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A New Method for Solving the Fully Z-Numbers Linear Programming Problems

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Abstract: Decisions are based on information. To be useful, information must be reliable. The concept of a Z-number relates to the issue of reliability of the information. The fully Z-number linear programming problems (FZLPP) in which all the parameters, as well as the variables, are represented by fully Z-numbers is a good topic for readers. and in this study, we proposed a practical method to solve fully Z-numbers linear programming by using the fuzzy ranking method for constraints and converting objective function to a multi-objective function, and finding their optimal solution with Z-number.

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1. Introduction

Applications of fuzzy set theory can already be found in many different areas. One could probably classify those applications as follows: 1. Applications to mathematics, that is generalizations of traditional mathematics such as topology, graph theory, algebra, logic, and so on. 2. Applications to algorithms such as clustering methods, control algorithms, mathematical programming, and so on. 3. Applications to standard models such as "the

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transportation model," "inventory control models," "maintenance models," and so on. 4. Finally, applications to real-world problems of different kinds. The first type of "applications" will be covered by looking at fuzzy logic and approximate reasoning. The second type of application will be illustrated by considering fuzzy clustering, fuzzy linear programming, and fuzzy dynamic programming. The third type will be covered by looking at fuzzy versions of standard operations research models and multicriteria approaches. The fourth type, eventually, will be illustrated on the one hand by describing the operations research (OR) model. In fact, in conventional linear programming, the decision problem coefficients are generally determined by experts with precise values. However, in fuzzy environments, the assumption of accurate data by experts seems far-fetched. Nowadays, the decisions of humans are increasingly dependent on information than ever. But, most information is not deterministic and in this situation, a human can make a rational decision based on this uncertainty. This is a hard challenge for the decision-maker to design an intelligent system that makes a decision the same as the human. So, it was led to propose a new concept of decision making in a fuzzy environment by Bellman and Zadeh [1]. Here, it is worth mentioning that the concept of fuzzy mathematical programming was first suggested by Tanaka et al., in the framework of fuzzy decision-making [2]. The initial formulation of the fuzzy linear programming problem was proposed by Zimmermann [3]. Afterward, several models of fuzzy linear programming problems were presented and then several methods were suggested for solving them [4–10]. On the other hand, what is very important is that it is not sufficient to take into account only fuzziness when dealing with real-world imperfect information. The other essential property of information is its partial reliability. Indeed, any estimation of values of interest, be it precise or soft, is subject to the confidence in sources of information we deal with – knowledge, assumptions, intuition, envision, experience – which, in general, cannot completely cover the whole complexity of real-world phenomena. Thus, fuzziness from t one side and partial reliability from the other side are strongly associated with each other. To take into account this fact, L.A. Zadeh suggested the concept of a Z-number as a more adequate formal construct for a description of real-world information. A Z-number is an ordered pair $Z = (A, B)$ of fuzzy numbers used to describe a value of a variable X, where A is an imprecise constraint on values of X and B is an imprecise estimation of reliability of A and is considered as a value of probability measure of A. After introducing Z-numbers, Yager used these numbers for decision-making in a fuzzy environment [11]. Other researchers also used these Z-numbers to decide an ambiguous environment (for more information, see references [1,11-16]). Since the concept of Z-numbers is relatively new. Fuzzy sets, their theoretical aspects have not yet been identified. This paper facilitates this mentioned gap by developing a fully Z-number programming problem (FZLPP) model using the basic concepts of classic LP described in a fuzzy and Z-number condition. finally, an practical algorithm with Z-number solution is suggested to solve this model. The outline of this paper is as follows. In Section 2 we review some preliminaries of the fuzzy set theory, fuzzy calculation, and Z-number. In Section 3, mathematical model of a fuzzy problem is introduced. Also, the optimal solution for a fully Z-linear programming problem is presented using the ranking function. Section 4, describes the numerical result using the ranking function are presented. In the fifth section, a practical example is presented, Finally, section 6 brings the conclusion.

2. Preliminaries and methodology

This section presents a general outline of the theories that support our ideas.

2.1. *Fuzzy set theory*

A fuzzy set is defined by a membership function in which the numerical range of input and output values are the universe of discourse and the membership functions assign a membership degree from 0 to 1 for each data set. A fuzzy number on the R of the universe is defined as a convex and normal fuzzy set. A fuzzy set A is characterized on a universe X may be given as:

$$A=\{(x,\mu_A(x))|x\in X\}.$$

where μ_A is the membership function of A. The membership value $\mu_A(x)$describes the grade of belongingness of $x\in X$ in A [18].

Triangular Fuzzy Number can be defined by a triplet (l,m,u), where the membership can be defined as follow:

$$\mu_A(x)=\begin{cases}0 x\in(-\infty,l),\\ \dfrac{x-l}{m-l} x\in[l,m],\\ \dfrac{u-x}{u-m} x\in[m,u],\\ 0 x\in(u,+\infty).\end{cases}$$

Definition 2.1 [19] Let $\tilde{A}=(l,m,u)$ be a triangular fuzzy number. Then $\tilde{A}$ is called a non-negative fuzzy number if and only if $l\geq 0$ [21]

Definition 2.2 [19] Let $\tilde{A}=(l,m,u)$ be a triangular fuzzy number. Then $\tilde{A}$ is called an unrestricted fuzzy number if $l,m,u\in R$.

Definition 2.3 [19] Consider $\tilde{A}=(l,m,u)$ and $\tilde{B}=(l',m',u')$ as two triangular fuzzy numbers, then:

$$\tilde{A}\oplus\tilde{B}=(l,m,u)\oplus(l',m',u')=(l+l',m+m',u+u'),$$
$$\tilde{A}-\tilde{B}=(l,m,u)\oplus(-u',-m',-l')=(l-u',m-m',u-l'),$$

$$\tilde{A} \otimes \tilde{B} = (l, m, u) \otimes (l', m', u') = (ll', mm', uu'), \forall l \geq 0,$$
$$\tilde{A} \otimes \tilde{B} = (l, m, u) \otimes (l', m', u') = (lu', mm', ul'), \forall l < 0, u \geq 0,$$
$$\tilde{A} \otimes \tilde{B} = (l, m, u) \otimes (l', m', u') = (lu', mm', ul'), \forall u < 0.$$

Definition 2.4 [19] Consider $\tilde{A} = (l, m, u)$ and $\tilde{B} = (l', m', u')$ as two triangular fuzzy numbers. Then these numbers are equal if and only if $l = l', m = m', u = u'$.

