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Generalization of Dynamic Two Stage Models in DEA

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Abstract. Dynamic network data envelopment analysis (DNDEA) has attracted a lot of attention in recent years. On one hand the available models in DNDEA evaluating the performance of a DMU with interrelated processes during specified multiple periods but on the other hand they can only measure the efficiency of dynamic network structure when a supply chain structure present. For example, in the banking industry, profit is generated by using loan as a source of investment funds, and the generate profit can also be regarded as an added source. Existing models are not able to use output from different section in different period as inputs, for the next section. So, we propose a dynamic network DEA model involve two-stage structure in each period. Hence, this article is composite of two-stage and dynamic DEA that is called dynamic two-stage DEA (DTDEA) model. It has the advantages of both dynamic DEA and two-stage models. This model can be utilized to highlight those bank branch managers can be further analyzed for best practice benchmarking.

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1. Introduction

One of the important branches of operation research (OR), is data envelopment analysis (DEA) that introduced by Charnes and Cooper [3]. DEA was basically presented the efficiency of a set of decision making unites (DMUs) which consist of multiple input to produce multiple output. But many cases have to considered with internal structure, because find the internal relation help to have better analyzing of system. In other word, DMUs consist of multi-stage network structure. There is a set of intermediate products between the two-stage, which those are the output from the first stage that become input for second stage. For example, Wang et al. [24] found that information technology (IT) has effect on some process, such as banking system. Which has two process, capital collection and investment. Färe and Grosskopf [7] introduced the network DEA model. Kao and Hwang [10] describe a two-stage process where 24 non-life insurance companies. For further details and recent extension. See also Sexton and Lewis [17], Liang et al. [13], Castelli et al. [2], Tone and Tsutsui [23], Chen and Zhu [4], Chilingerian and Sherman [5].

However, the traditional DEA techniques are used to measure the performance of a DMU in particular period of time in a static framework. But when there are several periods with inter-temporal dependence between input and output levels, the overall efficiency must be measured in a new manner that's called dynamic DEA, that taking into account the

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inter-relationship among consecutive terms. Otherwise, the accounted results for efficiency measure are not correct explicitly.

Sengupta [16] has considered a dynamic DEA model with respect to shadow values of quasi-fixed input and their optimal cost frontier. Färe and Grosskopf [7] formulated some multi period linear programming problems by which more realistic production processes among periods can be expressed to their DEA model. Nemoto and Goto [14] and then Sueyoshi and Sekitani [20] presented models that were defined on dynamic production possibility set. Emrouznejad and Thanassoulis [6] considered particular of inter-temporal dependence between input-output levels. Jahanshahloo et al. [8] and Soleimani-damaneh [18,19] supply some algorithm that determine the return to scale and efficiency measure by non-convex dynamic DEA model. Tone and Tsutsui [21,22] introduced some dynamic DEA model based on SBM. In application work Kawaguchi et al. [12] investigated a model that estimates performance of Japan hospital with dynamic network DEA. Amirteimoori [1] presented a dynamic DEA model that calculates performance of total income according to performance income, in each period. Omrani and Soltanzadeh [15] presented a combination of the relational dynamic DEA model, introduce by Kao [9] and the relational network DEA model presented by Kao and Hwang [11] to present new dynamic network DEA model.

The propose model from Omrani and Soltanzadeh [15] and other models in DNDEA applied to use for problems with supply chain structure. In this model, section (i) in (t) th period, is connected only to section (i) in $(t+1)$ th period. This reason, cause in some case study these models not responsive, in the other word, surplus of section (i) in (t) th period not become input to section $(i-1)$ in $(t+1)$ th period. For example, in the bank, profit is the final output of section (i) in (t) th period addition to the bank resources of section $(i-1)$ in $(t+1)$ th period. This is a kind of problem that conventional models are not responded. The main goal for this paper is combination of the relational dynamic DEA model introduce by Nemoto and Goto [14] and the relational two-stage model presented by Chen and Zhu [4] to propose a model for solve these problems. In addition to the new model contain both DNDEA and two-stage models advantages, and help manager for better analyze in in all process over the entire time.

