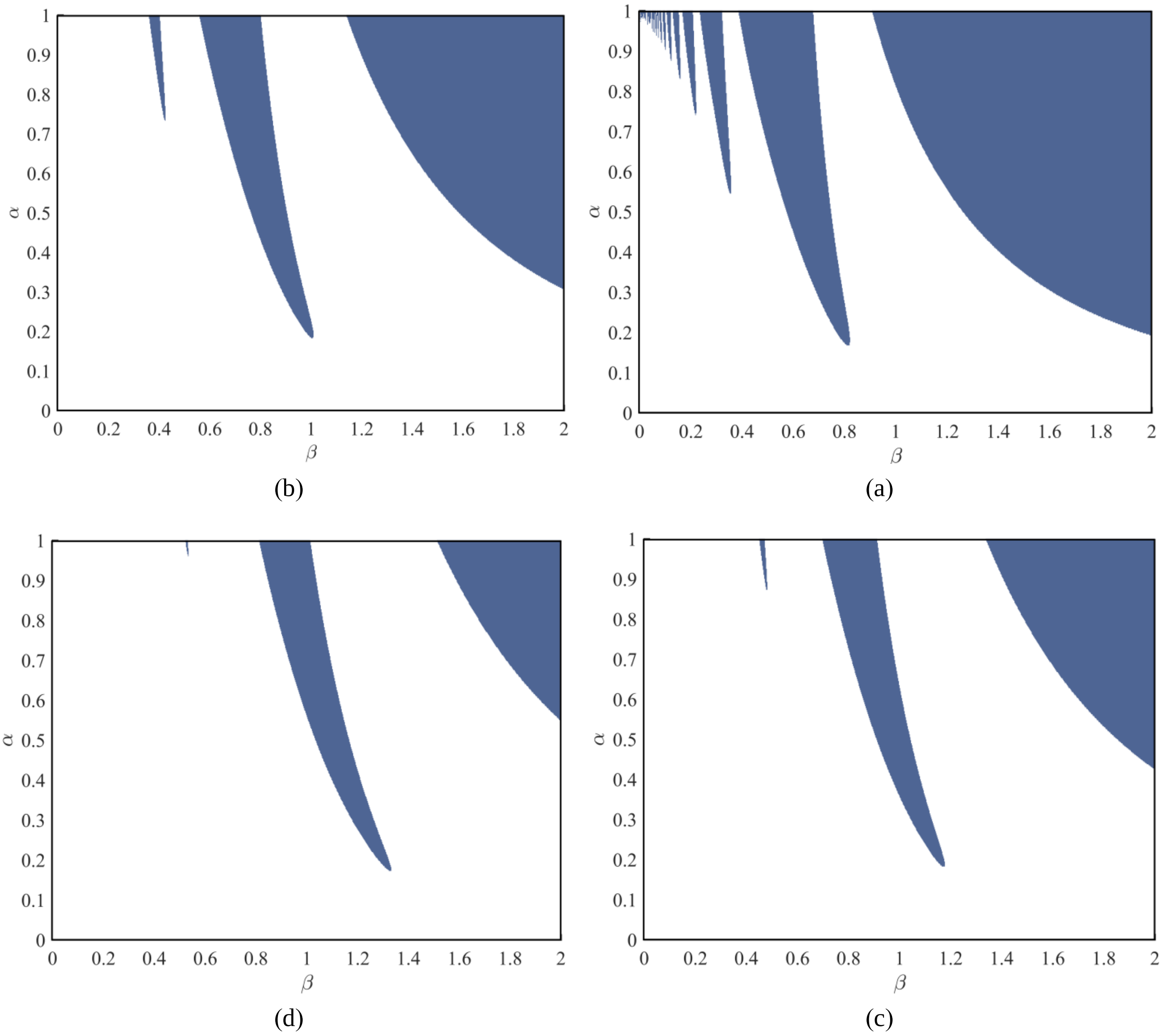


Fig. 4 The effect of the stiffness of the elastic medium on the stable and unstable regions for simple-fixed beam (a) $k^* = 0$ (b) $k^* = 0.5$ (c) $k^* = 1$ (d) $k^* = 1.5$

In Figs. 5-7, the effect of damping on the stability of the system is studied with dimensionless parameters $K^* = 0.5$, $\gamma = 0.1$ and $\lambda = 1$. Based on these figures, it is clear that the increase in bed viscosity has no effect on the large unstable areas, while it causes the unstable tongues to separate and move away from the axis and at the same time causes them to become thinner. In general, this result is in agreement with what was proposed for Mathieu's equation with the depreciation term in reference [13]. In addition to these cases, it can be seen in Fig. 8-a that the phenomenon of instability pockets has occurred for clamped-clamped beam, regardless of depreciation. This phenomenon occurs when the boundary curves of an instability tongue intersect each other in the plane of parameters [14].