Definition 2.5 [19] (fuzzy ranking) Consider $\tilde{A} = (l, m, u)$ and $\tilde{B} = (l', m', u')$ as two triangular fuzzy numbers. Then the fuzzy number $\tilde{A}$is bigger than the fuzzy number $\tilde{B}$ if and only if

$$\tilde{\boldsymbol{A}} = (\boldsymbol{l}, \boldsymbol{m}, \boldsymbol{u}) \leq \tilde{\boldsymbol{B}} = (\boldsymbol{l}', \boldsymbol{m}', \boldsymbol{u}') \Leftrightarrow \boldsymbol{l} \leq \boldsymbol{l}', \boldsymbol{m} \leq \boldsymbol{m}, \boldsymbol{u} \leq \boldsymbol{u}'.$$

2.2 *Z-number*

Decisions are based on information. To be useful, information must be reliable. The concept of a Z-number relates to the issue of reliability of the information. A Z-number, Z, has two components, Z= (A,B). The first component, A, is a restriction (constraint) on the values which a real-valued uncertain variable, X, is allowed to take. The second component, B, is a measure of reliability (certainty) of the first component. Typically, $A$and B are described in a natural language. The concept of a Z-number has the potential for many applications, especially in the realms of economics, decision analysis, risk assessment, prediction, anticipation, and rule-based characterization of imprecise functions and relations. In the real world, uncertainty is a pervasive phenomenon. Much of the information on which decisions are based is uncertain. Humans have a remarkable capability to make rational decisions based on information that is uncertain, imprecise, and/or incomplete. The formalization of this capability, at least to some degree, is a challenge that is hard to meet. It is this challenge that motivates the concepts and ideas outlined in this note. The concept of a restriction has greater generality than the concept of a constraint. A probability distribution is a restriction but is not a constraint [20]. A restriction may be viewed as a generalized constraint [21]. In this paper, the restriction and constraint of the term are used interchangeably.

The restriction

$$R(X){:}\, XisA,$$

is referred to as a possibilistic restriction (constraint), with A playing the role of the possibility distribution of X. More specifically,

$$R(X) = XisA \rightarrow poss(X = u) = \mu_A(u),$$

where μ_A is the membership function of A and u is a generic value of X. μ_A may be viewed as a constraint which is associated with R(X), meaning that $\mu_A(u)$ is the degree to which u satisfies the constraint. When X is a random variable, the probability distribution of X plays the role of a probabilistic restriction on X. A probabilistic restriction is expressed as follows:

$$R(X): Xisp$$

Where p is the probability density function of X. In this case,

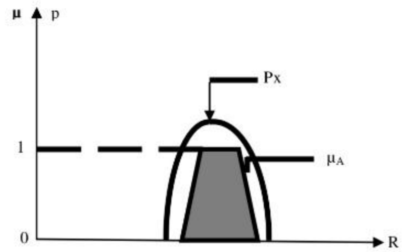
$$R(X): Xisp \rightarrow prob(u \leq X \leq u + du) = p(u)du.$$

Generally, the term ''restriction'' applies to X is R. Occasionally, ''restriction'' applies to R. Context serves to disambiguate the meaning of ''restriction.'' The ordered triple (X, A, B) is referred to as a Z-valuation. A Z-valuation is equivalent to an assignment statement, X is (A , B). X is an uncertain variable if A is not a singleton. In a related way, uncertain computation is a system of computation in which the objects of computation are not values of variables but restrictions on values of variables. In this paper, unless stated to the contrary, X is assumed to be a random variable. For convenience, A is referred to as a value of X, with the understanding that strictly speaking,A is not a value of X but a restriction on the values which X can take. The second component, B, is referred to as certainty. Closely related to certainty are the concepts of sureness, confidence, reliability, the strength of belief, probability, possibility, etc. When X is a random variable, certainty may be equated to probability. Informally, B may be interpreted as a response to the question: How sure are you that X is A? Typically, $A$and B are perception-based and are described in a natural language. A collection of Z-valuations is referred to as Z-information. It should be noted that much of everyday reasoning and decision-making is based, in effect, on Z-information. For purposes of computation, when A and B are described in a natural language, the meaning of A and B is precisiated (graduated) through association with membership functions, $\mu_A$and μ_B, respectively (Fig.1). The membership function of A, μ_A, may be elicited by asking a succession of questions of the form: To what degree does the number, a, fit your perception of A? Example: To what degree does 50 min fit your perception of about 45 min? The same applies to B. The fuzzy set, A, may be interpreted as the possibility distribution of X. The concept of a Z-number may be generalized in various ways. In particular, X may be assumed to take values in n R, in which case A is a Cartesian product of fuzzy numbers. Simple examples of Z-valuations are: (anticipated budget deficit, close to 2 million dollars, very likely) (the price of oil in the near future, significantly over 100 dollars/barrel, very likely)

If $X$is a random variable, then X is A represents a fuzzy event in R, the real line. The probability of this event, P, may be expressed as [22]:
wherep_X is the underlying (hidden) probability density of X. In effect, the Z-valuation (X, A, B) may be viewed as a restriction (generalized constraint) on Xdefined by:

$$Pr\,o\,b(XisA)isB.$$

What should be underscored is that in a Z-number, (A,B), the underlying probability distribution, p_X, is not known. What is known is a restriction on p_X which may be expressed as:



Figur 1. Membership function of A and probability density Function of X.

