International Journal of
Mathematical Modelling & Computations
Vol. 08, No. 02, Spring 2018, 101- 114

Purchasing Inventory Models for Deteriorating Items with Linear Demand and Shortages - in Third Order Equation

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Abstract. In this paper, a purchasing inventory model for deteriorating items is developed with a linear, positive trend in demand, allowing inventory shortages and backlogging. It is assumed that the goods in the inventory deteriorate over time at a constant rate θ . Two models are developed for two operational policies. The first policy covers the case that the inventory model with linear demand for deteriorative items and the second policy covers the case that the inventory model with linear demand for deteriorative items and shortages. Mathematical model is developed for each model to reduce the third order equation and the optimal cycle time and inventory lot size which minimizes the total cost is derived. Illustrative example is provided for each model. In each model, sensitivity analysis is performed to show how the optimal values of the policy variables in the model change as various model parameters are changed. The validation of result in this model was coded in Microsoft Visual Basic 6.0.

Received: 23 November 2017, Revised: 19 September 2018, Accepted: 29 February 2019.

Keywords: Inventory; Deteriorating; Linear demand; Cycle time; Shortages and sensitivity analysis.

1. Introduction

In formulating inventory models, two factors of the problem have been of growing interest to the researchers, one being the deterioration of items and the other being variation in the demand rate with time. Generally, deterioration is defined as decay, change or spoilage the prevent the item from being used for its original purpose. The classical inventory models assume constant demand an infinite planning horizon. The assumption is valid during the maturity phase of the product life cycle and for a finite period of time. In other phases of a product life cycle demand for the product may increase after is successful introduction into the market or decrease due to introduction of new competitors' products. However, in reality demand may not always be constant. The demand may be time dependent. In this paper, a purchasing inventory models for deteriorative items with linear demand, time value of money and shortages are considered. Economic order quantity (EOQ) models have been studied since Harries [8] presented the famous EOQ formulae. Dave and Patel [4] considered an EOQ model in which the demand rate is changing linearly with time and the deteriorating is assumed to be a constant fraction of the on hand inventory. Chund and Tsai (1997) considered the inventory replenishment policy over a fixed planning period for a deteriorating item having a deterministic demand pattern with a linear trend and

