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Modeling of Groundwater Flow over Sloping Beds in Response to Constant Recharge and Stream of Varying Water Level

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Abstract. This paper presents an analytical model characterizing unsteady groundwater flow in an unconfined aquifer resting on a sloping impervious bed. The aquifer is in contact with a constant water level at one end. The other end is in hydrological connectionwith a stream whose level is increasing form an initial level to a final level by a known exponentially function of time. During this process, the aquifer is vertically replenished by a constant downward recharge. The linearized Boussinesq equation is solved analytically using Laplace transform to obtain the closed form expression for hydraulic head distribution in the aquifer and flow rate at the stream-aquifer interface. The expressions derived in this study can handle the cases of upward sloping, horizontal bed conditions and sudden rise in stream water. The validity of linearization method adopted in this study is examined by solving the nonlinear equation by an explicit numerical scheme. Response of an aquifer to the variations in bed slope, recharge rate and rise rate of the stream water is illustrated with a numerical example. Sensitivity of the flow rate with various parameters is analyzed.

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Index to information contained in this paper

1. Introduction

Estimation of surface and groundwater interaction is an important hydrological investigation due to its key role in conjunctive management of groundwater resources.

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Several analytical and numerical models have been developed to predict the surface-groundwater interaction in stream-aquifer systems under varying hydrological conditions [11-14, 18]. Estimations of water table fluctuations in unconfined aquifer with surface infiltration have also been presented by several researchers [15, 17, 19-21].

The subsurface seepage flow in unconfined aquifers is usually formulated as a parabolic nonlinear Boussinesq equation which does not admit analytical solution. Therefore, the methodology adopted in most of the aforesaid models is to linearize the Boussinesq equation and develop analytical solution of the resulting equation. Although, the approximate analytical solution obtained in these studies provide useful insight in the flow process; however, the subsurface drainage over hillslope cannot be satisfactorily addressed with their results. Another perceived limitation is that they do not account for the gradual rise in the stream water.

The role of sloping bedrock in evolution of the phreatic surface has been underlined in numerous studies [8-10]. The results derived for horizontal aquifer may seriously underestimate or overestimate the actual results if applied to the sloping bed geometry. Approximation of groundwater flow over unconfined sloping beds based on extended Dupuit-Forchheimer assumption (streamlines are approximately parallel to the sloping impervious bed) can be expressed in the form of an advection-diffusion equation, popularly known as Boussinesq equation [1, 7]. Analytical solutions of linearized Boussinesq equation under constant or time-varying boundary conditions have been presented by a number of researchers [1- 6, 16]. It is worth mentioning that the efficiency of the linearization must be examined by validating the model results with either field data or by comparing the results with numerical solution of the nonlinear Boussinesq equation.

The present study is an attempt to quantify the groundwater-surface water interaction with more realistic approach. The mathematical model considered here deals with transient groundwater flow regime in a homogeneous unconfined aquifer of finite width overlying a downward sloping impervious bed owing to seepage from stream of varying water level and constant downward recharge. The aquifer is in contact with a constant piezometric level at one end and a stream of time varying water level at another end. The stream is considered to penetrate full thickness of the aquifer. Furthermore, the aquifer is replenished by a constant recharge. Efficiency of the linearization method is examined by solving the nonlinear Boussinesq equation by a fully explicit predictor-corrector numerical scheme. The effect of bed slope, recharge rate and stream rise rate on the water table fluctuation and flow mechanism is analyzed using a numerical example. The solution presented here can be applied to asymptotic scenarios of sudden or very slow rise in the stream water by assigning an appropriate value to the stream rise rate parameter.

2.Problem Formulation and Analytical Solution

As shown in Fig.1, we consider an unconfined aquifer overlaying an impermeable sloping bed with downward slopetanβ. The aquifer is in contact with a constant piezometric levelh_0 at its left end and a stream at the right end. The water in the stream is gradually rising from its initial level h_L to a final level h_0 by a known exponential decaying function of time$t$. Moreover, the aquifer is replenished vertically at a constant rate. If the variation in the hydraulic conductivityand specific yield of the aquifer with spatial coordinate is neglected, and the streamlines are considered to be nearly parallel to the impermeable bed (extended Dupuit-Forchheimer approach) then the groundwater flow in the aquifer can be characterized by the following nonlinear Boussinesq equation [10]

$$\frac{\partial}{\partial x}\left(h\frac{\partial h}{\partial x}\right)-tan\,\beta\frac{\partial h}{\partial x}+\frac{N\varepsilon(t)}{K\,cos^2\beta}=\frac{S}{K\,cos^2\beta}\frac{\partial h}{\partial t} \tag{1}$$

where $h(x,t)$ is the height of the water table measured above the impermeable slopingbed in the vertical direction. K and S respectively are the hydraulic conductivity and specific yield of the aquifer. N is the constant recharge rate. $\varepsilon(t)$ is a unit step function defined as follows:

$$\varepsilon(t) = \begin{cases} 0 \; if \; t \leq 0 \\ 1 \; if \; t > 0 \end{cases} \tag{2}$$