A subtle point is that B is a restriction on the probability measure of$A$rather than on the probability of A. Conversely, if B is a restriction on the probability of A rather than on the probability measure of A, then(A,B) is not a Z-number.[1]

2.3. *Converting a Z-Number into a Fuzzy Number*

In order to make more computations, the Z-number should be transformed into a usual fuzzy number. Kang et al. [24] presented an efficient and very easy to implement approach, called Kang et al.'s method, for turning a Z-number into a classical fuzzy number based on the fuzzy expectation. Kang et al.'s method is described as follows: Suppose a Z-number is Z =(A, B), where$A = \{(x, \mu_A(x)) | x \in [0,1]\} and B = \{(x, \mu_B(x)) | x \in [0,1]\}$, $\mu_A(x), \mu_B(x)$ are triangular membership functions. To convert a Z-number to a regular fuzzy number, the following three steps are suggested:

Step 1: First, convert the second part (reliability) into a crisp number.

$$\alpha = \frac{\int x\mu_B(x)dx}{\int \mu_B(x)dx}. \quad (1)$$

Step 2: The weighted Z-number can be defined as:

$$\tilde{z}^{\alpha} = \{(x, \mu_{A^{\alpha}}(x)) | \mu_{A^{\alpha}}(x) = \alpha\mu_A(x), x \in [0,1]\}. \quad (2)$$

α represents the weight of the reliability component of Z-number.

Step3: convert the irregular fuzzy number to a regular fuzzy number. The regular fuzzy set is given by

$$Z^{\alpha} = \{(x, \mu_{Z^{\alpha}}(x)) | \mu_Z(x) = \alpha\mu_A\left(\frac{x}{\sqrt{\alpha}}\right), x \in [0,1]\}. \quad (3)$$

3. Mathematical model of FZLP problem

As mentioned before, linear programming is one of the applied methods of research in operation. In the initial model, the values of the linear programming model parameters should be defined both properly and precisely. Nevertheless, in the real-world environment, this assumption is not conformed to reality and therefore will not be satisfied. In real problems, there is a lack of reliability in the parameters. In such a situation, the parameters of the linear programming problems should be inevitably presented as Z-numbers.

Definition 3.1. Let F(S) be the set of all Z-numbers.

The model

$$\begin{aligned} &Max[Z]^Z = ([C]^Z \times [X]^Z) \\ &s.t: [A]^Z \times [X]^Z (\leq = \geq) [B]^Z \\ &[X]^Z \geq [0]^Z, \end{aligned} \tag{4}$$

where $[A]^Z, [X]^Z, [B]^Z, [C]^z \in F(S)$ is called a full Z-linear programming problem (FZLPP) in which$[.]^Z$ shows valuation with Z-number. This model, $[A]^Z = [a_{ij}]^z$ shows the coefficients matrix that

$$\begin{aligned} [A]z &= (\tilde{a}_{ijA}, \tilde{a}_{ijB}), \\ \tilde{a}_{ijA} &= (l_{ijA}^a, m_{ijA}^a, u_{ijA}^a), \\ \tilde{a}_{ijB} &= (l_{ijB}^a, m_{ijB}^a, u_{ijB}^a), \end{aligned}$$

and $[C]^Z = [c_j]^Z$ is the vector of variables coefficients in the objective function that

$$\begin{aligned} [\boldsymbol{C_j}]^{\boldsymbol{Z}} &= (\tilde{\boldsymbol{c}}_{\boldsymbol{jA}}, \tilde{\boldsymbol{c}}_{\boldsymbol{jB}}), \\ \tilde{\boldsymbol{c}}_{\boldsymbol{jA}} &= (\boldsymbol{l_A^c}, \boldsymbol{m_A^c}, \boldsymbol{u_A^c}), \\ \tilde{\boldsymbol{c}}_{\boldsymbol{jB}} &= (\boldsymbol{l_B^c}, \boldsymbol{m_B^c}, \boldsymbol{u_B^c}), \end{aligned}$$

and $[B]^z = [b_i]^z$ shows a vector of the right-side numbers that

$$\begin{aligned} [B]^Z &= (\tilde{b}_{iA}, \tilde{b}_{iB}), \\ \tilde{b}_{iA} &= (l_{iA}^b, m_{iA}^b, u_{iA}^b), \\ \tilde{b}_{iB} &= (l_{iB}^b, m_{iB}^b, u_{iB}^b), \end{aligned}$$

and $[x]^Z = [x_{ij}]^Z$ are decision variables that

$$\begin{aligned} [x]^Z &= (\tilde{x}_{ijA}, \tilde{x}_{ijB}), \\ \tilde{x}_{ijA} &= (l_{ijA}^x, m_{ijA}^x, u_{ijA}^x), \\ \tilde{x}_{ijB} &= (l_{ijB}^x, m_{ijB}^x, u_{ijB}^x). \end{aligned}$$

In this paper, all parameters and variables are shown with Z-valuation so that the component related to their constraint is triangular fuzzy numbers and the component related to the reliability of their constraint component triangular fuzzy numbers.

Finding the optimal solution for FZLP using the ranking function:

Consider the FZLPP in (4), by substituting the mentioned cases in the previous part (4), the model can be rewritten to (5):

$$Max[z]^z = \sum_{j=1}^{n} (\tilde{c}_{jA}, \tilde{c}_{jB}) \otimes (\tilde{x}_{ijA}, \tilde{x}_{ijB})$$
$$s.t \sum_{j=1}^{n} (\tilde{a}_{ijA}, \tilde{a}_{ijB}) \otimes (\tilde{x}_{ijA}, \tilde{x}_{ijB}) \leq = \geq (\tilde{b}_{iA}, \tilde{b}_{iB})$$
$$\tilde{x}_{ijA}, \tilde{x}_{ijB} \geq 0. \qquad (5)$$

We can Convert Z- numbers into generalized fuzzy numbers. Then we have

$$MaxZ^\alpha = \sum_{j=1}^{n} (l^{\alpha c}_{jA}, m^{\alpha c}_{jA}, u^{\alpha c}_{jA}) \otimes (l^{\alpha x}_{ijA}, m^{\alpha x}_{ijA}, u^{\alpha x}_{ijA})$$
$$s.t \sum_{j=1}^{n} (l^{\alpha a}_{ijA}, m^{\alpha a}_{ijA}, u^{\alpha a}_{ijA}) \otimes (l^{\alpha x}_{ijA}, m^{\alpha x}_{ijA}, u^{x}_{ijA}) \leq (l^{\alpha b}_{iA}, m^{\alpha b}_{iA}, u^{\alpha b}_{iA})$$
$$l^{\alpha x}_{ijA} \leq m^{\alpha x}_{ijA} \leq u^{\alpha x}_{ijA}. \qquad (6)$$

