Numerical Solution of Integro-Differential Equation by using Chebyshev Wavelet Operational Matrix of Integration

M. A. Fariborzi Araghi[a]* , S. Daliri [b] and M. Bahmanpour [c]

[a,b] *Department of Mathematics, Islamic Azad University, Central Tehran Branch, P.O.Box 13185.768 Iran;*
[c] *Department of Mathematics, Sama Technical and Vocational Training College, Islamic Azad University, Khorasgan, Isfahan Branch, Iran.*

Abstract. In this paper, we propose a method to approximate the solution of a linear Fredholm integro-differential equation by using the Chebyshev wavelet of the first kind as basis. For this purpose, we introduce the first Chebyshev operational matrix of integration. Chebyshev wavelet approximating method is then utilized to reduce the integro-differential equation to a system of algebraic equations. Illustrative examples are included to demonstrate the advantages and applicability of the technique.

Index to information contained in this paper

1. Introduction

Wavelets theory is relatively new in mathematical researches. In recent years, wavelets have found their way into different fields of science and engineering. Wavelets permit the accurate representation of a variety of functions and operators. Orthogonal functions and polynomial series have been received considerable attention in dealing with various problems of dynamical systems. The main characteristic of this technique is that it reduces these problems to those of solving a system of algebraic equations, thus greatly simplifying the problems. The common wavelets which have been used to solve integral or integro-differential equations are Legendre, CAS, Haar and Chebyshev wavelets [3, 6].
The other approach is based on converting the underlying differential equations

---

*Corresponding author. Email: m_fariborzi@iauctb.ac.ir

into an integral equations through integration, approximating various signals involved in the equation truncated orthogonal series $\Psi(t)$ and using the operational matrix of integration P, to eliminate the integral operations. The matrix P is given by

$$\int_0^t \Psi(t')\, dt' = P\Psi(t) \qquad , \qquad 0 \leqslant t < 1$$

where $\Psi(t) = [\psi_0, \psi_1, \ldots, \psi_{n-1}]^T$ and the matrix M can be uniquely determined on the basis of the particular orthogonal functions. The elements $\psi_0, \psi_1, \ldots, \psi_{n-1}$ are the basis functions, orthogonal on the certain interval [0,1]

Gu and Jiang in [4] proposed the operational matrix of integration using Haar wavelets. In the sequel, Razzaghi and Yousefi in [5] used the Legendre wavelets to define the operational matrix of integration. Then, Danfu and Xufeng in [2] applied CAS wavelet operational matrix of integration in order to approximate the solution of integro-differential equations. Also, Babolian and Fattahzadeh in [1] suggested a method to solve the differential equations by using Chebyshev wavelet operational matrix of integration.

In this paper, we introduce a new numerical method to solve the following linear Fredholm integro-differential equation using the first kind Chebyshev wavelet operational matrix of integration.

$$\begin{cases} y'(t) = f(t) + y(t) + \int_0^1 k(t,s)y(s)\, d(s) & 0 \leqslant t < 1 \\ y(0) = y_0 \end{cases} \tag{1}$$

where the function $f(t) \in \mathcal{L}^2(0,1)$, the kernel $k(t,s) \in \mathcal{L}^2([0,1] \times [0,1])$ are known and $y(t)$ is the unknown function to be determined. The paper is organized as follows:

In Section 2, we describe the Chebyshev wavelets and its properties. The Chebyshev wavelets operational matrix of integration will be derived in Section 3. In Section 4, we propose a method to solve the linear Fredholm integro-differential equation (1) and approximate the unknown function . Finally, in Section 5, we solve some illustrative examples.

2. Chebyshev Wavelets and their Properties

2.1 *Chebyshev Wavelets*

Chebyshev wavelets $\psi_{m,n} = \psi(k,n,m,t)$ have four arguments,$n = 1, 2, \ldots, 2^{k-1}$, k can assume any positive integer, m is degree of Chebyshev polynomials of the first kind and denots the time.

