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On Increasing of Integration Rate of Elements of a Current Source Circuit

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Abstract. In this paper we introduce an approach to increase integration rate of elements of a current source circuit. Framework the approach we consider a hetero-structure with special configuration. Several specific areas of the heterostructure should be doped by diffusion or ion implantation. Annealing of dopant and/or radiation defects should be optimized.

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1. Introduction

An actual and intensively solving problems of solid state electronics is increasing of integration rate of elements of integrated circuits (*p*-*n*-junctions, their systems et al) [1,3,4,7,8,12-14]. Increasing of the integration rate leads to necessity to decrease their dimensions. To decrease the dimensions are using several approaches. They are widely using laser and microwave types of annealing of infused dopants. These types of annealing are also widely using for annealing of radiation defects, generated during ion implantation [2,5,11,15,26-28,30,32]. Using the approaches gives a possibility to increase integration rate of elements of integrated circuits through inhomogeneity of technological parameters due to generating in homogenous distribution of temperature. In this situation one can obtain decreasing dimensions of elements of integrated circuits [23] with account Arrhenius law [4,12]. Another approach to manufacture elements of integrated circuits with smaller dimensions is doping of heterostructure by diffusion or ion implantation [3,4,12]. However, in this case optimization of dopant and/or radiation defects is required [23].

In this paper we consider a heterostructure. The heterostructure consist of a substrate and several epitaxial layers. Some sections have been manufactured in the epitaxial layers. Further we consider doping of these sections by diffusion or ion implantation. The doping gives a possibility to manufacture field-effect transistors framework a current source circuit so as it is shown on Figure 1. The manufacturing gives a possibility to increase density of elements of the operational amplifier circuit [14]. After the considered doping dopant and/or radiation defects should be annealed. Framework the paper we analyzed dynamics of redistribution of dopant and/or radiation defects during their annealing. We

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introduce an approach to decrease dimensions of the element. However, it is necessary to complicate technological process.

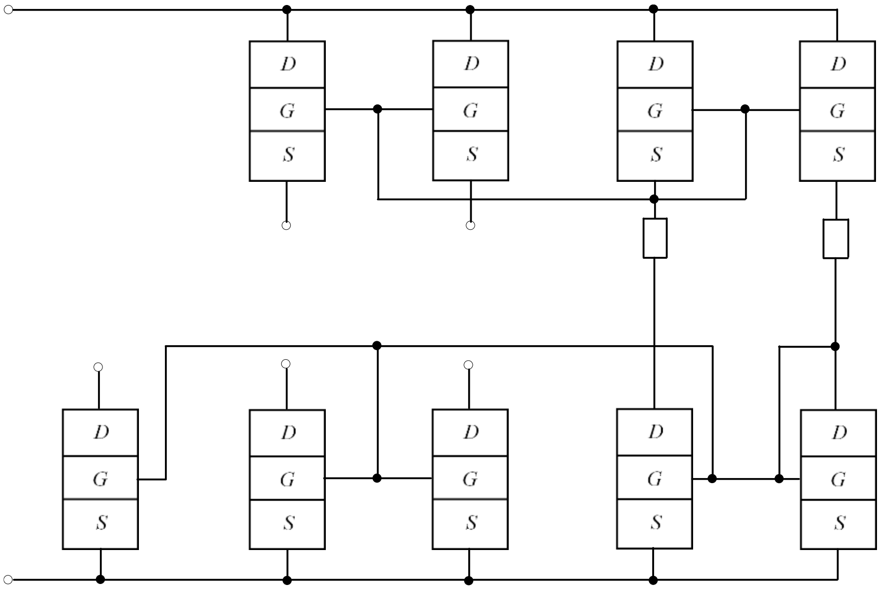


Figure 1. The considered cascaded-inverter [14].

2. Method of solution

In this section we determine spatio-temporal distributions of concentrations of infused and implanted dopants. To determine these distributions, we calculate appropriate solutions of the second Fick's law [4,12,23]:

$$\frac{\partial C(x,y,z,t)}{\partial t} = \frac{\partial}{\partial x}\left[D_C \frac{\partial C(x,y,z,t)}{\partial x}\right] + \frac{\partial}{\partial y}\left[D_C \frac{\partial C(x,y,z,t)}{\partial y}\right] + \frac{\partial}{\partial z}\left[D_C \frac{\partial C(x,y,z,t)}{\partial z}\right]. \tag{1}$$

Boundary and initial conditions for the equations are

$$\left.\frac{\partial C(x,y,z,t)}{\partial x}\right|_{x=0} = 0,\ \left.\frac{\partial C(x,y,z,t)}{\partial x}\right|_{x=L_x} = 0,\ \left.\frac{\partial C(x,y,z,t)}{\partial y}\right|_{y=0} = 0,\ \left.\frac{\partial C(x,y,z,t)}{\partial y}\right|_{x=L_y} = 0,$$

$$\left.\frac{\partial C(x,y,z,t)}{\partial z}\right|_{z=0} = 0,\ \left.\frac{\partial C(x,y,z,t)}{\partial z}\right|_{x=L_z} = 0,\ C(x,y,z,0) = f(x,y,z). \tag{2}$$

The function $C(x,y,z,t)$ describes the spatio-temporal distribution of concentration of dopant; T is the temperature of annealing; DC is the dopant diffusion coefficient. Value of dopant diffusion coefficient could be changed with changing materials of heterostructure, with changing temperature of materials (including annealing), with changing concentrations of dopant and radiation defects. We approximate dependences of dopant diffusion coefficient on parameters by the following relation with account results in [9,10,31]:

$$D_C = D_L\left(x,y,z,T\right)\left[1+\xi\frac{C^{\gamma}\left(x,y,z,t\right)}{P^{\gamma}\left(x,y,z,T\right)}\right]\left[1+\varsigma_1\frac{V\left(x,y,z,t\right)}{V^{*}}+\varsigma_2\frac{V^2\left(x,y,z,t\right)}{\left(V^{*}\right)^2}\right]. \tag{3}$$

Here the function $D_L\left(x,y,z,T\right)$ describes the spatial (in heterostructure) and temperature (due to Arrhenius law) dependences of diffusion coefficient of dopant. The function $P\left(x,y,z,T\right)$ describes the limit of solubility of dopant. Parameter $\gamma\in[1,3]$ describes average quantity of charged defects interacted with atom of dopant [10]. The function $V\left(x,y,z,t\right)$ describes the spatio-temporal distribution of concentration of radiation vacancies. Parameter V^{*} describes the equilibrium distribution of concentration of vacancies. The considered concentrational dependence of dopant diffusion coefficient has been described in details in [10]. It should be noted, that using diffusion type of doping did not generation radiation defects. In this situation $\zeta_1=\zeta_2=0$. We determine spatio-temporal distributions of concentrations of radiation defects by solving the following system of equations [9,31]:

$$\begin{aligned}
\frac{\partial I\left(x,y,z,t\right)}{\partial t} &= \frac{\partial}{\partial x}\left[D_I\left(x,y,z,T\right)\frac{\partial I\left(x,y,z,t\right)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_I\left(x,y,z,T\right)\frac{\partial I\left(x,y,z,t\right)}{\partial y}\right]\\
&+\frac{\partial}{\partial z}\left[D_I\left(x,y,z,T\right)\frac{\partial I\left(x,y,z,t\right)}{\partial z}\right]-k_{I,V}\left(x,y,z,T\right)I\left(x,y,z,t\right)V\left(x,y,z,t\right)\\
&-k_{I,I}\left(x,y,z,T\right)I^2\left(x,y,z,t\right),\\
\frac{\partial V\left(x,y,z,t\right)}{\partial t} &= \frac{\partial}{\partial x}\left[D_V\left(x,y,z,T\right)\frac{\partial V\left(x,y,z,t\right)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_V\left(x,y,z,T\right)\frac{\partial V\left(x,y,z,t\right)}{\partial y}\right]\\
&+\frac{\partial}{\partial z}\left[D_V\left(x,y,z,T\right)\frac{\partial V\left(x,y,z,t\right)}{\partial z}\right]-k_{I,V}\left(x,y,z,T\right)I\left(x,y,z,t\right)V\left(x,y,z,t\right)\\
&+k_{V,V}\left(x,y,z,T\right)V^2\left(x,y,z,t\right).
\end{aligned} \tag{4}$$

Boundary and initial conditions for these equations are

$$\left.\frac{\partial\rho\left(x,y,z,t\right)}{\partial x}\right|_{x=0}=0,\quad \left.\frac{\partial\rho\left(x,y,z,t\right)}{\partial x}\right|_{x=L_x}=0,\quad \left.\frac{\partial\rho\left(x,y,z,t\right)}{\partial y}\right|_{y=0}=0,\quad \left.\frac{\partial\rho\left(x,y,z,t\right)}{\partial y}\right|_{y=L_y}=0,$$

$$\left.\frac{\partial\rho\left(x,y,z,t\right)}{\partial z}\right|_{z=0}=0,\quad \left.\frac{\partial\rho\left(x,y,z,t\right)}{\partial z}\right|_{z=L_z}=0,\quad \rho(x,y,z,0)=f_{\rho}(x,y,z). \tag{5}$$

Here $\rho=I,V$. The function $I\left(x,y,z,t\right)$ describes the spatio-temporal distribution of concentration of radiation interstitials; $D_{\rho}\left(x,y,z,T\right)$ are the diffusion coefficients of point radiation defects; terms $V^2\left(x,y,z,t\right)$ and $I^2\left(x,y,z,t\right)$ correspond to generation divacancies and diinterstitials; $k_{I,V}\left(x,y,z,T\right)$ is the parameter of recombination of point radiation defects; $k_{I,I}\left(x,y,z,T\right)$ and $k_{V,V}\left(x,y,z,T\right)$ are the parameters of generation of simplest complexes of point radiation defects.

Further we determine distributions in space and time of concentrations of divacancies

$\Phi_V(x,y,z,t)$ and diinterstitials $\Phi_I(x,y,z,t)$ by solving the following system of equations [9,31]:

$$\frac{\partial\,\Phi_I(x,y,z,t)}{\partial\,t}=\frac{\partial}{\partial\,x}\left[D_{\Phi I}(x,y,z,T)\frac{\partial\Phi_I(x,y,z,t)}{\partial\,x}\right]+\frac{\partial}{\partial\,y}\left[D_{\Phi I}(x,y,z,T)\frac{\partial\Phi_I(x,y,z,t)}{\partial\,y}\right]$$
$$+\frac{\partial}{\partial\,z}\left[D_{\Phi I}(x,y,z,T)\frac{\partial\Phi_I(x,y,z,t)}{\partial\,z}\right]+k_{I,I}(x,y,z,T)I^2(x,y,z,t)$$
$$-k_I(x,y,z,T)I(x,y,z,t),$$

(6)

$$\frac{\partial\,\Phi_V(x,y,z,t)}{\partial\,t}=\frac{\partial}{\partial\,x}\left[D_{\Phi V}(x,y,z,T)\frac{\partial\Phi_V(x,y,z,t)}{\partial\,x}\right]+\frac{\partial}{\partial\,y}\left[D_{\Phi V}(x,y,z,T)\frac{\partial\Phi_V(x,y,z,t)}{\partial\,y}\right]$$
$$+\frac{\partial}{\partial\,z}\left[D_{\Phi V}(x,y,z,T)\frac{\partial\Phi_V(x,y,z,t)}{\partial\,z}\right]+k_{V,V}(x,y,z,T)V^2(x,y,z,t)$$
$$-k_V(x,y,z,T)V(x,y,z,t).$$

Boundary and initial conditions for these equations are

$$\left.\frac{\partial\Phi_\rho(x,y,z,t)}{\partial\,x}\right|_{x=0}=0,\quad \left.\frac{\partial\Phi_\rho(x,y,z,t)}{\partial\,x}\right|_{x=L_x}=0,\quad \left.\frac{\partial\Phi_\rho(x,y,z,t)}{\partial\,y}\right|_{y=0}=0,$$
$$\left.\frac{\partial\Phi_\rho(x,y,z,t)}{\partial\,y}\right|_{y=L_y}=0,\quad \left.\frac{\partial\Phi_\rho(x,y,z,t)}{\partial\,z}\right|_{z=0}=0,\quad \left.\frac{\partial\Phi_\rho(x,y,z,t)}{\partial\,z}\right|_{z=L_z}=0,$$
$$\Phi_I(x,y,z,0)=f_{\Phi I}(x,y,z),\quad F_V(x,y,z,0)=f_{FV}(x,y,z). \tag{7}$$

Here $D_{\Phi\rho}(x,y,z,T)$ are the diffusion coefficients of the above complexes of radiation defects; $k_I(x,y,z,T)$ and $k_V(x,y,z,T)$ are the parameters of decay of these complexes.