The fuzzy model (6) can be transformed by Definition 3, into the following model;

$$Max\tilde{Z}^\alpha = \sum_{j=1}^{n} (l^{\alpha c}_{jA} l^{\alpha x}_{ijA}, m^{\alpha c}_{jA} m^{\alpha x}_{ijA}, u^{\alpha c}_{jA} u^{\alpha x}_{ijA})$$
$$s.t \sum_{j=1}^{n} (l^{\alpha a}_{ijA} l^{\alpha x}_{ijA}, m^{\alpha a}_{ijA} m^{\alpha x}_{ijA}, u^{\alpha a}_{ijA} u^{\alpha x}_{ijA}) \leq (l^{\alpha b}_{iA}, m^{\alpha b}_{iA}, u^{\alpha b}_{iA})$$
$$l^{\alpha x}_{ijA} \geq 0 i = 1, \ldots, m$$
$$l^{\alpha x}_{ijA} \leq m^{\alpha x}_{ijA} \leq u^{\alpha x}_{ijA} i = 1, \ldots, m. \qquad (7)$$

With the ranking function in definition 5, we have the following model:

$$MaxZ^{\alpha} = \sum_{j=1}^{n} (l_{jA}^{\alpha c} l_{ijA}^{\alpha x}, m_{jA}^{\alpha c} m_{ijA}^{\alpha x}, u_{jA}^{\alpha c} u_{ijA}^{\alpha x})$$
$$s.t \sum_{j=1}^{n} l_{ijA}^{\alpha a} l_{ijA}^{\alpha x} \leq l_{iA}^{\alpha b} i = 1, \dots, m$$
$$\sum_{j=1}^{n} m_{ijA}^{\alpha a} m_{ijA}^{\alpha x} \leq m_{iA}^{\alpha b} i = 1, \dots, m$$
$$\sum_{j=1}^{n} u_{ijA}^{\alpha a} u_{ijA}^{\alpha x} \leq u_{iA}^{\alpha b} i = 1, \dots, m$$
$$l_{ijA}^{\alpha x} \geq 0 i = 1, \dots, m$$
$$l_{ijA}^{\alpha x} \leq m_{ijA}^{\alpha x} \leq u_{ijA}^{\alpha x} i = 1, \dots, m. \tag{8}$$

When the objective function will be in a maximum form, each objective function Z^{α} converts into three real objective functions in the form of the model (9). By these changes, the problem of Z linear programming with the objective function of $\tilde{Z}$, n variable, and m constraint converts into a programming problem with three targets, 3n variable, and 5m constraint. For this three-objective problem, each of the multi-objective methods of solving the problem can be used. However, according to the objective function to maximize the core, minimize the left spread, and maximize the right spread. So, another popular technique called the fuzzy programming method for solving fuzzy transportation problem (FTP) (8) will be explored [25].

$$MaxZ_2 = \sum_{j=1}^{n} m_{jA}^{c} m_{ijA}^{x}$$
$$MaxZ_3 = \sum_{j=1}^{n} (u_{jA}^{c} u_{ijA}^{x} - m_{jA}^{c} m_{ijA}^{x})$$
$$MinZ_1 = \sum_{j=1}^{n} (m_{jA}^{c} m_{ijA}^{x} - l_{jA}^{c} l_{ijA}^{x})$$
$$s.t \sum_{j=1}^{n} l_{ijA}^{\alpha a} l_{ijA}^{\alpha x} \leq l_{iA}^{\alpha b} i = 1, \dots, m$$
$$\sum_{j=1}^{n} m_{ijA}^{\alpha a} m_{ijA}^{\alpha x} \leq m_{iA}^{\alpha b} i = 1, \dots, m$$
$$\sum_{j=1}^{n} u_{ijA}^{\alpha a} u_{ijA}^{x} \leq u_{iA}^{\alpha b} i = 1, \dots, m$$

$$l_{ijA}^{\alpha x} \geq 0 i = 1, \ldots, m$$
$$l_{ijA}^{\alpha x} \leq m_{ijA}^{\alpha x} \leq u_{ijA}^{\alpha x} i = 1, \ldots, m. \quad (9)$$

To solve model (9), the positive ideal solution (PIS) and negative ideal solution (NIS) are obtained by solving the following linear programming problems:

$$z_1^{PLS} = Min \sum_{j=1}^{n} m_{jA}^{c} m_{ijA}^{x} - l_{jA}^{c} l_{ijA}^{x} z_1^{NLS} = Max \sum_{j=1}^{n} m_{jA}^{c} m_{ijA}^{x} - l_{jA}^{c} l_{ijA}^{x}$$
$$s.t constrainsofModel(9). s.t ConstrainsofModel(9).$$
$$z_2^{PLS} = Max \sum_{j=1}^{n} m_{jA}^{c} m_{ijA}^{x} \, z_2^{NLS} = Min \sum_{j=1}^{n} m_{jA}^{c} m_{ijA}^{x}$$
$$s.t ConstrainsofModel(9). s.t ConstrainsofModel(9).$$
$$z_3^{PLS} = Max \sum_{j=1}^{n} u_{jA}^{c} u_{ijA}^{x} - m_{jA}^{c} \, m_{ijA}^{x} z_3^{NLS} = Min \sum_{j=1}^{n} u_{jA}^{c} u_{ijA}^{x} - m_{jA}^{c} \, m_{ijA}^{x}$$
$$s.t. \quad ConstrainsofModel(9). s.t ConstrainsofModel(9). \quad (10)$$

