International Journal of
Mathematical Modelling & Computations
Vol. 09, No. 01, Winter 2019, 17- 37

Option Pricing on Commodity Prices Using Jump Diffusion Models

T. Berhane[a,*], M. Adam[b] and E. Haile[c]

[a,b,c]*Department of Mathematics, Bahir Dar University, P.O.B 79, Ethiopia.*

Abstract. In this paper, we aim at developing a model for option pricing to reduce the risks associated with Ethiopian commodity prices fluctuations. We used the daily closed Unwashed Nekempti grade 5 (ULK5) coffee and Whitish Wollega Sesame Seed Grade3 (WWSS3) prices obtained from Ethiopia commodity exchange (ECX) market to analyse the prices fluctuations.The natures of log-returns of the prices exhibit asymmetric heavy tails and high kurtosis. We used jump diffusion models for modeling and option pricing on commodity prices. The method of maximum likelihood is applied to estimate the parameters under the models. The root mean square error (RMSE) is used to test the goodness of fitting for the models to the data. This test indicates that the models fit the data well. The techniques of analytical and Monte Carlo simulation are used to find the call option pricing of the commodity prices. Based on the empirical results, we conclude that double exponential jump diffusion model is more efficient than Merton's model for modeling and option pricing of the commodity prices.

Received: 05 January 2019, Revised: 15 February 2019, Accepted: 11 March 2019.

Keywords: Jump diffusion model; Option pricing; Kurtosis; Skewness; Risk-neutral measure; Commodity prices.

Index to information contained in this paper

1. Introduction

In Ethiopia, the economy of the country is mainly dependent on the agricultural sector. It contributes 85% of the population for employment, 44% to the country's

*Corresponding author. Email: tesfahunb2002@gmail.com

gross domestic product (GDP) and 85% of the country's export earnings [28]. The country has a total land area of 111.5 million hectares. Out of which 66% of it is arable land. In 2015/2016, the total cultivated area is about 13 million hectares [14]. Coffee and Sesame, which are the major exported agricultural commodities, play vital roles for the growth of the Ethiopian economy. For instance, in 2012 fiscal year, Ethiopia was the major coffee producer and exporter in Africa and ranked tenth of the largest coffee exporters in the world. The country exported 3.2 million bags of coffee and accounted for 3 percent of internationally traded coffee in the same year [5]. Coffee, which covers the largest portion of total exports of the country, has high contributions to its GDP and the most crucial source of hard currency as it is pointed out by Ethiopia Revenue and Customs Authority(ERCA) [13]. Similarly, sesame, which is produced by both small and large scale farmers, is an important crop and export commodity. The total cultivated area of sesame, its production value and productivity during 2013 were 0.299 million hectares, 0.220 million tonnes and 0.735 tone per hectare, respectively; and the total area and production were increased by 61.23 % and 17.91 %, respectively [10]. It is the second largest export crop in Ethiopia next to coffee, and it accounts for over 90% of the value of oil seeds exports [15]. And the country has become the third largest exporter of sesame seed after India and Sudan [1]. Its annual total exported quantity increased from 50,000 to 150,000 tonnes, which is a threefold rise in eight years time [9]. The main importers of Ethiopian sesame are China, Israel, Turkey, Japan and other European countries [26].

Thus, some agricultural commodities like coffee, sesame, haricot bean, etc have great significance in earning the foreign currency to the country. To this effect, the Ethiopian ministry of agriculture has developed a master plan to enhance the productivity, quality and market efficiency of the commodities. Therefore, Ethiopia is leveled among the top commodity producer and exporter countries in the world as it is suggested by [1, 5]. The market price of cereal crops exhibits great variability and this leads to high price volatility in the country[25]. Agricultural markets also have been exposed to high price fluctuations which resulted in excessive risk [23]. This price fluctuation has an impact on the economy of the country as well as individuals whose life depend on agricultural products.

Therefore, it is of great significance to develop a model for option pricing to reduce the risks associated with the commodity prices fluctuations. The Black-Scholes model [6], which is based on Brownian motion and normal distribution, has been widely used to model the return of assets and to price financial option for almost four decades. However, many empirical evidences have recently shown the leptokurtic features that the return distribution of assets may have a higher peak and two asymmetric heavy tails than those of normal distribution, as well as an abnormality, often referred to as volatility smile, that is observed in option pricing [20, 22]. Many other models have been proposed in order to reflect the leptokurtic features under a market measure. However, this phenomena under risk-neutral measure leads to the volatility smile in option pricing. For instance, an affine jump diffusion model [12] was proposed to reflect the features. One of its special cases is the normal jump diffusion model[16], which was also used by Merton.

In Merton's model [24], the asset return follows a Brownian motion with drift punctuated by jumps arriving according to a compound Poisson process with constant intensity and with normally distributed jump sizes. Due to normality of the jump size distribution, Merton was able to obtain explicit analytical solutions for European style call options in this model. Kou [20] recently proposed a double exponential jump-diffusion model where jump sizes are double exponentially distributed. This model has a memoryless property inherited from the exponential distribution. This property explains the reason why analytical or approximated solutions for different option pricing problems are viable to this model. Thus, both Merton's jump diffusion and double exponential jump-diffusion models are proposed to reflect the asymmetric leptokurtic features of asset prices under a risk neutral measure leads to determine the call option pricing of some asset prices.

However, no study has been done on option pricing of Ethiopian agricultural commodity prices using jump diffusion models. Therefore, in this paper we used both the Merton's jump diffusion and double exponential jump diffusion models for modeling and call option pricing of commodity prices. Here, we also applied the method of maximum likelihood to estimate the parameters with the models. Based on the empirical results, we compared the models fitting them to the empirical data. More over, we used RMSE to test the validation of these models. Finally, we determined the call option pricing of Ethiopian agricultural commodity prices with analytical and Monte Carlo simulation techniques.

2. Analysis of ULK5 coffee and WWSS3 sesame prices data

We considered the daily closed prices of ULK5 coffee and WWSS3 sesame recorded from 31 May 2011 to 30 March 2018 and 8 November 2010 to 30 March 2018, respectively obtained at Ethiopia commodity exchange (ECX) market to study the prices movements. Ethiopian Birr(ETB) is used for the commodity market prices exchanges (Figures 1 and 2).

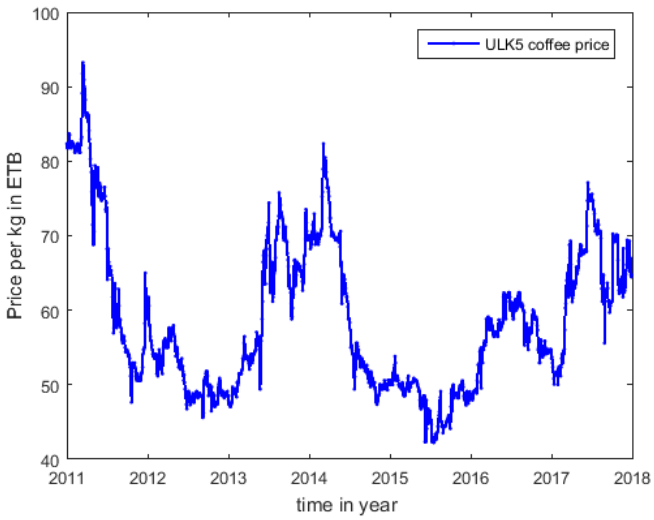

Figure 1. The ULK5 coffee price from 2011 to 2018.

We assume that S_t is to denote the Ethiopian commodity prices. We insight into the dynamic behavior of the prices by analyzing the log-return which is defined as $x_t = \Delta \ln(S_t) = \ln(S_t) - \ln S_{t-1}$. The graphs of log-return prices are plotted in Figures 3 and 4. These graphs illustrate that spikes are observed significantly in the empirical data. More over, the histograms of log-return prices along with the normal densities are plotted as shown in Figures 5 and 6. And the descriptive statistics of log-return prices of the commodities are displayed in Table 1. So, these figures and Table 1 show the existence of skewness, fat tails and high kurtosis in the empirical distribution of the commodity prices. These indicate that the log-returns of ULK5 and WWSS3 prices are not normally distributed.

