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Finite Population Single Server Batch Service Queue with Compulsory Server Vacation

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Abstract. A single server finite population queueing model with compulsory server vacation and with fixed batch service has been considered. For this model the system steady state probabilities are obtained. Some performance measures are calculated and numerical results are also given.

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1. Introduction

In most of the transportation problems the common feature one can see is that the arrival occurs singly but the service is in bulk. With this in mind, in the past many researchers worked on bulk service queues. For a detailed study one can see the book by Chaudhry and Templeton [1]. Most of the papers on queues with server vacation deals with infinite population models. But in real life the number of customers involved in such a queueing system is finite. For example the models related to computer and communication system the number of customers involved is finite. The finite population model with multiple server vacation and exhaustive service was studied by Takagi [2]. Takine and Hasegawa [3] have studied a multiple vacation model with gated service discipline and with infinite population. In this model instead of exhaustive service the gated service discipline with

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finite population has been considered by Langaris and Katsaros [4]. For different service and vacation policies one can see the book by Takagi [5].

In this article a single server finite population queueing model with compulsory server vacation and with batch service has been analyzed in section 2 using supplementary variable technique. For this model the probabilities corresponding to the number of customers in the queue when the server is busy, the server is on vacation and the server is idle are derived. Some performance measures are also calculated in section 3 and numerical results are given in section 4.

2. The Model and Analysis

The customers arrival process follows Poisson from the source of size N with parameter λ.The customers are served in batches of size M and the service time of successive batches follow a general distribution with density function $b(t)$ and with distribution function $B(t)$. Server vacation starts as soon as the service of a batch is completed. The duration of vacation period is assumed to be random with mean $\bar{v}$whose density function $v(t)$ and distribution function $V(t)$.

The following probability notations are introduced and the model is mathematically defined using these notations.

$P_n(x,t) =$ Pr {at time t the server is working and there are n customers in the queue, excluding a batch in service x is the elapsed service time of a batch in service}

$Q_n(x,t) =$ Pr {at time t there are n customers in the queue, and the server is on vacation x is the elapsed vacation time}

$Q(t) =$ Pr {at time t, the server is idle but available}

That is,

$P_n(x,t)dx =$ Pr $\{N(t) = n, x < S(t) \le x + dx)\}$,

$Q_n(x,t)dx =$ Pr $\{N(t) = n, x < V_0(t) \le x + dx)\}$,

$Q(t) =$ Pr $\{N(t) = n\}$= Pr { server is idle}, $n < M$, where $N(t)$ is number of customers in the queue (excluding the batch in service) at time t, $S(t)$is elapsed service time at time t and $V_0(t)$ is elapsed vacation time at time t.

The differential- difference equations governing the system are

$$\frac{d}{dx}P_n(x) + [\lambda(N-n-M) + \bar{s}(x)]P_n(x) = \lambda[N-(n-1)-M]P_{n-1}(x), 0 < n \le N-M \quad (1)$$

$$\frac{d}{dx}P_0(x) + [\lambda(N-M) + \bar{s}(x)]P_0(x) = 0 \quad (2)$$

$$\frac{d}{dx}Q_n(x) + [\lambda(N-n) + \bar{u}(x)]Q_n(x) = \lambda[N-(n-1)]Q_{n-1}(x), 0 < n \le N \quad (3)$$

$$\frac{d}{dx}Q_0(x) + [N\lambda + \bar{u}(x)]Q_0(x) = 0 \quad (4)$$

$$Q = \bar{v}\sum_{r=0}^{M-1} Q_r(0) \quad (5)$$

where $\bar{s}(x) = \frac{b(x)}{1-B(x)}$,$\bar{u}(x) = \frac{v(x)}{1-V(x)}$, $P_n(x) = \lim_{t\to\infty} P_n(x,t)$ and $Q_n(x) = \lim_{t\to\infty} Q_n(x,t)$.

The boundary conditions are

$$P_n(0) = \bar{v}Q_{n+M}(0), \quad 0 \le n \le N-M \quad (6)$$

$$Q_n(0) = \int_0^\infty P_n(x)\,\bar{s}(x)dx, \quad 0 \le n \le N-M \quad (7)$$

$$Q_n(0) = 0, \quad N-M+1 \le n \le N \quad (8)$$

We define, the probability generating function

$$P(z,x)=\frac{\sum_{n=0}^{N-M}P_n(x)z^{N-n-M}}{1-B(x)} \quad (9)$$

and

$$Q(z,x)=\frac{\sum_{n=0}^{N}Q_n(x)z^{N-n}}{1-V(x)} \quad (10)$$

Using equation (9) in (1) and (2), we get

$$\frac{\partial}{\partial x}P(z,x)+\lambda(z-1)\frac{\partial}{\partial z}P(z,x)=0 \quad (11)$$

Using equation (10) in (3) and (4), we get

$$\frac{\partial}{\partial x}Q(z,x)+\lambda(z-1)\frac{\partial}{\partial z}Q(z,x)=0 \quad (12)$$

Solving equation (11) in the usual way we obtain $P(z,x)=F\big((z-1)e^{-\lambda x}\big)$, where F is an unknown function. Putting $x=0$ we arrive at $F(y)=P(y+1,0)$ and so finally we get

$$P(z,x)=\sum_{n=0}^{N-M}d_n\left[1+(z-1)e^{-\lambda x}\right]^{N-n-M} \quad (13)$$

where $d_n=P_n(0)$. Solving equation (12) and (8), we get

$$Q(z,x)=\sum_{n=0}^{N-M}c_n\,[1+(z-1)e^{-\lambda x}]^{N-n} \quad (14)$$

where $c_n=Q_n(0)$.

The boundary conditions become

$$P(z,0)=\bar{v}\sum_{r=M}^{N-M}c_r\,z^{n-r} \quad (15)$$

$$Q(z,0)=z^M\int_0^\infty P(z,x)dB(x) \quad (16)$$

Putting $x=0$ in equation (13) and compare it with (15), we get

$$d_n=\begin{cases}\bar{v}c_{n+M}, & 0\le n\le N-2M\\ 0, & N-2M\le n\le N-M\end{cases} \quad (17)$$

Assuming $D=(d_0,d_1,\cdots,d_{N-2M},0,0,\cdots,0)$, $C=(c_M,c_{M+1},\cdots,c_{N-M},0,0,\cdots,0)$

Equation (17) can be written as

$$D=\bar{v}C \quad (18)$$

Using equations (13) and (14) in (16) and comparing the coefficients of $(z-1)$, we get

$$EP=D\left[\prod_{i=0}^{N-M}B^*(i\lambda)\right]P \quad (19)$$

where is $B^*(s)$the LST of $B(t)$.