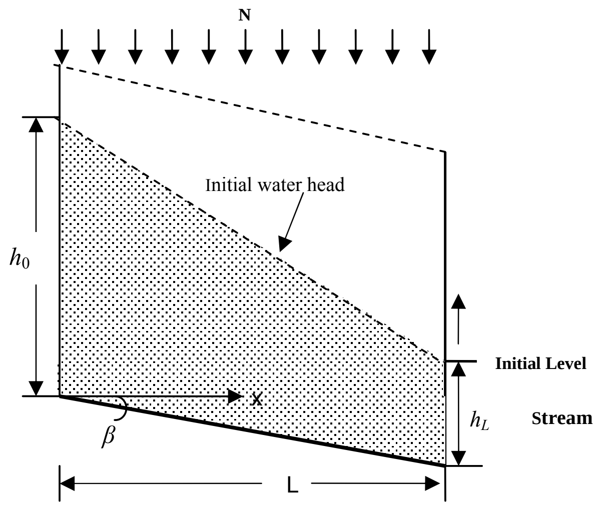

Fig. 1.Schematic diagram of an unconfined aquifer

The initial and boundary conditions are:

$$h(x, t = 0) = h_0 + \frac{h_0 - h_L}{L} x \tag{3}$$

$$h(x = 0, t) = h_0 \tag{4}$$

$$h(x = L, t) = h_0 - (h_0 - h_L)e^{-\lambda t} \tag{5}$$

where L is length of the aquifer. λ is a positive constant signifying the rate at which the water in the stream rises from its initial value h_L to a final level h_0. Equation (1) is a second order nonlinear parabolic partial differential equation which cannot be solved by analytical methods. However, an approximate analytical solution can be obtained by solving the corresponding linearized equation. In the present work, we adopt the linearization method as suggested by Marino [16]. Firstly, rewrite Equation (1) as

$$\frac{\partial^2 h}{\partial x^2} - \frac{tan\,\beta}{\overline{h}}\frac{\partial h}{\partial x} + \frac{N\varepsilon(t)}{K\,\overline{h}cos^2\,\beta} = \frac{S}{K\overline{h}\,cos^2\,\beta}\frac{\partial h}{\partial t} \tag{6}$$

where $\overline{h}$ is the mean saturated depth in the aquifer. The value of $\overline{h}$ is successively approximated using an iterative formula $\overline{h} = (h_0 + h_i)/2$, where h_0 is the initial water table height, and h_i is the varying water table height at time t at the end of which $\overline{h}$ is approximated. Equation (6) is further simplified using the following dimensionless variables and substitutions

$$H = \frac{h - h_0}{h_L - h_0},\ \ X = \frac{x}{L}, \tau = \frac{K\,\bar{h}\,cos^2\beta}{SL^2}t \tag{7}$$

Therefore, Equation (6) becomes

$$\frac{\partial^2 H}{\partial X^2} - \frac{L\,tan\,\beta}{\bar{h}}\frac{\partial H}{\partial X} + \frac{L^2 N\varepsilon(\tau)}{K\,\bar{h}(h_L - h_0)cos^2\beta} = \frac{\partial H}{\partial \tau} \tag{8}$$

where

$$\varepsilon(\tau) = \begin{cases} 0 \text{ if } \tau \le 0 \\ 1 \text{ if } \tau > 0 \end{cases} \tag{9}$$

Now, define the following parameters

$$\alpha = \frac{L\tan\beta}{2\bar{h}}, N_1 = \frac{L^2 N}{K\bar{h}(h_L - h_0)\cos^2\beta}, \\ \lambda_1 = \frac{SL^2}{K\bar{h}\cos^2\beta}\lambda \tag{10}$$

so that, equation (8) becomes

$$\frac{\partial^2 H}{\partial X^2} - 2\alpha\frac{\partial H}{\partial X} + N_1\varepsilon(\tau) = \frac{\partial H}{\partial \tau} \tag{11}$$

The initial and boundary conditions reduce to

$$H(X, \tau = 0) = X \tag{12}$$

$$H(X = 0, \tau) = 0 \tag{13}$$

$$H(X = 1, \tau) = 1 - e^{-\lambda_1 \tau} \tag{14}$$

Equation (11) along with conditions(12)–(14) is solved by Laplacetransform. Define the Laplace transform of $H(X, \tau)$ as follows:

$$L\{H(X, \tau): \tau \to s\} = \bar{H}(X, s) = \int_0^\infty H(X, \tau)e^{-s\tau}d\tau \tag{15}$$