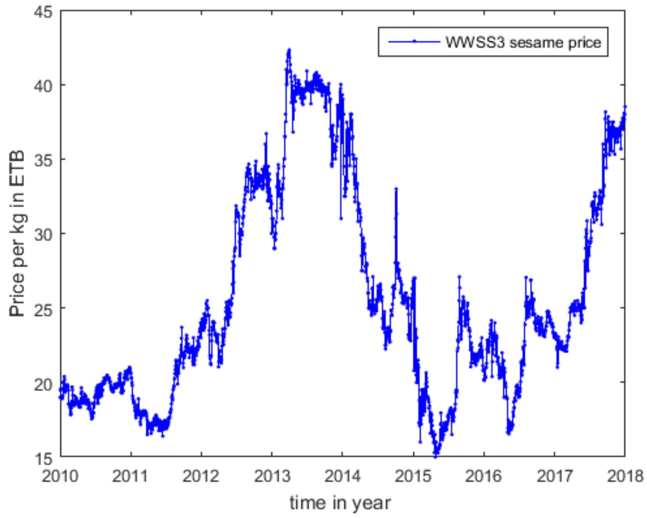

Figure 2. The WWSS3 sesame price from 2010 to 2018.

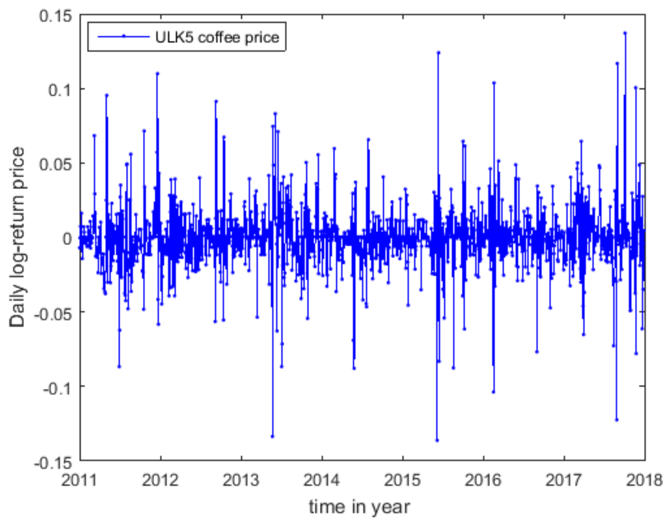

Figure 3. The log-return of ULK5 coffee price.

Table 1. Descriptive statistics of log-return of ULK5 and WWSS3 prices.

	Commodity prices	
Descriptive statistics	ULK5 coffee price	WWSS3 sesame price
Mean	-0.0001962854402620	0.0004295737604181
Standard Deviation	0.022644750485283	0.031385155727349
Skewness	0.072619510431776	-0.890291718668269
Kurtosis	10.737615556289899	15.643591825205396

3. Jump diffusion models

Based on the analysis of commodity prices data, namely the presence of skewness, fat tails and high kurtosis in the empirical distribution of the prices returns, the adequate models for such prices would be jump diffusion models [2]. In fact, Merton [24], recognizing the presence of jumps in asset prices and for more accurate option

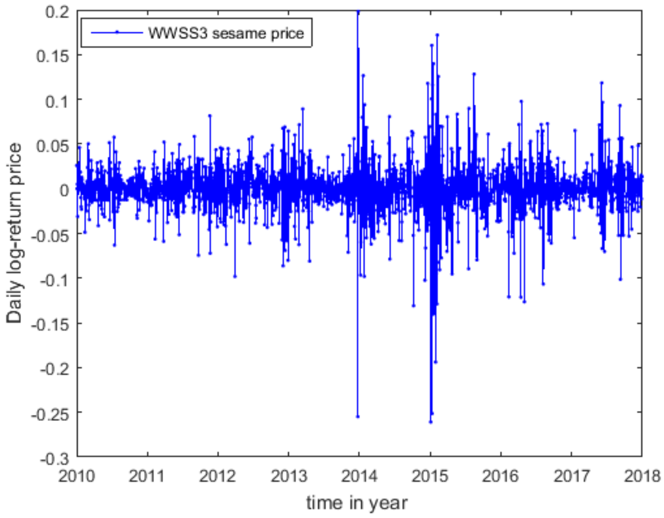

Figure 4. The log-return of WWSS3 sesame price.

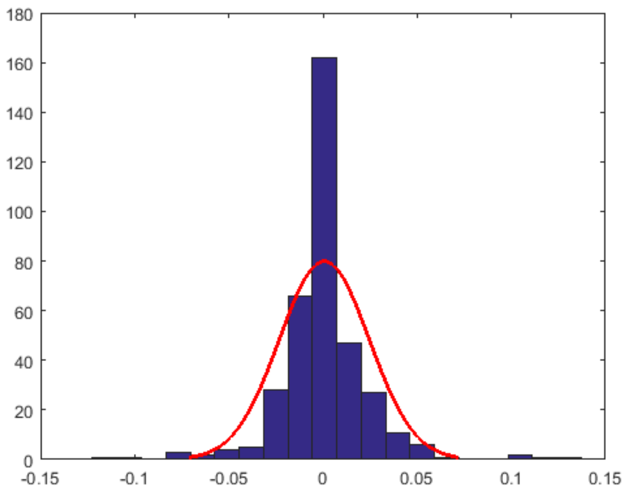

Figure 5. The histogram and normal density of daily log-return ULK5 coffee price from 2011 to 2018.

pricing, proposed modeling the prices as a jump diffusion process instead of a pure diffusion model. Pure diffusion based models could not adequately explain the smile effect in short-dated option prices and emphasized the importance of adding a jump component in modeling asset price dynamics [3]. Models with jumps generically lead to significant skews for short-term maturities. More generally, adding jumps to returns in a diffusion-based stochastic volatility model, the resulting model can generate sufficient variability and asymmetry in the short-term returns to match implied volatility skews for short-term maturities [2].

3.1 *Merton's jump diffusion model*

The commodity prices process S_t under Merton's jump diffusion model with the physical probability measure P is assumed to follow the stochastic differential equa-

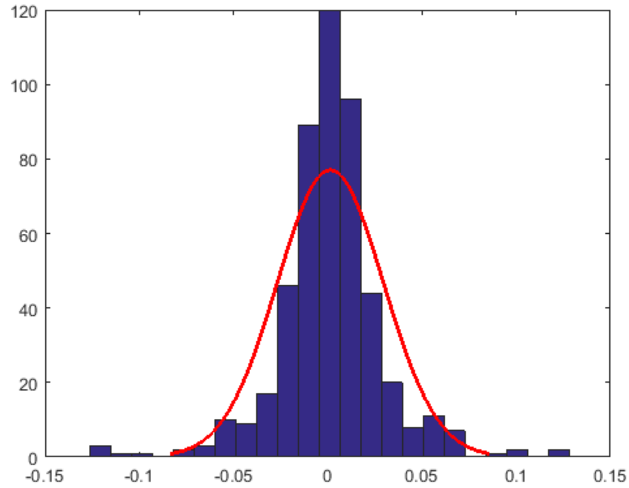

Figure 6. The histogram and normal density of daily log-return WWSS3 sesame price from 2010 to 2018.

tion

$$\frac{dS_t}{S_{t-}} = \mu dt + \sigma dB_t + (y_t - 1)dN_t, \tag{1}$$

where μ is the instantaneous expected return and σ is the instantaneous volatility of the price return. The continuous component is given by a standard Brownian motion, B_t, distributed as $dB_t \sim (0, dt)$. The discontinuity of the prices process is described by a Poisson counter N_t, characterized by its intensity, λ, and jump size y_t. The assumption is that the Brownian motion B_t, the Poisson process N_t and the jump size y_t are independent. The intensity of the Poisson process describes the mean number of arrivals of abnormal information per unit of time dt and is expressed as:

$$\text{Prob}[dN_t = 1] = \lambda dt, \text{ and } \text{Prob}[dN_t = 0] = 1 - \lambda dt. \tag{2}$$