Assuming $l=\prod_{i=0}^{N-M}B^*(i\lambda)$, $E=(c_0,c_1,c_2,\ldots c_{N-M})$

$$P = \begin{pmatrix} a_{1,1} & a_{1,2} & a_{1,3} & \cdots & a_{1,N-M-1} & a_{1,N-M} & a_{1,N-M+1} \\ a_{2,1} & a_{2,2} & a_{2,3} & \cdots & a_{2,N-M-1} & a_{2,N-M} & a_{2,N-M+1} \\ a_{3,1} & a_{3,2} & a_{3,3} & \cdots & a_{3,N-M-1} & a_{3,N-M} & a_{3,N-M+1} \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots \\ a_{N-M-2,1} & a_{N-M-2,2} & a_{N-M-2,3} & \cdots & a_{N-M-2,N-M-1} & a_{N-M-2,N-M} & a_{N-M-2,N-M+1} \\ a_{N-M-1,1} & a_{N-M-1,2} & a_{N-M-1,3} & \cdots & a_{N-M-1,N-M-1} & a_{N-M-1,N-M} & a_{N-M-1,N-M+1} \\ a_{N-M,1} & a_{N-M,2} & a_{N-M,3} & \cdots & a_{N-M,N-M-1} & a_{N-M,N-M} & a_{N-M,N-M+1} \\ a_{N-M+1,1} & a_{N-M+1,2} & a_{N-M+1,3} & \cdots & a_{N-M+1,N-M-1} & a_{N-M+1,N-M} & a_{N-M+1,N-M+1} \end{pmatrix}$$

where $a_{1,1} = \binom{N-M}{N-M}$, $a_{1,2} = \binom{N-M}{N-M-1}$, $a_{1,3} = \binom{N-M}{N-M-2}$, $a_{1,N-M-1} = \binom{N-M}{2}$, $a_{1,N-M} = \binom{N-M}{1}$, $a_{1,N-M+1} = \binom{N-M}{0}$, $a_{2,1} = \binom{N-M-1}{N-M-1}$, $a_{2,2} = \binom{N-M-1}{N-M-2}$, $a_{2,3} = \binom{N-M-1}{N-M-3}$, $a_{2,N-M-1} = \binom{N-M-1}{1}$, $a_{2,N-M} = \binom{N-M-1}{0}$, $a_{3,1} = \binom{N-M-2}{N-M-2}$,
$a_{3,2} = \binom{N-M-2}{N-M-3}$, $a_{3,3} = \binom{N-M-2}{N-M-4}$, $a_{3,N-M-1} = \binom{N-M-2}{0}$, $a_{N-M-2,1} = \binom{3}{3}$, $a_{N-M-2,2} = \binom{3}{2}$, $a_{N-M-2,3} = \binom{3}{1}$, $a_{N-M-1,1} = \binom{2}{2}$, $a_{N-M-1,2} = \binom{2}{1}$, $a_{N-M-1,3} = \binom{2}{0}$, $a_{N-M,1} = \binom{1}{1}$, $a_{N-M,2} = \binom{1}{0}$, $a_{N-M+1,1} = \binom{0}{0}$, $a_{2,N-M+1} = a_{3,N-M} = a_{3,N-M+1} = 0$,
$a_{N-M-2,N-M-1} = a_{N-M-2,N-M} = a_{N-M-2,N-M+1} = a_{N-M-1,N-M-1} = a_{N-M-1,N-M} = 0$,
$a_{N-M-1,N-M+1} = a_{N-M,3} = a_{N-M,N-M-1} = a_{N-M,N-M} = a_{N-M,N-M+1} = a_{N-M+1,2} = 0$,
$a_{N-M+1,3} = a_{N-M+1,N-M-1} = a_{N-M+1,N-M} = a_{N-M+1,N-M+1} = 0$.

Equation (19) can be written as

$$EP = DlP \tag{20}$$

Using equation (18) in (20), we get

$$E = \bar{v}lCI \tag{21}$$

where I = Identity matrix.

Assuming $N = kM + J; 0 \leq J \leq M-1; k \geq 0,$ solving equation (21), we get

$$c_i = \begin{cases} \dfrac{Q}{l^j \bar{v}^{j+1} M}; & \text{if } jM \leq i \leq (j+1)M-1;\ 0 \leq j \leq k-2; 0 \leq i \leq N-1 \\ 0; & \text{if } N-M+1 \leq i \leq N \end{cases} \tag{22}$$

Substituting c_i values in equation (18), we get

$$d_i = \begin{cases} \dfrac{Q}{(l\bar{v})^{j+1} M}; & \text{if } jM \leq i \leq (j+1)M-1;\ 0 \leq j \leq k-2; 0 \leq i \leq N-2M \\ 0; & \text{if } N-2M+1 \leq i \leq N \end{cases} \tag{23}$$

The normalization condition is

$$\sum_{n=0}^{N} Q_n(0) + \sum_{n=0}^{N-M} P_n(0) + Q = 1 \tag{24}$$

Substituting equations (22) and (23) in (24), we get

$$Q = \frac{\bar{v}}{(1+\bar{v})\left\{\sum_{j=0}^{k-2} \frac{1}{(l\bar{v})^j} + \frac{J+1}{M(l\bar{v})^{k-1}}\right\}} \tag{25}$$

which gives the probability that the server is idle and no one in the queue.