The Laplace transform of equation (11) yields

$$\frac{\partial^2 \bar{H}}{\partial X^2} - 2\alpha\frac{\partial \bar{H}}{\partial X} + \frac{N_1}{s} = \frac{\partial \bar{H}}{\partial \tau} - X \tag{16}$$

The general solution of equation (16) can be found using elementary methods. Moreover, the arbitrary constants contained in the general solution are obtained by using the Laplace transform of Equations (13)–(14) in it. We get

$$\overline{H}(X,s) = e^{\alpha X}\left[\left(\frac{2\alpha - N_1}{s^2}\right)\frac{sinh\left((1-X)\sqrt{\alpha^2+s}\right)}{sinh\left(\sqrt{\alpha^2+s}\right)} + e^{-\alpha}\left(\frac{2\alpha - N_1}{s^2} - \frac{\lambda_1}{s(s+\lambda_1)}\right)\frac{sinh\left(X\sqrt{\alpha^2+s}\right)}{sinh\left(\sqrt{\alpha^2+s}\right)}\right] + \frac{X}{s} - \frac{2\alpha}{s^2} + \frac{N_1}{s^2} \tag{17}$$

Inverse Laplace transform of equation (17) is obtained from calculus of residue. Define

$$L^{-1}\{\overline{H}(X,s)\} = H(X,\tau) = \frac{1}{2\pi i}\lim_{w\to\infty}\int_{c-iw}^{c+iw}\overline{H}(X,s)e^{s\tau}ds \tag{18}$$

where$c$ is an arbitrary positive number. The integration in equation (18) is performed along a line $s = c$ in the complex plane where $s = x + i\,y$. The real number c is chosen so that $s = c$ lies to the right of all singularities, but is otherwise arbitrary. The inverse Laplace transform of equation (17) yields

$$\begin{aligned} H(X,\tau) &= X - e^{-\alpha(1-X)}\left(1 - e^{-\lambda_1\tau}\right)\frac{sinh\left(X\sqrt{\alpha^2-\lambda_1}\right)}{sinh\left(\sqrt{\alpha^2-\lambda_1}\right)} \\ &- 2\pi(2\alpha - N_1)e^{\alpha X}\left\{\sum_{n=1}^{\infty}\frac{n\,sin(n\pi X)\left(1-e^{-(\alpha^2+n^2\pi^2)\tau}\right)}{(\alpha^2+n^2\pi^2)^2} - e^{-\alpha}\sum_{n=1}^{\infty}\frac{(-1)^n n\,sin(n\pi X)\left(1-e^{-(\alpha^2+n^2\pi^2)\tau}\right)}{(\alpha^2+n^2\pi^2)^2}\right\} \\ &- 2\pi\lambda_1 e^{-\alpha(1-X)}\sum_{n=1}^{\infty}\frac{(-1)^n n\,sin(n\pi X)\left(1-e^{-(\alpha^2+n^2\pi^2)\tau}\right)}{(\alpha^2+n^2\pi^2)(\alpha^2+n^2\pi^2-\lambda_1)} \end{aligned} \tag{19}$$

Equation (19) provides analytical expression for the water head distribution in the downward sloping aquifer under conditions mentioned in Equations (3)–(5). One can obtain the corresponding results for an upward sloping by replacing $\alpha$by – $\alpha$and for a horizontal bed by setting $\alpha \to 0$. Furthermore, equation (19) can also be used to predict the water head profiles when the rise in the stream is extremely rapid; similar to the case of flood like situation, by letting $\lambda \to\infty$. We obtain

$$\begin{aligned}
&H(X,\tau) \\
&= X - e^{-\alpha(1-X)}\frac{\sinh X}{\sinh 1} \\
&- 2\pi(2\alpha - N_1)e^{\alpha X}\left\{\sum_{n=1}^{\infty}\frac{n\sin(n\pi X)\left(1-e^{-(\alpha^2+n^2\pi^2)\tau}\right)}{(\alpha^2+n^2\pi^2)^2}\right. \\
&\left. - e^{-\alpha}\sum_{n=1}^{\infty}\frac{(-1)^n n\sin(n\pi X)\left(1-e^{-(\alpha^2+n^2\pi^2)\tau}\right)}{(\alpha^2+n^2\pi^2)^2}\right\} \\
&+ 2\pi e^{-\alpha(1-X)}\sum_{n=1}^{\infty}\frac{(-1)^n n\sin(n\pi X)\left(1-e^{-(\alpha^2+n^2\pi^2)\tau}\right)}{(\alpha^2+n^2\pi^2)}
\end{aligned} \tag{20}$$