In addition, based on the results of this section, the beam with fixed-fixed boundary conditions under moving and alternating axial mass loadings has a higher influence of the stiffness and damping of the substrate than the beam with simple-simple and simple-girder boundary conditions. Therefore, it can be concluded that choosing a suitable value for the stiffness and damping of the substrate leads to the prevention of inappropriate behavior of the system in practical applications.

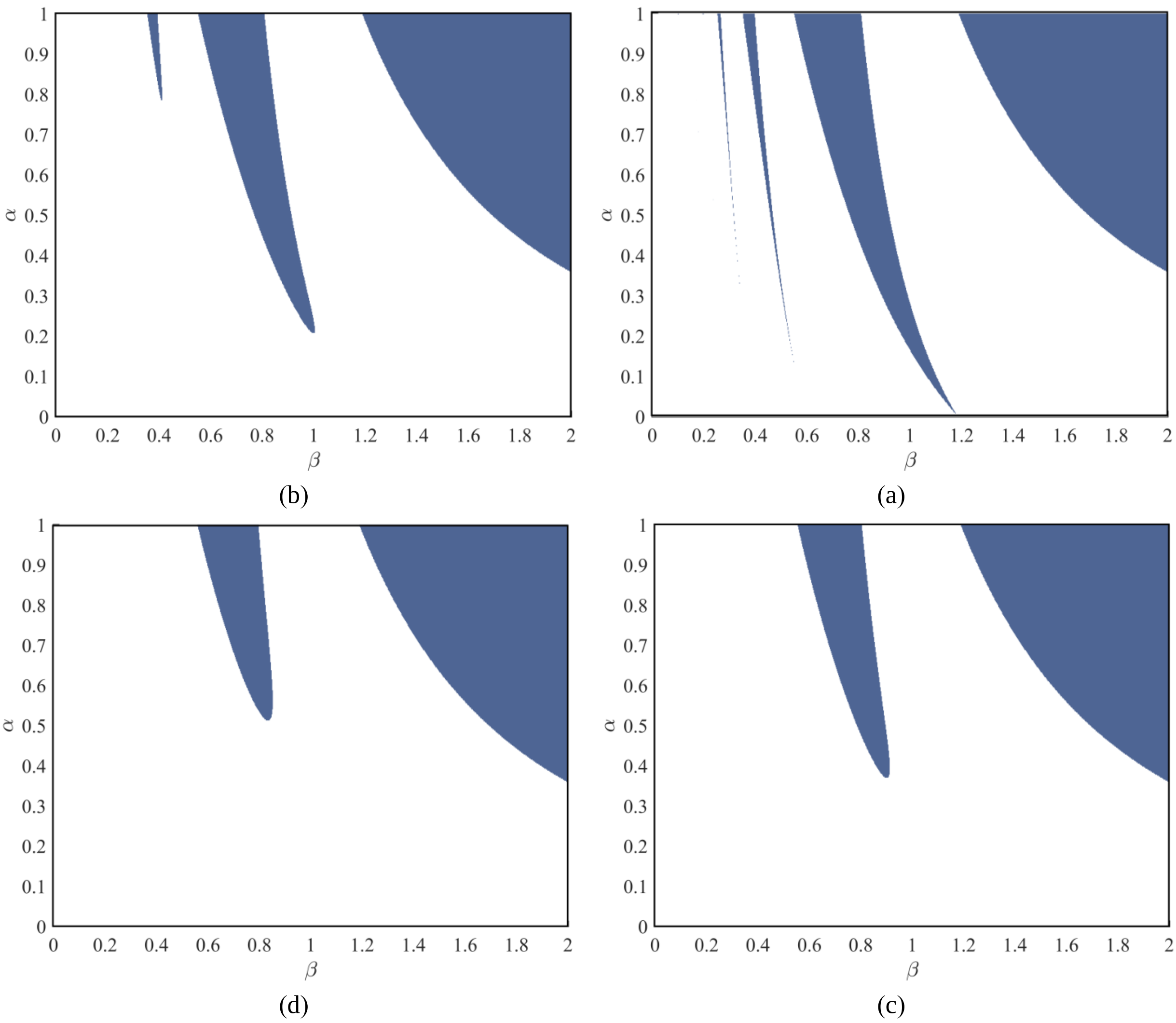


Fig. 5 the effect of damping on the stable and unstable regions of the simple-simple beam (a) $c^* = 0$ (b) $c^* = 0.1$ (c) $c^* = 0.2$ (d) $c^* = 0.3$