$$\psi_{n,m}(t) = \begin{cases} 2^{k/2}\tilde{T}_m(2^k t - 2n + 1) & \frac{n-1}{2^{k-1}} \leqslant t \leqslant \frac{n}{2^{k-1}} \\ 0 & otherwise \end{cases} \tag{2}$$

where ,

$$\tilde{T}_m(t) = \begin{cases} \frac{1}{\sqrt{\pi}} & m = 0 \\ \sqrt{\frac{2}{\pi}}T_m(t) & m > 0 \end{cases} \tag{3}$$

and $m = 0, 1 \ldots, M - 1, n = 1, 2 \ldots, 2^{k-1}$. In Equation (2) the coefficients are used for orthonormality. Here $T_m(t)$ are Chebyshev polynomials of the first kind of degree m which are orthogonal with respect to the weight function $\omega(t) = 1/\sqrt{1 - t^2}$, on [-1,1], and satisfy the following recursive formula:

$$T_0(t) = 1, \qquad T_1(t) = t, \qquad T_{m+1}(t) = 2tT_m(t) - T_{m-1}(t), \qquad m = 1, 2, \ldots$$

We should note that in dealing with Chebyshev wavelets the weight function $\omega(x)$ have to be dilated and translated as

$$\omega_n(t) = \omega(2^k t - 2n + 1)$$

to get orthogonal wavelets on the interval [0,1].

2.2 *Function approximation*

A function $f(t) \in \mathcal{L}^2_{\tilde{\omega}}[0, 1]$, (where $\tilde{\omega}(t) = \omega(2t - 1)$) may be expanded as

$$f(t) = \sum_{n=1}^{\infty} \sum_{m=0}^{\infty} c_{n,m} \psi_{n.m}(t), \tag{4}$$

where $c_{n,m} = (f(t), \psi_{n,m}(t))_{\omega_n}$, in which $(0,0)$ denotes the inner product in $\mathcal{L}^2_{\omega_n}[0, 1]$. If we consider truncated series in (3), we obtain

$$f(t) \simeq \sum_{n=1}^{2^{k-1}} \sum_{m=0}^{M-1} c_{n,m} \psi_{n,m}(t) = C^T \Psi(t), \tag{5}$$

where C and $\Psi(t)$ are $2^{k-1} M \times 1$ matrices given by

$$C = [c_{10}, c_{11}, \ldots, c_{1,M-1}, c_{20}, \ldots, c_{2,M-1}, \ldots, c_{2^{k-1},0}, \ldots, c_{2^{k-1},M-1}]^T \tag{6}$$
$$= [c_1, c_2, \ldots, c_{2^{k-1}M}]^T,$$
$$\Psi(t) = [\psi_{10}(t), \psi_{11}(t), \ldots, \psi_{1,M-1}(t), \ldots, \psi_{2^{k-1},0}(t), \ldots, \psi_{2^{k-1},M-1}(t)]^T \tag{7}$$
$$= [\psi_1, \psi_2, \ldots, \psi_{2^{k-1}M}]^T,$$

The integration of the vector can be obtained as

$$\int_0^t \Psi(t') \, dt' = P\Psi(t). \tag{8}$$

Our main purpose is to obtain the matrix P.

3. Chebyshev Wavelet Operational Matrix of Integration

In this section, we will derive the operational matrix P of integration which plays a great role in dealing with the problem of Ferdholm integro-differential equation.

First we construct the 6×6 matrix P for $M = 3$ and $k = 2$. In this case, there are six basis functions which are given by

$$\left. \begin{array}{l} \psi_{10} = \frac{2}{\sqrt{\pi}} \\ \psi_{11} = 2\sqrt{\frac{2}{\pi}}(4t - 1) \\ \psi_{12} = 2\sqrt{\frac{2}{\pi}}((4t - 1)^2 - 1) \end{array} \right\}, \qquad 0 \leqslant t \leqslant \frac{1}{2}, \qquad (9)$$

$$\left. \begin{array}{l} \psi_{20} = \frac{2}{\sqrt{\pi}} \\ \psi_{21} = 2\sqrt{\frac{2}{\pi}}(4t - 3) \\ \psi_{22} = 2\sqrt{\frac{2}{\pi}}((4t - 3)^2 - 1) \end{array} \right\}, \qquad \frac{1}{2} \leqslant t < 1, \qquad (10)$$

and from (7),

$$\Psi_6(t) = [\psi_{10}, \psi_{11}, \psi_{12}, \psi_{20}, \psi_{21}, \psi_{22}]^T. \qquad (11)$$