Comparing coefficients of 'z' in equations (9) and (13), we get

$$P_n(x) = [1 - B(x)] \sum_{i=0}^{n} d_i \binom{N-M-i}{N-M-n} e^{-\lambda(N-M-i)x} \left(e^{\lambda x} - 1\right)^{n-i};\ n = 0,1,2,\dots,N-M$$

$$\int_0^\infty P_n(x)\bar{s}(x)dx = \sum_{i=0}^{n} d_i \binom{N-M-i}{N-M-n} \sum_{j=0}^{n-i} (-1)^j \binom{n-i}{j} B^*[\lambda(N-M-n+j)]$$

$$P_n = \int_0^\infty P_n(x)dx$$

$$= \sum_{i=0}^{n} d_i \binom{N-M-i}{N-M-n} \sum_{r=0}^{n-i} (-1)^j \binom{n-i}{r} \frac{(-1)^r}{\lambda(N-M-n+r)} [1 - B^*[\lambda(N-M-n+r)] \quad (26)$$

$$n = 0,1,2,\dots,N-M$$

The steady state probabilities P_n can be obtained from equation (26) using (23) and (25).
Similarly comparing coefficients of 'z' in equations (10) and (14), we get

$$Q_n(x) = [1 - V(x)] \sum_{i=0}^{n} c_i \binom{N-i}{N-n} e^{-\lambda(N-i)x} \left(e^{\lambda x} - 1\right)^{n-i}; n = 0,1,2,\dots,N$$

$$\int_0^\infty Q_n(x)\bar{u}(x)dx = \sum_{i=0}^{n} c_i \binom{N-i}{N-n} \sum_{j=0}^{n-i} (-1)^j \binom{n-i}{j} V^*[\lambda(N-n+j)]$$

$$Q_n = \int_0^\infty Q_n(x)dx$$

$$= \sum_{i=0}^{n} c_i \binom{N-i}{N-n} \sum_{r=0}^{n-i} \binom{n-i}{r} \frac{(-1)^r}{\lambda(N-n+r)} [1 - V^*[\lambda(N-n+r)], n = 0,1,2,\dots,N \quad (27)$$

where$V^*(s)$ is the LST of $V(t)$.
Finally the steady state probabilities Q_n can be obtained from equation (27) using (22) and (25).

3. The Performance Measures

Using straightforward calculation the following performance measures have been obtained.
(i) The mean number of customers in the queue when the server is busy with a batch is

$$L_s = \sum_{n=0}^{N-M} P_n$$

(ii) The mean number of customers in the queue when the server is on vacation is

$$L_v = \sum_{n=0}^{N} nQ_n$$

(iii) The idle probability is

$$Q = \frac{\bar{v}}{(1+\bar{v})\left\{\sum_{j=0}^{k-2} \frac{1}{(l\bar{v})^j} + \frac{J+1}{M(l\bar{v})^{k-1}}\right\}}$$

4. The Numerical Results

Some numerical results are generated using the formulas in section 3 by fixing the source size as ($N =$) 30 and for the two batches M =3 and M =4. In addition the parameter arrival rate λ=0.1. The mean number of customers in the queue when the server is busy and when the server is on vacation are obtained by varying the vacation rate γ and by fixing the service rate μ as 10, 12, 14, 16 and 18, by calculating the probabilities Q, P_n's and Q_n's. These probabilities are presented in tables 1 to 7. The mean number of customers is drawn as graphs with respect to the vacation rate and is shown in figures 8 to 11. From the graphs it is clear that the mean functions are decreasing functions in vacation rate and are also convex in nature.

Table 1. The idle probability Q

N =30, M =3, λ=0.1			
γ	μ =10	μ =14	μ =18
1	3.60×10^{-14}	1.42×10^{-10}	1.55×10^{-08}
5	6.60×10^{-21}	2.92×10^{-17}	3.58×10^{-15}
10	7.10×10^{-24}	3.18×10^{-20}	3.97×10^{-18}
N =30, M =4, λ=0.1			
γ	μ =10	μ =14	μ =18
1	2.34 ×10^{-09}	4.09 ×10^{-07}	7.58× 10^{-06}
5	5.22 ×10^{-14}	9.69× 10^{-12}	1.93× 10^{-10}
10	4.47 ×10^{-16}	8.36×10^{-14}	1.68× 10^{-12}

Table 2. The steady state probabilities

N =30, M =3, λ=0.1, γ =1						
	μ =10		μ =14		μ =18	
n	P_n	Q_n	P_n	Q_n	P_n	Q_n
0	3.05×10^{-14}	3.0×10^{-15}	3.60×10^{-11}	1.19×10^{-11}	1.84×10^{-09}	1.29×10^{-09}
1	3.73×10^{-14}	5.38×10^{-15}	4.21×10^{-11}	2.13×10^{-11}	2.09×10^{-09}	2.31×10^{-09}
2	3.87×10^{-14}	7.26×10^{-15}	4.31×10^{-11}	2.88×10^{-11}	2.13×10^{-09}	3.12×10^{-09}
3	1.02×10^{-12}	1.10×10^{-13}	4.71×10^{-10}	1.84×10^{-10}	1.41×10^{-08}	1.27×10^{-08}
4	1.22×10^{-12}	1.90×10^{-13}	5.37×10^{-10}	3.05×10^{-10}	1.56×10^{-08}	2.01×10^{-08}
5	1.26×10^{-12}	2.52×10^{-13}	5.41×10^{-10}	3.98×10^{-10}	1.56×10^{-08}	2.58×10^{-08}
6	3.36×10^{-11}	3.86×10^{-12}	6.09×10^{-09}	2.53×10^{-09}	1.07×10^{-07}	1.02×10^{-07}
7	3.94×10^{-11}	6.60×10^{-12}	6.75×10^{-09}	4.19×10^{-09}	1.11×10^{-07}	1.64×10^{-07}
8	4.04×10^{-11}	8.58×10^{-12}	7.54×10^{-09}	5.12×10^{-09}	1.38×10^{-07}	1.77×10^{-07}
9	1.13×10^{-09}	1.37×10^{-10}	7.80×10^{-08}	3.81×10^{-08}	7.20×10^{-07}	1.17×10^{-06}
10	1.28×10^{-09}	2.29×10^{-10}	8.19×10^{-08}	5.29×10^{-08}	7.03×10^{-07}	8.70×10^{-07}
11	1.32×10^{-09}	2.98×10^{-10}	1.03×10^{-07}	7.51×10^{-08}	1.82×10^{-06}	1.89×10^{-06}
12	3.67×10^{-08}	4.84×10^{-09}	9.98×10^{-07}	4.60×10^{-07}	4.29×10^{-06}	4.05×10^{-06}
13	4.15×10^{-08}	8.01×10^{-09}	1.23×10^{-06}	6.55×10^{-07}	8.49×10^{-06}	1.94×10^{-06}
14	4.29×10^{-08}	1.04×10^{-08}	1.20×10^{-06}	1.55×10^{-06}	1.13×10^{-05}	7.42×10^{-05}
15	1.22×10^{-06}	1.74×10^{-07}	1.26×10^{-05}	5.62×10^{-06}	3.57×10^{-05}	8.07×10^{-05}
16	1.36×10^{-06}	2.85×10^{-07}	1.46×10^{-05}	1.42×10^{-05}	6.55×10^{-05}	4.27×10^{-04}
17	1.37×10^{-06}	3.56×10^{-07}	1.26×10^{-05}	8.89×10^{-06}	4.51×10^{-06}	3.48×10^{-04}
18	4.03×10^{-05}	6.38×10^{-06}	1.69×10^{-04}	9.93×10^{-05}	3.66×10^{-04}	7.14×10^{-04}
19	4.31×10^{-05}	1.01×10^{-05}	1.77×10^{-04}	1.44×10^{-04}	3.29×10^{-04}	2.06×10^{-04}
20	4.42×10^{-05}	1.23×10^{-05}	1.78×10^{-04}	1.77×10^{-04}	3.07×10^{-04}	4.86×10^{-04}
21	1.34×10^{-03}	2.37×10^{-04}	2.16×10^{-03}	1.40×10^{-03}	2.49×10^{-03}	3.98×10^{-03}
22	1.42×10^{-03}	3.62×10^{-04}	2.26×10^{-03}	2.08×10^{-03}	2.56×10^{-03}	5.48×10^{-03}
23	1.43×10^{-03}	4.28×10^{-04}	2.26×10^{-03}	2.42×10^{-03}	2.58×10^{-03}	5.26×10^{-03}
24	4.44×10^{-02}	9.02×10^{-03}	2.79×10^{-02}	2.07×10^{-02}	1.87×10^{-02}	3.16×10^{-02}