Hence, the linear membership function of

$$\mu_{\tilde{z}_1}(z_1) = \begin{cases} 1, z_1 < z_1^{PLS} \\ \frac{z_1^{NIS} - z_1}{z_1^{NIS} - z_1^{PIS}}, z_1^{PIS} < z_1 < z_1^{NIS} \\ 0, z_1 > z_1^{NIS} \end{cases} \quad (11)$$

$$\mu_{\tilde{z}_2}(z_2) = \begin{cases} 1, z_2 > z_2^{PLS} \\ \frac{z_2 - z_2^{NIS}}{z_1^{PIS} - z_1^{NIS}}, z_2^{NIS} < z_2 < z_2^{PIS} \\ 0, z_2 < z_2^{NIS} \end{cases} \quad (12)$$

$$\mu_{\tilde{z}_3}(z_3) = \begin{cases} 1, z_3 > z_3^{PLS} \\ \frac{z_2 - z_2^{NIS}}{z_1^{PIS} - z_1^{NIS}}, z_3^{NIS} < z_3 < z_3^{PIS} \\ 0, z_3 < z_3^{NIS}. \end{cases} \quad (13)$$

Finally, according to the fuzzy programming approach, the following model is solved:

$$max \, \alpha$$
$$s.t \mu_{\tilde{z}_i}(z_i) \geq \alpha, i = 1,2,3$$
$$Constra \, int \, s \, of ModelMaxn$$

$$s.t\mu_{\tilde{z}_i}(z_i) \geq n, i = 1,2,3$$
$$Constrains of Model(9) \quad (14)$$

By substituting the membership functions of (11)-(13) into the problem (14), the following problem is obtained:

$$max\ \alpha$$
$$s.t z_1 \leq z_1^{NIS} - (z_1^{NIS} - z_1^{PIS})\alpha,$$
$$z_2 \geq z_2^{NIS} + (z_2^{PIS} - z_2^{NIS})\alpha,$$
$$z_3 \geq z_3^{NIS} + (z_3^{PIS} - z_3^{NIS})\alpha,$$
$$Constra\ int\ s\ of Model().$$
$$Maxn$$
$$s.t z_1 \leq z_1^{NIS} - (z_1^{NIS} - z_1^{PIS})n,$$
$$z_2 \geq z_2^{NIS} + (z_2^{PIS} - z_2^{NIS})n,$$
$$z_3 \geq z_3^{NIS} + (z_3^{PIS} - z_3^{NIS})n,$$
$$Constrains of Model(9). \quad (15)$$

Example 3.1. Consider the following FZLP as the following:

$$Maxz = ((10,12,14),(0.25,0.35,0.45)) \otimes ((l_{1A}^x, m_{1A}^x, u_{1A}^x),(l_{1B}^x, m_{1B}^x, u_{1B}^x)) \oplus ((8,10,12),(0.41,0.51,0.61))$$
$$\otimes ((l_{2A}^x, m_{2A}^x, u_{2A}^x),(l_{2B}^x, m_{2B}^x, u_{2B}^x)) \oplus ((4,6,8),(0.32,0.52,0.72)) \otimes ((l_{3A}^x, m_{3A}^x, u_{3A}^x),(l_{3B}^x, m_{3B}^x, u_{3B}^x))$$
$$s.t((5,6,7),(0.75,0.85,0.95)) \otimes ((l_{1A}^x, m_{1A}^x, u_{1A}^x),(l_{1B}^x, m_{1B}^x, u_{1B}^x)) \oplus ((4,5,6),(0.32,0.52,0.72)) \otimes ((l_{2A}^x, m_{2A}^x, u_{2A}^x),(l_{2B}^x, m_{2B}^x, u_{2B}^x))$$
$$\oplus ((2,3,4),(0.41,0.51,0.61)) \otimes ((l_{3A}^x, m_{3A}^x, u_{3A}^x),(l_{3B}^x, m_{3B}^x, u_{3B}^x)) \leq ((20,24,26),(0.25,0.35,0.45))$$
$$((1,2,3),(0.41,0.51,0.61)) \otimes ((l_{1A}^x, m_{1A}^x, u_{1A}^x),(l_{1B}^x, m_{1B}^x, u_{1B}^x)) \oplus ((1,2,3),(0.5,0.75,1)) \otimes ((l_{2A}^x, m_{2A}^x, u_{2A}^x),(l_{2B}^x, m_{2B}^x, u_{2B}^x))$$
$$\oplus ((1,2,3),(0.32,0.52,0.72)) \otimes ((l_{3A}^x, m_{3A}^x, u_{3A}^x),(l_{3B}^x, m_{3B}^x, u_{3B}^x)) \leq ((10,12,14),(0.75,0.85,0.95))$$
$$((1,2,3),(0.41,0.51,0.61)) \otimes ((l_{1A}^x, m_{1A}^x, u_{1A}^x),(l_{1B}^x, m_{1B}^x, u_{1B}^x)) \oplus ((1,2,3),(0.5,0.75,1)) \otimes ((l_{2A}^x, m_{2A}^x, u_{2A}^x),(l_{2B}^x, m_{2B}^x, u_{2B}^x))$$
$$\oplus ((2,4,6),(0.25,0.35,0.45)) \otimes ((l_{3A}^x, m_{3A}^x, u_{3A}^x),(l_{3B}^x, m_{3B}^x, u_{3B}^x)) \leq ((12,16,24),(0.75,0.85,0.95)) \quad (16)$$

$$Max z = (5.9,7.08,8.26) \otimes (l_1^x, m_1^x, u_1^x) \oplus ((5.68,7.1,8.52) \otimes (l_2^x, m_2^x, u_2^x) \oplus (2.88,4.32,5.76) \otimes (l_3^x, m_3^x, u_3^x)$$

$$s.t (4.6,5.52,6.44) \otimes (l_1^x, m_1^x, u_1^x) \oplus (2.88,3.6,4.32) \otimes (l_2^x, m_2^x, u_2^x) \oplus (1.42,2.13,2.84) \otimes (l_3^x, m_3^x, u_3^x) \leq (11.8,14.16,15.34)$$

$$(0.71,1.42,2.13) \otimes (l_1^x, m_1^x, u_1^x) \oplus (0.86,1.72,2.58) \otimes (l_2^x, m_2^x, u_2^x) \oplus (0.71,1.44,2.16) \otimes (l_3^x, m_3^x, u_3^x) \leq (9.2,11.04,12.88)$$

$$(0.71,1.42,2.13) \otimes (l_1^x, m_1^x, u_1^x) \oplus (0.86,1.72,2.58) \otimes (l_2^x, m_2^x, u_2^x) \oplus (1.18,2.36,3.54) \otimes (l_3^x, m_3^x, u_3^x) \leq (11.04,14.72,22.08) \quad (17)$$