By integrating (9) and (10) from 0 to t and representing it to the matrix form, we obtain

$$\int_0^t \psi_{10}(t') \, dt' = \begin{cases} \frac{2}{\sqrt{\pi}}t, & 0 \leqslant t \leqslant \frac{1}{2}, \\ \frac{1}{\sqrt{\pi}}, & \frac{1}{2} \leqslant t \leqslant 1, \end{cases}$$
$$= [\frac{1}{4}, \frac{1}{4\sqrt{2}}, 0, \frac{1}{2}, 0, 0]\Psi_6(t).$$

$$\int_0^t \psi_{11}(t') \, dt' = \begin{cases} 2\sqrt{\frac{2}{\pi}}(2t^2 - t), & 0 \leqslant t \leqslant \frac{1}{2} \\ 0, & \frac{1}{2} \leqslant t \leqslant 1, \end{cases}$$
$$= [-\frac{1}{8\sqrt{2}}, 0, \frac{1}{16}, 0, 0, 0]\Psi_6(t).$$

$$\int_0^t \psi_{12}(t') \, dt' = \begin{cases} 2\sqrt{\frac{2}{\pi}}(\frac{32}{3}t^3 - 8t^2 + t), & 0 \leqslant t \leqslant \frac{1}{2}, \\ -\frac{\sqrt{\frac{2}{\pi}}}{3}, & \frac{1}{2} \leqslant t \leqslant 1, \end{cases}$$
$$= [-\frac{1}{6\sqrt{2}}, -\frac{1}{8}, 0, -\frac{1}{3\sqrt{2}}, 0, 0]\Psi_6(t).$$

$$\int_0^t \psi_{20}(t') \, dt' = \begin{cases} 0, & 0 \leqslant t \leqslant \frac{1}{2}, \\ \frac{2}{\sqrt{\pi}}t - \frac{1}{\sqrt{\pi}}, & \frac{1}{2} \leqslant t \leqslant 1, \end{cases}$$
$$= [0, 0, 0, \frac{1}{4}, \frac{1}{4\sqrt{2}}, 0]\Psi_6(t).$$

$$\int_0^t \psi_{21}(t')\,dt' = \begin{cases} 0, & 0 \leqslant t \leqslant \frac{1}{2}, \\ 2\sqrt{\frac{2}{\pi}}(2t^2 - 3t) + 1, & \frac{1}{2} \leqslant t \leqslant 1, \end{cases}$$

$$= [0, 0, 0, -\frac{1}{8\sqrt{2}}, 0, -\frac{1}{16}]\Psi_6(t).$$

$$\int_0^t \psi_{22}(t')\,dt' = \begin{cases} 0, & 0 \leqslant t \leqslant \frac{1}{2}, \\ 2\sqrt{\frac{2}{\pi}}\left((\frac{32}{3}t^3 - 24t^2 + 17t) - \frac{23}{6}\right), & \frac{1}{2} \leqslant t \leqslant 1, \end{cases}$$

$$= [0, 0, 0, -\frac{1}{6\sqrt{2}}, -\frac{1}{8}, 0]\Psi_6(t).$$

Thus

$$\int_0^t \Psi_6(t')\,dt' = P_{6\times6}\Psi_6(t), \tag{12}$$

where

$$P_{6\times6} = \frac{1}{4}\begin{bmatrix} 1 & \frac{1}{\sqrt{2}} & 0 & 2 & 0 & 0 \\ -\frac{1}{2\sqrt{2}} & 0 & \frac{1}{4} & 0 & 0 & 0 \\ -\frac{\sqrt{2}}{3} & -\frac{1}{2} & 0 & -\frac{2\sqrt{2}}{3} & 0 & 0 \\ 0 & 0 & 0 & 1 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & -\frac{1}{2\sqrt{2}} & 0 & \frac{1}{4} \\ 0 & 0 & 0 & -\frac{\sqrt{2}}{3} & -\frac{1}{2} & 0 \end{bmatrix}.$$

In (11) and (12) the subscript of $P_{6\times6}$ and Ψ_6 denote the dimension. In fact the matrix $P_{6\times6}$ can be written as

$$P_{6\times6} = \frac{1}{4}\begin{bmatrix} L_{3\times3} & F_{3\times3} \\ 0_{3\times3} & L_{3\times3} \end{bmatrix}$$

where

$$L_{3\times3} = \begin{bmatrix} 1 & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2\sqrt{2}} & 0 & \frac{1}{4} \\ -\frac{\sqrt{2}}{3} & -\frac{1}{2} & 0 \end{bmatrix} \quad \text{and} \quad F_{3\times3} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 0 \\ -\frac{2\sqrt{2}}{3} & 0 & 0 \end{bmatrix}$$