25	1.31×10^{-03}	1.03×10^{-02}	5.89×10^{-04}	2.92×10^{-02}	3.08×10^{-04}	4.33×10^{-02}
26	2.58×10^{-05}	1.48×10^{-02}	8.34×10^{-06}	3.29×10^{-02}	3.28×10^{-06}	4.84×10^{-02}
27		3.56×10^{-01}		3.16×10^{-01}		2.77×10^{-01}
28		8.89×10^{-02}		7.90×10^{-02}		6.92×10^{-02}
29		1.62×10^{-02}		1.44×10^{-02}		1.26×10^{-02}

Table 3. The steady state probabilities

$N=30,\ M=3,\ \lambda=0.1,\ \gamma=5$						
	$\mu=10$		$\mu=14$		$\mu=18$	
n	P_n	Q_n	P_n	Q_n	P_n	Q_n
0	2.80×10^{-20}	1.38×10^{-21}	3.69×10^{-17}	6.07×10^{-18}	2.13×10^{-15}	7.46×10^{-16}
1	3.42×10^{-20}	1.92×10^{-21}	4.31×10^{-17}	8.46×10^{-18}	2.42×10^{-15}	1.04×10^{-15}
2	3.55×10^{-20}	2.12×10^{-21}	4.41×10^{-17}	9.37×10^{-18}	2.46×10^{-15}	1.15×10^{-15}
3	4.63×10^{-18}	2.31×10^{-19}	2.38×10^{-15}	4.03×10^{-16}	8.03×10^{-14}	2.91×10^{-14}
4	5.57×10^{-18}	3.16×10^{-19}	2.74×10^{-15}	5.48×10^{-16}	8.99×10^{-14}	3.94×10^{-14}
5	5.75×10^{-18}	3.46×10^{-19}	2.79×10^{-15}	6.00×10^{-16}	9.08×10^{-14}	4.30×10^{-14}
6	7.66×10^{-16}	3.89×10^{-17}	1.54×10^{-13}	2.65×10^{-14}	3.01×10^{-12}	1.12×10^{-12}
7	9.05×10^{-16}	5.21×10^{-17}	1.74×10^{-13}	3.55×10^{-14}	3.33×10^{-12}	1.49×10^{-12}
8	9.29×10^{-16}	5.64×10^{-17}	1.78×10^{-13}	3.82×10^{-14}	3.39×10^{-12}	1.59×10^{-12}
9	1.27×10^{-13}	6.53×10^{-15}	9.91×10^{-12}	1.75×10^{-12}	1.13×10^{-10}	4.31×10^{-11}
10	1.47×10^{-13}	8.57×10^{-15}	1.11×10^{-11}	2.28×10^{-12}	1.23×10^{-10}	5.51×10^{-11}
11	1.50×10^{-13}	9.18×10^{-15}	1.12×10^{-11}	2.44×10^{-12}	1.26×10^{-10}	5.89×10^{-11}
12	2.10×10^{-11}	1.10×10^{-12}	6.39×10^{-10}	1.16×10^{-10}	4.23×10^{-09}	1.66×10^{-09}
13	2.39×10^{-11}	1.41×10^{-12}	7.05×10^{-10}	1.47×10^{-10}	4.57×10^{-09}	2.09×10^{-09}
14	2.43×10^{-11}	1.50×10^{-12}	7.11×10^{-10}	1.57×10^{-10}	4.61×10^{-09}	2.24×10^{-09}
15	3.48×10^{-09}	1.86×10^{-10}	4.13×10^{-08}	7.65×10^{-09}	1.59×10^{-07}	6.37×10^{-08}
16	3.88×10^{-09}	2.32×10^{-10}	4.48×10^{-08}	9.53×10^{-09}	1.70×10^{-07}	7.98×10^{-08}

17	3.93×10^{-09}	2.43×10^{-10}	4.50×10^{-08}	9.95×10^{-09}	1.70×10^{-07}	8.21×10^{-08}
18	5.77×10^{-07}	3.14×10^{-08}	2.67×10^{-06}	5.07×10^{-07}	5.97×10^{-06}	2.48×10^{-06}
19	6.30×10^{-07}	3.81×10^{-08}	2.84×10^{-06}	6.13×10^{-07}	6.27×10^{-06}	2.98×10^{-06}
20	6.34×10^{-07}	3.94×10^{-08}	2.85×10^{-06}	6.34×10^{-07}	6.29×10^{-06}	3.09×10^{-06}
21	9.57×10^{-05}	5.33×10^{-06}	1.72×10^{-04}	3.37×10^{-05}	2.24×10^{-04}	9.59×10^{-05}
22	1.02×10^{-04}	6.24×10^{-06}	1.80×10^{-04}	3.94×10^{-05}	2.32×10^{-04}	1.12×10^{-04}
23	1.02×10^{-04}	6.39×10^{-06}	1.81×10^{-04}	4.03×10^{-05}	2.33×10^{-01}	1.15×10^{-04}
24	1.59×10^{-02}	9.06×10^{-04}	1.11×10^{-02}	2.45×10^{-03}	8.41×10^{-03}	3.73×10^{-03}
25	4.67×10^{-04}	1.02×10^{-03}	2.35×10^{-04}	2.53×10^{-03}	1.39×10^{-04}	4.19×10^{-03}
26	9.25×10^{-05}	1.03×10^{-03}	3.31×10^{-06}	2.56×10^{-03}	1.53×10^{-06}	4.24×10^{-03}
27		1.54×10^{-01}		1.50×10^{-01}		1.45×10^{-01}
28		8.91×10^{-03}		8.67×10^{-03}		8.39×10^{-03}
29		3.49×10^{-04}		3.40×10^{-04}		3.29×10^{-04}