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shortages. Skouri and Papachristos [17] developed an inventory model for the deterioration of items occurs at a fixed rate independent of time. The model allows for partially backlogging and the backlogging rate is an exponentially decreasing, time-dependent function specified by a parameter. Ghosh and Chaudhuri [6] developed an inventory model for a deteriorating item, a quadratic time-vary demand and shortages in inventory. A two-parameter weibull distribution is taken to represent the time of deterioration. Ghosh and Chaudhuri [5] developed an EOQ model over a finite time-horizon for a deteriorating item with a quadratic, time-dependent demand, allowing shortages in inventory and the rate of deterioration is taken to be time-proportional and it is assumed that shortage occur in every cycle. Ajanta Roy [14] developed a deterministic inventory model when the deterioration rate is time proportional and demand rate is a function of selling price and holding cost is time dependent. Mingbao Cheng and Guequing Wang [2] developed an inventory model for deteriorating items with trapezoidal type demand rate that is the demand rate is a piecewise linearly function. It is proposed an inventory, replenishment policy for this type of inventory model. The numerical solution of the model is obtained and also examined. Biswaranjan Mandal [11] considered in which it is depleted not only by demand but also by deterioration. The Weibull distribution, which is capable of representing constant, increasing and decreasing rate of deterioration, is used to represent the distribution of the time to deterioration. Kripasindhu Chauduri et al. [1] an economic order quantity inventory problem is discussed over a finite time for deteriorating items with shortages, where the demand rate is of the ramp-type. It is assumed that a constant fraction of the on-hand inventory deteriorates per unit time and time value of money and the effects of inflation are taken into account. Mingbao Cheng et al. [3] considered an inventory model for time-dependent deteriorating items with trapezoidal type demand rate and partial backlogging. Ghosh et al. [7] considered an optimal inventory replenishment policy for a deteriorating items time-quadratic demand and time-dependent partial backlogging which depends on the length of the waiting time for the next replenishment over a finite time horizon and variable replenishment cycle. Vinod Kumar Mishra et al. [13] considered a deterministic inventory model with time-dependent demand and time-varying holding cost where deteriorating is time proportional and the model considered here allows shortages and the demand is partially backlogged. Sana et al. [16] analyzed a single-period newspaper inventory model to define the optimum order quantity where the consumers flinching occur and depend on holding cost. Shortages are allowed and partially backlogged. Sana and De [15] developed an economic order quantity model for fuzzy variables with promotional effort and selling price dependent demand. They observed that demand rate decreases over time during shortage period. Manna and Manna [12] developed an order level inventory system for deteriorating items with demand rate as a ramp type function of time. The finite production rate is proportional to the demand rate and the deterioration rat is independent of time. The unit production cost is inversely proportional to the demand rate. Shortages are not considered in this model. Islam et al. [9] develops a time dependent inventory model on the basis of constant production rate and market demands which are exponentially decreasing. It advances in quest of total average optimum cost considering those products which have finite shelf-life. The model also considers the small amount of decay. Without having any sort of backlogs, production starts. Reaching at the desired level of inventories, it stops production. After that due to demands along with the deterioration of the items it initiates its depletion and after certain periods the inventory gets zero. The decay of the products is level dependent. Kaliraman, et al. [10] considered a two warehouse inventory model for deteriorating items with exponential demand rate and permissible delay in payment. Shortages are not considered and deteriorating rate is constant. In this model, one warehouse is rented and the other is owned. The rented warehouse is provided with better facility for the stock than the owned warehouse but it is charged more. The objective of this model is to find the best replenishment policies for minimizing the total appropriate inventory cost. Tripathi et al. [18] established inventory model with exponential time-dependent demand and time- dependent time-dependent deterioration. Shortages are allowed. In this paper, a purchasing inventory models for deteriorating items with linear demand, time value of money and shortages are considered and the optimum solution is derived in higher order equations. Three models are developed for three operational policies. The first policy covers the case that the inventory model with linear demand, the second policy covers the case that the inventory model with linear demand with time value of money and third policy covers the case that the inventory model with linear demand with shortages. Mathematical model is developed for each model and the optimal cycle time and inventory lot size which minimizes the total cost is derived. The sensitivity analysis and illustrative example is provided for each model. The validation of result in this model was coded in Microsoft Visual Basic 6.0. The remaining of the paper is organized as follows: Section 2 presents the assumptions and notations. Section 3 is for problem formulation and numerical examples. Finally, the paper summarizes and concludes in section 4.

2. Assumptions and notations

2-1. Assumptions

1) The initial inventory level is zero, 2) The demand rate is linear ($a+bt$) where $a>0$, $b\neq 0$ at time t and it is continuous function of time. Here, a and b are constants and "a" stands for the initial demand and "*b*" is positive trend in demand, 3) The deteriorating rate is constant, 4) The planning horizon is finite, 5) Lead time is zero, 6) There is no repair or replacement of the deteriorated items, 7) Shortages are considered in this model.

2-2. Notations

The following notations are used in our analysis.

1. D – Demand rate in units per unit time
2. Q^* – Optimal inventory size
3. θ – Rate of deteriorative.
4. C_d – Deterioration cost per unit
5. C_0 – Ordering cost/order
6. B – Number of shortages in units
7. C_S – Shortages cost per unit
8. C_h – Holding cost per unit/time

9. T_1 – The time during which the inventory is building up
10. T – Cycle time
11. TC – Total cost

3. Mathematical models

3-1. Purchasing inventory model for deteriorating item with linear demand

This model is developed a deteriorating inventory model in which demand is linear function of time that is $a+bt$, where $a>0,\ b\neq 0,$ at time t and "a" stands for the initial demand and "b" is positive trend. Let Q be the units of item arrive at the inventory system at the beginning of each cycle. The inventory level decreases due to demand and deteriorating till it becomes zero in the interval (0, T). The total process is repeated. The inventory level at different instants of time is shown in Figure 1.