When abnormal information arrives, the commodity prices jump from S_{t-} (limit from left) to $S_t = y_t S_{t-}$. The percentage change is measured by $(y_t - 1)$. The price S_t presents log-normal jumps y_t on each random time t which represents the moments of jumping of a Poisson process [18, 19]. Introduction of the jump diffusion model adds three extra parameters $(\beta, \delta^2, \lambda)$ to the Black-Sholes process model which contains two parameters (μ, σ^2). Merton assumes that the log-price jump size Y_t=ln(y_t) is normal random variables. Using Ito's lemma, the log-price return process becomes:

$$dln(S_t) = (\mu - \frac{1}{2}\sigma^2)dt + \sigma dB_t + Y_t dN_t. \tag{3}$$

Discretized over $[t, t + \Delta t]$, the model takes the form:

$$\Delta ln(S_t) = (\mu - \frac{1}{2}\sigma^2)\Delta t + \sigma \Delta B_t + \sum_{j=0}^{\Delta N_t} Y_j, \tag{4}$$

where $\Delta B_t = B_{t+\Delta t} - B_t \sim N(0, \Delta t)$ and $\Delta N_t = N_{t+\Delta t} - N_t$ is the number of jumps occurring during the time interval over $(t, t+\Delta t)$ and Y_t are independently and identically distributed as $Y_t \sim N(\beta, \delta^2)$ with probability density

$$f_{Y_t}(y) = \frac{1}{\sqrt{2\pi\delta^2}} \exp\left[- \frac{(y-\beta)^2}{2\delta^2} \right], y \epsilon \mathbb{R}. \tag{5}$$

The log-return, $x_t = \Delta ln(S_t)$, therefore includes the sum of two independent components: a diffusion component with drift and a jump component. The probability density of x_t can be expressed [2] as:

$$f_{\Delta t}(x) = \sum_{n=0}^{\infty} \frac{(\lambda\Delta t)^n e^{-\lambda \Delta t}}{n!} \left[\frac{1}{\sqrt{(2\pi(\sigma^2\Delta t + n\delta^2))}} exp\left(- \frac{(x - (\mu - \frac{1}{2}\sigma^2)\Delta t - n\beta)^2}{2(\sigma^2 \Delta t + n\delta^2)} \right) \right], \tag{6}$$

with $n = 0, 1, 2, ...$ Putting $\Delta t = 1$, that is the time interval is $(t, t+1)$, the density function becomes

$$f(x) = \sum_{n=0}^{\infty} \frac{(\lambda)^n e^{-\lambda}}{n!} \left[\frac{1}{\sqrt{2\pi(\sigma^2 + n\delta^2)}} exp\left(- \frac{(x - (\mu - \frac{1}{2}\sigma^2) - n\beta)^2}{2(\sigma^2 + n\delta^2)} \right) \right]. \tag{7}$$

3.2 *Double exponential jump diffusion (DEJD) model*

The double exponential jump diffusion model, which is used to model the commodity prices, consists of two parts. That is a continuous part driven by a geometric Brownian motion, and a jump part, with the logarithm of jump sizes having a double exponential distribution and the jump times corresponding to the event times of a Poisson process. Thus, under the physical probability measure P, the dynamic behavior of commodity prices is assumed to follow the stochastic differential equation

$$\frac{dS_t}{S_{t-}} = \mu_1 dt + \sigma_1 dB_t + d\left(\sum_{i=1}^{N_t} (\upsilon_i - 1) \right), \tag{8}$$

where B_t is a standard Brownian motion, N_t is a Poisson process with rate λ_1, and $\{\upsilon_t\}$ is sequence of independent identically distributed (i.i.d.) nonnegative random variables such that $V_t = log(\upsilon_t)$ has an asymmetric double exponential distribution with density
$f_{V_t}(y) = p\,\eta_1 e^{-\eta_1 y} 1_{\{y \geq 0\}} + q\,\eta_2 e^{\eta_2 y} 1_{\{y<0\}}$, $\eta_1 > 1, \eta_2 > 0$, where $p, q \geq 0$, $p + q = 1$ are constants and represent the probabilities of upward and downward jumps respectively. This can be also written as:

$$\log(\upsilon_t) = V_t := \begin{cases} \xi^+ \text{ with probability } p \\ -\xi^- \text{ with probability q}, \end{cases} \tag{9}$$

where ξ^+ and ξ^- exponentially random variables with mean $\frac{1}{\eta_1}$ and $\frac{1}{\eta_2}$ respectively. The random variables N_t, B_t and V_t are letting to be independent and identically distributed in the model. It is assumed that the drift μ_1 and the volatility σ_1 are constants, while the Brownian motion and jumps are one- dimensional random variables [20].

The solution of the stochastic differential equation (8) using Ito's lemma can be expressed as:

$$S_t = S_0 \exp\left((\mu_1 - \frac{\sigma_1^2}{2})t + \sigma_1 B_t + \sum_{j=0}^{N_t} V_j \right), \tag{10}$$

where $\mathrm{E}[\mathrm{V_t}] = \frac{\mathrm{p}}{\eta_1} - \frac{\mathrm{q}}{\eta_2}$, $\mathrm{Var}(\mathrm{V_t}) = \mathrm{pq}\left(\frac{1}{\eta_1} + \frac{1}{\eta_2}\right)^2 + \left(\frac{\mathrm{p}}{\eta_1^2} + \frac{\mathrm{q}}{\eta_2^2}\right)$ and $\mathrm{E}[\upsilon_\mathrm{t}] = \mathrm{E}[\mathrm{e}^{\mathrm{V_t}}] = \left(\mathrm{p}\frac{\eta_1}{\eta_1 - 1} + \mathrm{q}\frac{\eta_2}{\eta_2+1}\right)$, $\eta_1 > 1$, $\eta_2 > 0$. Here, $\eta_1 > 1$ guarantees for $\mathrm{E}[\upsilon_\mathrm{t}] < \infty$ and $\mathrm{E}[\mathrm{S_t}] < \infty$. This means that the average upward jump cannot be greater than 100%, which is quite reasonable [21]. Based on (10), the rate of price return over the time interval Δt is given by:

$$\begin{aligned} \frac{\Delta S_t}{S_t} &= \frac{S_{t+\Delta t}}{S_t} - 1 \\ &= \exp\left((\mu_1 - \frac{\sigma_1^2}{2})\Delta t + \sigma_1 \Delta B_t + \sum_{j=0}^{\Delta N_t} V_j \right) - 1. \end{aligned}$$

If Δt becomes small enough, by neglecting the terms with order higher than Δt, the daily price return can be approximated in distribution using expansion $e^x \approx 1 + x + \frac{x^2}{2}$ by

$$\frac{\Delta S_t}{S_t} \approx \mu_1 \Delta t + \sigma_1 Z \sqrt{\Delta t} + \alpha V_t, \tag{11}$$

where Z is standard normal distribution and α is Bernoulli random variable with $P(\alpha = 1) = \lambda \Delta t$, $P(\alpha = 0) = 1 - \lambda \Delta t$ and V_t is given by (9). The density function g of the right side of (11) which is an approximation of the commodity prices return $\frac{\Delta S_t}{S_t}$ is given by:

$$\begin{aligned} g(x) = \frac{1 - \lambda_1 \Delta t}{\sigma_1 \sqrt{\Delta t}} \phi\left(\frac{1 - \lambda_1 \Delta t}{\sigma_1 \sqrt{\Delta t}}\right) + \lambda_1 \Delta t \Bigg[& p\eta_1 e^{\frac{\sigma_1^2 \eta_1^2 \Delta t}{2}} e^{-(x - \mu_1 \Delta t)\eta_1} \\ \times \Phi\left(\frac{x - \mu_1 \Delta t - \sigma_1^2 \eta_1 \Delta t}{\sigma_1 \sqrt{\Delta t}}\right) & + q\eta_2 e^{\frac{\sigma_1^2 \eta_2^2 \Delta t}{2}} e^{-\left(x - \mu_1 \Delta t\right)\eta_2} \\ \times \Phi\left(\frac{x - \mu_1 \Delta t + \sigma_1^2 \eta_2 \Delta t}{\sigma_1 \sqrt{\Delta t}}\right) \Bigg]. \end{aligned} \tag{12}$$