Table 4. The steady state probabilities

$N=30,\ M=3,\ \lambda=0.1,\ \gamma=10$						
	$\mu=10$		$\mu=14$		$\mu=18$	
n	P_n	Q_n	P_n	Q_n	P_n	Q_n
0	6.02×10^{-23}	1.82×10^{-24}	8.04×10^{-20}	8.15×10^{-21}	4.73×10^{-18}	1.02×10^{-18}
1	7.35×10^{-23}	2.26×10^{-24}	9.39×10^{-20}	1.01×10^{-20}	5.37×10^{-18}	1.26×10^{-18}
2	7.64×10^{-23}	2.36×10^{-24}	9.61×10^{-20}	1.06×10^{-20}	5.45×10^{-18}	1.32×10^{-18}
3	1.99×10^{-20}	6.02×10^{-22}	1.04×10^{-17}	1.06×10^{-18}	3.55×10^{-16}	7.73×10^{-17}
4	2.39×10^{-20}	7.35×10^{-22}	1.20×10^{-17}	1.29×10^{-18}	3.98×10^{-16}	9.42×10^{-17}
5	2.47×10^{-20}	7.64×10^{-22}	1.22×10^{-17}	1.34×10^{-18}	4.03×10^{-16}	9.79×10^{-17}
6	6.58×10^{-18}	1.99×10^{-19}	1.34×10^{-15}	1.37×10^{-16}	2.67×10^{-14}	5.85×10^{-15}
7	7.79×10^{-18}	2.39×10^{-19}	1.52×10^{-15}	1.65×10^{-16}	2.95×10^{-14}	7.03×10^{-15}
8	8.00	2.47	1.55×10^{-15}	1.70	2.99	7.25

	$\times10^{-18}$	$\times10^{-19}$		$\times10^{-16}$	$\times10^{-14}$	$\times10^{-15}$
9	2.18×10^{-15}	6.58×10^{-17}	1.73×10^{-13}	1.78×10^{-14}	2.00×10^{-12}	4.43×10^{-13}
10	2.53×10^{-15}	7.79×10^{-17}	1.93×10^{-13}	2.11×10^{-14}	2.19×10^{-12}	5.23×10^{-13}
11	2.59×10^{-15}	8.00×10^{-17}	1.96×10^{-13}	2.16×10^{-14}	2.21×10^{-12}	5.37×10^{-13}
12	7.22×10^{-13}	2.18×10^{-14}	2.23×10^{-11}	2.31×10^{-12}	1.50×10^{-10}	3.36×10^{-11}
13	8.23×10^{-13}	2.53×10^{-14}	2.46×10^{-11}	2.69×10^{-12}	1.62×10^{-10}	3.89×10^{-11}
14	8.36×10^{-13}	2.59×10^{-14}	2.48×10^{-11}	2.74×10^{-12}	1.63×10^{-10}	3.99×10^{-11}
15	2.39×10^{-10}	7.22×10^{-12}	2.88×10^{-09}	3.01×10^{-10}	1.13×10^{-08}	2.55×10^{-09}
16	2.67×10^{-10}	8.23×10^{-12}	3.12×10^{-09}	3.42×10^{-10}	1.20×10^{-08}	2.90×10^{-09}
17	2.70×10^{-10}	8.36×10^{-12}	3.14×10^{-09}	3.48×10^{-10}	1.21×10^{-08}	2.95×10^{-09}
18	7.93×10^{-08}	2.39×10^{-09}	3.72×10^{-07}	3.91×10^{-08}	8.46×10^{-07}	1.93×10^{-07}
19	8.66×10^{-08}	2.67×10^{-09}	3.97×10^{-07}	4.36×10^{-08}	8.90×10^{-07}	2.16×10^{-07}
20	8.73×10^{-08}	2.70×10^{-09}	3.98×10^{-07}	4.41×10^{-08}	8.92×10^{-07}	2.18×10^{-07}
21	2.63×10^{-08}	7.93×10^{-07}	4.80×10^{-05}	5.08×10^{-06}	6.35×10^{-05}	1.47×10^{-05}
22	2.81×10^{-05}	8.66×10^{-07}	5.03×10^{-05}	5.55×10^{-06}	6.59×10^{-05}	1.60×10^{-05}
23	2.82×10^{-05}	8.73×10^{-07}	5.04×10^{-05}	5.59×10^{-06}	6.60×10^{-05}	1.61×10^{-05}
24	8.75×10^{-03}	2.63×10^{-04}	6.21×10^{-03}	6.62×10^{-04}	4.77×10^{-03}	1.12×10^{-03}
25	2.57×10^{-04}	2.81×10^{-04}	1.13×10^{-04}	7.06×10^{-04}	7.87×10^{-05}	1.19×10^{-03}
26	5.09×10^{-06}	2.82×10^{-04}	1.86×10^{-06}	7.08×10^{-04}	8.70×10^{-07}	1.19×10^{-03}
27		8.75×10^{-02}		8.62×10^{-02}		8.48×10^{-02}
28		2.57×10^{-03}		2.54×10^{-03}		2.50×10^{-03}
29		5.09×10^{-05}		5.02×10^{-05}		4.94×10^{-05}