3.Determination of Flow Rate and Steady State Profile

The flow rate$q(x,t)$ in the aquifer is defined as

$$q(x,t) = -Kh\left(\frac{\partial h}{\partial x} - \tan\beta\right) \tag{21}$$

andat the stream-aquifer interface, the flow rate is

$$q_{x=L} = -K\left\{h_0 - (h_0 - h_L)e^{-\lambda t}\right\}\left[\left(\frac{\partial h}{\partial x}\right)_{x=L} - \tan\beta\right] \tag{22}$$

Invoking Equation (20) in Equation (22), the expressions for flow rate at the stream-aquifer interface are obtained as follows:

$$\begin{aligned}
q_{x=L} = \frac{K}{L}\Bigg[1 - 2\pi^2(2\alpha & \\
&- N_1)\left\{e^{\alpha}\sum_{n=1}^{\infty}\frac{(-1)^n n^2\left(1-e^{-(\alpha^2+n^2\pi^2)\tau}\right)}{(\alpha^2+n^2\pi^2)^2}\right. \\
&\left. - \sum_{n=1}^{\infty}\frac{n^2\left(1-e^{-(\alpha^2+n^2\pi^2)\tau}\right)}{(\alpha^2+n^2\pi^2)^2}\right\} \\
&- 2\pi^2\lambda_1\sum_{n=1}^{\infty}\frac{n^2\left(1-e^{-(\alpha^2+n^2\pi^2)\tau}\right)}{(\alpha^2+n^2\pi^2)(\alpha^2+n^2\pi^2-\lambda_1)} \\
&- \left(1-e^{-\lambda_1\tau}\right)\left\{\alpha + \frac{\sqrt{\alpha^2-\lambda_1}}{\tanh\sqrt{\alpha^2-\lambda_1}}\right\} \\
&+ \frac{L\tan\beta}{h_0 - h_L}\Bigg] \\
&\times (h_0 - h_L)\left\{h_0 + (h_L - h_0)e^{-\lambda\tau}\right\}
\end{aligned} \tag{23}$$

It is worth noting that despite continuous recharge, the profiles of water head attain a steady state value for large value of time. The expressions for steady-state water head and flow rate at stream-aquifer interface can be obtained by setting $t \to \infty$ in Equations (20) and (23), yielding

$$
\begin{aligned}
&h^* \\
&= \frac{x}{L} - e^{-\alpha(1-x/L)} \frac{sinh\left(\left(\frac{x}{L}\right)\sqrt{\alpha^2-\lambda_1}\right)}{sinh\left(\sqrt{\alpha^2-\lambda_1}\right)} \\
&+ 2\pi(2\alpha - N_1)e^{\alpha(x/L)} \left\{\sum_{n=1}^{\infty} \frac{n\, sin(n\pi x/L)}{(\alpha^2+n^2\pi^2)^2}\right. \\
&\left. - e^{-\alpha} \sum_{n=1}^{\infty} \frac{(-1)^n n\, sin(n\pi x/L)}{(\alpha^2+n^2\pi^2)^2}\right\} \\
&- 2\pi\lambda_1 e^{-\alpha(1-x/L)} \sum_{n=1}^{\infty} \frac{(-1)^n n\, sin(n\pi x/L)}{(\alpha^2+n^2\pi^2)(\alpha^2+n^2\pi^2-\lambda_1)} \\
&\times (h_0 - h_L)h_0
\end{aligned}
\tag{24}
$$

$$
\begin{aligned}
&q^* \\
&= \frac{K}{L}\left[1\right. \\
&- 2\pi^2(2\alpha - N_1)\left\{e^{\alpha}\sum_{n=1}^{\infty}\frac{(-1)^n n^2}{(\alpha^2+n^2\pi^2)^2}\right. \\
&\left. - \sum_{n=1}^{\infty}\frac{n^2}{(\alpha^2+n^2\pi^2)^2}\right\} \\
&- 2\pi^2\lambda_1 \sum_{n=1}^{\infty}\frac{n^2}{(\alpha^2+n^2\pi^2)(\alpha^2+n^2\pi^2-\lambda_1)} \\
&\left. - \left\{\alpha + \frac{\sqrt{\alpha^2-\lambda_1}}{tanh\sqrt{\alpha^2-\lambda_1}}\right\} + \frac{L\, tan\,\beta}{h_0 - h_L}\right] \times (h_0 - h_L)h_0
\end{aligned}
\tag{25}
$$