Finally, according to the problem (14), we should solve the following

$$\begin{aligned}
&\boldsymbol{Min 7.08m_1^x + 7.1m_2^x + 4.32m_3^x - 5.9l_1^x - 5.68l_2^x - 2.88l_3^x} \\
&\boldsymbol{Max 7.08m_1^x + 7.1m_2^x + 4.32m_3^x} \\
&\boldsymbol{Max 8.26u_1^x + 8.52u_2^x + 5.76u_3^x - 7.08m_1^x - 7.1m_2^x - 4.32m_3^x} \\
&\boldsymbol{s.t 4.6l_1^x + 2.888l_2^x + 1.42l_3^x \leq 11.8} \\
&\boldsymbol{5.52m_1^x + 3.6m_2^x + 2.13m_3^x \leq 14.16} \\
&\boldsymbol{6.446u_1^x + 4.32u_2^x + 2.846u_3^x \leq 15.34} \\
&\boldsymbol{0.71l_1^x + 0.868l_2^x + 0.71l_3^x \leq 9.2} \\
&\boldsymbol{1.42m_1^x + 1.72m_2^x + 1.44m_3^x \leq 11.04} \\
&\boldsymbol{2.13u_1^x + 2.58u_2^x + 2.16u_3^x \leq 12.88} \\
&\boldsymbol{0.71l_1^x + 0.868l_2^x + 1.18l_3^x \leq 11.04} \\
&\boldsymbol{1.42m_1^x + 1.72m_2^x + 2.36m_3^x \leq 14.72} \\
&\boldsymbol{2.13u_1^x + 2.58u_2^x + 3.54u_3^x \leq 22.08} \\
&\boldsymbol{l_1^x \geq 0, m_1^x - l_1^x \geq 0, u_1^x - m_1^x \geq 0} \\
&\boldsymbol{l_2^x \geq 0, m_2^x - l_2^x \geq 0, u_2^x - m_2^x \geq 0} \\
&\boldsymbol{l_3^x \geq 0, m_3^x - l_3^x \geq 0, u_3^x - m_3^x \geq 0} \qquad (18)
\end{aligned}$$

$z_1^{PIS} = \text{Min } z_1 = 7.08m_1^x+7.1m_2^x+4.32m_3^x - 5.9l_1^x - 5.6l_2^x - 2.88l_3^x$ $s.t\, 4.6l_1^x+2.88l_2^x+1.42l_3^x \le 11.8$ $5.52m_1^x+3.6m_2^x+2.13m_3^x \le 14.16$ $6.44u_1^x+4.32u_2^x+2.84u_3^x \le 15.34$ $0.71l_1^x+0.86l_2^x+0.71l_3^x \le 9.2$ $1.42m_1^x+1.72m_2^x+1.44m_3^x \le 11.04$ $2.13u_1^x+2.58u_2^x+2.16u_3^x \le 12.88$ $0.71l_1^x+0.86l_2^x+1.18l_3^x \le 11.04$ $1.42m_1^x+1.72m_2^x+2.36m_3^x \le 14.72$ $2.13u_1^x+2.58u_2^x+3.54u_3^x \le 22.08$ $l_1^x>=0$, $m_1^x-l_1^x>=0$, $u_1^x-m_1^x>=0$ $l_2^x>=0$, $m_2^x-l_2^x>=0$, $u_2^x-m_2^x>=0$ $l_3^x>=0$, $m_3^x-l_3^x>=0$, $u_3^x-m_3^x>=0$	$z_1^{NIS} = \text{Max } z_1 = 7.08m_1^x+7.1m_2^x+4.32m_3^x - 5.9l_1^x - 5.6l_2^x - 2.88l_3^x$ $s.t\, 4.6l_1^x+2.88l_2^x+1.42l_3^x \le 11.8$ $5.52m_1^x+3.6m_2^x+2.13m_3^x \le 14.16$ $6.44u_1^x+4.32u_2^x+2.84u_3^x \le 15.34$ $0.71l_1^x+0.86l_2^x+0.71l_3^x \le 9.2$ $1.42m_1^x+1.72m_2^x+1.44m_3^x \le 11.04$ $2.13u_1^x+2.58u_2^x+2.16u_3^x \le 12.88$ $0.71l_1^x+0.86l_2^x+1.18l_3^x \le 11.04$ $1.42m_1^x+1.72m_2^x+2.36m_3^x \le 14.72$ $2.13u_1^x+2.58u_2^x+3.54u_3^x \le 22.08$ $l_1^x>=0$, $m_1^x-l_1^x>=0$, $u_1^x-m_1^x>=0$ $l_2^x>=0$, $m_2^x-l_2^x>=0$, $u_2^x-m_2^x>=0$ $l_3^x>=0$, $m_3^x-l_3^x>=0$, $u_3^x-m_3^x>=0$
$z_2^{PIS} = \text{Max } z_2 = 7.08m_1^x+7.1m_2^x+4.32m_3^x$ $s.t\, 4.6l_1^x+2.88l_2^x+1.42l_3^x \le 11.8$ $5.52m_1^x+3.6m_2^x+2.13m_3^x \le 14.16$ $6.44u_1^x+4.32u_2^x+2.84u_3^x \le 15.34$ $0.71l_1^x+0.86l_2^x+0.71l_3^x \le 9.2$ $1.42m_1^x+1.72m_2^x+1.44m_3^x \le 11.04$ $2.13u_1^x+2.58u_2^x+2.16u_3^x \le 12.88$ $0.71l_1^x+0.86l_2^x+1.18l_3^x \le 11.04$ $1.42m_1^x+1.72m_2^x+2.36m_3^x \le 14.72$ $2.13u_1^x+2.58u_2^x+3.54u_3^x \le 22.08$ $l_1^x>=0$, $m_1^x-l_1^x>=0$, $u_1^x-m_1^x>=0$ $l_2^x>=0$, $m_2^x-l_2^x>=0$, $u_2^x-m_2^x>=0$ $l_3^x>=0$, $m_3^x-l_3^x>=0$, $u_3^x-m_3^x>=0$	$z_2^{NIS} = \text{Min } z_2 = 7.08m_1^x+7.1m_2^x+4.32m_3^x$ $s.t\, 4.6l_1^x+2.88l_2^x+1.42l_3^x \le 11.8$ $5.52m_1^x+3.6m_2^x+2.13m_3^x \le 14.16$ $6.44u_1^x+4.32u_2^x+2.84u_3^x \le 15.34$ $0.71l_1^x+0.86l_2^x+0.71l_3^x \le 9.2$ $1.42m_1^x+1.72m_2^x+1.44m_3^x \le 11.04$ $2.13u_1^x+2.58u_2^x+2.16u_3^x \le 12.88$ $0.71l_1^x+0.86l_2^x+1.18l_3^x \le 11.04$ $1.42m_1^x+1.72m_2^x+2.36m_3^x \le 14.72$ $2.13u_1^x+2.58u_2^x+3.54u_3^x \le 22.08$ $l_1^x>=0$, $m_1^x-l_1^x>=0$, $u_1^x-m_1^x>=0$ $l_2^x>=0$, $m_2^x-l_2^x>=0$, $u_2^x-m_2^x>=0$ $l_3^x>=0$, $m_3^x-l_3^x>=0$, $u_3^x-m_3^x>=0$