Setting $\Delta t = 1$, this density function can also be written as:

$$\begin{aligned} g(x) = \frac{1 - \lambda_1}{\sigma_1} \phi\left(\frac{1 - \lambda_1}{\sigma_1}\right) + \lambda_1 \Bigg[& p\eta_1 e^{\frac{\sigma_1^2 \eta_1^2}{2}} e^{-(x - \mu_1)\eta_1} \times \Phi\left(\frac{x - \mu_1 - \sigma_1^2 \eta_1}{\sigma_1}\right) \\ & + q\eta_2 e^{\frac{\sigma_1^2 \eta_2^2}{2}} e^{-(x - \mu_1)\eta_2} \times \Phi\left(\frac{x - \mu_1 + \sigma_1^2 \eta_2}{\sigma_1}\right) \Bigg], \end{aligned} \tag{13}$$

where $\phi(.)$ is density function of standard normal and $\Phi(.)$ is its distribution function. Using Le'vy-Khintchine theorem, the characteristic function of the double exponential jump diffusion process of the log-return price $\Delta \ln(S_t) = X_{\Delta t}$ over the time interval $(t, t + 1)$ is represented by:

$$\phi_{X_{\Delta t}(u)} = \mathrm{E}[\mathrm{e}^{\mathrm{iuX_{\Delta t}}}]$$

$$= \exp\left[iu\mu_1 - \frac{\sigma_1^2 u^2}{2} + \lambda_1\left(\frac{p\eta_1}{\eta_1 - iu} + \frac{q\eta_2}{\eta_2 + iu} - 1\right)\right]. \tag{14}$$

4. Parameter estimation

The parameters vectors are denoted by $\theta = (\mu, \sigma, \beta, \delta, \lambda)$ and $\Theta = (\mu_1, \sigma_1, \eta_1, \eta_2, p, \lambda_1)$ which are associated with both ULK5 coffee and WWSS3 sesame prices processes. The method of maximum likelihood estimation (MLE) are applied to estimate the parameters, θ and Θ under Merton's jump diffusion and double exponential jump diffusion models by maximizing the likelihood functions specified in (7) and (14), respectively. We considered 70% of the log-return commodity prices for parameters estimation and 30% for testing purpose that is for validation of the models. We truncate the number of jumps at $n = 10$ to estimate the parameter values as it is pointed out by [4].

In this paper, the parameters are estimated with corresponding 95% confidence intervals up on using this method. Finally, the values that we obtain here are shown in Tables 2, 3, 4 and 5 below.

Table 2. Estimated parameters associated with ULK5 coffee price under Merton's model.

		95% Confidence Interval	
Parameters	Values	Lower bound	Upper bound
μ	0.000661044851657	-0.000362681038688	0.001684770742002
σ	0.008818424747690	0.007146008267170	0.010490841228210
β	-0.000050175378757	-0.003218325439856	0.003117974682342
δ	0.028015238285806	0.022536305022630	0.033494171548982
λ	0.492344773106976	0.300409690894311	0.684279855319641

Table 3. Estimated parameters associated with ULK5 coffee price under DEJD model.

		95% Confidence Interval	
Parameters	Values	Lower bound	Upper bound
μ_1	0.000230016229897	-0.001443295436320	0.001903327896113
σ_1	0.007215684412257	0.005335338788916	0.009096030035598
η_1	68.192289084277704	48.908243457080431	87.476334711474976
η_2	62.986124231755149	45.829123754834029	80.143124708676268
p	0.534295155560463	0.403372135894600	0.665218175226326
λ_1	0.891506177000199	0.530430204612231	1.252582149388168

Table 4. Estimated parameters associated with WWSS3 sesame price under Merton's model.

		95% Confidence Interval	
Parameters	Values	Lower bound	Upper bound
μ	-0.000693057074750	-0.001969722278931	0.000583608129431
σ	0.015418827195508	0.013441562090757	0.017396092300259
β	0.002703106257488	-0.003593139843385	0.008999352358361
δ	0.050146989514887	0.040240773037236	0.060053205992539
λ	0.294915979685713	0.178442965540220	0.411388993831205

Table 5. Estimated parameters associated with WWSS3 sesame price under DEJD model.

Parameters	Values	95% Confidence Interval	
		Lower bound	Upper bound
μ_1	0.000635640253018	-0.001496031483955	0.002767312650518
σ_1	0.013076820113627	0.010726005471729	0.015427636255556
η_1	32.817870116806390	23.945852466554875	41.689872510044857
η_2	44.691965648828784	31.652376544118237	57.731543372990402
p	0.405554265070454	0.273928643735723	0.537179790408564
λ_1	0.598874279773129	0.338214548299840	0.859533841874166

5. Goodness for fit

In order to assess the goodness of fit of Merton's and double exponential jump diffusion distributions to the dynamic behaviors of the ULK5 and WWSS3 prices , we used the method of non-parametric fit with normal kernel. We applied this technique to approximate the probability density functions of the models specified in (7) and (13) respectively. As a result, we compared the densities of log-returns of commodity prices to the densities of the simulated data with the same mean and variance. This shows that the models provide good fit for the data. However, double exponential jump diffusion model has better fitness to the empirical data than Merton's model as shown in Figures 7 and 8 below.

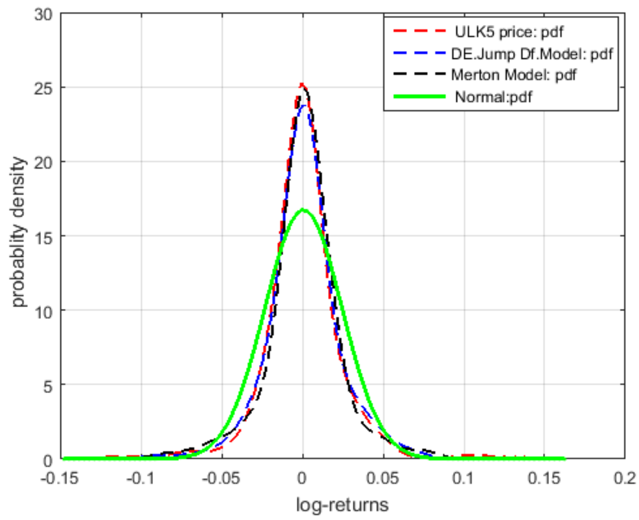

Figure 7. The probability density functions(pdf) of the models fitted with the log-return of ULK5 price.

6. Model simulation

In this section, we used the Euler discretized version of both Merton's and double exponential jump diffusion models to simulate ULK5 and WWSS3 prices. The discretized form of the Merton's model as specified in (4) over the time interval

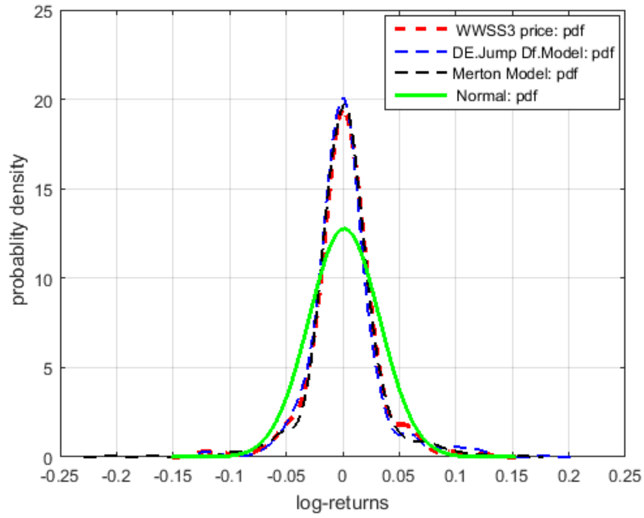

Figure 8. The probability density functions(pdf) of the models fitted with the log-return of WWSS3 price.

