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Analytical-Numerical Solution for Nonlinear Integral Equations of Hammerstein Type

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Abstract. Using the mean-value theorem for integrals we tried to solved the nonlinear integral equations of Hammerstein type . The mean approach is to obtain an initial guess with unknown coefficients for unknown function $y(x)$. The procedure of this method is so fast and don't need high cpu and complicated programming. The advantages of this method is that we can applied for those integral equations which have not the unique solution too.

Keywords: Nonlinear integral equations, Hammerstein equations, mean value theorem.

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1. Introduction

Consider the Hammerstein equations of the form

$$y(x) = f(x) + \int_a^b K(x,t)[y(t)]^n dt; \qquad x \in [a,b], \tag{1}$$

where f and K are given continuous and y is unknown function to be determined. Hammerstein integral equations arises in many fields of applied mathematics such as in study of the electro-magnetic fluid dynamics, reformulation of two-point boundary value problems with nonlinear boundary conditions [1, 2]. The integral equation (1) are the special case of the Hammerstein integral equations. Solution of this equation have been consider by many authors, in [26] the classical method of successive approximations has been introduced . In [23] rationalized Haar functions have been developed to approximate of the nonlinear Volterra-Fredholm-Hammerstein integral equations. Under mild differentiability conditions, the class of fourth-order iterations method has been developed by [9], the semilocal

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convergence of the method has been analyzed. Spline collocation method for solution of nonlinear Fredholm integral equations has been studied in [19], and the collocation method for Fredholm-Volterra integral equations with weakly kernels has been given in [10]. The method based on converting the given equation to an optimal control problem and then by using some concepts of measure theory, finally obtained a linear programming whose solution gives rise to the approximate solution of the integral equations has been developed in [6].

A variation of the Nystrom method was presented by [18]. A pseudospectral method was proposed by [8]. A new approach for numerical solution of (1) by using the globally defined Sinc basis functions has been developed in [25]. Degenerate kernel scheme developed in [27].

The cubic semiorthogonal compactly supported B-spline wavelets has been used in [20], and proved that the convergence of method is exponentially with rate of $O(2^{-4j})$. The methods based on interpolation suggested in [21]. The Galerkin and Collocation method are two commonly used methods for the numerical solution of the Hammerstein equations, many papers have appeared on these methods and a large part of the results presented in [11,12,15]. The properties of rationalized Haar function together with the Newton-Cotes nodes and Newton-Cotes integration method to solution of Volterra-Hammerstein integral equation used in [24]. The standard collocation method is given in [4]. A new collocation-type method for solution of (1) presented in [14].

The collocation method based on Daubechies wavelets and combined with the standard collocation method has been introduced in [22]. The wavelet basis Petrov-Galerkin method and the iterated Petrov-Galerkin method for a class of nonlinear Hammerstein equations was considered in [13]. The method in [3] is based on replacement of unknown function by truncated series of well known Chebyshev expansion of the function. In [1] by using chebyshev collocation method the nonlinear integral equation (1) has been solved. A Taylor expansion approach for solving nonlinear Volterra-Fredholm integral equations has been presented by [28]. In [5], homotopy perturbation method (HPM) is used to solve nonlinear integral equation.

In [16,17] a semiorthogonal wavelet generated by a linear B-spline functions have been used to solve Fredholm integro-differential equations, and nonlinear Fredholm-Hammerstein integral equations respectively.

In this paper by applying mean value theorem, we developed a scheme which produced an initial guess with unknown coefficients for unknown function $y(x)$ in equation (1). The main idea is to reduce the equation (1) to a set of algebraic equations with unknown coefficients that the equations would solve very easily and quickly. Finally, by solving several examples we demonstrate the accuracy and the efficiency of the proposed method.