Table 5. The steady state probabilities

N =30, M =4, λ=0.1, γ =1						
	μ =10		μ =14		μ =18	
n	P_n	Q_n	P_n	Q_n	P_n	Q_n
0	1.18×10^{-09}	1.47×10^{-10}	6.54×10^{-08}	2.56×10^{-08}	5.92×10^{-07}	4.74×10^{-07}
1	1.44×10^{-09}	2.63×10^{-10}	7.61×10^{-08}	4.59×10^{-08}	6.69×10^{-07}	8.51×10^{-07}
2	1.49×10^{-09}	3.55×10^{-10}	7.78×10^{-08}	6.19×10^{-08}	6.79×10^{-07}	1.15×10^{-06}
3	1.50×10^{-09}	4.27×10^{-10}	7.81×10^{-08}	7.45×10^{-08}	6.81×10^{-07}	1.38×10^{-06}
4	3.13×10^{-08}	4.46×10^{-09}	7.22×10^{-07}	3.58×10^{-07}	3.95×10^{-06}	4.43×10^{-06}
5	3.70×10^{-08}	7.56×10^{-09}	8.18×10^{-07}	5.74×10^{-07}	4.39×10^{-06}	6.74×10^{-06}
6	3.87×10^{-08}	9.96×10^{-09}	8.09×10^{-07}	7.46×10^{-07}	4.18×10^{-06}	8.62×10^{-06}
7	3.54×10^{-08}	1.22×10^{-08}	1.02×10^{-06}	9.50×10^{-07}	5.08×10^{-06}	1.14×10^{-05}
8	8.39×10^{-07}	1.24×10^{-07}	8.14×10^{-06}	3.72×10^{-06}	2.33×10^{-05}	2.23×10^{-05}
9	8.71×10^{-07}	2.52×10^{-07}	5.86×10^{-06}	1.36×10^{-05}	1.84×10^{-05}	1.70×10^{-04}
10	1.10×10^{-06}	2.31×10^{-07}	9.20×10^{-06}	8.73×10^{-07}	9.47×10^{-05}	8.28×10^{-05}
11	1.34×10^{-06}	3.55×10^{-07}	1.65×10^{-06}	1.57×10^{-05}	1.00×10^{-04}	1.75×10^{-04}
12	2.05×10^{-05}	3.40×10^{-06}	1.08×10^{-04}	5.99×10^{-06}	2.52×10^{-04}	5.97×10^{-04}
13	2.78×10^{-05}	4.75×10^{-06}	1.14×10^{-04}	1.37×10^{-04}	4.88×10^{-04}	3.66×10^{-03}
14	1.70×10^{-05}	1.48×10^{-05}	2.65×10^{-05}	1.33×10^{-03}	1.57×10^{-03}	2.30×10^{-02}
15	3.55×10^{-05}	6.64×10^{-06}	3.87×10^{-04}	2.62×10^{-03}	3.83×10^{-03}	5.04×10^{-02}
16	5.54×10^{-04}	1.44×10^{-04}	5.19×10^{-04}	6.79×10^{-03}	2.69×10^{-03}	1.15×10^{-01}
17	6.38×10^{-04}	1.20×10^{-04}	1.31×10^{-03}	7.55×10^{-03}	4.96×10^{-03}	1.52×10^{-01}
18	6.18×10^{-04}	2.65×10^{-04}	4.41×10^{-04}	7.87×10^{-03}	1.74×10^{-03}	1.16×10^{-01}
19	6.30×10^{-04}	1.86×10^{-04}	1.28×10^{-04}	6.60×10^{-03}	4.98×10^{-03}	1.38×10^{-01}
20	1.50×10^{-02}	3.21×10^{-03}	9.37×10^{-03}	7.77×10^{-03}	5.18×10^{-03}	1.02×10^{-01}
21	1.59×10^{-02}	4.98×10^{-03}	1.05×10^{-02}	1.43×10^{-02}	6.05×10^{-03}	8.04×10^{-02}
22	1.60×10^{-02}	5.93×10^{-03}	1.06×10^{-02}	1.02×10^{-02}	8.40×10^{-03}	3.82×10^{-02}
23	6.19×10^{-04}	6.37×10^{-03}	2.69×10^{-04}	5.33×10^{-03}	1.97×10^{-04}	1.25×10^{-01}
24	1.81×10^{-05}	1.02×10^{-01}	4.01×10^{-06}	9.61×10^{-02}	3.56×10^{-05}	6.46×10^{-02}
25	2.72×10^{-07}	1.46×10^{-01}	9.71×10^{-07}	1.36×10^{-01}	7.19×10^{-06}	1.05×10^{-01}
26		1.65×10^{-01}		1.54×10^{-01}		1.54×10^{-01}
27		5.08×10^{-02}		4.71×10^{-02}		4.42×10^{-02}
28		1.27×10^{-02}		1.19×10^{-02}		1.37×10^{-02}
29		2.31×10^{-03}		2.15×10^{-03}		2.11×10^{-03}

Table 6. The steady state probabilities

N =30, M =4, λ=0.1, γ =5						
	μ =10		μ =14		μ =18	
n	P_n	Q_n	P_n	Q_n	P_n	Q_n
0	1.32	8.15	7.74×10^{-12}	1.51	7.54	3.02