$(t, t+\Delta t)$ can be expressed as:

$$ln(S_{t+\Delta t}) = ln(S_t) + (\mu - \frac{1}{2}\sigma^2)\Delta t + \sigma \Delta B_t + \sum_{j=0}^{\Delta N_t} Y_j, \tag{15}$$

where ΔB_t=$\sqrt{\Delta t}Z$ and $Z \sim N(0,1)$. Putting Δt =1, we obtain

$$\ln(S_{t+1}) = \ln(S_t) + (\mu - \frac{1}{2}\sigma^2) + \sigma Z + \sum_{j=0}^{N_t} Y_j. \tag{16}$$

This implies that

$$S_{t+1} = S_t exp\Big((\mu - \frac{1}{2}\sigma^2) + \sigma Z + \sum_{j=0}^{N_t} Y_j\Big). \tag{17}$$

Similarly, the discretized form of double exponential jump diffusion model specified in (8) over the time interval $(t, t+\Delta t)$ and using Ito's lemma, it is given by:

$$\ln(S_{t+\Delta t}) = \ln(S_t) + (\mu_1 - \frac{1}{2}\sigma_1^2)\Delta t + \sigma_1 \Delta B_t + \sum_{j=0}^{\Delta N_t} V_j, \tag{18}$$

where ΔB_t=$\sqrt{\Delta t}Z$ and $Z \sim N(0,1)$. Setting Δt =1, we get

$$\ln(S_{t+1}) = \ln(S_t) + (\mu_1 - \frac{1}{2}\sigma_1^2) + \sigma_1 Z + \sum_{j=0}^{N_t} V_j. \tag{19}$$

This expression can be also written as :

$$S_{t+1} = S_t exp\Big((\mu_1 - \frac{1}{2}\sigma_1^2) + \sigma_1 Z + \sum_{j=0}^{N_t} V_j \Big). \tag{20}$$

We used the models specified in (16), (17), (19) and (20) for the simulation of commodity prices. The fitted values from the simulations are plotted against the observed prices as shown in Figures 9, 10,11 and 12.

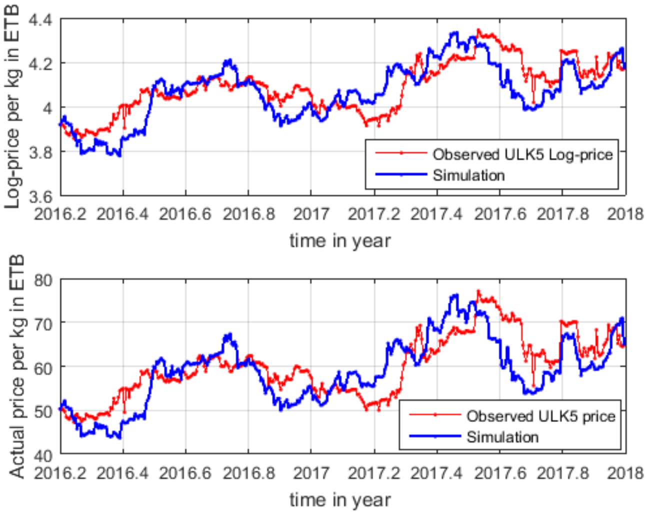

Figure 9. Simulated price fitted to ULK5 log-price(Upper panel) and ULK5 price (Lower panel) under Merton's model.

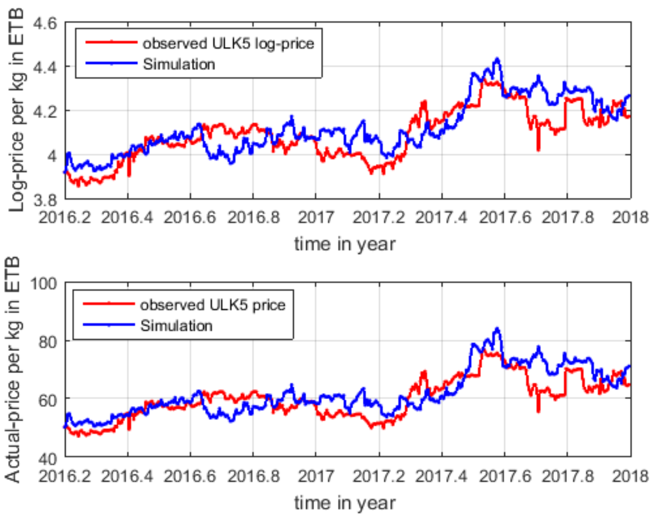

Figure 10. Simulated price fitted to ULK5 log-price(Upper panel) and ULK5 price (Lower panel) under DEJD model.

In this paper, the root mean square error(RMSE) is used to validate the performance of the models and we obtained the results as indicated in Table 6 below.

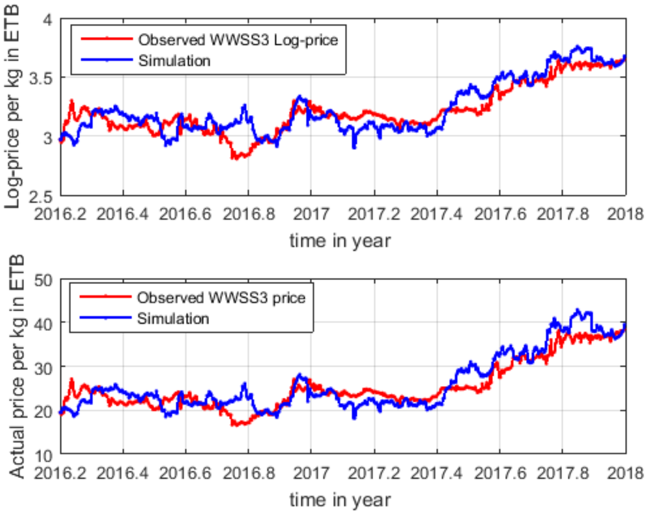

Figure 11. Simulated price fitted to WWSS3 log-price(Upper panel) and WWSS3 price (Lower panel) under Merton's model.

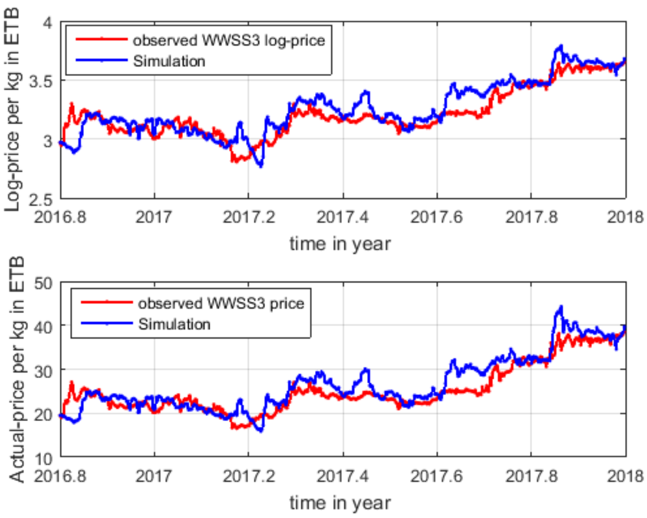

Figure 12. Simulated price fitted to WWSS3 log-price(Upper panel) and WWSS3 price (Lower panel) under DEJD model.

Table 6. RMSE values under DEJD and Merton's models.

Commodity prices	RMSE with DEJD model	RMSE with Merton's model
ULK5 coffee price	0.084568621148379	0.091967707613244
WWSS3 sesame price	0.108818914438138	0.115413737637904

7. Option pricing

7.1 *Option pricing using Merton's Jump diffusion model*

In this section, Merton's jump diffusion model are used to determine the call option prices for ULK5 and WWSS3 prices. However, this model which is to the contrary of Black-Scholes model is incomplete. So, there are many possible choices to define a risk neutral measure Q equivalent to the physical probability measure P such that the discounted price, $e^{-rt}S_t$ is a martingale where r is a risk free interest rate. More over, in order to make the discounted price is a martingale, the drift parameter μ must be set to $\mu = r - \lambda\kappa$ defining the risk neutral measure. The stochastic differential equation which represents the dynamics of the prices can be expressed under the risk neutral measure Q as:

$$\frac{dS_t}{S_{t-}} = (r - \lambda\kappa)dt + \sigma dB_t^Q + (y_t - 1)dN_t, \tag{21}$$

where B_t^Q is a standard Brownian motion under a risk neutral probability measure. The expected relative price change E$[\frac{dS_t}{S_t}]$ from the jump part dN_t with the change of time dt is $\lambda\kappa dt$ since E$[y_t - 1] = \exp(\beta + \frac{\delta^2}{2}) - 1 = \kappa$. This is the predictable part of the jump. This is why the instantaneous expected return under the risk neutral probability measure rdt is adjusted by $-\lambda\kappa dt$ in the drift term of the jump diffusion process to make the jump part unpredictable innovation.