4.Numerical Solution of the Non-linear Equation

To assess the efficiency of linearization technique, the non-linear Boussinesq equation is solved numerically by using Mac Cormack scheme. For this, Equation (1) is written as

$$
\frac{\partial h}{\partial t} = C_1 \frac{\partial}{\partial x}\left(h\frac{\partial h}{\partial x}\right) - C_2 \frac{\partial h}{\partial x} + \frac{N}{S} \tag{26}
$$

where, $C_1 = (K\cos^2\beta)/S$ and$C_1 = (K\sin 2\beta)/2S$. Mac Cormack scheme is a predictor corrector scheme in which the predicted value of *h* is obtained by replacing the spatial and temporal derivatives by forward difference, i.e.

$$
\begin{aligned}
h^*_{k,n+1} = h_{k,n} + C_1 \frac{\Delta t}{(\Delta x)^2} & \left[h_{k+1,n}\left(h_{k+1,n} - h_{k,n}\right)\right. \\
& \left. - h_{k,n}\left(h_{k,n} - h_{k-1,n}\right)\right] \\
& - C_2 \frac{\Delta t}{\Delta x}\left(h_{k+1,n} - h_{k,n}\right) + \frac{\mathrm{N}}{\mathrm{S}}\Delta \mathrm{t}
\end{aligned}
\tag{27}
$$

The corrector is obtained by replacing the space derivative by backward differences, whereas the time derivative is still approximated by forward difference

$$h_{k,n+1}^{**} = h_{k,n} + C_1 \frac{\Delta t}{(\Delta x)^2} \left[h_{k,n+1}^{*} \left(h_{k+1,n+1}^{*} - h_{k,n+1}^{*} \right) - h_{k-1,n+1}^{*} \left(h_{k,n+1}^{*} - h_{k-1,n+1}^{*} \right) \right] - C_2 \frac{\Delta t}{\Delta x} \left(h_{k,n+1}^{*} - h_{k-1,n+1}^{*} \right) + \frac{\mathrm{N}}{\mathrm{S}} \Delta \mathrm{t} \tag{28}$$

The final value of $h_{k,n+1}$ is given as an arithmetic mean of $h_{k,n+1}^{*}$ and $h_{k,n+1}^{**}$, i.e.

$$h_{k,n+1} = \frac{1}{2} \left[h_{k,n} + h_{k,n+1}^{*} - C_2 \frac{\Delta t}{\Delta x} \left(h_{k,n+1}^{*} - h_{k-1,n+1}^{*} \right) + C_1 \frac{\Delta t}{(\Delta x)^2} \left\{ h_{k,n+1}^{*} \left(h_{k+1,n+1}^{*} - h_{k,n+1}^{*} \right) - h_{k-1,n+1}^{*} \left(h_{k,n+1}^{*} - h_{k-1,n+1}^{*} \right) \right\} \right] + \frac{\mathrm{N}}{\mathrm{S}} \Delta \mathrm{t} \tag{29}$$

The initial and boundary conditions are discretized as

$$h_{k,1} = h_0 - \frac{h_0 - h_L}{L} x_k \tag{30}$$

$$h_{1,n+1} = h_0 \tag{31}$$

$$h_{L,n+1} = h_0 - (h_0 - h_L) e^{\lambda t_{n+1}} \tag{32}$$

Numerical experiments reveal that the method is stable if $\frac{A_1 \Delta t}{(\Delta x)^2} \leq 0.06$ and $\frac{A_2 \Delta t}{\Delta x} \leq 0.09$

5. Discussion of Results

To demonstrate the applicability of the closed form solution given by Equations (19) and (23), we consider an aquifer with $\boldsymbol{L}$ = 150 m. Other hydrological parameters are: $\boldsymbol{K}$ = 3.6 m/h, $\boldsymbol{S}$ = 0.34, $\boldsymbol{h_0}$ = 5 m, $\boldsymbol{h_L}$ = 2 m, $\boldsymbol{N}$ = 4 mm/hr and $\boldsymbol{\lambda}$ = 0.054 per hr. Numerical experiments were carried out, and it is found that the first 50 terms of the summation series satisfactorily approximates the final value. The profiles of transient water head $\boldsymbol{h}$ obtained from equation (19) for $\boldsymbol{\beta}$ = 3 deg are plotted in Fig. 2 (continuous curves).