2. The Presented Method

We consider the nonlinear Fredholm-Hammerstein integral equations in (1), by using mean value theorem we know that there exists a constant $c \in [a, b]$ such that:

$$\int_a^b K(x,t)[y(t)]^n dt = (b-a)K(x,c)[y(c)]^n; \qquad x \in [a,b]. \tag{2}$$

Substituting (2) in (1) we obtain

$$y(x) = f(x) + (b-a)K(x,c)[y(c)]^n; \qquad x \in [a,b], \tag{3}$$

and

$$y(x) = f(x) + c'K(x,c); \qquad x \in [a,b], \tag{4}$$

where $(b-a)y(c)^n = c'$. In the case, when the kernel in the equation (1) is separable namely in the following form

$$K(x,t) = k_1(x)k_2(t), \tag{5}$$

then by using (5) in (4) we obtain

$$y(x) = f(x) + c'k_1(x)k_2(c); \qquad x \in [a,b], \tag{6}$$

finally we have

$$y(x) = f(x) + d_1.k_1(x) = f(x) + g(x,d_1); \qquad x \in [a,b], \tag{7}$$

where $d_1 = c'k_2(c)$ is unknown and by estimate it we can obtain the solution of the integral equation (1). Now if the kernel is not separable then we can rewrite the equation (4) in the following form

$$y(x) = f(x) + g(x,d_1,d_2); \qquad x \in [a,b], \tag{8}$$

where constants d_1 and d_2 are unknowns and these values can be estimated so that we can obtain the solution of the integral equation (1). For convenience we consider the equation (4) in the form equation (8). For estimate the unknown coefficients we substitute the equation (8) in the equation (1) and we get

$$y(x) = f(x) + g(x,d_1,d_2) = f(x) + \int_a^b K(x,t)[f(t) + g(t,d_1,d_2)]^n dt, \tag{9}$$

then by simplifying we get

$$g(x,d_1,d_2) = \int_a^b K(x,t)[f(t) + g(t,d_1,d_2)]^n dt, \tag{10}$$

where $g(x,d_1,d_2)$ and $K(x,t)$ and $f(x)$ are known functions and constants d_1 and d_2 in both side of equation (10) are unknown.
If $n = 1$ in the equation (1) then the unknowns can be obtained easily with equation (10) and If $n > 1$ and the kernel is separable then the equation (10) lead to the equation of degree n and if the kernel is not separable then the equation (10) lead to the system of two nonlinear equations and two unknowns in the following form

$$\begin{cases} g(a,d_1,d_2) = \int_a^b K(a,t)[f(t) + g(t,d_1,d_2)]^n dt, \\ g(b,d_1,d_2) = \int_a^b K(b,t)[f(t) + g(t,d_1,d_2)]^n dt, \end{cases}$$

by solving this system the unknowns d_1 and d_2 can be determined and by substitute in (8) we can obtain the solution of nonlinear integral equation (1).

3. Applications

We consider some examples, to test the method and show the efficiency and accuracy of the presented method. The arising nonlinear systems have been solved by Newton's Method.

Example Consider the following nonlinear integral equation in [3]

$$y(x) = x^3 - (6-2e)e^x + \int_0^1 e^{(x+t)} y(t)dt, \qquad 0 \leqslant x \leqslant 1, \tag{11}$$

where kernel is separable in the form (5) and by using mean value theorem we get

$$y(x) = x^3 - (6-2e)e^x + e^c y(c)e^x, \qquad 0 \leqslant x \leqslant 1, \tag{12}$$

where $c \in [0,1]$, by substituting $e^c y(c) = d_1$, we get $g(x, d_1) = d_1 e^x$ that d_1 is unknown and must be determined, then we have

$$y(x) = x^3 - (6-2e)e^x + d_1 e^x, \qquad 0 \leqslant x \leqslant 1, \tag{13}$$

and by using the equation (10) we get

$$\int_0^1 e^{(x+t)}(t^3 - (6-2e)e^t + d_1 e^t)dt = d_1 e^x, \tag{14}$$

then we have

$$9 - 3e - 3e^2 + e^3 + \frac{1}{2}(e^2 - 1)d_1 = d_1, \tag{15}$$

and finally we get $d_1 = 6 - 2e$ and by substituting in equation (13) we obtain the exact solution $y(x) = x^3$ for the integral equation (11). Babolian et al showed that the maximum absolute errors in Adomion Decomposition and the presented method in [3] are 1.02×10^{-3} and 0.797×10^{-2} respectively and the minimum absolute errors are 0.02×10^{-9} and 0.145×10^{-8} respectively.