Most of the works in dynamic systems, calculate overall efficiency and specific period efficiencies, separately. dynamic network DEA models have supply chain structure that overproduction in section (i) of period (t) cannot transfer as input as section $(i-1)$ of period $(t+1)$ and it is not suitable for our case study. Therefore, we present a new model that is a combination of dynamic and two-stage structure. This new model is called dynamic two-stage DEA (DTDEA) that solves this issue and also it has both dynamic DEA and two-stage models advantages.

2. Definition

2.1. *Black box model*

In traditional DEA models, DMUs use inputs to produce output and ignore internal relations of sub processes, for this reason called black box models (Figure 1).

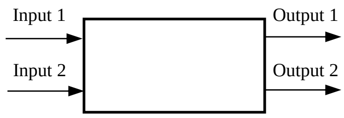

Figure 1. An example of black-box system.

2.2. *Two-stage models*

Simplest structure of network DEA models is two-stage that is, for each DMU_j ($j = 1, \dots, n$), all inputs (x_{ij}, $i = 1, \dots, m$) in stage 1, produce intermediate products (z_{dj}, $d = 1, \dots, D$) and then, in stage 2, z_{dj} as input product final output (y_{rj}, $r = 1, \dots, s$). For DMU_o ($o \in j = 1, \dots, n$), E_o^1, E_o^2 are respectively performance of stage 1 and stage 2, according to CCR input-oriented model (Figure 2).

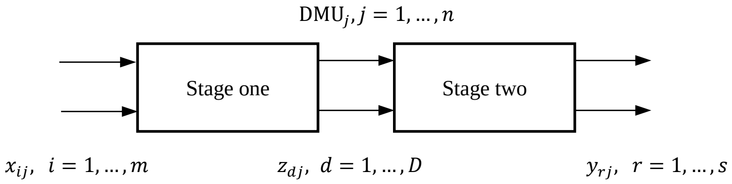

Figure 2. Simplest structure of two-stage network DEA models.

E_o^1 is

$$
\begin{aligned}
&\max \ \frac{\sum_{d=1}^{D} w_d z_{do}}{\sum_{i=1}^{m} v_i x_{io}} \\
&\text{s.t.} \ \ \frac{\sum_{d=1}^{D} w_d z_{dj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1, \quad j = 1, \dots, n, \\
&\qquad w_d \ge 0, \quad d = 1, \dots, D, \\
&\qquad v_i \ge 0, \quad i = 1, \dots, m.
\end{aligned}
$$

And E_o^2 is

$$
\begin{aligned}
&E_o^2 = \max \ \frac{\sum_{r=1}^{S} u_r y_{ro}}{\sum_{d=1}^{D} \widehat{w}_d z_{do}} \\
&\text{s.t.} \ \ \frac{\sum_{r=1}^{S} u_r y_{rj}}{\sum_{d=1}^{D} \widehat{w}_d z_{dj}} \le 1, \quad j = 1, \dots, n, \\
&\qquad \widehat{w}_d \ge 0, \quad d = 1, \dots, D, \\
&\qquad u_r \ge 0, \quad r = 1, \dots, s.
\end{aligned}
$$

So, if put $w_d = \widehat{w}_d$ (Kao and Hwang [10]), final efficiency is equal to $E_o^1 * E_o^2$ and so final model is

$$
\begin{aligned}
&\max \ E_o^1 * E_o^2 = \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}} \\
&\text{s.t.} \ \ \mathrm{E}_j^1 \le 1, \quad j = 1, \dots, n, \\
&\qquad \mathrm{E}_j^2 \le 1, \quad j = 1, \dots, n, \\
&\qquad w_d = \widehat{w}_d, \quad d = 1, \dots, D.
\end{aligned}
$$