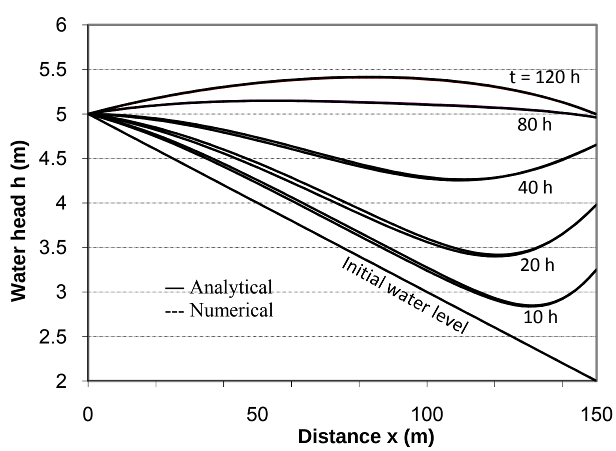

Fig. 2.Comparison of analytical and numerical solution for β = 3 deg

Numerical solution of the non-linear Boussinesq equation (1) using Mac Cormack scheme for the same data set is also presented in Fig. 2 (dotted curves). Close agreement between the numerical and analytical solutions demonstrates the efficiency of the linearization method adopted in this study. Comparison of water head height obtained from numerical and analytical solutions is presented in Table 1. It is observed that the analytical solution slightly underestimates the actual results.

Table1. Comparison of analytical (anal.) andnumerical (num.) results

x(m)	t=20h		t=40h		t=80h		t=120h	
	anal.	num.	anal.	num.	anal.	num.	anal.	num.
20	4.8216	4.8466	4.9321	4.9492	5.0956	5.0964	5.175	5.1775
40	4.5558	4.6001	4.7919	4.8213	5.14	5.1403	5.3005	5.3046
60	4.2301	4.2853	4.6054	4.6380	5.1479	5.1462	5.3791	5.3838
80	3.879	3.9341	4.4116	4.4387	5.1336	5.1297	5.4103	5.4153
100	3.5611	3.6002	4.2742	4.2905	5.1081	5.1032	5.3891	5.3942
120	3.3997	3.4185	4.2817	4.2846	5.0731	5.069	5.3036	5.3079
140	3.6527	3.6525	4.49	4.4845	5.0119	5.0108	5.1293	5.1312

Relative percentage difference (RPD) between numerical and analytical values of water head with the numerical solution is analyzed. It is observed that the RPD is maximum in the middle part of the aquifer and negligible near the interfaces $x = 0$ and $x = L$. In order to obtain a validity range of the analytical results, the range of RPD for different values of sloping angle β is presented in Table 2.

Table2. Range of RPD between analytical and numerical solution for $\lambda = 0.054$ h^{-1}

β	Range of RPD
5	– 0.0713 to 0.5232
3	– 0.1322 to 0.5455
0	– 0.1822 to 0.5692
–3	– 0.2014 to 0.6227
–5	– 0.2511 to 0.6532

Average distance between the analytical and numerical solutions is also calculated using $L2$ norm which is defined as follows:

$$\left(\frac{1}{\sqrt{L}}\right)\|h_n - h_a\| = \left(\frac{1}{\sqrt{L}}\right)\left[\int_{x=0}^{L}\{h_{num} - h_{ana}\}^2 dx\right]^{1/2} \tag{33}$$

where$h_{num}$and h_{ana}respectively denote the numerical and analytical solutions. The $L2$ norm for $\beta = 3$ deg, $\lambda = 0.054\text{h}^{-1}$,$t = 20, 40, 80$ and 120 hrs is presented in Table 3.

Table3. Average distance between analytical and numerical solution

t (h)	Average distance using $L2$ Norm
20	0.01344
40	0.02107
80	0.01074
120	0.00528

Fig. 3 presents the comparison of transient water head profiles for $\boldsymbol{\beta = 3}$ and 5 deg. As time progresses, the water table grows in the aquifer. Continuous recharge causes groundwater mound which finally stabilizes due to inflow-outflow equilibrium. Since, steeper downhill slope allows more water to from the left end; such aquifers exhibit relatively higher growth the phreatic surface. This phenomenon is evident from Fig. 3 wherein the water table profiles in an aquifer with 5 deg bed slope are higher than the corresponding profiles of 3 deg bed slope.