We calculate distributions of concentrations of point radiation defects in space and time by recently elaborated approach [23]. The approach based on transformation of approximations of diffusion coefficients in the following form:

$$D_\rho(x,y,z,T)=D_{0\rho}[1+\varepsilon_\rho g_\rho(x,y,z,T)],$$

where $D_{0\rho}$ are the average values of diffusion coefficients, $0\le\varepsilon_\rho<1$, $|g_\rho(x,y,z,T)|\le 1$, $\rho=I,V$. We also used analogous transformation of approximations of parameters of recombination of point defects and parameters of generation of their complexes: $k_{I,V}(x,y,z,T)=k_{0I,V}[1+\varepsilon_{I,V}g_{I,V}(x,y,z,T)]$, $k_{I,I}(x,y,z,T)=k_{0I,I}[1+\varepsilon_{I,I}g_{I,I}(x,y,z,T)]$ and $k_{V,V}(x,y,z,T)=k_{0V,V}[1+\varepsilon_{V,V}g_{V,V}(x,y,z,T)]$, where $k_{0\rho1,\rho2}$ are the their average values, $0\le\varepsilon_{I,V}<1$, $0\le\varepsilon_{I,I}<1$, $0\le\varepsilon_{V,V}<1$, $|g_{I,V}(x,y,z,T)|\le 1$, $|g_{I,I}(x,y,z,T)|\le 1$, $|g_{V,V}(x,y,z,T)|\le 1$. Let us introduce the following dimensionless variables:

$$\tilde I(x,y,z,t)=I(x,y,z,t)/I^*,\ \tilde V(x,y,z,t)=V(x,y,z,t)/V^*,\ \omega=L^2k_{0I,V}\big/\sqrt{D_{0I}D_{0V}},$$
$$\Omega_\rho=L^2k_{0\rho,\rho}\big/\sqrt{D_{0I}D_{0V}},\ \vartheta=\sqrt{D_{0I}D_{0V}}\,t/L^2,\ \chi=x/L_x,\ \eta=y/L_y,\ \phi=z/L_z.$$

The introduction leads to transformation of equations (4) and conditions (5) to the following form:

$$\frac{\partial \tilde{I}(\chi,\eta,\varphi,\vartheta)}{\partial \vartheta}=\frac{D_{0I}}{\sqrt{D_{0I}D_{0V}}}\frac{\partial}{\partial \chi}\left\{\left[1+\varepsilon_I g_I(\chi,\eta,\varphi,T)\right]\frac{\partial \tilde{I}(\chi,\eta,\varphi,\vartheta)}{\partial \chi}\right\}$$
$$+\frac{\partial}{\partial \eta}\left\{\left[1+\varepsilon_I g_I(\chi,\eta,\varphi,T)\right]\frac{\partial \tilde{I}(\chi,\eta,\varphi,\vartheta)}{\partial \eta}\right\}\frac{D_{0I}}{\sqrt{D_{0I}D_{0V}}}$$
$$+\frac{D_{0I}}{\sqrt{D_{0I}D_{0V}}}\frac{\partial}{\partial \varphi}\left\{\left[1+\varepsilon_I g_I(\chi,\eta,\varphi,T)\right]\frac{\partial \tilde{I}(\chi,\eta,\varphi,\vartheta)}{\partial \varphi}\right\}$$
$$-\tilde{I}(\chi,\eta,\varphi,\vartheta)\,\omega\left[1+\varepsilon_{I,V}g_{I,V}(\chi,\eta,\varphi,T)\right]\tilde{V}(\chi,\eta,\varphi,\vartheta)$$
$$-\Omega_I\left[1+\varepsilon_{I,I}g_{I,I}(\chi,\eta,\varphi,T)\right]\tilde{I}^2(\chi,\eta,\varphi,\vartheta),$$

(8)

$$\frac{\partial \tilde{V}(\chi,\eta,\varphi,\vartheta)}{\partial \vartheta}=\frac{D_{0V}}{\sqrt{D_{0I}D_{0V}}}\frac{\partial}{\partial \chi}\left\{\left[1+\varepsilon_V g_V(\chi,\eta,\varphi,T)\right]\frac{\partial \tilde{V}(\chi,\eta,\varphi,\vartheta)}{\partial \chi}\right\}$$
$$+\frac{\partial}{\partial \eta}\left\{\left[1+\varepsilon_V g_V(\chi,\eta,\varphi,T)\right]\frac{\partial \tilde{V}(\chi,\eta,\varphi,\vartheta)}{\partial \eta}\right\}\frac{D_{0V}}{\sqrt{D_{0I}D_{0V}}}$$
$$+\frac{D_{0V}}{\sqrt{D_{0I}D_{0V}}}\frac{\partial}{\partial \varphi}\left\{\left[1+\varepsilon_V g_V(\chi,\eta,\varphi,T)\right]\frac{\partial \tilde{V}(\chi,\eta,\varphi,\vartheta)}{\partial \varphi}\right\}$$
$$-\tilde{I}(\chi,\eta,\varphi,\vartheta)\,\omega\left[1+\varepsilon_{I,V}g_{I,V}(\chi,\eta,\varphi,T)\right]\tilde{V}(\chi,\eta,\varphi,\vartheta)$$
$$-\Omega_V\left[1+\varepsilon_{V,V}g_{V,V}(\chi,\eta,\varphi,T)\right]\tilde{V}^2(\chi,\eta,\varphi,\vartheta).$$

$$\left.\frac{\partial \tilde{\rho}(\chi,\eta,\varphi,\vartheta)}{\partial \chi}\right|_{\chi=0}=0,\quad \left.\frac{\partial \tilde{\rho}(\chi,\eta,\varphi,\vartheta)}{\partial \chi}\right|_{\chi=1}=0,\quad \left.\frac{\partial \tilde{\rho}(\chi,\eta,\varphi,\vartheta)}{\partial \eta}\right|_{\eta=0}=0,$$
$$\left.\frac{\partial \tilde{\rho}(\chi,\eta,\varphi,\vartheta)}{\partial \eta}\right|_{\eta=1}=0,\quad \left.\frac{\partial \tilde{\rho}(\chi,\eta,\varphi,\vartheta)}{\partial \varphi}\right|_{\varphi=0}=0,\quad \left.\frac{\partial \tilde{\rho}(\chi,\eta,\varphi,\vartheta)}{\partial \varphi}\right|_{\varphi=1}=0, \tag{9}$$
$$\tilde{\rho}(\chi,\eta,\varphi,\vartheta)=\frac{f_\rho(\chi,\eta,\varphi,\vartheta)}{\rho^*}.$$

We determine solutions of Equations (8) with conditions (9) framework recently introduced approach [23], i.e., as the power series

$$\tilde{\rho}(\chi,\eta,\varphi,\vartheta)=\sum_{i=0}^{\infty}\varepsilon_\rho^i\sum_{j=0}^{\infty}\omega^j\sum_{k=0}^{\infty}\Omega_\rho^k\tilde{\rho}_{ijk}(\chi,\eta,\varphi,\vartheta). \tag{10}$$

Substitution of the series (10) into Equations (8) and conditions (9) gives us possibility to obtain equations for initial-order approximations of concentration of point defects $\tilde{I}_{000}(\chi,\eta,\varphi,\vartheta)$ and $\tilde{V}_{000}(\chi,\eta,\varphi,\vartheta)$ and corrections for them $\tilde{I}_{ijk}(\chi,\eta,\varphi,\vartheta)$ and $\tilde{V}_{ijk}(\chi,\eta,\varphi,\vartheta)$, $i\geq 1$, $j\geq 1$, $k\geq 1$. The equations are presented in the Appendix. Solutions of the equations could be obtained by standard Fourier approach [6,25,29]. The solutions are presented in the Appendix.

Now we calculate distributions of concentrations of simplest complexes of point radiation defects in space and time. To determine the distributions, we transform approximations of diffusion coefficients in the following form:

$$D_{\Phi\rho}\left(x,y,z,T\right)=D_{0\Phi\rho}[1+e_{\Phi\rho}g_{\Phi\rho}\left(x,y,z,T\right)],$$

where $D_{0\Phi_\rho}$ are the average values of diffusion coefficients. In this situation the Equations (6) could be written as

$$\frac{\partial\,\Phi_I\left(x,y,z,t\right)}{\partial\,t}=D_{0\Phi I}\,\frac{\partial}{\partial\,x}\left\{\left[1+\varepsilon_{\Phi I}g_{\Phi I}\left(x,y,z,T\right)\right]\frac{\partial\,\Phi_I\left(x,y,z,t\right)}{\partial\,x}\right\}$$

$$+k_{I,I}\left(x,y,z,T\right)\,I^2\left(x,y,z,t\right)+D_{0\Phi I}\,\frac{\partial}{\partial\,y}\left\{\left[1+\varepsilon_{\Phi I}g_{\Phi I}\left(x,y,z,T\right)\right]\frac{\partial\,\Phi_I\left(x,y,z,t\right)}{\partial\,y}\right\}$$

$$+D_{0\Phi I}\,\frac{\partial}{\partial\,z}\left\{\left[1+\varepsilon_{\Phi I}g_{\Phi I}\left(x,y,z,T\right)\right]\frac{\partial\,\Phi_I\left(x,y,z,t\right)}{\partial\,z}\right\}-k_I\left(x,y,z,T\right)I\left(x,y,z,t\right),$$

$$\frac{\partial\,\Phi_V\left(x,y,z,t\right)}{\partial\,t}=D_{0\Phi V}\,\frac{\partial}{\partial\,x}\left\{\left[1+\varepsilon_{\Phi V}g_{\Phi V}\left(x,y,z,T\right)\right]\frac{\partial\,\Phi_V\left(x,y,z,t\right)}{\partial\,x}\right\}$$

$$+k_{I,I}\left(x,y,z,T\right)\,I^2\left(x,y,z,t\right)+D_{0\Phi V}\,\frac{\partial}{\partial\,y}\left\{\left[1+\varepsilon_{\Phi V}g_{\Phi V}\left(x,y,z,T\right)\right]\frac{\partial\,\Phi_V\left(x,y,z,t\right)}{\partial\,y}\right\}$$

$$+D_{0\Phi V}\,\frac{\partial}{\partial\,z}\left\{\left[1+\varepsilon_{\Phi V}g_{\Phi V}\left(x,y,z,T\right)\right]\frac{\partial\,\Phi_V\left(x,y,z,t\right)}{\partial\,z}\right\}-k_I\left(x,y,z,T\right)\,I\left(x,y,z,t\right).$$

Farther we determine solutions of above equations as the following power series:

$$\Phi_\rho\left(x,y,z,t\right)=\sum_{i=0}^{\infty}\varepsilon_{\Phi\rho}^i\Phi_{\rho i}\left(x,y,z,t\right). \tag{11}$$

Now we used the series (11) into Equations (6) and appropriate boundary and initial conditions. The using gives the possibility to obtain equations for initial-order approximations of concentrations of complexes of defects $\Phi_{\rho 0}\left(x,y,z,t\right)$, corrections for them $\Phi_{\rho i}\left(x,y,z,t\right)$ (for them $i\geq 1$) and boundary and initial conditions for them. We remove equations and conditions to the Appendix. Solutions of the equations have been calculated by standard Fourier approach [6,25,29] and presented in the Appendix.

Now we calculate distribution of concentration of dopant in space and time by using the approach, which was used for analysis of radiation defects. To use the approach, we consider following transformation of approximation of dopant diffusion coefficient:

$$D_L\left(x,y,z,T\right)=D_{0L}[1+\varepsilon_L g_L\left(x,y,z,T\right)],$$

where D_{0L} is the average value of dopant diffusion coefficient, $0\leq\varepsilon_L<1$, $\left|g_L\left(x,y,z,T\right)\right|\leq 1$. Farther we consider solution of (1) as the following series:

$$C\left(x,y,z,t\right)=\sum_{i=0}^{\infty}\varepsilon_L^i\sum_{j=1}^{\infty}\xi^j C_{ij}\left(x,y,z,t\right)..$$

Using the relation into (1) and conditions (2) leads to obtaining equations for the functions $C_{ij}\left(x,y,z,t\right)$ $(i\geq 1,\,j\geq 1)$, boundary and initial conditions for them. The equations are presented in the Appendix. Solutions of the equations have been calculated by standard Fourier approach [6,25,29]. The solutions are presented in the Appendix.

We analyzed distributions of concentrations of dopant and radiation defects in space and time analytically by using the second-order approximations on all parameters, which have been used in appropriate series. Usually the second-order approximations are enough good approximations to make qualitative analysis and to obtain quantitative results. All analytical results have been checked by numerical simulation.

It should be noted, that one can consider Fourier approach directly for solution of equations (1), (4) and (6). As an alternative approach it could be used approach, based on integral transformations [6,25]. But only few solutions have been calculated by using these approaches. The considered framework this paper combination of approaches gives a possibility to obtain solution of equations (1), (4) and (6) in more common form. It should be also noted, that crosslinking of solutions on interfaces between layers of heterostructure [6,25] could be used for ideal step-wise interface. Framework the paper we used more common approach, which could be used for both: step-wise and smooth interfaces between layers of heterostructure.

3. Discussion

In this section we analyzed spatio-temporal distributions of concentrations of dopants. Figure 2 shows typical spatial distributions of concentrations of dopants in neighborhood of interfaces of heterostructures. We calculate these distributions of concentrations of dopants under the following condition: value of dopant diffusion coefficient in doped area is larger, than value of dopant diffusion coefficient in nearest areas. In this situation one can find increasing of compactness of field-effect transistors with increasing of homogeneity of distribution of concentration of dopant at one time. Changing relation between values of dopant diffusion coefficients leads to opposite result (see Figure 3).

It should be noted, that framework the considered approach one shall optimize annealing of dopant and/or radiation defects. To do the optimization we used recently introduced criterion [16-22,24]. The optimization based on approximation real distribution by step-wise function $\psi(x,y,z)$ (see Figure 4). Farther the required values of optimal annealing time have been calculated by minimization the following mean-squared error

$$U=\frac{1}{L_xL_yL_z}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}\left[C(x,y,z,\Theta)-\psi(x,y,z)\right]d\,z\,d\,y\,d\,x. \tag{12}$$

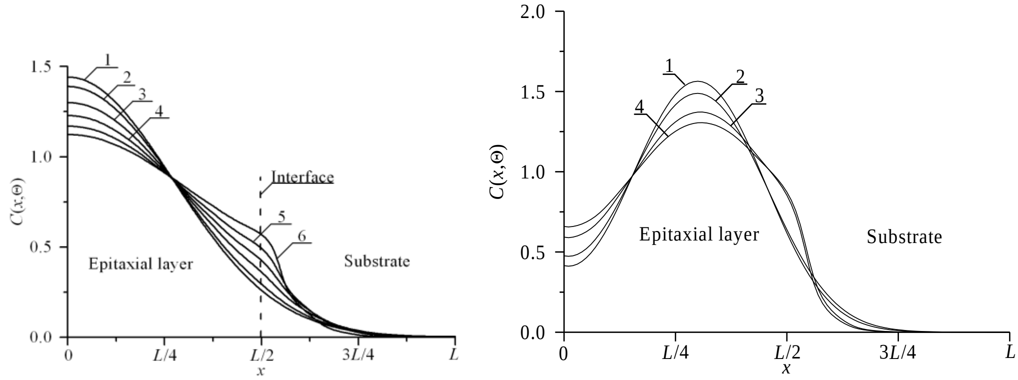
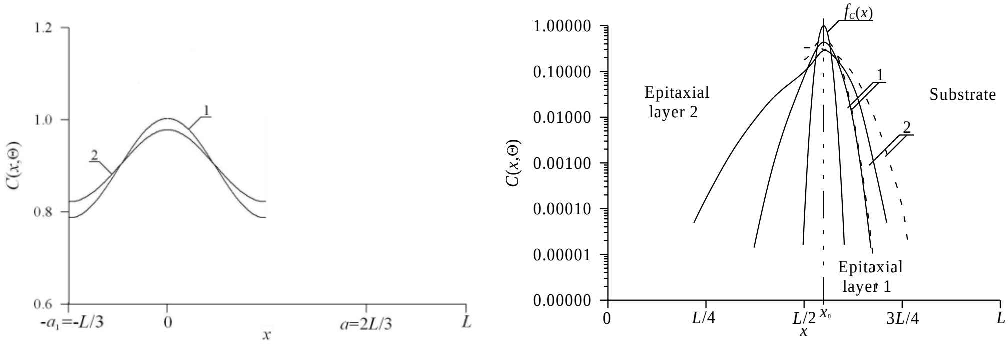

(a) (b)

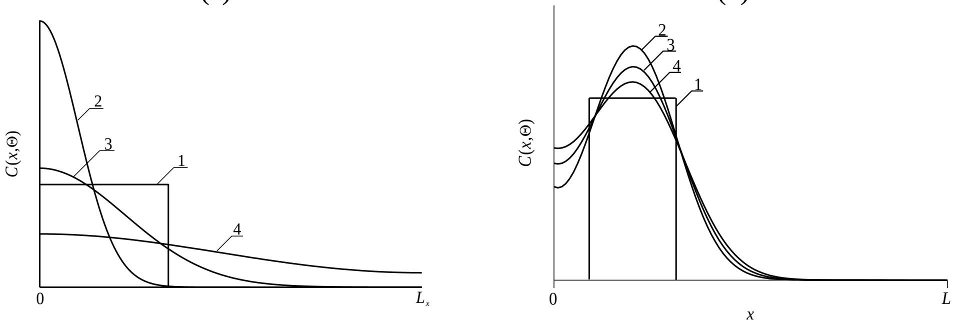
Figure 2. (a) Dependences of concentration of dopant, infused in heterostructure from Figure 1, on coordinate in direction, which is perpendicular to interface between epitaxial layer substrate. Difference between values of dopant diffusion coefficient in layers of heterostructure increases with increasing of number of curves. Value of dopant diffusion coefficient in the epitaxial layer is larger, than value of dopant diffusion coefficient in the substrate.
(b) Dependences of concentration of dopant, implanted in heterostructure from Figure 1, on coordinate in direction, which is perpendicular to interface between epitaxial layer substrate.