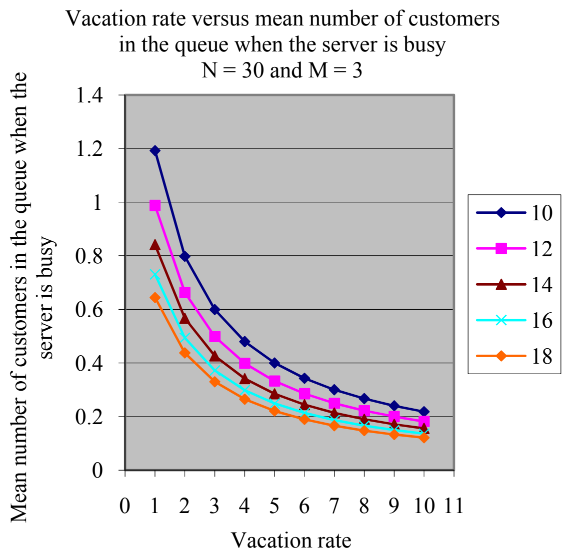
	$\times10^{-13}$	$\times10^{-15}$		$\times10^{-12}$	$\times10^{-11}$	$\times10^{-11}$
1	1.60×10^{-13}	1.14×10^{-14}	9.01×10^{-12}	2.11×10^{-12}	8.53×10^{-11}	4.20×10^{-11}
2	1.66×10^{-13}	1.26×10^{-14}	9.21×10^{-12}	2.34×10^{-12}	8.65×10^{-11}	4.66×10^{-11}
3	1.67×10^{-13}	1.31×10^{-14}	9.25×10^{-12}	2.42×10^{-12}	8.68×10^{-11}	4.83×10^{-11}
4	1.73×10^{-11}	1.10×10^{-12}	4.22×10^{-10}	8.54×10^{-11}	2.48×10^{-09}	1.04×10^{-09}
5	2.06×10^{-11}	1.48×10^{-12}	4.82×10^{-10}	1.15×10^{-10}	2.76×10^{-09}	1.39×10^{-09}
6	2.12×10^{-11}	1.62×10^{-12}	4.88×10^{-10}	1.26×10^{-10}	2.76×10^{-09}	1.53×10^{-09}
7	2.09×10^{-11}	1.72×10^{-12}	5.14×10^{-10}	1.40×10^{-10}	2.88×10^{-09}	1.77×10^{-09}
8	2.28×10^{-09}	1.47×10^{-10}	2.30×10^{-08}	4.74×10^{-09}	8.10×10^{-08}	3.44×10^{-08}
9	2.63×10^{-09}	1.95×10^{-10}	2.54×10^{-08}	6.46×10^{-09}	8.77×10^{-08}	4.95×10^{-08}
10	2.71×10^{-09}	2.00×10^{-10}	2.60×10^{-08}	5.13×10^{-09}	9.79×10^{-08}	1.71×10^{-08}
11	2.76×10^{-09}	2.03×10^{-10}	2.71×10^{-08}	5.35×10^{-09}	7.35×10^{-08}	2.16×10^{-08}
12	2.99×10^{-07}	1.98×10^{-08}	1.25×10^{-06}	2.70×10^{-07}	2.67×10^{-06}	1.21×10^{-06}
13	3.39×10^{-07}	2.52×10^{-08}	1.37×10^{-06}	3.30×10^{-07}	2.79×10^{-06}	1.33×10^{-06}
14	3.42×10^{-07}	2.70×10^{-08}	1.36×10^{-06}	3.66×10^{-07}	2.99×10^{-06}	1.53×10^{-06}
15	3.45×10^{-07}	2.53×10^{-08}	1.43×10^{-06}	7.48×10^{-08}	3.31×10^{-06}	3.93×10^{-06}
16	3.94×10^{-05}	2.67×10^{-06}	6.81×10^{-05}	1.56×10^{-05}	8.70×10^{-05}	4.93×10^{-05}
17	4.33×10^{-05}	3.29×10^{-06}	7.31×10^{-05}	1.77×10^{-05}	9.26×10^{-05}	3.41×10^{-05}
18	4.37×10^{-05}	3.45×10^{-06}	7.33×10^{-05}	2.15×10^{-05}	9.30×10^{-05}	8.75×10^{-05}
19	4.37×10^{-05}	3.44×10^{-06}	7.34×10^{-05}	1.70×10^{-05}	9.26×10^{-05}	1.09×10^{-05}
20	5.19×10^{-03}	3.61×10^{-04}	3.71×10^{-03}	8.57×10^{-04}	2.87×10^{-03}	1.44×10^{-03}
21	5.53×10^{-03}	4.28×10^{-04}	3.89×10^{-03}	1.01×10^{-03}	2.97×10^{-03}	1.59×10^{-03}
22	5.55×10^{-03}	4.40×10^{-04}	3.90×10^{-03}	1.04×10^{-03}	2.98×10^{-03}	1.71×10^{-03}
23	$2.16\times$	$4.41\times$	$1.09\times$	$1.04\times$	6.50×10^{-05}	$1.64\times$

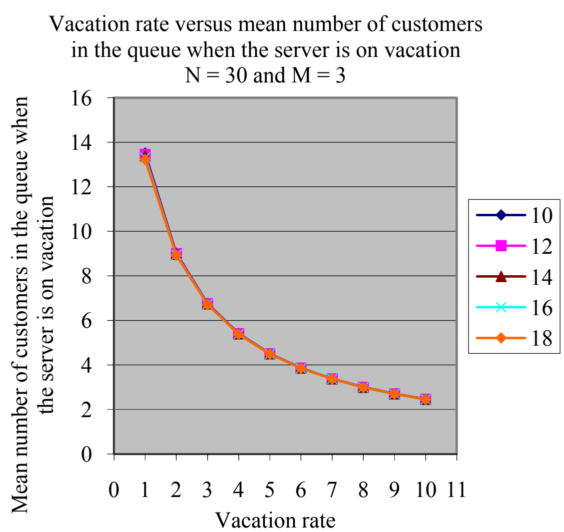
	10^{-04}	10^{-04}	10^{-04}	10^{-03}		10^{-03}
24	6.33×10^{-06}	4.91×10^{-02}	2.28×10^{-06}	4.85×10^{-02}	9.90×10^{-07}	4.78×10^{-02}
25	9.62×10^{-08}	5.53×10^{-02}	2.18×10^{-08}	5.45×10^{-02}	4.37×10^{-08}	5.36×10^{-02}
26		5.60×10^{-02}		5.52×10^{-02}		5.43×10^{-02}
27		4.23×10^{-03}		4.17×10^{-03}		4.10×10^{-03}
28		2.44×10^{-04}		2.40×10^{-04}		2.37×10^{-04}
29		9.62×10^{-06}		9.52×10^{-05}		9.32×10^{-06}