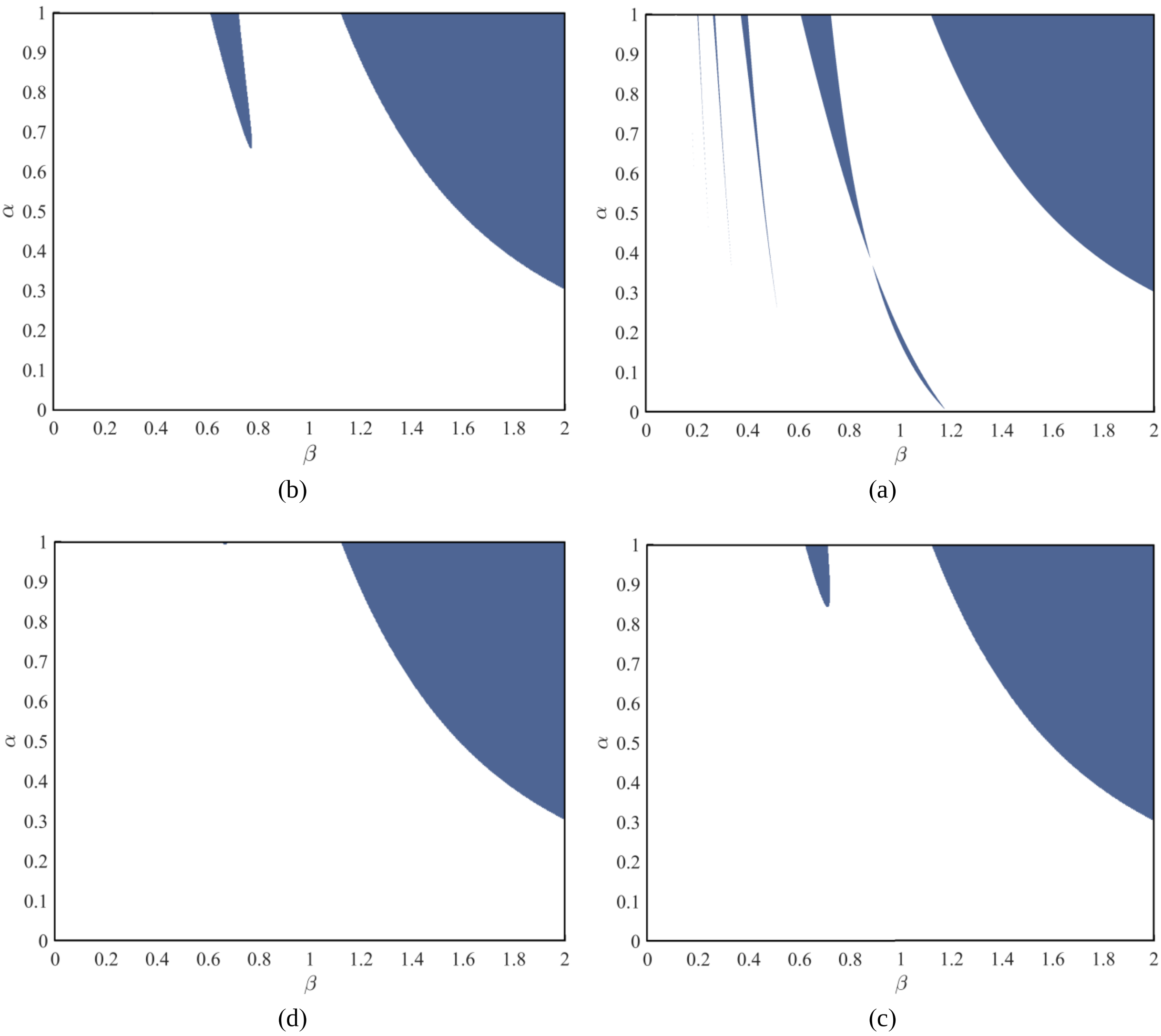
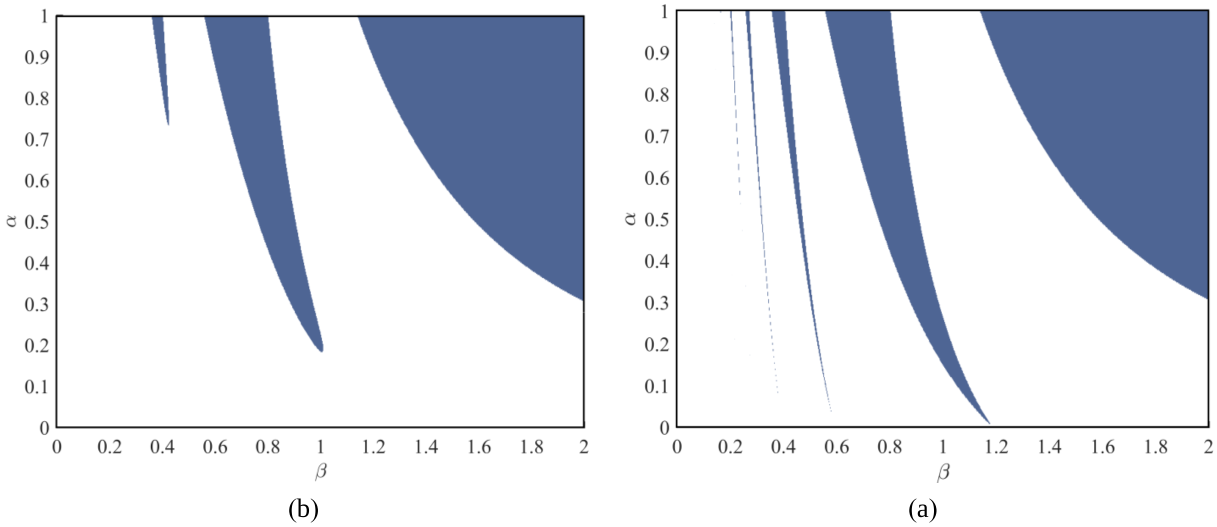
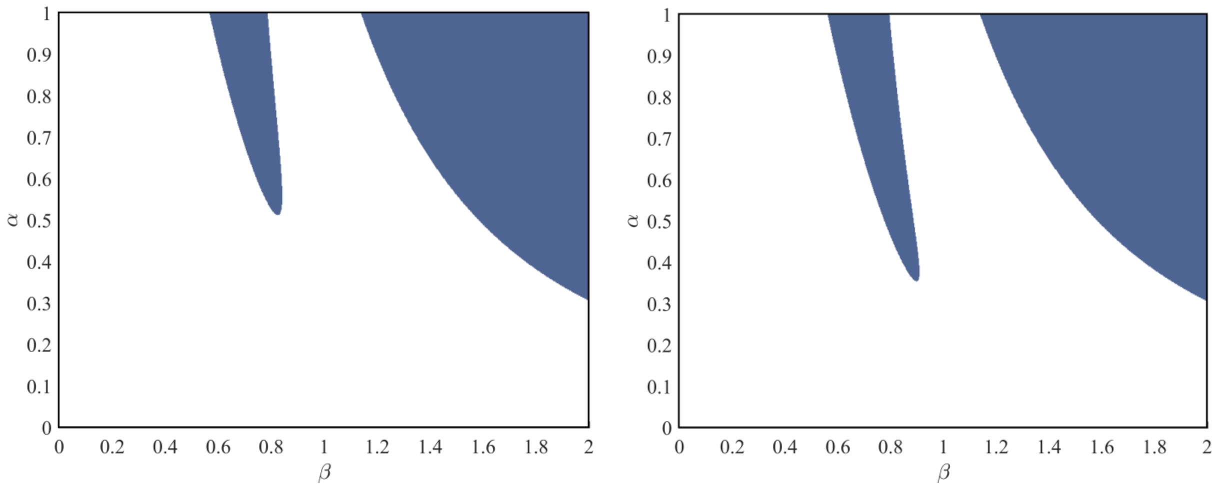