The above model can be change to linear programming as follows:

$$
\begin{aligned}
\max \quad & \textstyle\sum_{r=1}^{s} u_r y_{ro} \\
\text{s.t.} \quad & \textstyle\sum_{r=1}^{s} u_r y_{rj} - \sum_{d=1}^{D} w_d z_{dj} \leq 0, \quad j = 1, \dots, n, \\
& \textstyle\sum_{d=1}^{D} w_d z_{dj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \quad j = 1, \dots, n, \\
& \textstyle\sum_{i=1}^{m} v_i x_{io} = 1, \\
& w_d \geq 0, \quad d = 1, \dots, D, \\
& v_i \geq 0, \quad i = 1, \dots, m, \\
& u_r \geq 0, \quad r = 1, \dots, S.
\end{aligned}
$$

But one of problem in the above model when happen that intermediate products have different goals, in this case according to Chen and Zhu [4] the final model is

$$
\begin{aligned}
\min \quad & w_1\alpha - w_2\beta \\
\text{s.t.} \quad & \textstyle\sum_{j=1}^{n} \lambda_j x_{ij} \leq \alpha x_{io}, \quad i = 1, \dots, m, \\
& \textstyle\sum_{j=1}^{n} \lambda_j z_{dj} \leq \bar{z}_{do}, \quad d = 1, \dots, D, \\
& \textstyle\sum_{j=1}^{n} \lambda_j = 1, \quad j = 1, \dots, n, \\
& \textstyle\sum_{j=1}^{n} \mu_j z_{dj} \leq \bar{z}_{do}, \quad d = 1, \dots, D, \\
& \textstyle\sum_{j=1}^{n} \mu_j y_{rj} \leq \beta y_{ro}, \quad r = 1, \dots, R, \\
& \textstyle\sum_{j=1}^{n} \mu_j = 1, \quad j = 1, \dots, n, \\
& \lambda_j, \mu_j \geq 0, \quad j = 1, \dots, n, \\
& \alpha \leq 1, \ \beta \geq 1.
\end{aligned}
$$

2.3. *Dynamic DEA model*

Dynamic structure is general form of series structure that each period has a network structure.

2.4. *Proposed model*

The main contribution of this paper is combination of the Dynamic DEA model introduced by Nemoto and Goto [14] and the two-stage model by Chen and Zhu [4]. The proposed Dynamic Two-stage DEA (DTDEA) model is applied to calculate the efficiency scores of Iranian Saderat Bank branch. In this paper we develop a new structure of dynamic DEA model. We present a model that is able to transfer over production in section (i) of period (t) as input as to section $(i-1)$ of period $(t+1)$.

3. Formulation for dynamic two-stage DEA model

For measuring score of efficiency, we deal with n DMUs $(j = 1, \dots, n)$ consist of two-stage structure at T time periods $(t = 1, \dots, T)$. The observed data are as follows (Figure 3):

x_{tij} is ith input of DMU_j in the period t.

k_{tfj} is fth quasi-fixed input of DMU_j at the end of the period t.

y_{trj} is rth good production of DMU_j in the period t.

z_{tdj} is dth intermediate production of DMU_j in the period t.

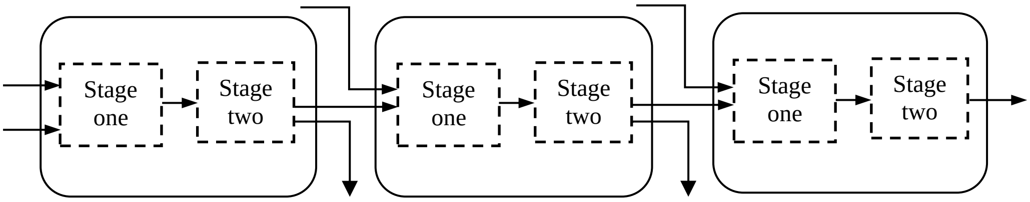

Figure 3. Structure of dynamic two-stage DEA models.