Difference between values of dopant diffusion coefficient in layers of heterostructure increases with increasing of number of curves. Value of dopant diffusion coefficient in the epitaxial layer is larger than value of dopant diffusion coefficient in the substrate. Curve 1 corresponds to homogenous sample and annealing time $\Theta = 0.0048(L_x^2 + L_y^2 + L_z^2) / D_0$. Curve 2 corresponds to homogenous sample and annealing time $\Theta = 0.0057(L_x^2 + L_y^2 + L_z^2) / D_0$. Curves 3 and 4 correspond to heterostructure from Figure 1; annealing times $\Theta = 0.0048(L_x^2 + L_y^2 + L_z^2) / D_0$ and $\Theta = 0.0057(L_x^2 + L_y^2 + L_z^2) / D_0$, respectively.



(a) (b)

Figure 3. (a) Distributions of concentration of dopant, infused in average section of epitaxial layer of heterostructure from Figure 1 in direction parallel to interface between epitaxial layer and substrate of heterostructure. Difference between values of dopant diffusion coefficients increases with increasing of number of curves. Value of dopant diffusion coefficient in this section is smaller, than value of dopant diffusion coefficient in nearest sections. (b) Calculated distributions of implanted dopant in epitaxial layers of heterostructure. Solid lines are spatial distributions of implanted dopant in system of two epitaxial layers. Dushed lines are spatial distributions of implanted dopant in one epitaxial layer. Annealing time increases with increasing of number of curves.



(a) (b)

Figure 4. (a) Distributions of concentration of infused dopant in depth of heterostructure from Figure 1 for different values of annealing time (curves 2-4) and idealized step-wise approximation (curve 1). Increasing of number of curve corresponds to increasing of annealing time. (b) Distributions of concentration of implanted dopant in depth of heterostructure from Figure 1 for different values of annealing time (curves 2-4) and idealized step-wise approximation (curve 1). Increasing of number of curve corresponds to increasing of annealing time.

We show optimal values of annealing time as functions of parameters on Figure 5. It is known, that standard step of manufactured ion-doped structures is annealing of radiation defects. In the ideal case after finishing the annealing dopant achieves interface between layers of heterostructure. If the dopant has no enough time to achieve the interface, it is practicably to anneal the dopant additionally. The Figure 5(b) shows the described dependences of optimal values of additional annealing time for the same parameters as for Figure 5(a) Necessity to anneal radiation defects leads to smaller values of optimal annealing of implanted dopant in comparison with optimal annealing time of infused dopant.

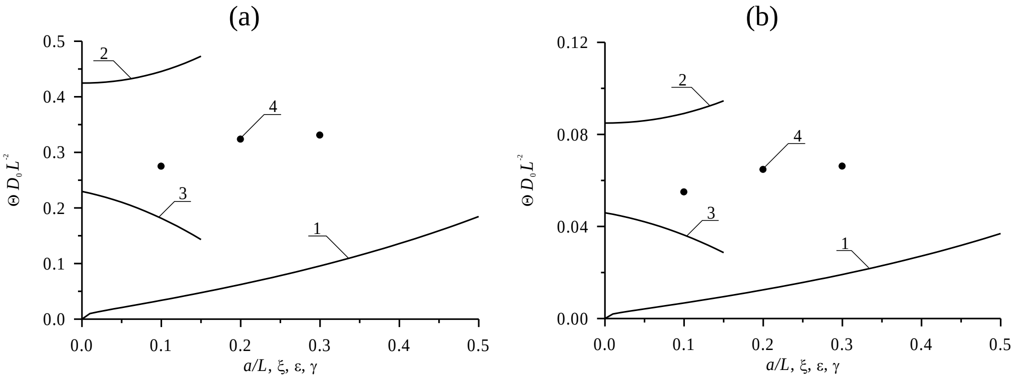


Figure 5. (a) Dimensionless optimal annealing time of infused dopant as a function of several parameters. Curve 1 describes the dependence of the annealing time on the relation *a/L* and $\xi = \gamma = 0$ for equal to each other values of dopant diffusion coefficient in all parts of heterostructure. Curve 2 describes the dependence of the annealing time on value of parameter ε for *a/L*=1/2 and $\xi = \gamma = 0$. Curve 3 describes the dependence of the annealing time on value of parameter ξ for *a/L*=1/2 and $\varepsilon = \gamma = 0$. Curve 4 describes the dependence of the annealing time on value of parameter γ for *a/L*=1/2 and $\varepsilon = \xi = 0$.

(b) Dimensionless optimal annealing time of implanted dopant as a function of several parameters. Curve 1 describes the dependence of the annealing time on the relation *a/L* and $\xi = \gamma = 0$ for equal to each other values of dopant diffusion coefficient in all parts of heterostructure. Curve 2 describes the dependence of the annealing time on value of parameter ε for *a/L*=1/2 and $\xi = \gamma = 0$. Curve 3 describes the dependence of the annealing time on value of parameter ξ for *a/L*=1/2 and $\varepsilon = \gamma = 0$. Curve 4 describes the dependence of the annealing time on value of parameter γ for *a/L*=1/2 and $\varepsilon = \xi = 0$.

It should be noted, that using inhomogeneity of heterostructure leads to increasing density of elements of the considered current source. At the same time using laser and microwave types [2,5,11,15,26-28,30,32] of annealing also leads to inhomogeneity of doping materials due to inhomogeneity of temperature and Arrhenius law. However, using inhomogeneity of heterostructure gives a possibility to obtain larger increasing density of the considered elements.

4. Conclusions

In this paper we introduce an approach to increase integration rate of element of a current source circuit. The approach gives us possibility to decrease area of the elements with smaller increasing of the element's thickness.

References

[1] A. O. Ageev, A. E. Belyaev, N. S. Boltovets, V. N. Ivanov, R. V. Konakova, Ya. Ya. Kudrik, P. M. Litvin, V. V. Milenin and A. V. Sachenko, Au-TiB$_x$ -*n*-6*H*-SiC Schottky barrier diodes: Specific features of charge transport in rectifying and nonrectifying contacts, Semiconductors, **43 (7)** (2009) 865–871.
[2] O. V. Aleksandrov, A. O. Zakhar'in, N. A. Sobolev, E. I. Shek, M. I. Makoviichuk and E. O. Parshin, Formation

of donor centers upon annealing of dysprosium-and holmium-implanted silicon, Semiconductors, **32 (9)** (1998) 921–923.

[3] A. G. Alexenko and I. I. Shagurin, Microcircuitry, Radio and communication, Moscow, (1990).

[4] N. A. Avaev, Yu. E. Naumov and V. T. Frolkin, Basis of Microelectronics, Radio and Communication, Moscow, (1991).

[5] Yu. V. Bykov, A. G. Yeremeev, N. A. Zharova, I. V. Plotnikov, K. I. Rybakov, M. N. Drozdov, Yu. N. Drozdov and V. D. Skupov, Diffusion processes in semiconductor structures during microwave annealing, Radiophysics and Quantum Electronics, **46** (2003) 749–755.

[6] H. S. Carslaw and J. C. Jaeger, Conduction of Heat in Solids, Oxford University Press, (1964).

[7] S. A. Chachuli, P. N. A. Fasyar, N. Soin, N. M. Kar and N. Yusop, Pareto ANOVA analysis for CMOS 0.18 µm two-stage Op-amp, Materials Science in Semiconductor Processing, **24** (2014) 9–14.

[8] D. Fathi, B. Forouzandeh and N. Masoumi, New enhanced noise analysis in active mixers in nanoscale technologies, Nano, **4 (4)** (2009) 233–238.

[9] Z. Yu. Gotra, Technology of Microelectronic Devices, Radio and Communication, Moscow, (1991).

[10] V. V. Kozlivsky, Modification of Semiconductors by Proton Beams, Nauka, Sant-Peterburg, (2003).

[11] M. J. Kumar and T. V. Singh, Quantum confinement effect in strained silicon mosfets, International Journal of Nanoscience, **7 (2-3)** (2008) 81–84.

[12] V. I. Lachin and N. S. Savelov, Electronics, Phoenix, Rostov-na-Donu, (2001).

[13] Z. Li, J. Waldron, T. Detchprohm, C. Wetzel, R. F. Karlicek and Jr. T. P. Chow, Monolithic integration of light-emitting diodes and power metal-oxide-semiconductor channel high-electron-mobility transistors for light-emitting power integrated circuits in GaN on sapphire substrate, Applied Physics Letters, **102 (19)** (2013) 192107, doi:10.1063/1.4807125.

[14] M. Nenadovic, G. Fischer and N. Fiebig, A 32 ppm/°C temperature-compensated operational amplifier for application in medical device tracking, Analog Integrated Circuits and Signal Processing, **87** (2016) 117–127.

[15] K. K. Ong, K. L. Pey, P. S. Lee, A. T. S. Wee, X. C. Wang and Y. F. Chong, Dopant distribution in the recrystallization transient at the maximum melt depth induced by laser annealing, Applied Physics Letters, **89 (17)** (2006) 172111, doi:10.1063/1.2364834.

[16] E. L. Pankratov, Dopant diffusion dynamics and optimal diffusion time as influenced by diffusion-coefficient nonuniformity, Russian Microelectronics, **36 (1)** (2007) 33–39.

[17] E. L. Pankratov, On approach to optimize manufacturing of bipolar heterotransistors framework circuit of an operational amplifier to increase their integration rate. Influence mismatch-induced stress, Journal of Computational and Theoretical Nanoscience, **14 (10)** (2017) 4885–4899.

[18] E. L. Pankratov, On optimization of manufacturing of two-phase logic circuit based on heterostructures to increase density of their elements. Influence of miss-match induced stress, Advanced Science, Engineering and Medicine, **9 (9)** (2017) 787–801 (2017).

[19] E. L. Pankratov, Redistribution of a dopant during annealing of radiation defects in a multilayer structure by laser scans for production of an implanted-junction rectifier, International Journal of Nanoscience, **7 (4-5)** (2008) 187–197.

[20] E. L. Pankratov and E. A. Bulaeva, An analytical approach for analysis and optimization of formation of field-effect heterotransistors, Multidiscipline Modeling in Materials and Structures, **12 (4)** (2016) 578–604.

[21] E. L. Pankratov and E. A. Bulaeva, An approach to increase the integration rate of planar drift heterobipolar transistors, Materials Science in Semiconductor Processing, **34** (2015) 260–268.

[22] E. L. Pankratov and E. A. Bulaeva, An approach to manufacture a heterobipolar transistors in thin film structures. On the method of optimization, International Journal of Micro-Nano Scale Transport, **4 (1)** (2014) 17–32.

[23] E. L. Pankratov and E. A. Bulaeva, Doping of materials during manufacture p-n-junctions and bipolar transistors. Analytical approaches to model technological approaches and ways of optimization of distributions of dopants, Reviews in Theoretical Science, **1 (1)** (2013) 58–82.

[24] E. L. Pankratov, E. A. Bulaeva, On increasing of density of transistors in a hybrid cascaded multilevel inverter, Multidiscipline Modeling in Materials and Structures, **13 (4)** (2017) 664–677.

[25] A. I. Raichenko, Mathematical theory of diffusion in applications, Naukova Dumka, Kiev, (1981).

[26] J. G. Reynolds, C. L. Reynolds, Jr. A. Mohanta, J. F. Muth, J. E. Rowe, H. O. Everitt and D. E. Aspnes, Shallow acceptor complexes in p-type ZnO, Applied Physics Letters, **102 (15)** (2013) 152114, doi:10.1063/1.4802753.

[27] P. Sinsermsuksakul, K. Hartman, S. B. Kim, J. Heo, L. Sun, H. H. Park, R. Chakraborty, T. Buonassisi, R. G. Gordon. Applied Physics Letters, **102 (5)** (2013) 053901, doi:10.1063/1.4789855.

[28] S. T. Șișianu, T. S. Șișianu and S. K. Railyan, Shallow *p-n* junctions formed in silicon using pulsed photon annealing, Semiconductors, **36 (5)** (2002) 581-587.

[29] A. N. Tikhonov and A. A. Samarskii, The Mathematical Physics Equations, Nauka, Moscow (1972).

[30] J.-H. Tsai, S.-Y. Chiu, W.-S. Lour and D.-F. Guo, High-performance InGaP/GaAs *pnp* δ-doped heterojunction bipolar transistor, Semiconductors, **43 (7)** (2009) 939–942.

[31] V. L. Vinetskiy and G. A. Kholodar, Radiative Physics of Semiconductors, Naukova Dumka, Kiev, (1979).

[32] H. T. Wang, L. S. Tan and E. F. Chor, Pulsed laser annealing of Be-implanted GaN, Journal of Applied Physics, **98 (9)** (2006) 094901, doi:10.1063/1.2120893.