For general case, we have

$$\int_0^t \Psi(t')\,\mathrm{d}t' = P\Psi(t) \tag{13}$$

where $\Psi(t)$ is given in (7) and P is a $2^{k-1}M \times 2^{k-1}M$ matrix given by

$$P = \frac{1}{2^k}\begin{bmatrix} L & F & F & \dots & F \\ 0 & L & F & \dots & F \\ 0 & 0 & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & L & F \\ 0 & 0 & \dots & 0 & L \end{bmatrix}$$

where F and L are $M \times M$ matrices. In this case,

a) If $M = 2$ we have

$$F = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix} \quad \text{and} \quad L = \begin{bmatrix} 1 & \dfrac{1}{\sqrt{2}} \\ -\dfrac{1}{2\sqrt{2}} & 0 \end{bmatrix}$$

b) If $M = 3, 5, 7, \dots$ we have

$$F = \begin{bmatrix} 2 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 \\ -\frac{2\sqrt{2}}{3} & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 \\ -\frac{2\sqrt{2}}{15} & 0 & \cdots & 0 & 0 \\ \vdots & 0 & \cdots & 0 & 0 \\ -\frac{2\sqrt{2}}{(M-4)(M-2)} & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 \\ -\frac{2\sqrt{2}}{(M-2)M} & 0 & \cdots & 0 & 0 \end{bmatrix}$$

and

$$L = \begin{bmatrix} 1 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ -\frac{1}{2\sqrt{2}} & 0 & \frac{1}{4} & 0 & 0 & 0 & \cdots & 0 & 0 \\ -\frac{\sqrt{2}}{3} & -\frac{1}{2} & 0 & \frac{1}{6} & 0 & 0 & \cdots & 0 & 0 \\ \frac{\sqrt{2}}{8} & 0 & -\frac{1}{4} & 0 & \frac{1}{8} & 0 & \cdots & 0 & 0 \\ -\frac{\sqrt{2}}{15} & 0 & 0 & -\frac{1}{6} & 0 & \frac{1}{10} & \cdots & 0 & 0 \\ \frac{\sqrt{2}}{24} & 0 & 0 & 0 & -\frac{1}{8} & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \cdots & \vdots & \vdots \\ \frac{\sqrt{2}}{(M-3)(M-1)} & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & \frac{1}{2M-2} \\ -\frac{\sqrt{2}}{(M-2)M} & 0 & 0 & 0 & 0 & 0 & \cdots & -\frac{1}{2(M-2)} & 0 \end{bmatrix}$$

c) If $M = 4, 6, 8, \cdots$ we have

$$F = \begin{bmatrix} 2 & 0 \cdots 0 \, 0 \\ 0 & 0 \cdots 0 \, 0 \\ -\frac{2\sqrt{2}}{3} & 0 \cdots 0 \, 0 \\ 0 & 0 \cdots 0 \, 0 \\ -\frac{2\sqrt{2}}{15} & 0 \cdots 0 \, 0 \\ \vdots & \vdots \cdots \vdots \, \vdots \\ -\frac{2\sqrt{2}}{(M-5)(M-3)} & 0 \cdots 0 \, 0 \\ 0 & 0 \cdots 0 \, 0 \\ -\frac{2\sqrt{2}}{(M-3)(M-1)} & 0 \cdots 0 \, 0 \\ 0 & 0 \cdots 0 \, 0 \end{bmatrix}$$

and

$$L = \begin{bmatrix} 1 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ -\frac{1}{2\sqrt{2}} & 0 & \frac{1}{4} & 0 & 0 & 0 & \cdots & 0 & 0 \\ -\frac{\sqrt{2}}{3} & -\frac{1}{2} & 0 & \frac{1}{6} & 0 & 0 & \cdots & 0 & 0 \\ \frac{\sqrt{2}}{8} & 0 & -\frac{1}{4} & 0 & \frac{1}{8} & 0 & \cdots & 0 & 0 \\ -\frac{\sqrt{2}}{15} & 0 & 0 & -\frac{1}{6} & 0 & \frac{1}{10} & \cdots & 0 & 0 \\ \frac{\sqrt{2}}{24} & 0 & 0 & 0 & -\frac{1}{8} & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \cdots & \vdots & \vdots \\ -\frac{\sqrt{2}}{(M-3)(M-1)} & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & \frac{1}{2M-2} \\ \frac{\sqrt{2}}{(M-2)M} & 0 & 0 & 0 & 0 & 0 & \cdots & -\frac{1}{2(M-2)} & 0 \end{bmatrix}$$

The integration of the product of two Chebyshev wavelet function vectors is obtained as:

$$I = \int_0^1 \Psi(t)\Psi(t)^T \, \mathrm{d}t \tag{14}$$

where I is the identity matrix.