Fig. 6 the effect of damping on the stable and unstable regions of the fixed-fixed beam (a) $c^* = 0$ (b) $c^* = 0.1$ (c) $c^* = 0.2$ (d) $c^* = 0.3$





(d) (c)

Fig. 7 the effect of damping on the stable and unstable regions of the simple-fixed beam (a) $c^* = 0$ (b) $c^* = 0.1$ (c) $c^* = 0.2$ (d) $c^* = 0.3$

4- Conclusion

The dynamic stability of transverse vibrations of Euler-Bernoulli beams with different boundary conditions and located on an elastic bed under moving and alternating axial loads was carried out. The Coriolis and centripetal acceleration components were included in deriving the motion equations. The stability plane of the system was extracted in the mass-velocity plane of the passing loads. The influence of the boundary conditions, the stiffness and damping effect of the substrate, as well as the amplitude of the axial force on the parametric regions of the stability plane were investigated. Some of the results obtained in this research are as follows:

- The unstable regions belonging to simple-fixed boundary conditions are slightly wider than simple-simple boundary conditions; however, the stable region associated with fixed-fixed boundary conditions is wider than other cases.
- The use of clamp supports for two ends of the beam exposed to the passage of moving masses and periodic axial force has high reliability in dynamic stability.
- The stiffness of the substrate has a positive effect on the stable areas of the parameter plane, so that with its increase, the unstable areas are transferred to higher speeds of the passing mass, and the beam will be stable for larger amounts of the mass and speed of the passing loads.
- An increase in bed wear has no effect on the large unstable areas, while it causes the unstable tongues to separate and move away from the β axis and at the same time causes them to become thinner.

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