The solution of the stochastic differential equation specified in (21) can be written as:

$$S_t = S_0 \exp\left((r - \frac{\sigma^2}{2} - \lambda\kappa)t + \sigma B_t^Q + \sum_{i=1}^{N_t} Y_i\right), 0 \leq t \leq T, \tag{22}$$

where Y_t=ln(y_t) is the log-return price jump size. Assuming that the jumps are log-normally distributed that is $Y_t \sim N(\beta, \delta^2)$. The prices of European style call options for the given strike price K, spot price S_0 at time t_0 and the terminal price S_T at maturity time T can be expressed [8, 24] as:

$$C(S_0, T) = \sum_{n=0}^{\infty} \frac{(\hat{\lambda}T)^n e^{-\hat{\lambda}T}}{n!} E^Q\left[e^{-rT}(S_T - K)^+ | S_t = S_0\right] \tag{23}$$

$$C(S_0, T) = \sum_{n=0}^{\infty} \frac{(\hat{\lambda}T)^n e^{-\hat{\lambda}T}}{n!} \left[S_0\Phi(d_{1,n}) - Ke^{-r_nT}\Phi(d_{2,n})\right], \tag{24}$$

where $\hat{\lambda} = \lambda(1 + \kappa)$, $\sigma_n = \sqrt{\sigma^2 + \frac{n\delta^2}{T}}$, $r_n = r - \lambda\kappa + n\frac{ln(1+\kappa)}{T}$, $d_{1,n} = \frac{ln\frac{S_0}{K} + (r_n + \frac{\sigma^2}{2})T}{\sigma_n\sqrt{T}}$, $d_{2,n} = d_{1,n} - \sigma_n\sqrt{T}$.

7.2 *Option pricing using double exponential jump diffusion model*

In this section , the double exponential jump diffusion model are used to find the call option prices for ULK5 coffee and WWSS3 sesame prices. However, this model is not complete because of its jump component. We considered the rational expectations arguments with a hyperbolic absolute risk aversion type utility function for the representative agent, as it is suggested by [22]. So, one can choose a risk neutral probability measure $\hat{Q}$ equivalent to the physical probability measure P so that the equilibrium price of an option is given by the rational expectation of the discounted option payoff with the risk neutral measure. The commodity price S_t still follows a double exponential jump diffusion process under the risk neutral probability measure. The stochastic differential equation which describes the dynamic behavior of the prices under this measure is given by:

$$\frac{dS_t}{S_{t-}} = (r - \hat{\lambda}_1\hat{\zeta})dt + \sigma_1 d\hat{B}_t + d\bigg(\sum_{i=1}^{\hat{N}_t}(\hat{\upsilon}_i - 1)\bigg). \tag{25}$$

Letting X_t= $\ln(S_t/S_0)$ and using Ito's lemma, the log-return commodity prices over the time interval $(0,t)$, can be written as:

$$X_t = \left(r - \frac{\sigma_1^2}{2} - \hat{\lambda}_1\hat{\zeta}\right)t + \sigma_1\hat{B}_t + \sum_{i=1}^{\hat{N}_t}\hat{V}_i,\ X_0 = 0, \tag{26}$$

where $\hat{B}_t$ is a standard normal Brownian motion, $\hat{N}_t$ is a poison process with intensity $\hat{\lambda}_1$ and the log jump sizes $\hat{V}_i$ which forms a sequence of random variables with a new double exponential density function $f\hat{(y)}$ are under the measure $\hat{Q}$. More precisely, this function can be written as: $f\hat{(y)} = \hat{p}\,\hat{\eta_1}e^{-\hat{\eta_1}y}1_{\{y\geq 0\}} + \hat{q}\,\hat{\eta_2}e^{\hat{\eta_2}y}1_{\{y<0\}}$ where $\hat{p},\ \hat{q} \geqslant 0,\ \hat{p}+\hat{q} = 1,\ \hat{\lambda}_1 > 0,\ \hat{\eta_1} > 1,\ \hat{\eta_2} > 0$ are constants and $\hat{\zeta} := \mathrm{E}[\hat{\upsilon}_\mathrm{t}] - 1 = \hat{\mathrm{p}}\frac{\hat{\eta_1}}{\hat{\eta_1}-1} + \hat{\mathrm{q}}\frac{\hat{\eta_2}}{\hat{\eta_2}+1} - 1$, is the expected relative jump size in the double exponential jump diffusion model under $\hat{Q}$. For simplicity, we omit the superscript $\hat{}$ in the parameters and processes as we focus on option pricing. Here, it is assumed that the sources of randomness, N_t, B_t and V_t are independent and identically distributed under Q.

In this paper, we also used the method of Monte Carlo(MC) simulation to obtain an approximate solution to the call option prices. This method is one of the most popular numerical method for pricing financial options because of the current advances in applying the tool [7, 27]. The method is mainly used to find an approximate solution to a complex financial problem, particularly European-style and exotic options for which no analytical pricing formula is available [11]. A Monte Carlo method is a technique that involves using random numbers and probability to solve problems and simulates paths for asset prices [17]. Since, the dynamic behavior of the prices are modeled by double exponential jump diffusion model under a risk neutral measure, we used this method to find the expectation of the discount payoff of commodity prices.

We considered call options giving an opportunity for the holder the right to buy the commodity at a fixed price K and specified time T in the future. If at time T the commodity price S_T exceeds the strike price K , the holder exercises the option for a profit of $(S_T - K)^+$, other wise the option expires worthless. Thus, the payoff to the option holder at time T is given by $(S_T - K)^+$=max$\,(S_T - K,\ 0)$. The solution which represents the daily return of the commodity prices as specified in (25) can be expressed by:

$$S_t = S_0\exp\left[\left(r - \frac{\sigma^2}{2} - \lambda_1\zeta\right)t + \sigma_1 B_t + \sum_{i=1}^{N_t}V_i\right],\ 0 \leqslant t \leqslant T. \tag{27}$$

It is assumed that S_0 is the current price of the commodity at $t = 0$, the random variable $B_t = \sqrt{T}Z,\ Z \sim N(0,\,1)$ and the log jump size Υ_t having double exponential distributed. So, the terminal price over the time interval $[0,\ T]$ can be represented by:

$$S_T = S_0 \exp\left[\left(r - \frac{\sigma^2}{2} - \lambda_1\zeta\right)T + \sigma_1 Z\sqrt{T} + \sum_{i=1}^{N_T} V_i\right]. \tag{28}$$

We considered the first 10,000 sample paths simulation of the price S_T and we used the following algorithm to estimate the expectation $E\left[e^{-rT}\left(S_T - K\right)^+\right]$:
Take $n = 10,000$;
for $j = 1, ..., n$,
generate Z_i and V_i from the respective distribution,
set $S_T(j) = S_0 \exp\left[\left(r - \frac{\sigma_1^2}{2} - \lambda_1\zeta\right)T + \sigma Z(j)\sqrt{T} + \sum_{i=1}^{N_T(j)} V_i\right]$,
set $C(j) = e^{-rT}\max\left(S_T - K, 0\right)$ and
set $\hat{C}_n = \frac{C_1 + C_2 + C_3 + ... + C_n}{n}$.
For $n \geqslant 1$, the estimator $C\hat{(n)}$ is unbiased. That is its expectation, $\mathrm{E}[\hat{\mathrm{C}}_\mathrm{n}] = \mathrm{C} \equiv \mathrm{E}\left[\mathrm{e}^{-\mathrm{rT}}\left(\mathrm{S_T} - \mathrm{K}\right)^+\right]$ and it is consistent meaning that as $n \longrightarrow \infty$, $\hat{C}_n \longrightarrow C$ with probability 1.

Thus, the call option prices on ULK5 coffee and WWSS3 sesame are computed at maturity time $T = 0.25728$, under Merton's jump diffusion and double exponential diffusion models with different strike prices K, spot prices 47 and 31 ETB per kg, respectively, risk neutral interest rate $r = 0.07$ and the corresponding parameters as shown in figures and tables below.

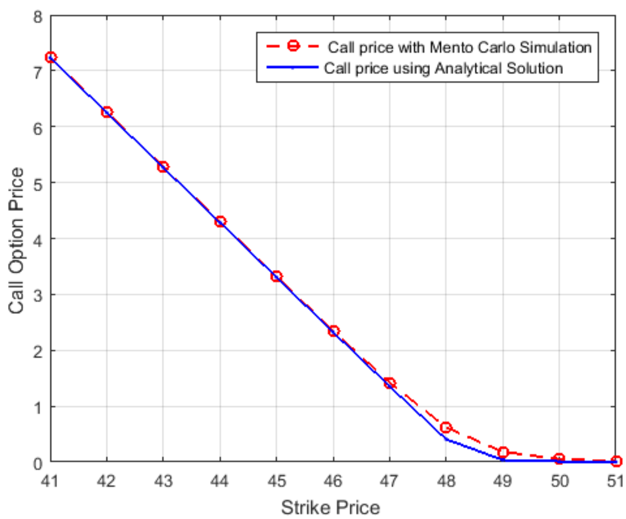

Figure 13. Call option price of ULK5 coffee using Merton's model.