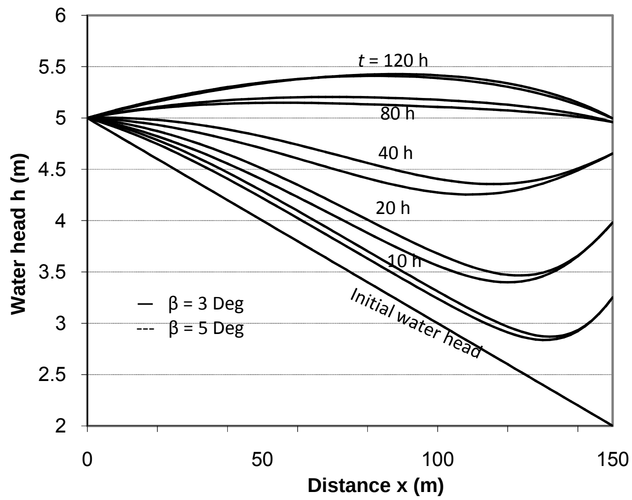

Fig. 3.Transient profiles of water head for β = 3 and 5 deg

Considering a fixed point x in the domain with distance ratio x/L = 0.75, we plot in Fig. 4 the water head at this point against time t for various values ofλ. It can be observed from this figure that the rise rate of the stream water plays an important role in determining the transient profiles of free surface as well as its stabilization level. A fast rising stream reduces the outflow at stream-aquifer interface, leading to growth in the water table.

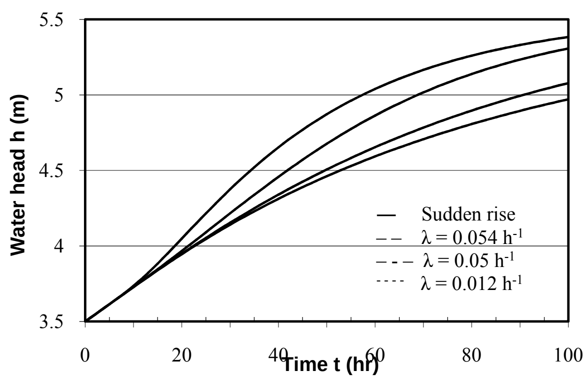

Fig.4.Variation in water head height at x = 75 m

Using equation (23), the flow rates at interface $x = L$ is plotted in Fig. 5. The initial flow rate at the interface $x = L$ are given by

$$q_{x=L} = Kh_L\left[\frac{(h_0 - h_L)}{L} + \tan\beta\right] \tag{34}$$

Clearly, $q_{x=L}$ varies with bed slope. At initial stages, the stream-aquifer interface experiences flow along positive direction of x-axis. As time progresses, the outflow at $x = L$ reaches a minimum level and increases thereafter. A reasonably long aquifer with small bed slope experiences a temporary inflow at this interface. For large value of time, the flow rate attains steady state value given by Equation (25). It is worth noting that stabilization of flow rates leads to convergence of water table to a final value. Variation in $q_{x=L}$ with recharge rate N and stream rise rate λ are plotted in Fig. 6 and Fig. 7 respectively. It is clear from this figure that flow rate at stream-aquifer interface increases with recharge. Furthermore, stream rise rate plays an important role in determining the inflow-outflow at the stream aquifer interface. It is established that a fast rising stream may allow its water to enter into the aquifer (bank storage mechanism).

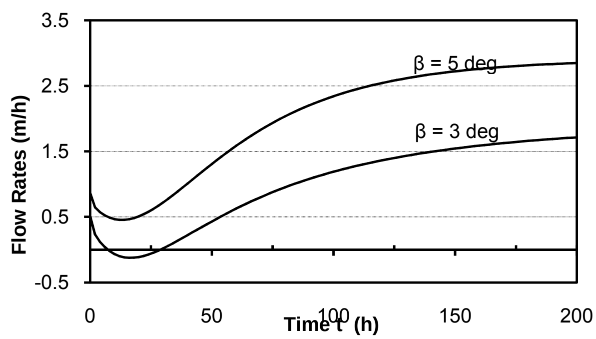

Fig. 5.Flow rate at the stream-aquifer interface for β = 3 and 5 deg

6. **Conclusions**

This study focuses on three major issues: (i) derive analytical expressions for water head and flow rate in an unconfined sloping aquifer due to continuous recharge and stream-varying water level, (ii) examine the efficiency and validity range of the linearization of method, and (ii) analyze the response of an aquifers to varying hydrological parameters. The linearized Boussinesq equation characterizing the transient groundwater flow is solved using Laplace transform technique, and the corresponding nonlinear equation is solved numerically by a fully explicit predictor-corrector scheme. Numerical experiments carried out in this study indicate that the evolution and stabilization of the phreatic surface and interaction of water between the stream and aquifer depend significantly on the bed slope and other aquifer parameters.