$$z_3^{PIS} = \text{Max } z_3 = 8.26u_1^x + 8.52u_2^x + 5.76u_3^x - 7.08m_1^x - 7.1m_2^x - 4.32m_3^x$$

$$\begin{aligned}
s.t \quad & 4.6l_1^x + 2.88l_2^x + 1.42l_3^x \le 11.8 \\
& 5.52m_1^x + 3.6m_2^x + 2.13m_3^x \le 14.16 \\
& 6.44u_1^x + 4.32u_2^x + 2.84u_3^x \le 15.34 \\
& 0.71l_1^x + 0.86l_2^x + 0.71l_3^x \le 9.2 \\
& 1.42m_1^x + 1.72m_2^x + 1.44m_3^x \le 11.04 \\
& 2.13u_1^x + 2.58u_2^x + 2.16u_3^x \le 12.88 \\
& 0.71l_1^x + 0.86l_2^x + 1.18l_3^x \le 11.04 \\
& 1.42m_1^x + 1.72m_2^x + 2.36m_3^x \le 14.72 \\
& 2.13u_1^x + 2.58u_2^x + 3.54u_3^x \le 22.08 \\
& l_1^x >= 0,\ m_1^x - l_1^x >= 0,\ u_1^x - m_1^x >= 0 \\
& \tilde{l}_2^x >= 0,\ \tilde{m}_2^x - l_2^x >= 0,\ u_2^x - m_2^x >= 0 \\
& \tilde{l}_3^x >= 0,\ \tilde{m}_3^x - \tilde{l}_3^x >= 0,\ \tilde{u}_3^x - \tilde{m}_3^x >= 0
\end{aligned}$$

$$z_3^{NIS} = \text{Min} z_3 = 8.26u_1^x + 8.52u_2^x + 5.76u_3^x - 7.08$$

$$\begin{aligned}
s.t \quad & 4.6l_1^x + 2.88l_2^x + 1.42l_3^x \le 11.8 \\
& 5.52m_1^x + 3.6m_2^x + 2.13m_3^x \le 14.16 \\
& 6.44u_1^x + 4.32u_2^x + 2.84u_3^x \le 15.34 \\
& 0.71l_1^x + 0.86l_2^x + 0.71l_3^x \le 9.2 \\
& 1.42m_1^x + 1.72m_2^x + 1.44m_3^x \le 11.04 \\
& 2.13u_1^x + 2.58u_2^x + 2.16u_3^x \le 12.88 \\
& 0.71l_1^x + 0.86l_2^x + 1.18l_3^x \le 11.04 \\
& 1.42m_1^x + 1.72m_2^x + 2.36m_3^x \le 14.72 \\
& 2.13u_1^x + 2.58u_2^x + 3.54u_3^x \le 22.08 \\
& l_1^x > 0,\ m_1^x - l_1^x >= 0,\ u_1^x - m_1^x >= 0 \\
& l_2^x >= 0,\ m_2^x - l_2^x >= 0,\ u_2^x - m_2^x >= 0 \\
& l_3^x >= 0,\ m_3^x - l_3^x >= 0,\ u_3^x - m_3^x >= 0
\end{aligned}$$

By the graphic technique, we obtain $z_1^{PIS} = 0, z_1^{NIS} = 28.5703, z_2^{PIS} = 28.5703, z_2^{NIS} = 0, z_3^{PIS} = 31.1121, z_3^{NIS} = -27.9266$ thus, we solve the following problem with regard to the problem (15);

$$\begin{aligned}
& \text{Max}=n; \\
& s.t\, 7.08m_1^x + 7.1m_2^x + 4.32m_3^x - 5.9l_1^x - 5.68l_2^x - 2.88l_3^x \le 28.5703 - 28.5703n \\
& 7.08m_1^x + 7.1m_2^x + 4.32m_3^x \ge 28.5703n \\
& 8.26u_1^x + 8.52u_2^x + 5.76u_3^x - 7.08m_1^x - 7.1m_2^x - 4.32m_3^x \\
& \qquad \ge -27.9266 + 31.1121n + 27.9266n \\
& 4.6l_1^x + 2.88l_2^x + 1.42l_3^x \le 11.8 \\
& 5.52m_1^x + 3.6m_2^x + 2.13m_3^x \le 14.16 \\
& 6.44u_1^x + 4.32u_2^x + 2.84u_3^x \le 15.34 \\
& 0.71l_1^x + 0.86l_2^x + 0.71l_3^x \le 9.2 \\
& 1.42m_1^x + 1.72m_2^x + 1.44m_3^x \le 11.04 \\
& 2.13u_1^x + 2.58u_2^x + 2.16u_3^x \le 12.88 \\
& 0.71l_1^x + 0.86l_2^x + 1.18l_3^x \le 11.04 \\
& 1.42m_1^x + 1.72m_2^x + 2.36m_3^x \le 14.72 \\
& 2.13u_1^x + 2.58u_2^x + 3.54u_3^x \le 22.08 \\
& l_1^x >= 0, m_1^x - l_1^x >= 0, u_1^x - m_1^x >= 0 \\
& l_2^x >= 0, m_2^x - l_2^x >= 0,\ u_2^x - m_2^x >= 0 \\
& l_3^x >= 0,\ m_3^x - l_3^x >= 0, u_3^x - m_3^x >= 0
\end{aligned}$$