Example Consider the following nonlinear integral equation in [5]

$$y(x) = xln(x+1) - \frac{55}{108}x + \frac{1}{3}ln2(\frac{8}{3}x + 2 - xln2) - \frac{241}{576} + \frac{1}{2}\int_0^1 (x-t)[y(t)]^2 dt, \tag{16}$$

where $0 \leqslant x \leqslant 1$ and $n > 1$ with the exact solution $y(x) = xln(x+1)$, by using mean value theorem we get

$$y(x) = xln(x+1) - \frac{55}{108}x + \frac{1}{3}ln2(\frac{8}{3}x + 2 - xln2) - \frac{241}{576} + \frac{1}{2}(x-c)[y(c)]^2, \tag{17}$$

where $c \in [0,1]$, by substituting $\frac{1}{2}(x-c)[y(c)]^2 = d_1 x + d_2$, we get $g(x, d_1, d_2) =$

$d_1x + d_2$ that d_1, d_2 are unknowns and must be determined, then we have

$$\begin{aligned} y(x) &= xln(x+1) - \frac{55}{108}x + \frac{1}{3}ln2(\frac{8}{3}x + 2 - xln2) - \frac{241}{576} + d_1x + d_2 \\ &= xln(x+1) - 0.053279436734263903555198x \\ &+ 0.043695342595519095167043 + d_1x + d_2, \qquad 0 \leqslant x \leqslant 1, \end{aligned} \tag{18}$$

and by using the equation (10) we get

$$\frac{1}{2}\int_0^1 (x-t)[tln(t+1) - \frac{55}{108}t + \frac{1}{3}ln2(\frac{8}{3}t + 2 - tln2) - \frac{241}{576} + d_1t + d_2]^2 dt = d_1x + d_2, \tag{19}$$

then we have

$$\begin{cases} .041329613300250285731960d_1 + .078647422579863135157289d_2 \\ +.010611012395529242039038 + \frac{1}{4}d_2^2 + \frac{1}{24}d_1^2 + .\frac{1}{6}d_1d_2 = d_1 + d_2 \\ \\ -.147078588348273722500250d_1 - .188408201648524008232210d_2 \\ -.044035502213481952045408 - \frac{1}{3}d_1d_2 - \frac{1}{4}d_2^2 - \frac{1}{8}d_1^2 = d_2 \end{cases}$$

and by solving this nonlinear system with the Newton's method we get

$$\begin{aligned} d_1 &= 0.053279436734263903555223, \\ d_2 &= -0.043695342595519095167064, \end{aligned} \tag{20}$$

by substituting (20) in equation (18) we obtain

$$y(x) = xln(x+1) + 2.50713 \times 10^{-23}x - 2.07035 \times 10^{-23}, \tag{21}$$

hance the coefficients of the second and third term in (21) is very small and can be neglected then we obtain the exact solution $y(x) = xln(x+1)$ of the integral equation (16). In [5] the best result is reported in the following form

$$y_5^* = xln(x+1) + 8.600785 \times 10^{-8}x - 2.667965 \times 10^{-8}, \tag{22}$$

and the minimum absolute error is 1.8073×10^{-7}, this shows that our method is more effective than the method on [5].

Example Consider the following nonlinear integral equation of Hammerstein type in [25]

$$y(x) = 1 - \frac{5}{12}x + \int_0^1 xt[y(t)]^2 dt, \qquad 0 \leqslant x \leqslant 1, \tag{23}$$

where kernel is separable in the form (5) and by using mean value theorem we get

$$y(x) = 1 - \frac{5}{12}x + xc[y(c)]^2, \qquad 0 \leqslant x \leqslant 1, \tag{24}$$

where $c \in [0,1]$, by substituting $c[y(c)]^2 = d_1$, we get $g(x, d_1) = d_1x$ that d_1 is unknown and must be determined, then we have

$$y(x) = 1 - \frac{5}{12}x + d_1 x, \qquad 0 \leqslant x \leqslant 1, \tag{25}$$

and by using the equation (10) we get

$$\int_0^1 xt(1 - \frac{5}{12}t + d_1 t)^2 dt = d_1 x, \tag{26}$$

then we have

$$\frac{1}{2} + \frac{1}{4}(d_1 - \frac{5}{12})^2 + \frac{2}{3}(d_1 - \frac{5}{12}) = d_1, \tag{27}$$