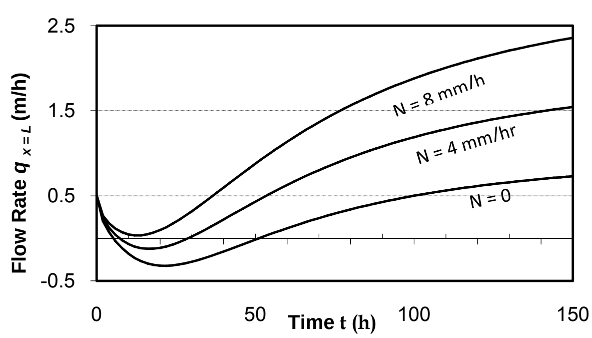

Fig. 6.Variation in flow rate at stream-aquifer interface with N

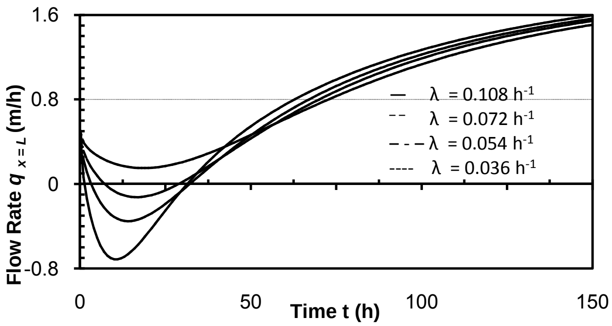

Fig. 7.Variation in flow rate at stream-aquifer interface with λ

7. References

[1] Akylas, E., Koussis,A.D., Response of sloping unconfined aquifer to stage changes in adjacent stream. I. Theoretical analysis and derivation of system response functions. J. Hydrol,**338** (2007) 85–95.

[2] Bansal, R.K., Das, S.K., The effect of bed slope on water head and flow rate at the interfaces between the stream and groundwater: analytical study. J HydrolEng,**14**(**8**) (2009) 832–838.

[3] Bansal, R.K., Das,S.K. Water table fluctuations in a sloping aquifer: analytical expressions for water exchange between stream and groundwater. J Porous Media ,**13**(**4**)(2010) 365–374.

[4] Bansal, R.K. Das S.K.An analytical study of water table fluctuations in unconfined aquifers due to varying bed slopes and spatial location of the recharge basin. J HydrolEng, **15** (2010) 909–917.

[5] Bansal, R.K., Das S.K., Response of an unconfined sloping aquifer to constant recharge and seepage from the stream of varying water level.Water Resour. Manag,**25** (2011) 893–911.

[6] Bansal R.K., Groundwater fluctuations in sloping aquifers induced by time-varying replenishment and seepage from a uniformly rising stream. Trans. Porous Med,**94** (2012) 817-826.

[7] Behzadi,S.S., Numerical solution of Boussinesq equation using modified Adomian decomposition and homotopy analysis methods. Int. J. of Mathematical Modeling & Computations, **1(1)** (2011) 45–58.

[8] Boufadel, M.C. PeridierV., Exact analytical expressions for the piezometric profile and water exchange between the stream and groundwater during and after a uniform rise of the stream level. Water Resour Res. Doi: 10.1029/2001WR000780, 2002.

[9] Brutsaert, W., The unit response of groundwater outflow from a hillslope. Water Resour Res,**30**(**10**) (1994) 2759–2763.

[10]Chapman T.G., Modelling groundwater flow over sloping beds. Water Resour Res,**16**(**6**) (1980) 1114–1118.

[11]Gill,M.A., Bank storage characteristic of a finite aquifer due to sudden rise and fall in the river level. J Hydro, **l76**(1985) 133–142.

[12]Hantush,M.S., MS Wells near streams with semipervious bed. J Geophys Res,**3**(**1**) (1965), 227–234.

[13]Higgins,D.T., Unsteady drawdown in a two-dimensional water table aquifer. J Irrig Drain Div, Proc Am SocCivEng 106(IR3) (1980)237–251.

[14] Latinopoulos,P., Periodic recharge of finite aquifers from rectangular areas. Adv Water Resour, **7** (1984) 137–140.

[15]Latinopoulos,P., A boundary element approach for modeling groundwater movement.Adv Water Resour,**9** (1986) 171–177.

[16]Marino M.A., Rise and decline of the water table induced by vertical recharge. J. Hydrol,**23** (1974) 289–298.

[17]Moench, A.F., Barlow P.M., Aquifer response to stream-stage and recharge variations. I. Analytical step-response functions. J Hydrol,**230** (2000) 192–210.

[18]PY Polubarinova-Kochina “Theory of groundwater movement,” Princeton University Press, N.J. 1962.

[19] Rai, S.N. Singh, R.N., On the prediction of groundwater mound due to transient recharge from rectangular area. Water ResourManag,**10** (1986) 189–198.

[20]Upadhyaya, A., Chauhan H.S., Water table rise in sloping aquifer due to canal seepage and constant recharge. J Irrg Drain Eng,**128** (2002) 160–167.

[21]Verhoest, N.E.C.,Troach,P.A., Some analytical solution of the linearized Boussinesq equation with recharge for a sloping aquifer. Water Resour Res,**36**(**3**) (2000)793–800.