8. Comparison of models

We used both Merton's jump diffusion and double exponential jump diffusion models for modeling and call option pricing of the commodity prices. These models are applied to describe the dynamic behaviors of the prices to capture the asymmetric heavy tails and high kurtosis of the prices. However, the double exponential jump diffusion model is more pronounced to reflect these phenomena than Merton's model as shown in Figures 9, 10, 11 and 12. The RMSE and the method of non-parametric fit are used to test the goodness for fitting to the empirical data.

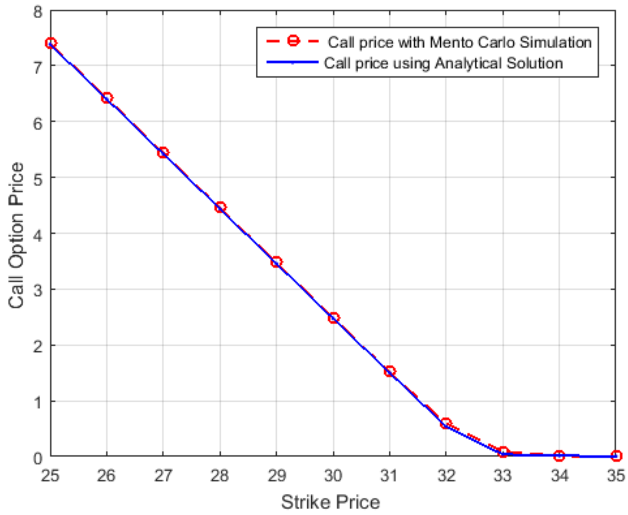

Figure 14. Call option price of WWSS3 sesame using Merton's model.

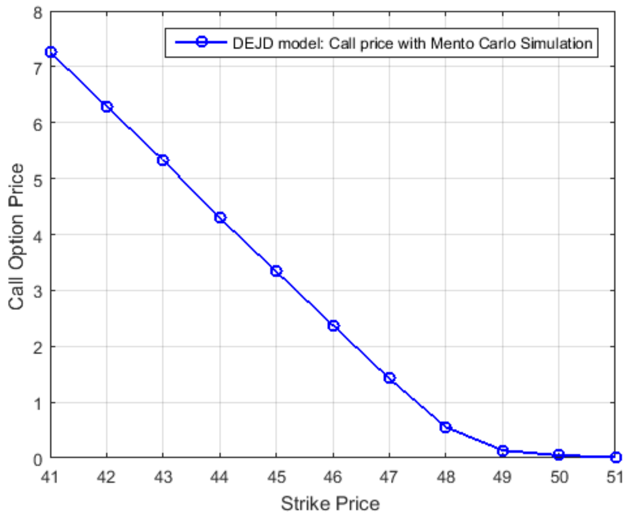

Figure 15. Call option prices of ULK5 coffee using DEJD model.

The tests indicate that these two models perform well. However, double exponential jump diffusion model shows good performance than Merton's model as indicated in Table 6 and Figures 7 and 8. Moreover, the call option prices under double exponential jump diffusion model over estimated both in-the money and out-of-the money when we compared to Merton's model as shown in Tables 7 and 8 and Figures 17 and 18. Finally, from the empirical results, we conclude that the double exponential jump diffusion model is more suitable for modeling and option pricing of the commodity prices.

9. Results and discussion

In this paper, we used the daily closed ULK5 coffee and WWSS3 sesame prices recorded from November 8, 2010 to 30 March 2018 and May 31, 2011 to 30 March 2018, respectively obtained at the Ethiopian Commodity Exchange (ECX). The method of maximum likelihood is being used to estimate the parameters with

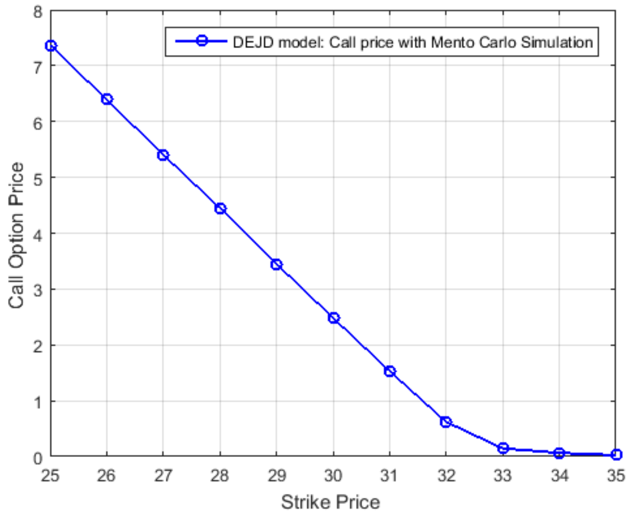

Figure 16. Call option price of WWSS3 sesame using DEJD model.

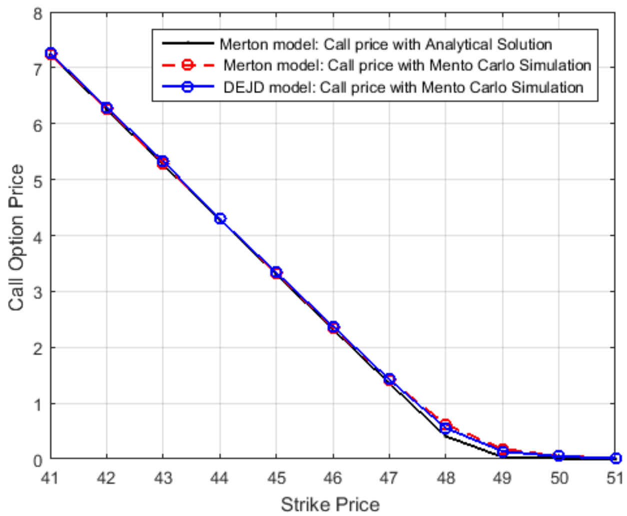

Figure 17. Call option prices of ULK5 coffee using DEJD and Merton's models.

Merton's jump diffusion and double exponential jump diffusion models as shown in Tables 2, 3, 4 and 5, respectively. These results indicate that the dynamics of the commodity prices process were influenced by both diffusion and jump components; however, the prices were dominated by a jump component with large discontinuities occurring at high intensity. The high volatility of the jump component reflects the presence of jumps of large magnitude and was in accordance with excess kurtosis in the empirical distribution of the data.

We used both models to simulate the commodity prices as shown Figures 9, 10, 11 and 12. In order to test the validity of these models, we applied the root mean square error (RMSE). The values, which are obtained under these models, are shown in Table 6. Furthermore, the method of non-parametric fit with normal kernel is used to plot the graphs of the probability density functions of the models to assess the goodness of fit of distributions of the models to the dynamics behaviors of the prices as shown in Figures 5 and 6. Analytical and MC simulation methods under Merton's model and MC simulation technique with double exponential jump diffusion model are used to find the call option pricing of the commodity prices as

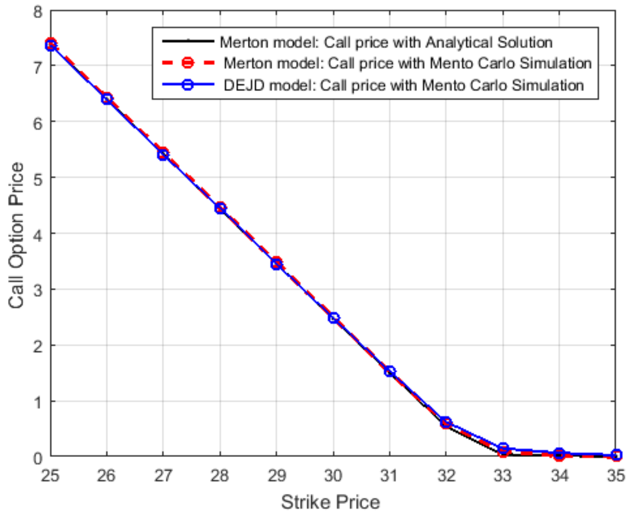

Figure 18. Call option prices of WWSS3 sesame using DEJD and Merton's models.