A branch puts (x_{tij}, x_{tfj}) into both the production process and the investment activity to yield (y_{trj}, k_{tfj}). Denote to Figure 3, the proposed model is:

$$
\begin{aligned}
\min \quad & Z = \sum_{t=1}^{T} \zeta_t \left(w_t(\theta^t - \beta^t)\right) \\
\text{s.t.} \quad & \sum_{j=1}^{n} \lambda_{tj}\, x_{tij} \leq \theta^t x_{tio}, \quad t = 1, \dots, T,\ i = 1, \dots, m, \\
& \sum_{j=1}^{n} \lambda_{1j}\, \bar{k}_{01j} \leq \theta^t k_{01o}, \\
& \sum_{j=1}^{n} \lambda_{tj}\, z_{tpj} \geq \widetilde{w}_{tpo}, \quad t = 1, \dots, T,\ p = 1, \dots, f, \\
& \sum_{j=1}^{n} \mu_{tj}\, z_{tpj} \leq \widetilde{w}_{tpo}, \quad t = 1, \dots, T,\ p = 1, \dots, f, \\
& \sum_{j=1}^{n} \mu_{tj}\, k_{tlj} \geq \beta^t k_{tlo}, \quad t = 1, \dots, T,\ l = 1, \dots, d, \\
& \sum_{j=1}^{n} \mu_{tj}\, y_{trj} \geq \beta^t y_{tro}, \quad t = 1, \dots, T,\ r = 1, \dots, s, \\
& \sum_{j=1}^{n} \lambda_{tj} = 1, \quad t = 1, \dots, T, \\
& \sum_{j=1}^{n} \mu_{tj} = 1, \quad t = 1, \dots, T, \\
& \theta^t \leq 1, \quad t = 1, \dots, T, \\
& \beta^t \geq 1, \quad t = 1, \dots, T, \\
& \lambda_{tj}, \mu_{tj}, x_{tij}, y_{trj}, k_{tlj} \geq 0, \\
& t = 1, \dots, T, \quad i = 1, \dots, m, \quad l = 1, \dots, d, \quad r = 1, \dots, s, \quad j = 1, \dots, n.
\end{aligned}
$$

According to (1) and (2), the above formula can be rewritten as follows:

$$
\begin{aligned}
\min \quad & Z = \sum_{t=1}^{T} \zeta_t \left(w_t(\theta^t - \beta^t)\right) \\
\text{s.t.} \quad & \sum_{j=1}^{n} \lambda_{tj}\, x_{tij} \leq \theta^t x_{tio}, \quad t = 1, \dots, T,\ i = 1, \dots, m, \\
& \sum_{j=1}^{n} \lambda_{1j}\, \bar{k}_{01j} \leq \theta^t k_{01o}, \\
& \sum_{j=1}^{n} \mu_{tj}\, z_{tpj} \leq \sum_{j=1}^{n} \lambda_{tj}\, z_{tpj}, \quad t = 1, \dots, T,\ p = 1, \dots, f, \\
& \sum_{j=1}^{n} \mu_{tj}\, k_{tlj} \geq \beta^t k_{tlo}, \quad t = 1, \dots, T,\ l = 1, \dots, d, \\
& \sum_{j=1}^{n} \mu_{tj}\, y_{trj} \geq \beta^t y_{tro}, \quad t = 1, \dots, T,\ r = 1, \dots, s, \\
& \sum_{j=1}^{n} \lambda_{tj} = 1, \quad t = 1, \dots, T, \\
& \sum_{j=1}^{n} \mu_{tj} = 1, \quad t = 1, \dots, T,
\end{aligned}
$$

$$\theta^t \le 1, \quad t = 1, \dots, T,$$

$$\beta^t \ge 1, \quad t = 1, \dots, T,$$

$$\lambda_{tj}, \mu_{tj}, x_{tij}, y_{trj}, k_{tlj} \ge 0,$$

$$t = 1, \dots, T, \quad i = 1, \dots, m, \quad l = 1, \dots, d, \quad r = 1, \dots, s, \quad j = 1, \dots, n.$$

4. Conclusion

In this paper, a new model was presented by combining two-stage and dynamic DEA. This new model is called dynamic two-stage DEA (DTDEA) and also it has both dynamic DEA and two-stage models advantages.