Appendix

Equations for the functions $\tilde{I}_{ijk}(\chi,\eta,\varphi,\vartheta)$ and $\tilde{V}_{ijk}(\chi,\eta,\varphi,\vartheta)$, $i\geq 0,\ j\geq 0,\ k\geq 0$ and conditions for them

$$\frac{\partial \tilde{I}_{000}(\chi,\eta,\varphi,\vartheta)}{\partial \vartheta}=\sqrt{\frac{D_{0I}}{D_{0V}}}\left[\frac{\partial^2 \tilde{I}_{000}(\chi,\eta,\varphi,\vartheta)}{\partial \chi^2}+\frac{\partial^2 \tilde{I}_{000}(\chi,\eta,\varphi,\vartheta)}{\partial \eta^2}+\frac{\partial^2 \tilde{I}_{000}(\chi,\eta,\varphi,\vartheta)}{\partial \varphi^2}\right],$$

$$\frac{\partial \tilde{V}_{000}(\chi,\eta,\varphi,\vartheta)}{\partial \vartheta}=\sqrt{\frac{D_{0V}}{D_{0I}}}\left[\frac{\partial^2 \tilde{V}_{000}(\chi,\eta,\varphi,\vartheta)}{\partial \chi^2}+\frac{\partial^2 \tilde{V}_{000}(\chi,\eta,\varphi,\vartheta)}{\partial \eta^2}+\frac{\partial^2 \tilde{V}_{000}(\chi,\eta,\varphi,\vartheta)}{\partial \varphi^2}\right],$$

$$\begin{aligned}\frac{\partial \tilde{I}_{i00}(\chi,\vartheta)}{\partial \vartheta}&=\sqrt{\frac{D_{0I}}{D_{0V}}}\left[\frac{\partial^2 \tilde{I}_{i00}(\chi,\eta,\varphi,\vartheta)}{\partial \chi^2}+\frac{\partial^2 \tilde{I}_{i00}(\chi,\eta,\varphi,\vartheta)}{\partial \eta^2}+\frac{\partial^2 \tilde{I}_{i00}(\chi,\eta,\varphi,\vartheta)}{\partial \varphi^2}\right]\\&+\sqrt{\frac{D_{0I}}{D_{0V}}}\left\{\frac{\partial}{\partial \chi}\left[g_I(\chi,\eta,\varphi,T)\frac{\partial \tilde{I}_{i-100}(\chi,\eta,\varphi,\vartheta)}{\partial \chi}\right]\right.\\&+\frac{\partial}{\partial \eta}\left[g_I(\chi,\eta,\varphi,T)\frac{\partial \tilde{I}_{i-100}(\chi,\eta,\varphi,\vartheta)}{\partial \eta}\right]\\&\left.+\frac{\partial}{\partial \varphi}\left[g_I(\chi,\eta,\varphi,T)\frac{\partial \tilde{I}_{i-100}(\chi,\eta,\varphi,\vartheta)}{\partial \varphi}\right]\right\},\ i\geq 1,\end{aligned}$$

$$\begin{aligned}\frac{\partial \tilde{V}_{i00}(\chi,\vartheta)}{\partial \vartheta}&=\sqrt{\frac{D_{0V}}{D_{0I}}}\left[\frac{\partial^2 \tilde{V}_{i00}(\chi,\eta,\varphi,\vartheta)}{\partial \chi^2}+\frac{\partial^2 \tilde{V}_{i00}(\chi,\eta,\varphi,\vartheta)}{\partial \eta^2}+\frac{\partial^2 \tilde{V}_{i00}(\chi,\eta,\varphi,\vartheta)}{\partial \varphi^2}\right]\\&+\frac{\partial}{\partial \chi}\left[g_V(\chi,\eta,\varphi,T)\frac{\partial \tilde{V}_{i-100}(\chi,\eta,\varphi,\vartheta)}{\partial \chi}\right]\sqrt{\frac{D_{0V}}{D_{0I}}}\\&+\sqrt{\frac{D_{0V}}{D_{0I}}}\frac{\partial}{\partial \eta}\left[g_V(\chi,\eta,\varphi,T)\frac{\partial \tilde{V}_{i-100}(\chi,\eta,\varphi,\vartheta)}{\partial \eta}\right]\\&+\frac{\partial}{\partial \varphi}\left[g_V(\chi,\eta,\varphi,T)\frac{\partial \tilde{V}_{i-100}(\chi,\eta,\varphi,\vartheta)}{\partial \varphi}\right]\sqrt{\frac{D_{0V}}{D_{0I}}},\ i\geq 1,\end{aligned}$$

$$\begin{aligned}\frac{\partial \tilde{I}_{010}(\chi,\eta,\varphi,\vartheta)}{\partial \vartheta}&=\sqrt{\frac{D_{0I}}{D_{0V}}}\left[\frac{\partial^2 \tilde{I}_{010}(\chi,\eta,\varphi,\vartheta)}{\partial \chi^2}+\frac{\partial^2 \tilde{I}_{010}(\chi,\eta,\varphi,\vartheta)}{\partial \eta^2}+\frac{\partial^2 \tilde{I}_{010}(\chi,\eta,\varphi,\vartheta)}{\partial \varphi^2}\right]\\&-\left[1+\varepsilon_{I,V}g_{I,V}(\chi,\eta,\varphi,T)\right]\tilde{I}_{000}(\chi,\eta,\varphi,\vartheta)\,\tilde{V}_{000}(\chi,\eta,\varphi,\vartheta),\end{aligned}$$

$$\begin{aligned}\frac{\partial \tilde{V}_{010}(\chi,\eta,\varphi,\vartheta)}{\partial \vartheta}&=\sqrt{\frac{D_{0V}}{D_{0I}}}\left[\frac{\partial^2 \tilde{V}_{010}(\chi,\eta,\varphi,\vartheta)}{\partial \chi^2}+\frac{\partial^2 \tilde{V}_{010}(\chi,\eta,\varphi,\vartheta)}{\partial \eta^2}+\frac{\partial^2 \tilde{V}_{010}(\chi,\eta,\varphi,\vartheta)}{\partial \varphi^2}\right]\\&-\left[1+\varepsilon_{I,V}g_{I,V}(\chi,\eta,\varphi,T)\right]\tilde{I}_{000}(\chi,\eta,\varphi,\vartheta)\tilde{V}_{000}(\chi,\eta,\varphi,\vartheta),\end{aligned}$$

$$\begin{aligned}\frac{\partial \tilde{I}_{020}(\chi,\eta,\varphi,\vartheta)}{\partial \vartheta}&=\sqrt{\frac{D_{0I}}{D_{0V}}}\left[\frac{\partial^2 \tilde{I}_{020}(\chi,\eta,\varphi,\vartheta)}{\partial \chi^2}+\frac{\partial^2 \tilde{I}_{020}(\chi,\eta,\varphi,\vartheta)}{\partial \eta^2}+\frac{\partial^2 \tilde{I}_{020}(\chi,\eta,\varphi,\vartheta)}{\partial \varphi^2}\right]\\&-\left[1+\varepsilon_{I,V}g_{I,V}(\chi,\eta,\varphi,T)\right]\left[\tilde{I}_{010}(\chi,\eta,\varphi,\vartheta)\,\tilde{V}_{000}(\chi,\eta,\varphi,\vartheta)\right.\\&\left.+\tilde{I}_{000}(\chi,\eta,\varphi,\vartheta)\tilde{V}_{010}(\chi,\eta,\varphi,\vartheta)\right],\end{aligned}$$

$$\frac{\partial \tilde{V}_{020}(\chi,\eta,\varphi,\vartheta)}{\partial \vartheta}=\sqrt{\frac{D_{0I}}{D_{0V}}}\left[\frac{\partial^2\tilde{V}_{020}(\chi,\eta,\varphi,\vartheta)}{\partial \chi^2}+\frac{\partial^2\tilde{V}_{020}(\chi,\eta,\varphi,\vartheta)}{\partial \eta^2}+\frac{\partial^2\tilde{V}_{020}(\chi,\eta,\varphi,\vartheta)}{\partial \varphi^2}\right]$$
$$-\left[1+\varepsilon_{I,V}g_{I,V}(\chi,\eta,\varphi,T)\right]\left[\tilde{I}_{010}(\chi,\eta,\varphi,\vartheta)\tilde{V}_{000}(\chi,\eta,\varphi,\vartheta)\right.$$
$$\left.+\tilde{I}_{000}(\chi,\eta,\varphi,\vartheta)\tilde{V}_{010}(\chi,\eta,\varphi,\vartheta)\right],$$

$$\frac{\partial \tilde{I}_{001}(\chi,\eta,\varphi,\vartheta)}{\partial \vartheta}=\sqrt{\frac{D_{0I}}{D_{0V}}}\left[\frac{\partial^2\tilde{I}_{001}(\chi,\eta,\varphi,\vartheta)}{\partial \chi^2}+\frac{\partial^2\tilde{I}_{001}(\chi,\eta,\varphi,\vartheta)}{\partial \eta^2}+\frac{\partial^2\tilde{I}_{001}(\chi,\eta,\varphi,\vartheta)}{\partial \varphi^2}\right]$$
$$-\left[1+\varepsilon_{I,I}g_{I,I}(\chi,\eta,\varphi,T)\right]\tilde{I}^2_{000}(\chi,\eta,\varphi,\vartheta),$$

$$\frac{\partial \tilde{V}_{001}(\chi,\eta,\varphi,\vartheta)}{\partial \vartheta}=\sqrt{\frac{D_{0V}}{D_{0I}}}\left[\frac{\partial^2\tilde{V}_{001}(\chi,\eta,\varphi,\vartheta)}{\partial \chi^2}+\frac{\partial^2\tilde{V}_{001}(\chi,\eta,\varphi,\vartheta)}{\partial \eta^2}+\frac{\partial^2\tilde{V}_{001}(\chi,\eta,\varphi,\vartheta)}{\partial \varphi^2}\right]$$
$$-\left[1+\varepsilon_{I,I}g_{I,I}(\chi,\eta,\varphi,T)\right]\tilde{V}^2_{000}(\chi,\eta,\varphi,\vartheta),$$

$$\frac{\partial \tilde{I}_{110}(\chi,\eta,\varphi,\vartheta)}{\partial \vartheta}$$
$$=\sqrt{\frac{D_{0I}}{D_{0V}}}\left[\frac{\partial^2\tilde{I}_{110}(\chi,\eta,\varphi,\vartheta)}{\partial \chi^2}+\frac{\partial^2\tilde{I}_{110}(\chi,\eta,\varphi,\vartheta)}{\partial \eta^2}+\frac{\partial^2\tilde{I}_{110}(\chi,\eta,\varphi,\vartheta)}{\partial \varphi^2}\right]$$
$$+\sqrt{\frac{D_{0I}}{D_{0V}}}\left\{\frac{\partial}{\partial \chi}\left[g_I(\chi,\eta,\varphi,T)\frac{\partial \tilde{I}_{010}(\chi,\eta,\varphi,\vartheta)}{\partial \chi}\right]+\frac{\partial}{\partial \eta}\left[g_I(\chi,\eta,\varphi,T)\frac{\partial \tilde{I}_{010}(\chi,\eta,\varphi,\vartheta)}{\partial \eta}\right]\right.$$
$$\left.+\frac{\partial}{\partial \varphi}\left[g_I(\chi,\eta,\varphi,T)\frac{\partial \tilde{I}_{010}(\chi,\eta,\varphi,\vartheta)}{\partial \varphi}\right]\right\}$$
$$-\left[\tilde{I}_{100}(\chi,\eta,\varphi,\vartheta)\tilde{V}_{000}(\chi,\eta,\varphi,\vartheta)+\tilde{I}_{000}(\chi,\eta,\varphi,\vartheta)\tilde{V}_{100}(\chi,\eta,\varphi,\vartheta)\right]\left[1+\varepsilon_{I,I}g_{I,I}(\chi,\eta,\varphi,T)\right],$$

$$\frac{\partial \tilde{V}_{110}(\chi,\eta,\varphi,\vartheta)}{\partial \vartheta}$$
$$=\sqrt{\frac{D_{0V}}{D_{0I}}}\left[\frac{\partial^2\tilde{V}_{110}(\chi,\eta,\varphi,\vartheta)}{\partial \chi^2}+\frac{\partial^2\tilde{V}_{110}(\chi,\eta,\varphi,\vartheta)}{\partial \eta^2}+\frac{\partial^2\tilde{V}_{110}(\chi,\eta,\varphi,\vartheta)}{\partial \varphi^2}\right]$$
$$+\sqrt{\frac{D_{0V}}{D_{0I}}}\left\{\frac{\partial}{\partial \chi}\left[g_V(\chi,\eta,\varphi,T)\frac{\partial \tilde{V}_{010}(\chi,\eta,\varphi,\vartheta)}{\partial \chi}\right]+\frac{\partial}{\partial \eta}\left[g_V(\chi,\eta,\varphi,T)\frac{\partial \tilde{V}_{010}(\chi,\eta,\varphi,\vartheta)}{\partial \eta}\right]\right.$$
$$\left.+\frac{\partial}{\partial \varphi}\left[g_V(\chi,\eta,\varphi,T)\frac{\partial \tilde{V}_{010}(\chi,\eta,\varphi,\vartheta)}{\partial \varphi}\right]\right\}$$
$$-\left[1+\varepsilon_{V,V}g_{V,V}(\chi,\eta,\varphi,T)\right]\left[\tilde{V}_{100}(\chi,\eta,\varphi,\vartheta)\,\tilde{I}_{000}(\chi,\eta,\varphi,\vartheta)+\tilde{V}_{000}(\chi,\eta,\varphi,\vartheta)\tilde{I}_{100}(\chi,\eta,\varphi,\vartheta)\right],$$

$$\frac{\partial \tilde{I}_{002}(\chi,\eta,\varphi,\vartheta)}{\partial \vartheta}=\sqrt{\frac{D_{0I}}{D_{0V}}}\left[\frac{\partial^2\tilde{I}_{002}(\chi,\eta,\varphi,\vartheta)}{\partial \chi^2}+\frac{\partial^2\tilde{I}_{002}(\chi,\eta,\varphi,\vartheta)}{\partial \eta^2}+\frac{\partial^2\tilde{I}_{002}(\chi,\eta,\varphi,\vartheta)}{\partial \varphi^2}\right]$$
$$-\left[1+\varepsilon_{I,I}g_{I,I}(\chi,\eta,\varphi,T)\right]\tilde{I}_{001}(\chi,\eta,\varphi,\vartheta)\tilde{I}_{000}(\chi,\eta,\varphi,\vartheta),$$

$$\frac{\partial \tilde{V}_{002}(\chi,\eta,\varphi,\vartheta)}{\partial \vartheta}=\sqrt{\frac{D_{0V}}{D_{0I}}}\left[\frac{\partial^2 \tilde{V}_{002}(\chi,\eta,\varphi,\vartheta)}{\partial \chi^2}+\frac{\partial^2 \tilde{V}_{002}(\chi,\eta,\varphi,\vartheta)}{\partial \eta^2}+\frac{\partial^2 \tilde{V}_{002}(\chi,\eta,\varphi,\vartheta)}{\partial \varphi^2}\right]$$
$$-\left[1+\varepsilon_{V,V}\,g_{V,V}(\chi,\eta,\varphi,E)\right]\tilde{V}_{001}(\chi,\eta,\varphi,\vartheta)\tilde{V}_{000}(\chi,\eta,\varphi,\vartheta),$$