After computations with Lingo, we have the following optimal solution:

$$n = 0.6738$$
$$m_1^x = m_2^x = l_1^x = l_2^x = u_1^x = u_2^x = 0$$
$$l_3^x = 4.4567, m_3^x = 4.4567, u_3^x = 5.4014$$

Solving using the method proposed we get the optimum solution as z= ((12.83,19.25,31.11),0.76). where 0.76 is the center of gravity of the second component of the z-number, which can be expressed as follows

Z=((12.83,19.25,31.11),(0.66,0.76,0.86)).

4. Numerical example

In this section, the presented example in the Kumar [25], Here is a case in point of a mixture of a motor vehicle for the journey to demonstrate the process of the projected loom. There are three dissimilar choices, namely train, auto, and bus. Take the three most important criteria (an outlay, trip time, soothe) into reflection. For every automobile, according to the meticulous case, the outlay is the mainly momentous aspect, which can be described by means of the linguistic variable" Very High", and the sureness of the outlay is also very sturdy, described by means of linguistic variable" Very High". Also, the linguistic variable " High" and the sureness of the outlay is also sturdy, described by means of linguistic variable " High"., and the linguistic variable " medium" and the sureness of the outlay is also medium, described by means of linguistic variable " medium". Likewise, the trip time and the soothe can also be described by means of linguistic beneath the idea of Z-number. The linguistic criteria assessment of the three motor vehicles can be gained described in Table 1.

Table1. Linguistic criterion table.

	Outlay (Rupees)	Trip time(min)	soothe
Train	((9,10,12), (VH))	((70,100,120), (M))	((4,5,6), (H))
Auto	((20,24,25), (H))	((60,70,100), (VH)	((7,8,10), (H))
Bus	((15,15,15), (H))	((70,80,90), (H))	((1,4,7), (H))

Using to the membership function, the linguistic erratic can be renewed to mathematical assessment, which is described as Table 2.

Table2: Mathematical assessment table

	Outlay (Rupees)	Trip time(min)	soothe
Train	((9,10,12), (0.75,1,1))	((70,100,120), (0.25,0.5,0.75))	((4,5,6), (0.5,0.75,1))
Auto	((20,24,25), (0.5,0.75,1))	((60,70,100), (0.75,1,1)	((7,8,10), (0,5,0.75,1))
Bus	((15,15,15), (0.5,0.75,1))	((70,80,90), (0.5,0.75,1))	((1,4,7), (0.5,0.75,1))

Now, simplify the fuzzy information to evade convolution of exact operations in the assessment progression, which is described as Table 3.

Table3: Modified decision matrix with Z-number

	Outlay (Rupees)	Trip time(min)	soothe
Train	((0.10,0.11,0.14), (0.75,1,1))	((0.16,0.23,0.27), (0.25,0.5,0.75))	((0.13,0.16,0.19), (0.5,0.75,1))
Auto	((0.23,0.27,0.28), (0.5,0.75,1))	((0.14,0.16,0.23), (0.75,1,1)	((0.22,0.25,0.32), (0,5,0.75,1))
Bus	((0.17,0.17,0.17), (0.5,0.75,1))	((0.16,0.18,0.20), (0.5,0.75,1))	((0.03,0.13,0.22), (0.5,0.75,1))

According to Eqs. (4) and (6) we have:

$$max\, z = (0.09,0.10,0.13) \otimes (l_1^x, m_1^x, u_1^x) \oplus (0.19,0.23,0.24) \otimes (l_2^x, m_2^x, u_2^x) \oplus (0.14,0.14,0.14) \otimes (l_3^x, m_3^x, u_3^x)$$
$$s.t (0.11,0.16,0.18) \otimes (l_1^x, m_1^x, u_1^x) \oplus (0.13,0.15,0.22) \otimes (l_2^x, m_2^x, u_2^x) \oplus (0.11,0.15,0.17) \otimes (l_3^x, m_3^x, u_3^x) \leq (0.48,0.72,0.97)$$
$$(0.11,0.13,0.16) \otimes (l_1^x, m_1^x, u_1^x) \oplus (0.18,0.21,0.27) \otimes (l_2^x, m_2^x, u_2^x) \oplus (0.02,0.11,0.18) \otimes (l_3^x, m_3^x, u_3^x) \leq (0.24,0.48,0.72)$$

Hence:

$$(l_1^x, m_1^x, u_1^x) = (l_3^x, m_3^x, u_3^x) = (0,0,0), (l_2^x, m_2^x, u_2^x) = (0.81,1.64,2.66)$$

Solving based on the proposed aigorithm, we get the optimum solution as z= ((0.155, 0.379, 0.639),0.75). where 0.75 is the center of gravity of the second component of the Z-number, which can be expressed as follows

Z=((0.155,0.379,0.639),(0.65,0.75,0.85)).

The answer obtained in reference [25] is 0.65.

The advantage of our method compared to Kang's method [15] is that Kang's method is not able to calculate z by Z- numbers. Therefore, first, the problem of linear programming based on z-numbers becomes a real problem and then it solves the problem. But we use our proposed method to solve the problem based on z numbers so that the value of z is based on Z numbers.

5. Conclusion

In this paper, a new method is represented to solve FZLP problems. The suggested Z-number-based LP model is closer to real-world optimization problems because of its ability to more adequately capture imprecise and partially reliable data.in fact, in the multi-objective functions approach to solving the fuzzy Z-linear programming problems, each objective function converts into three real objective functions. For these three-objective problems, we used the FTP method for solving problems. The main advantage of the modified solution approach is that the obtained fuzzy optimal solution is a non-negative fuzzy number. Another interesting topic for future work is to develop the proposed approach for solving DEA models and supply chain with Z-number.

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