References

[1] A. Amirteimoori, Data envelopment analysis in dynamic framework, Applied Mathematics and Computation, **191** (2006) 21-28.
[2] L. Castelli, R. Pesenti and W. Ukovich, DEA-like models for the efficiency evaluation of hierarchically structured units, European Journal of Operational Research, **154 (2)** (2004) 465-476.
[3] A. Charnes, W. W. Cooper and E. Rhodes, Measuring the efficiency of decision making units, European Journal of Operational Research, **2** (1978) 429-444.
[4] Y. Chen and J. Zhu, Measuring information technology's indirect impact on firm performance, Information Technology and Management, **5** (2004) 9-22.
[5] J. A. Chilingerian and H. D. Sherman, Health Care Applications, In: W. W. Cooper, L. M. Seiford and J. Zhu (eds), Handbook on Data Envelopment Analysis, International Series in Operations Research & Management Science, vol 71. Springer, Boston, MA, (2004).
[6] A. Emrouznejad and E. Thanassoulis, A mathematical model for dynamic efficiency using data envelopment analysis, Applied Mathematics and Computation, **160** (2005) 363-378.
[7] R. Färe and S. Grosskopf, Intertemporal Production Frontiers: With Dynamic DEA, Kluwer Academic Publishers, (1996).
[8] G. R. Jahanshahloo, M. Soleimani-damaneh and M. Reshadi, On the Pareto (dynamically) efficient path, International Journal of Computer Mathematics, **83** (2006) 629-633.
[9] C. Kao, Dynamic data envelopment analysis: a relational analysis, European Journal of Operational Research, **227** (2013) 325-330.
[10] C. Kao and S. N. Hwang, Efficiency decomposition in two-stage data envelopment analysis: An application to non-life insurance companies in Taiwan, European Journal of Operational Research, **185** (2008) 418-429.
[11] C. Kao and S. N. Hwang, Efficiency measurement for network systems: IT impact on firm performance, Decision Support Systems, **48** (2010) 437-446.
[12] H. Kawaguchi, K. Tone and M. Tsutsui, Estimation of the efficiency of Japanese hospitals using a dynamic and network data envelopment analysis model, Health Care Management Science, **17** (2014) 101-112.
[13] L. Liang, W. D. Cook and J. Zhu, DEA models for two-stage processes: game approach and efficiency decomposition. Naval Research Logistics, **55** (2008) 643-653.
[14] J. Nemoto and M. Goto, Dynamic data envelopment analysis: modeling intertemporal behavior of a firm in the presence of productive inefficiencies, Economics Letters, **64** (1999) 51-56.
[15] H. Omrani and E. Soltanzadeh, Dynamic DEA models with network structure: An application for Iranian airlines, Journal of Air Transport Management, **57 (2016)** 52-61.
[16] J. Sengupta, Dynamics of Data Envelopment Analysis: Theory of System Efficiency, Springer Netherlands, (1995).
[17] T. R. Sexton and H. F. Lewis, Two-stage DEA: An application to major league baseball, Journal of Productivity Analysis, **19** (2003) 227-249.
[18] M. Soleimani-damaneh, An enumerative algorithm for solving nonconvex dynamic DEA models, Optimization Letters, **7** (2013) 101-115.
[19] M. Soleimani-damaneh, Another approach to estimating RTS in dynamic DEA, Journal of Productivity Analysis, **39** (2013) 75-81.
[20] T. Sueyoshi and K. Sekitani, Returns to scale in dynamic DEA, European Journal of operational Research, **161 (2)** (2005) 536-544.
[21] K. Tone and M. Tsutsui, Dynamic DEA: A slacks-based measure approach, Omega, **38** (2010) 145-156.
[22] K. Tone and M. Tsutsui, Dynamic DEA with network structure: A slacks-based measure approach, Omega, **42** (2014) 124-131.
[23] K. Tone and M. Tsutsui, Network DEA: a slack-based measure approach, European Journal of operational Research, **197** (2009) 243-252.
[24] C. H. Wang, R. D. Gopal and S. Zionts, Use of data envelopment analysis in assessing information technology impact on firm performance, Annals of Operations Research, **73** (1997) 191-213.