$$\frac{\partial \tilde{I}_{101}(\chi,\eta,\varphi,\vartheta)}{\partial \vartheta}$$
$$=\sqrt{\frac{D_{0I}}{D_{0V}}}\left[\frac{\partial^2 \tilde{I}_{101}(\chi,\eta,\varphi,\vartheta)}{\partial \chi^2}+\frac{\partial^2 \tilde{I}_{101}(\chi,\eta,\varphi,\vartheta)}{\partial \eta^2}+\frac{\partial^2 \tilde{I}_{101}(\chi,\eta,\varphi,\vartheta)}{\partial \varphi^2}\right]$$
$$+\sqrt{\frac{D_{0I}}{D_{0V}}}\left\{\frac{\partial}{\partial \chi}\left[g_I(\chi,\eta,\varphi,T)\frac{\partial \tilde{I}_{001}(\chi,\eta,\varphi,\vartheta)}{\partial \chi}\right]+\frac{\partial}{\partial \eta}\left[g_I(\chi,\eta,\varphi,T)\frac{\partial \tilde{I}_{001}(\chi,\eta,\varphi,\vartheta)}{\partial \eta}\right]\right.$$
$$\left.+\frac{\partial}{\partial \varphi}\left[g_I(\chi,\eta,\varphi,T)\frac{\partial \tilde{I}_{001}(\chi,\eta,\varphi,\vartheta)}{\partial \varphi}\right]\right\}$$
$$-\left[1+\varepsilon_I\,g_I(\chi,\eta,\varphi,T)\right]\tilde{I}_{100}(\chi,\eta,\varphi,\vartheta)\tilde{V}_{000}(\chi,\eta,\varphi,\vartheta),$$

$$\frac{\partial \tilde{V}_{101}(\chi,\eta,\varphi,\vartheta)}{\partial \vartheta}$$
$$=\sqrt{\frac{D_{0V}}{D_{0I}}}\left[\frac{\partial^2 \tilde{V}_{101}(\chi,\eta,\varphi,\vartheta)}{\partial \chi^2}+\frac{\partial^2 \tilde{V}_{101}(\chi,\eta,\varphi,\vartheta)}{\partial \eta^2}+\frac{\partial^2 \tilde{V}_{101}(\chi,\eta,\varphi,\vartheta)}{\partial \varphi^2}\right]$$
$$+\sqrt{\frac{D_{0V}}{D_{0I}}}\left\{\frac{\partial}{\partial \chi}\left[g_V(\chi,\eta,\varphi,T)\frac{\partial \tilde{V}_{001}(\chi,\eta,\varphi,\vartheta)}{\partial \chi}\right]+\frac{\partial}{\partial \eta}\left[g_V(\chi,\eta,\varphi,T)\frac{\partial \tilde{V}_{001}(\chi,\eta,\varphi,\vartheta)}{\partial \eta}\right]\right.$$
$$\left.+\frac{\partial}{\partial \varphi}\left[g_V(\chi,\eta,\varphi,T)\frac{\partial \tilde{V}_{001}(\chi,\eta,\varphi,\vartheta)}{\partial \varphi}\right]\right\}$$
$$-\left[1+\varepsilon_V\,g_V(\chi,\eta,\varphi,T)\right]\tilde{I}_{000}(\chi,\eta,\varphi,\vartheta)\tilde{V}_{100}(\chi,\eta,\varphi,\vartheta),$$

$$\frac{\partial \tilde{I}_{011}(\chi,\eta,\varphi,\vartheta)}{\partial \vartheta}=\sqrt{\frac{D_{0I}}{D_{0V}}}\left[\frac{\partial^2 \tilde{I}_{011}(\chi,\eta,\varphi,\vartheta)}{\partial \chi^2}+\frac{\partial^2 \tilde{I}_{011}(\chi,\eta,\varphi,\vartheta)}{\partial \eta^2}+\frac{\partial^2 \tilde{I}_{011}(\chi,\eta,\varphi,\vartheta)}{\partial \varphi^2}\right]$$
$$-\tilde{I}_{010}(\chi,\eta,\varphi,\vartheta)\left[1+\varepsilon_{I,I}\,g_{I,I}(\chi,\eta,\varphi,T)\right]\tilde{I}_{000}(\chi,\eta,\varphi,\vartheta)$$
$$-\left[1+\varepsilon_{I,V}\,g_{I,V}(\chi,\eta,\varphi,T)\right]\tilde{I}_{001}(\chi,\eta,\varphi,\vartheta)\,\tilde{V}_{000}(\chi,\eta,\varphi,\vartheta),$$

$$\frac{\partial \tilde{V}_{011}(\chi,\eta,\varphi,\vartheta)}{\partial \vartheta}=\sqrt{\frac{D_{0V}}{D_{0I}}}\left[\frac{\partial^2 \tilde{V}_{011}(\chi,\eta,\varphi,\vartheta)}{\partial \chi^2}+\frac{\partial^2 \tilde{V}_{011}(\chi,\eta,\varphi,\vartheta)}{\partial \eta^2}+\frac{\partial^2 \tilde{V}_{011}(\chi,\eta,\varphi,\vartheta)}{\partial \varphi^2}\right]$$
$$-\tilde{V}_{010}(\chi,\eta,\varphi,\vartheta)\left[1+\varepsilon_{V,V}\,g_{V,V}(\chi,\eta,\varphi,T)\right]\tilde{V}_{000}(\chi,\eta,\varphi,\vartheta)$$
$$-\left[1+\varepsilon_{I,V}\,g_{I,V}(\chi,\eta,\varphi,t)\right]\tilde{I}_{000}(\chi,\eta,\varphi,\vartheta)\,\tilde{V}_{001}(\chi,\eta,\varphi,\vartheta),$$

$$\left.\frac{\partial \tilde{\rho}_{ijk}(\chi,\eta,\varphi,\vartheta)}{\partial \chi}\right|_{x=0}=0,\quad \left.\frac{\partial \tilde{\rho}_{ijk}(\chi,\eta,\varphi,\vartheta)}{\partial \chi}\right|_{x=1}=0,\quad \left.\frac{\partial \tilde{\rho}_{ijk}(\chi,\eta,\varphi,\vartheta)}{\partial \eta}\right|_{\eta=0}=0,$$

$$\left.\frac{\partial\tilde{\rho}_{ijk}(\chi,\eta,\varphi,\vartheta)}{\partial\eta}\right|_{\eta=1}=0,\quad \left.\frac{\partial\tilde{\rho}_{ijk}(\chi,\eta,\varphi,\vartheta)}{\partial\varphi}\right|_{\varphi=0}=0,\quad \left.\frac{\partial\tilde{\rho}_{ijk}(\chi,\eta,\varphi,\vartheta)}{\partial\varphi}\right|_{\varphi=1}=0,$$

$$\tilde{\rho}_{000}(\chi,\eta,\varphi,0)=f_{\rho}(\chi,\eta,\varphi)/\rho^{*},\ \ \tilde{\rho}_{ijk}(\chi,\eta,\varphi,0)=0,\ \ (i\geq 1,\ j\geq 1,\ k\geq 1).$$

Solutions of the above equations could be written as

$$\tilde{\rho}_{000}(\chi,\eta,\varphi,\vartheta)=\frac{1}{L}+\frac{2}{L}\sum_{n=1}^{\infty}F_{n\rho}c(\chi)c(\eta)c(\varphi)e_{n\rho}(\vartheta),$$

where

$$F_{n\rho}=\frac{1}{\rho^{*}}\int_{0}^{1}\cos(\pi n u)\int_{0}^{1}\cos(\pi n v)\int_{0}^{1}\cos(\pi n w)\,f_{n\rho}(u,v,w)\,d\,w\,d\,v\,d\,u,$$

$$c_{n}(\chi)\ =cos(p\,n\chi),$$

$$e_{nI}(\vartheta)=\exp\left(-\pi^{2}n^{2}\vartheta\sqrt{D_{0V}/D_{0I}}\right),$$

$$e_{nV}(\vartheta)=\exp\left(-\pi^{2}n^{2}\vartheta\sqrt{D_{0I}/D_{0V}}\right);$$

$$\begin{aligned}
&\tilde{I}_{i00}(\chi,\eta,\varphi,\vartheta)=\\
&-2\pi\sqrt{\frac{D_{0I}}{D_{0V}}}\sum_{n=1}^{\infty}n\,c_{n}(\chi)c(\eta)c(\varphi)e_{nI}(\vartheta)\int_{0}^{\vartheta}e_{nI}(-\tau)\int_{0}^{1}s_{n}(u)\int_{0}^{1}c_{n}(v)\\
&\qquad\times\int_{0}^{1}\frac{\partial\tilde{I}_{i-100}(u,v,w,\tau)}{\partial u}c_{n}(w)g_{I}(u,v,w,T)\,d\,w\,d\,v\,d\,u\,d\,\tau\\
&-2\pi\sqrt{\frac{D_{0I}}{D_{0V}}}\sum_{n=1}^{\infty}n\,c_{n}(\chi)c(\eta)c(\varphi)e_{nI}(\vartheta)\int_{0}^{\vartheta}e_{nI}(-\tau)\int_{0}^{1}c_{n}(u)\int_{0}^{1}s_{n}(v)\\
&\qquad\times\int_{0}^{1}c_{n}(w)g_{I}(u,v,w,T)\frac{\partial\tilde{I}_{i-100}(u,v,w,\tau)}{\partial v}\,d\,w\,d\,v\,d\,u\,d\,\tau\\
&-2\pi\sqrt{\frac{D_{0I}}{D_{0V}}}\sum_{n=1}^{\infty}n\,c_{n}(\chi)c(\eta)c(\varphi)e_{nI}(\vartheta)\int_{0}^{\vartheta}e_{nI}(-\tau)\int_{0}^{1}c_{n}(u)\int_{0}^{1}c_{n}(v)\\
&\qquad\times\int_{0}^{1}s_{n}(w)g_{I}(u,v,w,T)\frac{\partial\tilde{I}_{i-100}(u,v,w,\tau)}{\partial w}\,d\,w\,d\,v\,d\,u\,d\,\tau,\ \ i\geq 1,\\
&\tilde{V}_{i00}(\chi,\eta,\varphi,\vartheta)=\\
&-2\pi\sqrt{\frac{D_{0V}}{D_{0I}}}\sum_{n=1}^{\infty}n\,c_{n}(\chi)c(\eta)c(\varphi)e_{nV}(\vartheta)\int_{0}^{\vartheta}e_{nI}(-\tau)\int_{0}^{1}s_{n}(u)\int_{0}^{1}c_{n}(v)\\
&\qquad\times\int_{0}^{1}g_{V}(u,v,w,T)c_{n}(w)\frac{\partial\tilde{V}_{i-100}(u,\tau)}{\partial u}\,d\,w\,d\,v\,d\,u\,d\,\tau\\
&-\sqrt{\frac{D_{0V}}{D_{0I}}}\sum_{n=1}^{\infty}n\,c_{n}(\chi)c(\eta)c(\varphi)e_{nV}(\vartheta)\int_{0}^{\vartheta}e_{nI}(-\tau)\int_{0}^{1}c_{n}(u)\int_{0}^{1}s_{n}(v)\\
&\qquad\times 2\pi\int_{0}^{1}c_{n}(w)\,g_{V}(u,v,w,T)\frac{\partial\tilde{V}_{i-100}(u,\tau)}{\partial v}\,d\,w\,d\,v\,d\,u\,d\,\tau
\end{aligned}$$

$$-2\pi\sqrt{\frac{D_{0V}}{D_{0I}}}\sum_{n=1}^{\infty} n\,c_n(\chi)c(\eta)c(\varphi)e_{nV}(\vartheta)\times\int_0^{\vartheta} e_{nI}(-\tau)\int_0^1 c_n(u)\int_0^1 c_n(v)$$

$$\times\int_0^1 s_n(w)\,g_V(u,v,w,T)\frac{\partial \tilde{V}_{i-100}(u,\tau)}{\partial w}\,d\,w\,d\,v\,d\,u\,d\,\tau,\quad i\geq 1,$$

where $s_n(\chi)=\sin(pn\chi)$;

$$\tilde{\rho}_{010}(\chi,\eta,\varphi,\vartheta)=-2\sum_{n=1}^{\infty} c_n(\chi)c_n(\eta)c_n(\varphi)e_{n\rho}(\vartheta)\int_0^{\vartheta} e_{n\rho}(-\tau)\int_0^1 c_n(u)\int_0^1 c_n(v)\int_0^1 c_n(w)$$

$$\times\left[1+\varepsilon_{I,V}\,g_{I,V}(u,v,w,T)\right]\tilde{I}_{000}(u,v,w,\tau)\,\tilde{V}_{000}(u,v,w,\tau)\,d\,w\,d\,v\,d\,u\,d\,\tau,$$

$$\tilde{\rho}_{020}(\chi,\eta,\varphi,\vartheta)=-2\sqrt{\frac{D_{0I}}{D_{0V}}}\sum_{n=1}^{\infty} c_n(\chi)c_n(\eta)\,c_n(\varphi)e_{n\rho}(\vartheta)\int_0^{\vartheta} e_{n\rho}(-\tau)\int_0^1 c_n(u)\int_0^1 c_n(v)\int_0^1 c_n(w)$$

$$\times\left[1+\varepsilon_{I,V}\,g_{I,V}(u,v,w,T)\right]\left[\tilde{I}_{010}(u,v,w,\tau)\tilde{V}_{000}(u,v,w,\tau)+\tilde{I}_{000}(u,v,w,\tau)\right.$$

$$\left.\times\tilde{V}_{010}(u,v,w,\tau)\right]d\,w\,d\,v\,d\,u\,d\,\tau,$$

$$\tilde{\rho}_{001}(\chi,\eta,\varphi,\vartheta)=-2\sum_{n=1}^{\infty} c_n(\chi)c_n(\eta)c_n(\varphi)e_{n\rho}(\vartheta)\int_0^{\vartheta} e_{n\rho}(-\tau)\int_0^1 c_n(u)\int_0^1 c_n(v)$$

$$\times\int_0^1 c_n(w)\left[1+\varepsilon_{\rho,\rho}\,g_{\rho,\rho}(u,v,w,T)\right]\tilde{\rho}_{000}^2(u,v,w,\tau)\,d\,w\,d\,v\,d\,u\,d\,\tau,$$

$$\tilde{\rho}_{002}(\chi,\eta,\varphi,\vartheta)=-2\sum_{n=1}^{\infty} c_n(\chi)c_n(\eta)c_n(\varphi)e_{n\rho}(\vartheta)\int_0^{\vartheta} e_{n\rho}(-\tau)\int_0^1 c_n(u)\int_0^1 c_n(v)$$