4.   Solution of the Linear Fredholm Integro-Differential Equation

Consider the linear Fredholm integro-differential equation given by (1). We approximate $y'(t)$, $f(t)$ and $k(t, s)$ by using Chebyshev wavelet space as follows:

$$y'(t) \simeq Y'^T \Psi(t), \quad y(0) \simeq Y_0^T \Psi(t), \quad f(t) \simeq X^T \Psi(t) \quad and \quad k(t, s) \simeq \Psi^T(t) K \Psi(t),$$

where Y', Y_0 and X are $2^{k-1}M \times 1$ matrices whose coefficients are defined similarly to (6). Also, K is a $2^{k-1}M \times 2^{k-1}M$ matrix with the following elements:

$$K_{ij} = (\psi_i(t), (k(t, s), \psi_j(s))) \quad , \quad i, j = 1, 2, \ldots, 2^{k-1}M .$$

Then from (13), we have

$$y(t) = \int_0^t y'(s)\,d(s) + y(0) \simeq \int_0^t Y'^T \Psi(s)\,d(s) + Y_0^T \Psi(t) = Y'^T P \Psi(t) + Y_0^T \Psi(t)$$

$$= (Y'^T P + Y_0^T)\Psi(t)$$

Substituting into (1) we have

$$\Psi(t)^T Y' = \Psi(t)^T X + \Psi(t)^T (P^T Y' + Y_0) + \int_0^1 \Psi(t)^T K \Psi(s) \Psi(s)^T (P^T Y' + Y_0)\,d(s)$$

By (14),

$$\Psi(t)^T Y' = \Psi(t)^T X + \Psi(t)^T (P^T Y' + Y_0) + \Psi(t)^T K (P^T Y' + Y_0)$$

or

$$(I - KP^T - P^T)Y' = KY_0 + Y_0 + X \tag{15}$$

By solving this linear system we can get the vector Y'. Thus,

$$Y^T = Y'^T P + Y_0^T \qquad or \qquad y(t) = Y^T \Psi(t) \tag{16}$$

5.   Numerical Examples

In this section, we consider three integro-differential equations. We apply the system of equations in (15) and (16). The programs have been provided by Mathematica. y and $\tilde{y}$ in tables denote the exact solution and the numerical solution, respectively.

Example 5.1 Let

$$\begin{cases} y'(x) = \int_0^1 \sin[4\pi x + 2\pi t]\, y(t)\,dt + y(x) - \cos(2\pi x) - 2\pi \sin(2\pi x) - \frac{1}{2}\sin(4\pi x) \\ y(0) = 1 \end{cases}$$
$$\tag{17}$$

with exact solution $y(x) = \cos(2\pi x)$.
We solve (15) by using the mentioned method in Section 4, with $P_{12\times12}$. Table 1 shows the numerical results of the Equation (17) using Chabyshev wavelet method are better than the results of CAS wavelet method.

Example 5.2 Let

$$\begin{cases} y'(x) = xe^x + e^x - x + \int_0^1 xy(t)\,dt \\ y(0) = 0 \end{cases} \tag{18}$$

with exact solution $y(x) = xe^x$.

In this example, we solve (15) by using the Chebyshev wavelet method with $P_{10\times10}$. Table 2 shows that the numerical results of the example in Chabyshev wavelet method are better than the results of the Legendre wavelet method.

Example 5.3 Let

$$\begin{cases} y^{'}(x) = 1 - \frac{1}{3}x + \int_0^1 xty(t)\,\mathrm{d}t \\ y(0) = 0 \end{cases} \tag{19}$$

with exact solution $y(x) = x$.