and finally we obtain two constants $d_1 = \frac{3}{4}$ and $d_1' = \frac{17}{12}$ that by substituting in equation (25) we obtain two solutions of the equation (23) in the following forms:

$$y_1(x) = 1 + \frac{x}{3}, \qquad and \qquad y_2(x) = 1 + x. \tag{28}$$

But in [25] just $y(x) = 1 + \frac{x}{3}$ has been reported and the maximum and minimum absolute errors in the solution are 4.79192×10^{-3} and 6.52101×10^{-10} respectively.

Example We Consider the following nonlinear integral equation of Hammerstein type in [1]

$$y(x) = x^2 - \frac{8}{15}x - \frac{7}{6} + \int_0^1 (x + t)[y(t)]^2 dt, \qquad 0 \leqslant x \leqslant 1, \tag{29}$$

by using mean value theorem we get

$$y(x) = x^2 - \frac{8}{15}x - \frac{7}{6} + (x + c)[y(c)]^2, \qquad 0 \leqslant x \leqslant 1, \tag{30}$$

where $c \in [0, 1]$, by substituting $(x + c)[y(c)]^2 = d_1 x + d_2$, we get $g(x, d_1, d_2) = d_1 x + d_2$ that d_1, d_2 are unknowns and must be determined, then we have

$$y(x) = x^2 - \frac{8}{15}x - \frac{7}{6} + d_1 x + d_2, \qquad 0 \leqslant x \leqslant 1, \tag{31}$$

and by using the equation (10) we get

$$\int_0^1 (x + t)[t^2 - \frac{8}{15}t - \frac{7}{6} + d_1 t + d_2]^2 dt = d_1 x + d_2, \tag{32}$$

then we have

$$\begin{cases} \frac{2897}{5400} - \frac{29}{45}d_1 - \frac{46}{45}d_2 + \frac{1}{4}d_1^2 + \frac{2}{3}d_1 d_2 + \frac{1}{2}d_2^2 = d_2 \\ \frac{9559}{5400} - \frac{5}{3}d_1 - \frac{29}{9}d_2 + \frac{7}{12}d_1^2 + \frac{5}{3}d_1 d_2 + \frac{3}{2}d_2^2 = d_1 + d_2 \end{cases}$$

by solving this nonlinear system we get $d_1 = \frac{8}{15}$ and $d_2 = \frac{1}{6}$ and by substituting in (31) we obtain $y(x) = x^2 - 1$ that it is the exact solution of integral equation (29).

Example By Considering the following nonlinear integral equation in [28]

$$y(x) = \frac{5}{6}x^2 - \frac{8}{105}x - 1 + \int_0^1 (x^2t + xt^2)[y(t)]^2 dt, \qquad 0 \leqslant x \leqslant 1, \tag{33}$$

and using mean value theorem we have

$$y(x) = \frac{5}{6}x^2 - \frac{8}{105}x - 1 + (cx^2 + c^2x)[y(c)]^2, \qquad 0 \leqslant x \leqslant 1, \tag{34}$$

where $c \in [0, 1]$, by substituting $(cx^2 + c^2x)[y(c)]^2 = d_1x^2 + d_2x$, we get

$$y(x) = \frac{5}{6}x^2 - \frac{8}{105}x - 1 + d_1x^2 + d_2x, \qquad 0 \leqslant x \leqslant 1, \tag{35}$$

and by using the equation (10) we get

$$\int_0^1 (x^2t + xt^2)[\frac{5}{6}t^2 - \frac{8}{105}t - 1 + d_1t^2 + d_2t]^2 dt = d_1x^2 + d_2x, \tag{36}$$

we have

$$\begin{cases} \frac{1}{6}d_1^2 + \frac{1}{4}d_2^2 - \frac{398}{1575}d_1 - \frac{13}{35}d_2 + \frac{2}{5}d_1d_2 + \frac{59779}{264600} = d_1 \\ \frac{1}{7}d_1^2 + \frac{1}{5}d_2^2 - \frac{59}{315}d_1 - \frac{398}{1575}d_2 + \frac{1}{3}d_1d_2 + \frac{77593}{661500} = d_2 \end{cases}$$

By solving we obtain $d_1 = \frac{1}{6}$ and $d_2 = \frac{8}{105}$ and by substituting in (35) we obtain $y(x) = x^2 - 1$ that it is the exact solution of integral equation (33).