$$\times\int_0^1 c_n(w)\left[1+\varepsilon_{\rho,\rho}\,g_{\rho,\rho}(u,v,w,T)\right]\tilde{\rho}_{001}(u,v,w,\tau)\,\tilde{\rho}_{000}(u,v,w,\tau)\,d\,w\,d\,v\,d\,u\,d\,\tau,$$

$$\tilde{I}_{110}(\chi,\eta,\varphi,\vartheta)$$

$$=-2\pi\sqrt{\frac{D_{0I}}{D_{0V}}}\sum_{n=1}^{\infty} n\,c_n(\chi)c_n(\eta)c_n(\varphi)e_{nI}(\vartheta)\int_0^{\vartheta} e_{nI}(-\tau)\int_0^1 s_n(u)\int_0^1 c_n(v)$$

$$\times\int_0^1 c_n(u)\,g_I(u,v,w,T)\frac{\partial \tilde{I}_{i-100}(u,v,w,\tau)}{\partial u}\,d\,w\,d\,v\,d\,u\,d\,\tau$$

$$-2\pi\sqrt{\frac{D_{0I}}{D_{0V}}}\sum_{n=1}^{\infty} n\,c_n(\chi)c_n(\eta)c_n(\varphi)e_{nI}(\vartheta)\int_0^{\vartheta} e_{nI}(-\tau)\int_0^1 c_n(u)\int_0^1 s_n(v)$$

$$\times\int_0^1 c_n(u)\,g_I(u,v,w,T)\frac{\partial \tilde{I}_{i-100}(u,v,w,\tau)}{\partial v}\,d\,w\,d\,v\,d\,u\,d\,\tau$$

$$-2\pi\sqrt{\frac{D_{0I}}{D_{0V}}}\sum_{n=1}^{\infty} n\,e_{nI}(\vartheta)\int_0^{\vartheta} e_{nI}(-\tau)\int_0^1 c_n(u)\int_0^1 c_n(v)$$

$$\times\int_0^1 s_n(u)\,g_I(u,v,w,T)\frac{\partial \tilde{I}_{i-100}(u,v,w,\tau)}{\partial w}\,d\,w\,d\,v\,d\,u\,d\,\tau\,c_n(\chi)c_n(\eta)c_n(\varphi)$$

$$-2\sum_{n=1}^{\infty} c_n(\chi) e_{nI}(\vartheta) c_n(\eta) c_n(\varphi) \int_0^{\vartheta} e_{nI}(-\tau) \int_0^1 c_n(u) \int_0^1 c_n(v) \int_0^1 c_n(v) \left[1+\varepsilon_{I,V} g_{I,V}(u,v,w,T)\right]$$

$$\times \left[\tilde{I}_{100}(u,v,w,\tau) \tilde{V}_{000}(u,v,w,\tau) + \tilde{I}_{000}(u,v,w,\tau) \tilde{V}_{100}(u,v,w,\tau)\right] d\,w\,d\,v\,d\,u\,d\,\tau,$$

$$\tilde{V}_{110}(\chi,\eta,\varphi,\vartheta)$$

$$= -2\pi \sqrt{\frac{D_{0V}}{D_{0I}}} \sum_{n=1}^{\infty} n\, c_n(\chi) c_n(\eta) c_n(\varphi) e_{nV}(\vartheta) \int_0^{\vartheta} e_{nV}(-\tau) \int_0^1 s_n(u) \int_0^1 c_n(v)$$

$$\times \int_0^1 c_n(u) g_V(u,v,w,T) \frac{\partial \tilde{V}_{i-100}(u,v,w,\tau)}{\partial u} d\,w\,d\,v\,d\,u\,d\,\tau$$

$$-2\pi \sqrt{\frac{D_{0V}}{D_{0I}}} \sum_{n=1}^{\infty} n\, c_n(\chi) c_n(\eta) c_n(\varphi) e_{nV}(\vartheta) \int_0^{\vartheta} e_{nV}(-\tau) \int_0^1 c_n(u) \int_0^1 s_n(v)$$

$$\times \int_0^1 c_n(u) g_V(u,v,w,T) \frac{\partial \tilde{V}_{i-100}(u,v,w,\tau)}{\partial v} d\,w\,d\,v\,d\,u\,d\,\tau$$

$$-2\pi \sqrt{\frac{D_{0V}}{D_{0I}}} \sum_{n=1}^{\infty} n\, e_{nV}(\vartheta) \int_0^{\vartheta} e_{nV}(-\tau) \int_0^1 c_n(u) \int_0^1 c_n(v)$$

$$\times \int_0^1 s_n(u) g_V(u,v,w,T) \frac{\partial \tilde{V}_{i-100}(u,v,w,\tau)}{\partial w} d\,w\,d\,v\,d\,u\,d\,\tau\, c_n(\chi) c_n(\eta) c_n(\varphi)$$

$$-2\sum_{n=1}^{\infty} c_n(\chi) e_{nI}(\vartheta) c_n(\eta) c_n(\varphi) \int_0^{\vartheta} e_{nV}(-\tau) \int_0^1 c_n(u) \int_0^1 c_n(v) \int_0^1 \left[1+\varepsilon_{I,V}\ \ g_{I,V}(u,v,w,T)\right]$$

$$\times c_n(w) \left[\tilde{I}_{100}(u,v,w,\tau) \tilde{V}_{000}(u,v,w,\tau) + \tilde{I}_{000}(u,v,w,\tau)\, \tilde{V}_{100}(u,v,w,\tau)\right] d\,w\,d\,v\,d\,u\,d\,\tau,$$

$$\tilde{I}_{101}(\chi,\eta,\varphi,\vartheta)$$

$$= -2\pi \sqrt{\frac{D_{0I}}{D_{0V}}} \sum_{n=1}^{\infty} n\, c_n(\chi) c_n(\eta) c_n(\varphi) e_{nI}(\vartheta) \int_0^{\vartheta} e_{nI}(-\tau) \int_0^1 s_n(u) \int_0^1 c_n(v)$$

$$\times \int_0^1 g_I(u,v,w,T) c_n(w) \frac{\partial \tilde{I}_{001}(u,v,w,\tau)}{\partial u} d\,w\,d\,v\,d\,u\,d\,\tau$$

$$-2\pi \sqrt{\frac{D_{0I}}{D_{0V}}} \sum_{n=1}^{\infty} n\, c_n(\chi)\, c_n(\eta)\, c_n(\varphi)\, e_{nI}(\vartheta)$$

$$\times \int_0^1 s_n(v) \int_0^1 c_n(w) g_I(u,v,w,T) \frac{\partial \tilde{I}_{001}(u,v,w,\tau)}{\partial v} d\,w\,d\,v\,d\,u\,d\,\tau$$

$$-2\pi \sqrt{\frac{D_{0I}}{D_{0V}}} \sum_{n=1}^{\infty} n\, e_{nI}(\vartheta) c_n(\chi) c_n(\eta) c_n(\varphi) \int_0^{\vartheta} e_{nI}(-\tau) \int_0^1 c_n(u) \int_0^1 c_n(v)$$

$$\times \int_0^1 s_n(w) g_I(u,v,w,T) \frac{\partial \tilde{I}_{001}(u,v,w,\tau)}{\partial w} d\,w\,d\,v\,d\,u\,d\,\tau$$

$$-2\sum_{n=1}^{\infty} c_n(\chi) c_n(\eta) c_n(\varphi) e_{nI}(\vartheta) \int_0^{\vartheta} e_{nI}(-\tau) \int_0^1 c_n(u) \int_0^1 c_n(v)$$

$$\times \int_0^1 c_n(w) \left[1+\varepsilon_{I,V} g_{I,V}(u,v,w,T)\right] \tilde{I}_{100}(u,v,w,\tau)\, \tilde{V}_{000}(u,v,w,\tau)\, d\,w\,d\,v\,d\,u\,d\,\tau,$$

$$\tilde{V}_{101}(\chi,\eta,\varphi,\vartheta)=-2\pi\sqrt{\frac{D_{0V}}{D_{0I}}}\sum_{n=1}^{\infty}n\,c_n(\chi)c_n(\eta)c_n(\varphi)e_{nV}(\vartheta)\int_0^{\vartheta}e_{nV}(-\tau)\int_0^1 s_n(u)\int_0^1 c_n(v)$$

$$\times\int_0^1 g_V(u,v,w,T)c_n(w)\frac{\partial\,\tilde{V}_{001}(u,v,w,\tau)}{\partial u}d\,w\,d\,v\,d\,u\,d\,\tau$$

$$-2\pi\sqrt{\frac{D_{0V}}{D_{0I}}}\sum_{n=1}^{\infty}n\,c_n(\chi)\,c_n(\eta)\,c_n(\varphi)\,e_{nI}(\vartheta)\int_0^{\vartheta}e_{nV}(-\tau)\int_0^1 c_n(u)\times\int_0^1 s_n(v)$$

$$\times\int_0^1 c_n(w)g_I(u,v,w,T)\frac{\partial\,\tilde{I}_{001}(u,v,w,\tau)}{\partial v}d\,w\,d\,v\,d\,u\,d\,\tau$$

$$-2\pi\sqrt{\frac{D_{0I}}{D_{0V}}}\sum_{n=1}^{\infty}n\,e_{nI}(\vartheta)c_n(\chi)c_n(\eta)c_n(\varphi)\int_0^{\vartheta}e_{nV}(-\tau)\int_0^1 c_n(u)\int_0^1 c_n(v)$$

$$\times\int_0^1 s_n(w)g_V(u,v,w,T)\frac{\partial\tilde{V}_{001}(u,v,w,\tau)}{\partial w}d\,w\,d\,v\,d\,u\,d\,\tau$$

$$-2\sum_{n=1}^{\infty}c_n(\chi)c_n(\eta)c_n(\varphi)e_{nV}(\vartheta)\int_0^{\vartheta}e_{nV}(-\tau)\int_0^1 c_n(u)\int_0^1 c_n(v)$$

$$\times\int_0^1 c_n(w)\left[1+\varepsilon_{I,V}g_{I,V}(u,v,w,T)\right]\tilde{I}_{100}(u,v,w,\tau)\,\tilde{V}_{000}(u,v,w,\tau)\,d\,w\,d\,v\,d\,u\,d\,\tau,$$

$$\tilde{I}_{011}(\chi,\eta,\varphi,\vartheta)=-2\sum_{n=1}^{\infty}c_n(\chi)c_n(\eta)c_n(\varphi)e_{nI}(\vartheta)\int_0^{\vartheta}e_{nI}(-\tau)\int_0^1 c_n(u)\int_0^1 c_n(v)$$

$$\times\int_0^1 c_n(w)\Big\{\tilde{I}_{000}(u,v,w,\tau)\left[1+\varepsilon_{I,I}g_{I,I}(u,v,w,T)\right]\tilde{I}_{010}(u,v,w,\tau)$$

$$+\left[1+\varepsilon_{I,V}g_{I,V}(u,v,w,T)\right]\tilde{I}_{001}(u,v,w,\tau)\tilde{V}_{000}(u,v,w,\tau)\Big\}d\,w\,d\,v\,d\,u\,d\,\tau,$$

$$\tilde{V}_{011}(\chi,\eta,\varphi,\vartheta)=-2\sum_{n=1}^{\infty}c_n(\chi)c_n(\eta)c_n(\varphi)e_{nV}(\vartheta)\int_0^{\vartheta}e_{nV}(-\tau)\int_0^1 c_n(u)\int_0^1 c_n(v)$$

$$\int_0^1 c_n(w)\Big\{\tilde{I}_{000}(u,v,w,\tau)\left[1+\varepsilon_{I,I}g_{I,I}(u,v,w,T)\right]\tilde{I}_{010}(u,v,w,\tau)$$

$$+\left[1+\varepsilon_{I,V}g_{I,V}(u,v,w,T)\right]\tilde{I}_{001}(u,v,w,\tau)\tilde{V}_{000}(u,v,w,\tau)\Big\}d\,w\,d\,v\,d\,u\,d\,\tau.$$

Equations for functions $\Phi_{\rho i}(x,y,z,t)$, $i\geq 0$, to describe concentrations of simplest complexes of radiation defects.

$$\frac{\partial\,\Phi_{I0}(x,y,z,t)}{\partial\,t}=D_{0\Phi I}\left[\frac{\partial^2\,\Phi_{I0}(x,y,z,t)}{\partial\,x^2}+\frac{\partial^2\,\Phi_{I0}(x,y,z,t)}{\partial\,y^2}+\frac{\partial^2\,\Phi_{I0}(x,y,z,t)}{\partial\,z^2}\right]$$

$$+k_{I,I}(x,y,z,T)I^2(x,y,z,t)-k_I(x,y,z,T)I(x,y,z,t),$$

$$\frac{\partial\,\Phi_{V0}(x,y,z,t)}{\partial\,t}=D_{0\Phi V}\left[\frac{\partial^2\,\Phi_{V0}(x,y,z,t)}{\partial\,x^2}+\frac{\partial^2\,\Phi_{V0}(x,y,z,t)}{\partial\,y^2}+\frac{\partial^2\,\Phi_{V0}(x,y,z,t)}{\partial\,z^2}\right]$$

$$+k_{V,V}(x,y,z,T)V^2(x,y,z,t)-k_V(x,y,z,T)V(x,y,z,t),$$

$$\frac{\partial\,\Phi_{I\,i}(x,y,z,t)}{\partial\,t}=D_{0\Phi I}\left[\frac{\partial^2\Phi_{I\,i}(x,y,z,t)}{\partial\,x^2}+\frac{\partial^2\Phi_{I\,i}(x,y,z,t)}{\partial\,y^2}+\frac{\partial^2\Phi_{I\,i}(x,y,z,t)}{\partial\,z^2}\right]$$
$$+D_{0\Phi I}\left\{\frac{\partial}{\partial\,x}\left[g_{\Phi I}(x,y,z,T)\frac{\partial\,\Phi_{I\,i-1}(x,y,z,t)}{\partial\,x}\right]+\frac{\partial}{\partial\,y}\left[g_{\Phi I}(x,y,z,T)\frac{\partial\,\Phi_{I\,i-1}(x,y,z,t)}{\partial\,y}\right]\right.$$
$$\left.+\frac{\partial}{\partial\,z}\left[g_{\Phi I}(x,y,z,T)\frac{\partial\,\Phi_{I\,i-1}(x,y,z,t)}{\partial\,z}\right]\right\},i\ge 1,$$

$$\frac{\partial\,\Phi_{V\,i}(x,y,z,t)}{\partial\,t}=D_{0\Phi V}\left[\frac{\partial^2\Phi_{V\,i}(x,y,z,t)}{\partial\,x^2}+\frac{\partial^2\Phi_{V\,i}(x,y,z,t)}{\partial\,y^2}+\frac{\partial^2\Phi_{V\,i}(x,y,z,t)}{\partial\,z^2}\right]$$
$$+D_{0\Phi V}\left\{\frac{\partial}{\partial\,x}\left[g_{\Phi V}(x,y,z,T)\frac{\partial\,\Phi_{V\,i-1}(x,y,z,t)}{\partial\,x}\right]+\frac{\partial}{\partial\,y}\left[g_{\Phi V}(x,y,z,T)\frac{\partial\,\Phi_{V\,i-1}(x,y,z,t)}{\partial\,y}\right]\right.$$
$$\left.+\frac{\partial}{\partial\,z}\left[g_{\Phi V}(x,y,z,T)\frac{\partial\,\Phi_{V\,i-1}(x,y,z,t)}{\partial\,z}\right]\right\},\ i\ge 1.$$