Table 7. Estimated call option prices on ULK5 coffee using DEJD and Merton's models.

		Merton's model	
K	DEJDM with MC simulation	Analytical technique	MC simulation
41	7.264791662876689	7.232965235570052	7.245094309864723
42	6.291444232548717	6.250814601359072	6.263041028149838
43	5.327862397204506	5.268669863373756	5.281116246684560
44	4.297120412950489	4.286568735586035	4.299868404289132
45	3.338320400778561	3.304800344263846	3.321186541099049
46	2.378471730455367	2.325107171239929	2.351911294765969
47	1.423144228442257	1.353862310655529	1.419274547858670
48	0.553928550857992	0.407059682575624	0.620283677263872
49	0.138984417480561	0.036201384486724	0.183069077997915
50	0.047129599044087	0.010970827175518	0.053087018362515
51	0.014646168941873	0.002555029173606	0.014406482515673

Table 8. Estimated call option prices on WWSS3 sesame using DEJD and Merton's models.

		Merton's model	
K	DEJDM with MC simulation	Analytical technique	MC simulation
25	7.419546734552747	7.382879414647297	7.404663818978261
26	6.433370561513205	6.400728916712499	6.422512322113714
27	5.464061509417212	5.418584993725071	5.440360825249139
28	4.461329599573344	4.436492384201657	4.458209328384564
29	3.451803285354543	3.454746068178997	3.476057831519984
30	2.484107429428337	2.474653050603823	2.494250169913052
31	1.531357181346600	1.499797614445632	1.517550866500188
32	0.620600729035908	0.537884001637778	0.594420314971845
33	0.145735504412863	0.039078550273871	0.091981135597093
34	0.066423197447607	0.015119962564113	0.014336528629418
35	0.025767345556120	0.005197162389270	0.001978940727499

indicated in Figures 13, 14, 15, 16, 17 and 18. The estimated values of call option prices of the commodities are displayed on Tables 7 and 8. Lastly, the comparison between the graphs of call price functions under Merton's jump diffusion and the double exponential jump diffusion models are indicated in Figures 17 and 18.

10. Conclusion

The ULK5 coffee and WWSS3 sesame prices are characterized by large fluctuations in values. The natures of log-returns of the prices display asymmetric heavy tails and high kurtosis. We applied both the Merton's jump diffusion and double exponential jump diffusion models to capture the dynamic behaviors of the commodity prices. The method of maximum likelihood is used to estimate the parameters under the models. The method of non-parametric fit with normal kernel is used to approximate the probability density functions of the models to assess the goodness of fit of distributions of the models to the empirical data. We used the root mean square error to test the goodness of fit the models to the observed prices. The results indicate that the models perform well. However, double exponential jump diffusion model has performed more efficiently than the Merton's model. We developed models for modeling and option pricing to reduce the risk associated with the prices fluctuations. Both analytical and Monte Carlo simulation techniques are applied to find the call option pricing of the commodity prices with Merton's model. Similarly, we used the method of Monte Carlo simulation under double exponential jump diffusion model to determine the call option prices of the commodity prices. Finally, we compare the call prices under these models. From the empirical results, we conclude that double exponential jump diffusion model is better than Merton's model for modeling and option pricing of ULK5 coffee and WWSS3 sesame prices.

References

[1] D. Alemu and G. W. Meijerink, Sesame Traders and the Ethiopia Commodity Exchange: An Overview with Focus on Transaction Costs and Risks, Addis Ababa, VC4PPD Report no. 8, (2010).

[2] H. Askari and N. Krichene, Oil price dynamics (2002-2006), Energy Economics, **30 (5)** (2008) 2134-2153.

[3] G. Bakshi, C. Cao and Z. Chen, Empirical performance of alternative pricing models, Journal of Finance, **52** (1997) 2003-2049.

[4] C. A. Ball and W. N. Torous, On jumps in common stock prices and their impact on call option pricing, Journal of Finance, **40 (1)** (1985) 155-173.

[5] M. Bart, T. Seneshaw, K. Tadesse and N. Yaw, Structural and Performance of Ethiopians Coffee Export Sector, International Food Policy Research Institutes (IFPRI), working paper 6, (2014).

[6] F. Black and M. Scholes, The pricing of options and corporate liabilities, Journal of Political Economy, **81 (3)** (1973) 637-654.

[7] J. M. Charnes, Using Simulation for Option Pricing, in Winter Simulation Conference, Orlando, USA, (2000).

[8] R. Cont and P. Tank, Financial Modelling With Jump Processes, Chapman and Hall/CRC, UK, (2004).

[9] Central Statistical Agency(CSA), Agricultural Sample Survey 2006-07, Addis Ababa, (2007).

[10] Central Statistical Agency(CSA), Agricultural Sample Survey 2012/2013 (2005 E.C.), Report on Area and Production of Crops (Private Peasant Holdings, Meher Season), Statistical Bulletin, Addis Ababa, (2013).

[11] R. A. DeFusco, D. W. McLeavey, J. E. Pinto and D. E. Runkle, Quantitative Methods for Investment Analysis, United Book Press, Inc., Baltimore, MD., USA, (2001).

[12] D. Duffie, J. Pan and K. Singleton, Transform analysis and option pricing for affine jump-diffusions, Econometrica, **68** (2000) 1343-1376.

[13] Ethiopia Revenue and Customs Authority (ERCA), Import and Export Database, ASCCUDA++ software, (2013).

[14] A. Eyasu, The Crop Sector in Ethiopia Current and Future Prospects, international agro-industry forum, Addis Ababa, (2018).

[15] FAO Statistical Database,Trade. Rome: Food and Agriculture Organization, (2012). http://faostat.fao.org//

[16] F. B. Hanson and J. J. Westman, Stochastic analysis of jump-diffusions for financial log-return

processes, In B. Pasik-Duncan (Ed.), Proceedings of Stochastic Theory and Control Workshop, Lecture Notes in Control and Information Sciences, 280, Springer-Verlag, New York, (2002) 169-184.
[17] Q. Jia, Pricing American Options using Monte Carlo Methods, U.U.D.M. Project Report, Department of Mathematics, Uppsala University, (2009).
[18] E. Jondeau, S. H. Poon and M. Rockinger, Financial Modeling under Non-Gaussian Distributions, Springer, London, UK, (2007).
[19] P. Jorion, On jump processes in the foreign exchange and stock markets, Review of Financial Studies, **1 (4)** (1988) 427-445.
[20] S. G. Kou, A jump-diffusion model for option pricing, Management Science, **48 (8)** (2002) 1086-1101.
[21] S. G. Kou, Jump-Diffusion Models for Asset Pricing in Financial Engineering, In J. R. Birge and V. Linetsky (Eds.), Financial Engineering, Handbooks in OR and MS, **15** (2008) 73-116.
[22] S. G. Kou and H. Wang, Option pricing Under a Double Exponential Jump Diffusion Model, Management Science, **50 (9)** (2004) 1178-1192.
[23] H. Manuel, L. Solomon and R. Shahidur, Agriculture in an integrated world: The Ethiopian Commodity Exchange and the coffee market: Are local prices more integrated to global markets?, international conference of agricultural economics, **211732** (2015) 1-48.
[24] R. C. Merton, Option pricing when underlying stock returns are discontinuous, Journal of Financial Economics, **3** (1976) 125-144.
[25] S. Rashid, Staple food prices in Ethiopia: Variation in staple food prices: Causes, consequence, and policy options, African agricultural marketing project, **58552** (2010) 1-14.
[26] D. Soresa, Baseline survey of sesame trade arrangements, costs and risks in Ethiopia draft report for Sesame Value Chain for Pro-Poor Development Projects, Addis Ababa, Wageningen UR., Mark Speer, (2009).
[27] X. Tian, K. Benkrid and X. Gu, High Performance Monte-Carlo Based Option Pricing on FPGAs, Engineering Letters, **16 (9)** (2008) 434-442.
[28] United nations development programme (UNDP), Agricultural Growth and Transformation: Strengthening National Capacity through Sustainable Increases in Agricultural Production and Productivity, Ethiopia, (2016) 1-2.