Example In continuance we consider $n = 3$ in the equation (1) and the following integral equation [21,22,25]

$$y(x) = e^{(x+1)} - \int_0^1 e^{(x-2t)}[y(t)]^3 dt, \qquad 0 \leqslant x \leqslant 1, \tag{37}$$

by using mean value theorem for this equation we have

$$y(x) = e^{(x+1)} - e^{(x-2c)}[y(c)]^3 dt, \qquad 0 \leqslant x \leqslant 1, \tag{38}$$

where $c \in [0, 1]$, by substituting $e^{(x-2c)}[y(c)]^3 = d_1e^x$, we get

$$y(x) = e^{(x+1)} - d_1e^x = e^x(e - d_1), \qquad 0 \leqslant x \leqslant 1, \tag{39}$$

and we use to the equation (10) that we get

$$\int_0^1 e^{(x-2t)}[e^t(e - d_1)]^3 dt = d_1e^x, \tag{40}$$

then we obtain

$$(e - d_1)^3(e - 1) = d_1, \tag{41}$$

this equation has unique real solution $d_1 = e - 1$ and by substituting in (39) we obtain $y(x) = e^x$ that it is the exact solution of integral equation (37). you can see that this method is accurate and efficient toward different reference. however, this integral equation considered by several references. In [21,22,25] the minimum absolute errors are reported 4.8593×10^{-12}, 0.71385×10^{-9} and 4.85529×10^{-8} respectively.

Example we consider the following nonlinear integral equation in [5]

$$y(x) = sin(\pi x) + \frac{1}{5}\int_0^1 cos(\pi x)sin(\pi t)[y(t)]^3 dt, \qquad 0 \leqslant x \leqslant 1, \tag{42}$$

then there is a $c \in [0, 1]$ that

$$y(x) = sin(\pi x) + \frac{1}{5}cos(\pi x)sin(\pi c)[y(c)]^3, \qquad 0 \leqslant x \leqslant 1, \tag{43}$$

and by substituting $sin(\pi c)[y(c)]^3 = d_1$, we get

$$y(x) = sin(\pi x) + \frac{1}{5}d_1 cos(\pi x), \qquad 0 \leqslant x \leqslant 1, \tag{44}$$

and we use to the equation (10) that we get

$$\int_0^1 cos(\pi x)sin(\pi t)[sin(\pi t) + \frac{1}{5}d_1 cos(\pi t)]^3 dt = d_1 cos(\pi x), \tag{45}$$

then we obtain

$$\frac{3}{200}d_1^2 + \frac{3}{8} = d_1, \tag{46}$$

then by solving this equation we obtain two solutions $d_1 = \frac{100+5\sqrt{391}}{3}$ and $d_1' = \frac{100-5\sqrt{391}}{3}$ then by using equation (44) we obtain two solution for the nonlinear integral equation (42) in the forms:

$$y_1(x) = sin(\pi x) + \frac{20 + \sqrt{391}}{3}cos(\pi x), \tag{47}$$

and

$$y_2(x) = sin(\pi x) + \frac{20 - \sqrt{391}}{3}cos(\pi x). \tag{48}$$

But in [5] just $y(x) = sin(\pi x) + \frac{20-\sqrt{391}}{3}cos(\pi x)$ has been considered. In [5] the minimum absolute errors of approximation is 3.6765×10^{-7} and the error just in the point $x = 0.5$ is zero.

4. Conclusions

Solution of nonlinear integral equations are usually difficult, therefore many authors seek to obtain the approximate solution. In the present work, the approach based on mean value theorem has been developed for solving nonlinear integral equations of the Hammerstein type. The problem has been reduced to solving a system of nonlinear algebraic equations. The advantages of our method is that the arising nonlinear system is at most 2×2, by solving the several examples we obtain the exact solution and in comparison with references [1,3,5,21,22,25,28] our method is highly accurate.

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