Boundary and initial conditions for the functions takes the form

$$\left.\frac{\partial\Phi_{\rho i}(x,y,z,t)}{\partial\,x}\right|_{x=0}=0,\quad \left.\frac{\partial\Phi_{\rho i}(x,y,z,t)}{\partial\,x}\right|_{x=L_x}=0,\quad \left.\frac{\partial\Phi_{\rho i}(x,y,z,t)}{\partial\,y}\right|_{y=0}=0,$$
$$\left.\frac{\partial\Phi_{\rho i}(x,y,z,t)}{\partial\,y}\right|_{y=L_y}=0,\quad \left.\frac{\partial\Phi_{\rho i}(x,y,z,t)}{\partial\,z}\right|_{z=0}=0,\quad \left.\frac{\partial\Phi_{\rho i}(x,y,z,t)}{\partial\,z}\right|_{z=L_z}=0,\ \ i\ge 0,$$
$$\Phi_{\rho 0}(x,y,z,0)=f_{\Phi\rho}(x,y,z),\ \ \Phi_{\rho i}(x,y,z,0)=0,\ \ i\ge 1.$$

Solutions of the above equations could be written as

$$\Phi_{\rho 0}(x,y,z,t)=\frac{1}{L_xL_yL_z}+\frac{2}{L_xL_yL_z}\sum_{n=1}^{\infty}F_{n\Phi_\rho}c_n(x)c_n(y)c_n(z)e_{n\Phi_\rho}(t)$$
$$+\frac{2}{L}\sum_{n=1}^{\infty}n\,c_n(x)c_n(y)\,c_n(z)e_{\Phi_\rho n}(t)\int_0^t e_{\Phi_\rho n}(-\tau)\int_0^{L_x}c_n(u)\int_0^{L_y}c_n(v)$$
$$\times\int_0^{L_z}c_n(w)\left[k_{I,I}(u,v,w,T)I^2(u,v,w,\tau)-k_I(u,v,w,T)I(u,v,w,\tau)\right]d\,w\,d\,v\,d\,u\,d\,\tau,$$

where

$$F_{n\Phi_\rho}=\int_0^{L_x}c_n(u)\int_0^{L_y}c_n(v)\int_0^{L_z}c_n(w)\,f_{\Phi_\rho}(u,v,w)d\,w\,d\,v\,d\,u,$$
$$e_{n\Phi_\rho}(t)=\exp\left[-\pi^2n^2D_{0\Phi_\rho}t\left(L_x^{-2}+L_y^{-2}+L_z^{-2}\right)\right],\ \ c_n(x)\ =\cos(pnx/L_x);$$

$$\Phi_{\rho\,i}(x,y,z,t)=-\frac{2\pi}{L_x^2L_yL_z}\sum_{n=1}^{\infty}n\,c_n(x)\,c_n(y)\,c_n(z)\,e_{\Phi_\rho n}(t)\int_0^t e_{\Phi_\rho n}(-\tau)\int_0^{L_x}s_n(u)\int_0^{L_y}c_n(v)$$
$$\times\int_0^{L_z}g_{\Phi_\rho}(u,v,w,T)c_n(w)\frac{\partial\,\Phi_{I_\rho\,i-1}(u,v,w,\tau)}{\partial\,u}d\,w\,d\,v\,d\,u\,d\,\tau$$

$$-\frac{2\pi}{L_x L_y^2 L_z}\sum_{n=1}^{\infty} n\, c_n(x)\, c_n(y)\, c_n(z)\, e_{\Phi_\rho n}(t)\int_0^t e_{\Phi_\rho n}(-\tau)\times\int_0^t e_{\Phi_\rho n}(-\tau)\int_0^{L_x} c_n(u)\int_0^{L_y} s_n(v)$$

$$\times\int_0^{L_z} c_n(w)\, g_{\Phi_\rho}(u,v,w,T)\frac{\partial\Phi_{I_\rho\, i-1}(u,v,w,\tau)}{\partial v}\, d\,w\, d\,v\, d\,u\, d\tau$$

$$-\frac{2\pi}{L_x L_y L_z^2}\sum_{n=1}^{\infty} n\; e_{\Phi_\rho n}(t)\int_0^t e_{\Phi_\rho n}(-\tau)\int_0^{L_x} c_n(u)\int_0^{L_y} c_n(v)$$

$$\times\int_0^{L_z} s_n(w)\frac{\partial\Phi_{I_\rho\, i-1}(u,v,w,\tau)}{\partial\, w}\, g_{\Phi_\rho}(u,v,w,T)\; d\,w\, d\,v\, d\,u\, d\tau\, c_n(x)\; c_n(y)\; c_n(z),\;\; i\ge 1,$$

where $s_n(x)=\sin(pn\; x\,/\,L_x)$.

Equations for the functions $C_{ij}(x,y,z,t)$ $(i\ge 0,\, j\ge 0)$, boundary and initial conditions could be written as

$$\frac{\partial C_{00}(x,y,z,t)}{\partial t}=D_{0L}\frac{\partial^2 C_{00}(x,y,z,t)}{\partial x^2}+D_{0L}\frac{\partial^2 C_{00}(x,y,z,t)}{\partial y^2}+D_{0L}\frac{\partial^2 C_{00}(x,y,z,t)}{\partial z^2},$$

$$\frac{\partial C_{i0}(x,y,z,t)}{\partial t}=D_{0L}\left[\frac{\partial^2 C_{i0}(x,y,z,t)}{\partial x^2}+\frac{\partial^2 C_{i0}(x,y,z,t)}{\partial y^2}+\frac{\partial^2 C_{i0}(x,y,z,t)}{\partial z^2}\right]$$

$$+D_{0L}\frac{\partial}{\partial x}\left[g_L(x,y,z,T)\frac{\partial C_{i-10}(x,y,z,t)}{\partial x}\right]+D_{0L}\frac{\partial}{\partial y}\left[g_L(x,y,z,T)\frac{\partial C_{i-10}(x,y,z,t)}{\partial y}\right]$$

$$+D_{0L}\frac{\partial}{\partial z}\left[g_L(x,y,z,T)\frac{\partial C_{i-10}(x,y,z,t)}{\partial z}\right],\; i\ge 1,$$

$$\frac{\partial C_{01}(x,y,z,t)}{\partial t}=D_{0L}\frac{\partial^2 C_{01}(x,y,z,t)}{\partial x^2}+D_{0L}\frac{\partial^2 C_{01}(x,y,z,t)}{\partial y^2}+D_{0L}\frac{\partial^2 C_{01}(x,y,z,t)}{\partial z^2}$$

$$+D_{0L}\frac{\partial}{\partial x}\left[\frac{C_{00}^{\gamma}(x,y,z,t)}{P^{\gamma}(x,y,z,T)}\frac{\partial C_{00}(x,y,z,t)}{\partial x}\right]+D_{0L}\frac{\partial}{\partial y}\left[\frac{C_{00}^{\gamma}(x,y,z,t)}{P^{\gamma}(x,y,z,T)}\frac{\partial C_{00}(x,y,z,t)}{\partial y}\right]$$

$$+D_{0L}\frac{\partial}{\partial z}\left[\frac{C_{00}^{\gamma}(x,y,z,t)}{P^{\gamma}(x,y,z,T)}\frac{\partial C_{00}(x,y,z,t)}{\partial z}\right],$$

$$\frac{\partial C_{02}(x,y,z,t)}{\partial t}=D_{0L}\frac{\partial^2 C_{02}(x,y,z,t)}{\partial x^2}+D_{0L}\frac{\partial^2 C_{02}(x,y,z,t)}{\partial y^2}+D_{0L}\frac{\partial^2 C_{02}(x,y,z,t)}{\partial z^2}$$

$$+D_{0L}\left\{\frac{\partial}{\partial x}\left[C_{01}(x,y,z,t)\frac{C_{00}^{\gamma-1}(x,y,z,t)}{P^{\gamma}(x,y,z,T)}\frac{\partial C_{00}(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[C_{01}(x,y,z,t)\frac{C_{00}^{\gamma-1}(x,y,z,t)}{P^{\gamma}(x,y,z,T)}\right.\right.$$

$$\left.\left.\times\frac{\partial C_{00}(x,y,z,t)}{\partial y}\right]+\frac{\partial}{\partial z}\left[C_{01}(x,y,z,t)\frac{C_{00}^{\gamma-1}(x,y,z,t)}{P^{\gamma}(x,y,z,T)}\frac{\partial C_{00}(x,y,z,t)}{\partial z}\right]\right\}$$

$$+D_{0L}\left\{\frac{\partial}{\partial x}\left[\frac{C_{00}^{\gamma}(x,y,z,t)}{P^{\gamma}(x,y,z,T)}\frac{\partial C_{01}(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[\frac{C_{00}^{\gamma}(x,y,z,t)}{P^{\gamma}(x,y,z,T)}\frac{\partial C_{01}(x,y,z,t)}{\partial y}\right]\right.$$
$$\left.+\frac{\partial}{\partial z}\left[\frac{C_{00}^{\gamma}(x,y,z,t)}{P^{\gamma}(x,y,z,T)}\frac{\partial C_{01}(x,y,z,t)}{\partial z}\right]\right\},$$

$$\frac{\partial C_{11}(x,y,z,t)}{\partial t}=D_{0L}\frac{\partial^2 C_{11}(x,y,z,t)}{\partial x^2}+D_{0L}\frac{\partial^2 C_{11}(x,y,z,t)}{\partial y^2}+D_{0L}\frac{\partial^2 C_{11}(x,y,z,t)}{\partial z^2}+$$
$$+\left\{\frac{\partial}{\partial x}\left[C_{10}(x,y,z,t)\frac{C_{00}^{\gamma-1}(x,y,z,t)}{P^{\gamma}(x,y,z,T)}\frac{\partial C_{00}(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[C_{10}(x,y,z,t)\frac{C_{00}^{\gamma-1}(x,y,z,t)}{P^{\gamma}(x,y,z,T)}\right.\right.$$
$$\left.\left.\times\frac{\partial C_{00}(x,y,z,t)}{\partial y}\right]+\frac{\partial}{\partial z}\left[C_{10}(x,y,z,t)\frac{C_{00}^{\gamma-1}(x,y,z,t)}{P^{\gamma}(x,y,z,T)}\frac{\partial C_{00}(x,y,z,t)}{\partial z}\right]\right\}D_{0L}$$
$$+D_{0L}\left\{\frac{\partial}{\partial x}\left[\frac{C_{00}^{\gamma}(x,y,z,t)}{P^{\gamma}(x,y,z,T)}\frac{\partial C_{10}(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[\frac{C_{00}^{\gamma}(x,y,z,t)}{P^{\gamma}(x,y,z,T)}\frac{\partial C_{10}(x,y,z,t)}{\partial y}\right]\right.$$
$$\left.+\frac{\partial}{\partial z}\left[\frac{C_{00}^{\gamma}(x,y,z,t)}{P^{\gamma}(x,y,z,T)}\frac{\partial C_{10}(x,y,z,t)}{\partial z}\right]\right\}$$
$$+D_{0L}\left\{\frac{\partial}{\partial x}\left[g_L(x,y,z,T)\frac{\partial C_{01}(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[g_L(x,y,z,T)\frac{\partial C_{01}(x,y,z,t)}{\partial y}\right]\right.$$
$$\left.+\frac{\partial}{\partial z}\left[g_L(x,y,z,T)\frac{\partial C_{01}(x,y,z,t)}{\partial z}\right]\right\},$$

$$\left.\frac{\partial C_{ij}(x,y,z,t)}{\partial x}\right|_{x=0}=0,\quad \left.\frac{\partial C_{ij}(x,y,z,t)}{\partial x}\right|_{x=L_x}=0,\quad \left.\frac{\partial C_{ij}(x,y,z,t)}{\partial y}\right|_{y=0}=0,$$
$$\left.\frac{\partial C_{ij}(x,y,z,t)}{\partial y}\right|_{y=L_y}=0,\quad \left.\frac{\partial C_{ij}(x,y,z,t)}{\partial z}\right|_{z=0}=0,\quad \left.\frac{\partial C_{ij}(x,y,z,t)}{\partial z}\right|_{z=L_z}=0,\quad i\ge 0,\ j\ge 0,$$
$$C_{00}(x,y,z,0)=f_C(x,y,z),\quad C_{ij}(x,y,z,0)=0,\quad i\ge 1,\ j\ge 1.$$