Table 1. Comparison of Chebyshev wavelet and CAS wavelet in Example 1

x	Chebyshev-$P_{12\times12}$ $\|y - \tilde{y}\|$	CAS - $P_{12\times12}$ $\|y - \tilde{y}\|$
0.09	0.00059917	0.05078780
0.08	0.00276144	0.04767720
0.07	0.00521435	0.02589700
0.06	0.00664701	0.01486380
0.05	0.00696096	0.07351310
0.04	0.00607222	0.14761900
0.03	0.00391160	0.23355700
0.02	0.00042504	0.32673500
0.01	0.00442625	0.42187600
0.1	0.00474126	0.03691660
0.2	0.05276500	0.09556360
0.3	0.03453150	0.04916060
0.4	0.04278170	0.05737700
0.5	0.00244686	0.49269400
0.6	0.04624910	0.05097740
0.7	0.04743300	0.06504720
0.8	0.07903460	0.08221020
0.9	0.03978690	0.03469560

Table 2. Comparison of Chebyshev wavelet and Legendre wavelet in Example 2

x	Chebyshev-$P_{12\times12}$ $\|y - \tilde{y}\|$	Legendre-$P_{12\times12}$ $\|y - \tilde{y}\|$
0.09	0.003936730	0.156341000
0.08	0.003098730	0.138591000
0.07	0.002363070	0.115878000
0.06	0.001728820	0.087918000
0.05	0.001195080	0.054426900
0.04	0.000760973	0.015121000
0.03	0.000425633	0.030282900
0.02	0.000188235	0.082067400
0.01	0.000047972	0.140514000
0.1	0.004878020	0.169415000
0.2	0.020196300	0.106074000
0.3	0.047036800	0.107078000
0.4	0.086633600	0.175800000
0.5	0.140321000	0.339474000

Table 3. Comparison of Chebyshev, CAS and Legendre wavelets in Example 3

x	Chebyshev-$P_{12\times12}$ $\|y - \tilde{y}\|$	CAS - $P_{12\times12}$ $\|y - \tilde{y}\|$	Legendre-$P_{12\times12}$ $\|y - \tilde{y}\|$
0.09	0.00120024	0.02486320	0.07005790
0.08	0.00094507	0.02574420	0.05855910
0.07	0.00072107	0.02211350	0.04446370
0.06	0.00052791	0.01357090	0.02766580
0.05	0.00036531	0.00002468	0.00806295
0.04	0.00023295	0.01830220	0.01444370
0.03	0.00013055	0.04088650	0.03994960
0.02	0.00005781	0.06693760	0.06854620
0.01	0.00001445	0.09544690	0.10032200
0.1	0.00148687	0.02015420	0.07906980
0.2	0.00615451	0.00164509	0.05906060
0.3	0.01433670	0.00015358	0.06262440
0.4	0.02640410	0.02318070	0.11753000
0.5	0.04276500	0.12556300	0.05171500

We solve (15) by using the method mentioned in Section 4 with $P_{12\times12}$. Table 3 shows that the numerical results of the example in Chabyshev wavelet method are better than the results of Legendre wavelet and CAS wavelet methods

6. Conclusion

The aim of the presented work is to develop an efficient method for solving an integro-differential equation by reducing it into a set of algebraic equations. In this method, we used the first kind Chebyshev wavelet operational matrix of integration for solving the integro-differential equation.

Numerical examples showed that Chebyshev wavelet method can behave better than the other mentioned wavelets method and give better approximation in comparison with the other wavelets especially for small values of x.

References

[1] Babolian E., Fattahzadeh F. Numerical solution of differential equations by using Chebyshev wavelet operational matrix of integration, Appl. Math. Comput.,**188** (2007) 417-426.

[2] Danfu H., Xufeng S. Numerical solution of integro-differential equations by using CAS wavelet operational matrix of integration, Appl. Math. Comput., **194** (2007) 460-466.

[3] Fariborzi Araghi M. A., Bahmanpour M. Numerical Solution of Fredholm Integral Equation of the First Kind using Legendre, Chebyshev and CAS wavelets, Int. J. of Math. Sci. & Engg. Appls. (IJMSEA),**2(4)** (2008) 1-9.

[4] Gu J. S., Jiang W. S. The Haar wavelets operational matrix of integration, Int. J. Sys. Sci., **27** (1996) 623-628.

[5] Razzaghi M., Yousefi S. The Legendre wavelets operational matrix of integration, Int. J. Syst. Sci.,**32(4)** (2001) 495-502.

[6] Tavassoli Kajani M., Ghasemi M., Babolian E. Numerical solution of linear integro-differential equation by using sine-cosine wavelets, Appl. Math. Comput., **180** (2006) 569-574

[7] Tavassoli Kajani M., Hadi Vencheh A. Solving linear integro-differential equation with Legendre wavelets, Int. J. Comp. Math.,**81(6)** (2004) 719-726.