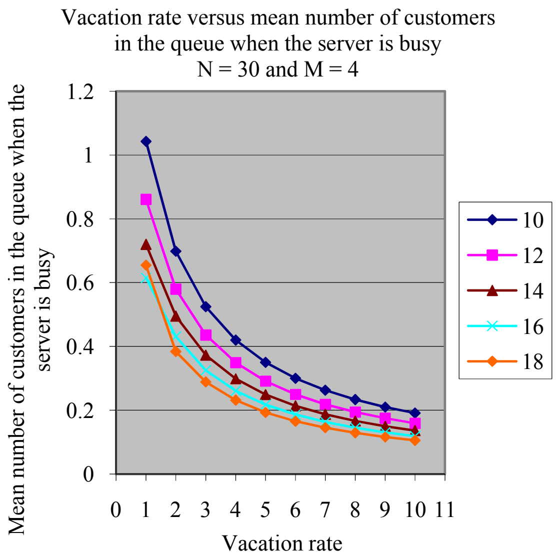
Table 7. The steady state probabilities

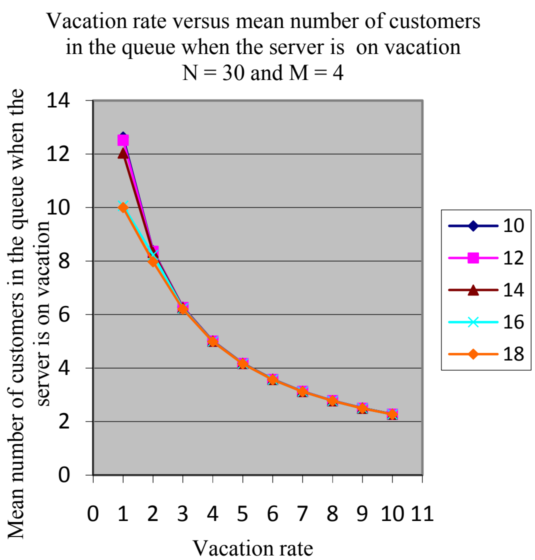
$N=30,\ M=4,\ \lambda=0.1,\ \gamma=10$						
	$\mu=10$		$\mu=14$		$\mu=18$	
n	P_n	Q_n	P_n	Q_n	P_n	Q_n
0	2.25×10^{-15}	8.60×10^{-17}	1.34×10^{-13}	1.61×10^{-14}	1.31×10^{-12}	3.23×10^{-13}
1	2.74×10^{-15}	1.07×10^{-16}	1.56×10^{-13}	1.99×10^{-14}	1.49×10^{-12}	4.01×10^{-13}
2	2.84×10^{-15}	1.12×10^{-16}	1.59×10^{-13}	2.09×10^{-14}	1.51×10^{-12}	4.19×10^{-13}
3	2.87×10^{-15}	1.13×10^{-16}	1.60×10^{-13}	2.10×10^{-14}	1.51×10^{-12}	4.23×10^{-13}
4	5.93×10^{-13}	2.26×10^{-14}	1.46×10^{-11}	1.77×10^{-12}	8.62×10^{-11}	2.15×10^{-11}
5	7.05×10^{-13}	2.74×10^{-14}	1.66×10^{-11}	2.14×10^{-12}	9.60×10^{-11}	2.61×10^{-11}
6	7.26×10^{-13}	2.84×10^{-14}	1.69×10^{-11}	2.22×10^{-12}	9.65×10^{-11}	2.71×10^{-11}
7	7.23×10^{-13}	2.88×10^{-14}	1.73×10^{-11}	2.26×10^{-12}	9.86×10^{-11}	2.76×10^{-11}
8	1.56×10^{-10}	5.93×10^{-12}	1.59×10^{-09}	1.94×10^{-10}	5.65×10^{-09}	1.43×10^{-09}
9	1.81×10^{-10}	7.04×10^{-12}	1.77×10^{-09}	2.30×10^{-10}	6.17×10^{-09}	1.69×10^{-09}
10	1.85×10^{-10}	7.23×10^{-12}	1.80×10^{-09}	2.31×10^{-10}	6.38×10^{-09}	1.63×10^{-09}
11	1.87×10^{-10}	7.26×10^{-12}	1.82×10^{-09}	2.38×10^{-10}	5.97×10^{-09}	1.80×10^{-09}
12	4.10×10^{-08}	1.56×10^{-09}	1.73×10^{-07}	2.13×10^{-08}	3.71×10^{-07}	9.55×10^{-08}
13	4.64×10^{-08}	1.81×10^{-09}	1.89×10^{-07}	2.42×10^{-08}	3.98×10^{-07}	1.02×10^{-07}
14	4.70×10^{-08}	1.86×10^{-09}	1.91×10^{-07}	2.66×10^{-08}	4.02×10^{-07}	1.41×10^{-07}
15	4.72×10^{-08}	1.86×10^{-09}	1.92×10^{-07}	2.52×10^{-08}	4.07×10^{-07}	1.02×10^{-07}
16	1.08×10^{-05}	4.10×10^{-07}	1.88×10^{-05}	2.36×10^{-06}	2.43×10^{-05}	6.38×10^{-06}
17	1.19×10^{-05}	4.64×10^{-07}	2.02×10^{-05}	2.64×10^{-06}	2.57×10^{-05}	7.15×10^{-06}
18	1.20×10^{-05}	4.70×10^{-07}	2.03×10^{-05}	2.66×10^{-06}	2.58×10^{-05}	7.05×10^{-06}
19	1.20×10^{-05}	4.72×10^{-07}	2.03×10^{-05}	2.71×10^{-06}	2.58×10^{-05}	7.55×10^{-06}
20	2.85×10^{-03}	1.08×10^{-04}	2.05×10^{-03}	2.57×10^{-04}	1.60×10^{-03}	4.20×10^{-04}
21	3.03×10^{-03}	1.19×10^{-04}	2.15×10^{-03}	2.82×10^{-04}	1.66×10^{-03}	4.63×10^{-04}
22	3.04×10^{-03}	1.20×10^{-04}	2.15×10^{-03}	2.85×10^{-04}	1.66×10^{-03}	4.66×10^{-04}
23	1.18×10^{-04}	1.20×10^{-04}	6.02×10^{-05}	2.85×10^{-04}	3.63×10^{-05}	4.66×10^{-04}
24	3.47×10^{-06}	2.85×10^{-02}	1.26×10^{-06}	2.83×10^{-02}	5.79×10^{-07}	2.80×10^{-02}
25	5.23×10^{-08}	3.03×10^{-02}	1.24×10^{-08}	3.01×10^{-02}	1.67×10^{-08}	2.99×10^{-02}
26		3.04×10^{-02}		3.02×10^{-02}		2.99×10^{-02}

27	1.18×10^{-03}	1.17×10^{-03}	1.16×10^{-03}
28	3.47×10^{-05}	3.45×10^{-05}	3.41×10^{-05}
29	5.23×10^{-07}	5.26×10^{-07}	5.31×10^{-07}









5. Conclusion

In this paper we analyzed the finite population a single server queue. The service is given in batches of fixed size and the server takes compulsory server vacation at each service completion epoch. The probabilities are obtained for this model by deriving the probability generating functions. For this model some performance measures are derived and some numerical examples are given by taking particular values to the parameter.

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