Functions $C_{ij}(x,y,z,t)$ $(i\ge 0,\ j\ge 0)$ could be approximated by the following series during solutions of the above equations:

$$C_{00}(x,y,z,t)=\frac{F_{0C}}{L_xL_yL_z}+\frac{2}{L_xL_yL_z}\sum_{n=1}^{\infty}F_{nC}c_n(x)c_n(y)c_n(z)e_{nC}(t),$$

where

$$e_{nC}(t)=\exp\left[-\pi^2n^2D_{0C}t\left(\frac{1}{L_x^2}+\frac{1}{L_y^2}+\frac{1}{L_z^2}\right)\right],$$

$$F_{nC}=\int_0^{L_x}c_n(u)\int_0^{L_y}c_n(v)\int_0^{L_z}f_C(u,v,w)\,c_n(w)\,d\,w\,d\,v\,d\,u;$$

$$\begin{aligned}
&C_{i0}\left(x,y,z,t\right)\\
&=-\frac{2\pi}{L_x^2L_yL_z}\sum_{n=1}^{\infty}nF_{nC}c_n\left(x\right)c_n\left(y\right)c_n\left(z\right)e_{nC}\left(t\right)\int\limits_0^t e_{nC}\left(-\tau\right)\int\limits_0^{L_x}s_n\left(u\right)\int\limits_0^{L_y}c_n\left(v\right)\\
&\qquad\times\int\limits_0^{L_z}c_n\left(w\right)g_L\left(u,v,w,T\right)\frac{\partial C_{i-10}\left(u,v,w,\tau\right)}{\partial u}d\,w\,d\,v\,d\,u\,d\,\tau\\
&-\frac{2\pi}{L_xL_y^2L_z}\sum_{n=1}^{\infty}nF_{nC}c_n\left(x\right)c_n\left(y\right)c_n\left(z\right)e_{nC}\left(t\right)\int\limits_0^t e_{nC}\left(-\tau\right)\int\limits_0^{L_x}c_n\left(u\right)\int\limits_0^{L_y}s_n\left(v\right)\\
&\qquad\times\int\limits_0^{L_z}c_n\left(v\right)g_L\left(u,v,w,T\right)\frac{\partial C_{i-10}\left(u,v,w,\tau\right)}{\partial v}d\,w\,d\,v\,d\,u\,d\,\tau\\
&-\frac{2\pi}{L_xL_yL_z^2}\sum_{n=1}^{\infty}nF_{nC}e_{nC}\left(t\right)\times c_n\left(x\right)c_n\left(y\right)c_n\left(z\right)\int\limits_0^t e_{nC}\left(-\tau\right)\int\limits_0^{L_x}c_n\left(u\right)\int\limits_0^{L_y}c_n\left(v\right)\\
&\qquad\times\int\limits_0^{L_z}s_n\left(v\right)g_L\left(u,v,w,T\right)\frac{\partial C_{i-10}\left(u,v,w,\tau\right)}{\partial w}d\,w\,d\,v\,d\,u\,d\,\tau,\ i\ge 1,
\end{aligned}$$

$$\begin{aligned}
&C_{01}\left(x,y,z,t\right)\\
&=-\frac{2\pi}{L_x^2L_yL_z}\sum_{n=1}^{\infty}nF_{nC}c_n\left(x\right)c_n\left(y\right)c_n\left(z\right)e_{nC}\left(t\right)\int\limits_0^t e_{nC}\left(-\tau\right)\int\limits_0^{L_x}s_n\left(u\right)\int\limits_0^{L_y}c_n\left(v\right)\int\limits_0^{L_z}c_n\left(w\right)\\
&\qquad\times\frac{C_{00}^{\gamma}\left(u,v,w,\tau\right)}{P^{\gamma}\left(u,v,w,T\right)}\frac{\partial C_{00}\left(u,v,w,\tau\right)}{\partial u}d\,w\,d\,v\,d\,u\,d\,\tau\\
&-\frac{2\pi}{L_xL_y^2L_z}\sum_{n=1}^{\infty}nF_{nC}c_n\left(x\right)c_n\left(y\right)c_n\left(z\right)e_{nC}\left(t\right)\int\limits_0^t e_{nC}\left(-\tau\right)\int\limits_0^{L_x}c_n\left(u\right)\int\limits_0^{L_y}s_n\left(v\right)\\
&\qquad\times\int\limits_0^{L_z}c_n\left(w\right)\frac{C_{00}^{\gamma}\left(u,v,w,\tau\right)}{P^{\gamma}\left(u,v,w,T\right)}\frac{\partial C_{00}\left(u,v,w,\tau\right)}{\partial v}d\,w\,d\,v\,d\,u\,d\,\tau\\
&-\frac{2\pi}{L_xL_yL_z^2}\sum_{n=1}^{\infty}n\,e_{nC}\left(t\right)F_{nC}c_n\left(x\right)c_n\left(y\right)c_n\left(z\right)\int\limits_0^t e_{nC}\left(-\tau\right)\int\limits_0^{L_x}c_n\left(u\right)\int\limits_0^{L_y}c_n\left(v\right)\\
&\qquad\times\int\limits_0^{L_z}s_n\left(w\right)\frac{C_{00}^{\gamma}\left(u,v,w,\tau\right)}{P^{\gamma}\left(u,v,w,T\right)}\frac{\partial C_{00}\left(u,v,w,\tau\right)}{\partial w}d\,w\,d\,v\,d\,u\,d\,\tau
\end{aligned}$$

$$\begin{aligned}
&C_{02}\left(x,y,z,t\right)\\
&=-\frac{2\pi}{L_x^2L_yL_z}\sum_{n=1}^{\infty}nF_{nC}c_n\left(x\right)c_n\left(y\right)c_n\left(z\right)e_{nC}\left(t\right)\int\limits_0^t e_{nC}\left(-\tau\right)\int\limits_0^{L_x}s_n\left(u\right)\int\limits_0^{L_y}c_n\left(v\right)\\
&\qquad\times\int\limits_0^{L_z}c_n\left(w\right)C_{01}\left(u,v,w,\tau\right)\frac{C_{00}^{\gamma-1}\left(u,v,w,\tau\right)}{P^{\gamma}\left(u,v,w,T\right)}\frac{\partial C_{00}\left(u,v,w,\tau\right)}{\partial u}d\,w\,d\,v\,d\,u\,d\,\tau\\
&-\frac{2\pi}{L_xL_y^2L_z}\sum_{n=1}^{\infty}F_{nC}c_n\left(x\right)c_n\left(y\right)n\,c_n\left(z\right)e_{nC}\left(t\right)\int\limits_0^t e_{nC}\left(-\tau\right)\int\limits_0^{L_x}c_n\left(u\right)\int\limits_0^{L_y}s_n\left(v\right)\\
&\qquad\times\int\limits_0^{L_z}c_n\left(w\right)C_{01}\left(u,v,w,\tau\right)\frac{C_{00}^{\gamma-1}\left(u,v,w,\tau\right)}{P^{\gamma}\left(u,v,w,T\right)}\frac{\partial C_{00}\left(u,v,w,\tau\right)}{\partial v}d\,w\,d\,v\,d\,u\,d\,\tau
\end{aligned}$$

$$-\frac{2\pi}{L_xL_yL_z^2}\sum_{n=1}^{\infty} n\,F_{nC}c_n(x)c_n(y)c_n(z)e_{nC}(t)\int_0^t e_{nC}(-\tau)\int_0^{L_x} c_n(u)\int_0^{L_y} c_n(v)$$

$$\times\int_0^{L_z} s_n(w)C_{01}(u,v,w,\tau)\frac{C_{00}^{\gamma-1}(u,v,w,\tau)}{P^{\gamma}(u,v,w,T)}\frac{\partial C_{00}(u,v,w,\tau)}{\partial w}d\,w\,d\,v\,d\,u\,d\,\tau$$

$$-\frac{2\pi}{L_x^2L_yL_z}\sum_{n=1}^{\infty} n\,c_n(x)\times F_{nC}c_n(y)c_n(z)e_{nC}(t)\int_0^t e_{nC}(-\tau)\int_0^{L_x} s_n(u)\int_0^{L_y} c_n(v)$$

$$\times\int_0^{L_z} c_n(w)C_{01}(u,v,w,\tau)\frac{\partial C_{00}(u,v,w,\tau)}{\partial u}\frac{C_{00}^{\gamma-1}(u,v,w,\tau)}{P^{\gamma}(u,v,w,T)}d\,w\,d\,v\,d\,u\,d\,\tau$$

$$-\frac{2\pi}{L_xL_y^2L_z}\sum_{n=1}^{\infty} n\,F_{nC}c_n(x)c_n(y)c_n(z)e_{nC}(t)\int_0^t e_{nC}(-\tau)\int_0^{L_x} c_n(u)\int_0^{L_y} s_n(v)$$

$$\times\int_0^{L_z} c_n(w)\,C_{01}(u,v,w,\tau)\frac{C_{00}^{\gamma-1}(u,v,w,\tau)}{P^{\gamma}(u,v,w,T)}\frac{\partial C_{00}(u,v,w,\tau)}{\partial v}d\,w\,d\,v\,d\,u\,d\,\tau$$

$$-\frac{2\pi}{L_xL_yL_z^2}\sum_{n=1}^{\infty} n\,F_{nC}c_n(x)c_n(y)c_n(z)e_{nC}(t)\int_0^t e_{nC}(-\tau)\int_0^{L_x} c_n(u)\int_0^{L_y} c_n(v)$$

$$\times\int_0^{L_z} s_n(w)C_{01}(u,v,w,\tau)\frac{C_{00}^{\gamma-1}(u,v,w,\tau)}{P^{\gamma}(u,v,w,T)}\frac{\partial C_{00}(u,v,w,\tau)}{\partial w}d\,w\,d\,v\,d\,u\,d\,\tau$$

$$-\frac{2\pi}{L_x^2L_yL_z}\sum_{n=1}^{\infty} F_{nC}c_n(x)c_n(y)c_n(z)e_{nC}(t)\int_0^t e_{nC}(-\tau)\int_0^{L_x} s_n(u)n\int_0^{L_y} c_n(v)$$

$$\times\int_0^{L_z} c_n(w)\frac{C_{00}^{\gamma}(u,v,w,\tau)}{P^{\gamma}(u,v,w,T)}\frac{\partial C_{01}(u,v,w,\tau)}{\partial u}d\,w\,d\,v\,d\,u\,d\,\tau$$

$$-\frac{2\pi}{L_xL_y^2L_z}\sum_{n=1}^{\infty} c_n(x)e_{nC}(t)F_{nC}c_n(y)\int_0^t e_{nC}(-\tau)\int_0^{L_x} c_n(u)\int_0^{L_y} s_n(v)$$

$$\times\int_0^{L_z} c_n(w)\frac{C_{00}^{\gamma}(u,v,w,\tau)}{P^{\gamma}(u,v,w,T)}\frac{\partial C_{01}(u,v,w,\tau)}{\partial v}d\,w\,d\,v\,d\,u\,d\,\tau\;n\,c_n(z)$$

$$-\frac{2\pi}{L_xL_yL_z^2}\sum_{n=1}^{\infty} n\,F_{nC}c_n(x)c_n(y)c_n(z)e_{nC}(t)\int_0^t e_{nC}(-\tau)\int_0^{L_x} c_n(u)\int_0^{L_y} c_n(v)$$

$$\times\int_0^{L_z} s_n(w)\frac{C_{00}^{\gamma}(u,v,w,\tau)}{P^{\gamma}(u,v,w,T)}\frac{\partial C_{01}(u,v,w,\tau)}{\partial w}d\,w\,d\,v\,d\,u\,d\,\tau,$$

$$C_{11}(x,y,z,t)$$

$$=-\frac{2\pi}{L_x^2L_yL_z}\sum_{n=1}^{\infty} n\,F_{nC}c_n(x)c_n(y)c_n(z)e_{nC}(t)\int_0^t e_{nC}(-\tau)\int_0^{L_x} s_n(u)\int_0^{L_y} c_n(v)$$

$$\times\int_0^{L_z} c_n(w)g_L(u,v,w,T)\frac{\partial C_{01}(u,v,w,\tau)}{\partial u}d\,w\,d\,v\,d\,u\,d\,\tau$$

$$-\frac{2\pi}{L_x L_y^2 L_z}\sum_{n=1}^{\infty} n F_{nC} c_n(x) c_n(y) c_n(z) e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^{L_x} c_n(u) \int_0^{L_y} s_n(v)$$

$$\times \int_0^{L_z} c_n(w) g_L(u,v,w,T) \frac{\partial C_{01}(u,v,w,\tau)}{\partial v} d\,w\,d\,v\,d\,u\,d\,\tau$$

$$-\frac{2\pi}{L_x L_y L_z^2}\sum_{n=1}^{\infty} n\, e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^{L_x} c_n(u) \int_0^{L_y} c_n(v)$$

$$\times \int_0^{L_z} s_n(w) g_L(u,v,w,T) \frac{\partial C_{01}(u,v,w,\tau)}{\partial w} d\,w\,d\,v\,d\,u\,d\,\tau\, F_{nC} c_n(x) c_n(y) c_n(z)$$

$$-\frac{2\pi}{L_x^2 L_y L_z}\sum_{n=1}^{\infty} F_{nC} c_n(x) c_n(y) c_n(z) e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^{L_x} s_n(u) \int_0^{L_y} c_n(v)\, n$$

$$\times \int_0^{L_z} c_n(w) \frac{C_{00}^{\gamma}(u,v,w,\tau)}{P^{\gamma}(u,v,w,T)} \frac{\partial C_{10}(u,v,w,\tau)}{\partial u} d\,w\,d\,v\,d\,u\,d\,\tau$$

$$-\frac{2\pi}{L_x L_y^2 L_z}\sum_{n=1}^{\infty} n F_{nC} c_n(x) c_n(y) c_n(z) e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^{L_x} c_n(u) \int_0^{L_y} s_n(v)$$

$$\times \int_0^{L_z} c_n(w) \frac{C_{00}^{\gamma}(u,v,w,\tau)}{P^{\gamma}(u,v,w,T)} \frac{\partial C_{10}(u,v,w,\tau)}{\partial v} d\,w\,d\,v\,d\,u\,d\,\tau$$

$$-\frac{2\pi}{L_x L_y L_z^2}\sum_{n=1}^{\infty} n F_{nC} c_n(x) c_n(y) c_n(z) e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^{L_x} c_n(u) \int_0^{L_y} c_n(v)$$

$$\times \int_0^{L_z} s_n(w) \frac{C_{00}^{\gamma}(u,v,w,\tau)}{P^{\gamma}(u,v,w,T)} \frac{\partial C_{10}(u,v,w,\tau)}{\partial w} d\,w\,d\,v\,d\,u\,d\,\tau$$

$$-\frac{2\pi}{L_x^2 L_y L_z}\sum_{n=1}^{\infty} n F_{nC} c_n(x) c_n(y) c_n(z) e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^{L_x} s_n(u) \int_0^{L_y} c_n(v)$$

$$\times \int_0^{L_z} c_n(w) C_{10}(u,v,w,\tau) \frac{C_{00}^{\gamma-1}(u,v,w,\tau)}{P^{\gamma}(u,v,w,T)} \frac{\partial C_{00}(u,v,w,\tau)}{\partial u} d\,w\,d\,v\,d\,u\,d\,\tau$$

$$-\frac{2\pi}{L_x L_y^2 L_z}\sum_{n=1}^{\infty} n F_{nC} c_n(x) c_n(y) c_n(z) e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^{L_x} c_n(u) \int_0^{L_y} s_n(v)$$

$$\times \int_0^{L_z} c_n(w) \frac{C_{00}^{\gamma-1}(u,v,w,\tau)}{P^{\gamma}(u,v,w,T)} \frac{\partial C_{00}(u,v,w,\tau)}{\partial v} C_{10}(u,v,w,\tau) d\,w\,d\,v\,d\,u\,d\,\tau$$

$$-\frac{2\pi}{L_x L_y L_z^2}\sum_{n=1}^{\infty} n F_{nC} c_n(x) c_n(y) c_n(z) e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^{L_x} c_n(u) \int_0^{L_y} c_n(v)$$

$$\times \int_0^{L_z} s_n(w) C_{10}(u,v,w,\tau) \frac{C_{00}^{\gamma-1}(u,v,w,\tau)}{P^{\gamma}(u,v,w,T)} \frac{\partial C_{00}(u,v,w,\tau)}{\partial w} d\,w\,d\,v\,d\,u\,d\,\tau,$$