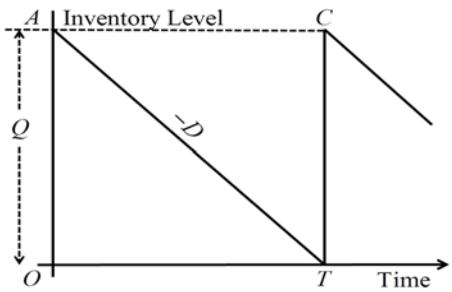

Figure 1. Purchasing inventory model for deteriorating items with linear demand.

$$\frac{d}{dt}I(t)+\theta I(t)=-(a+bt);\ \ 0<t<T \tag{1}$$

The boundary conditions are $I(0)=Q,\ I(T)=0$ (2)

The solution of equation (1) is

$$I(t)=\left(\frac{a+bT}{\theta}-\frac{b}{\theta^2}\right)e^{\theta(T-t)}-\frac{a+bt}{\theta}+\frac{b}{\theta^2} \tag{3}$$

From the equation (3) and boundary conditions

$$I(0)=Q\Rightarrow Q=\frac{D}{\theta}\left(e^{\theta T}-1\right) \tag{4}$$

Total cost: Total cost comprised of the sum of the setup cost, holding cost and deteriorating cost. They are grouped together after evaluating the above cost individually.

1. Setup cost = $\frac{C_0}{T}$ (5)

2. Holding cost = $\frac{C_h}{T}\left[\int_o^T I(t)dt\right]$ (6)

$$= \frac{C_h}{T}\int_o^T\left[\left(\frac{a+bT}{\theta}-\frac{b}{\theta^2}\right)e^{\theta(T-t)}-\frac{a+bt}{\theta}+\frac{b}{\theta^2}\right]dt$$

$$= \frac{C_h}{T}\left[\frac{-a}{\theta^2}-\frac{bT}{\theta^2}+\frac{b}{\theta^3}-\frac{aT}{\theta}-\frac{bT^2}{2\theta}+\frac{bT}{\theta^2} + \left(\frac{a}{\theta^2}+\frac{bT}{\theta^2}-\frac{b}{\theta^3}\right)\left(1+\theta T+\frac{\theta^2T^2}{2}+\frac{\theta^3T^3}{6}\right)\right]$$

Higher power of θ is discarded since the value of higher power of θ is very small.

$$\text{Holding cost} = \frac{C_k}{T}\left[\frac{aT^2}{2}+\frac{bT^3}{2}\right] = \frac{C_k}{2}(a+bT)T$$

$$\text{3. Deteriorating cost} = \frac{\theta C_d}{T}\left[\int_o^T I(t)dt\right] = \frac{C_h}{2}\left(a+bT\right)T \tag{7}$$

Therefore, total cost = setup cost + holding cost + deteriorating cost.

$$TC = \frac{C_0}{T}+\frac{(C_h+\theta C_d)(aT+bT^2)}{2} \tag{8}$$

Optimality: It can be easily shown that $TC(T)$ is a convex function in T. Hence, an optimal cycle time T can be calculated from

$$\frac{d}{dT}TC(T)=0 \text{ and } \frac{d^2}{dT^2}TC(T)>0$$

Differentiating the total cost equation (8) with respect to T, then

$$\frac{d}{dT}(TC)=\frac{-C_0}{T^2}+\frac{(C_h+\theta C_d)(a+2bT)}{2}=0$$

$$and \quad \frac{d^2}{dT^2}(TC)=\frac{2C_0}{T^3}+\frac{(C_h+\theta C_d)a}{2}>0$$

$$-C_0+\frac{(C_h+\theta C_d)}{2}\left(aT^2+2bT^3\right)=0$$

$$2bT^3+aT^2=\frac{2C_0}{C_h+\theta C_d}$$

$$2bT^3+aT^2-\frac{2C_0}{C_h+\theta C_d}=0 \tag{9}$$

which is the equation for optimum solution in third order equation.

Note: When $b=0$ and $a=D$ then $T=\sqrt{\frac{2C_0}{D(C_h+\theta C_d)}}$ which is the standard inventory model.

Numerical Example:

$D=8000$, $C_0=100$, $C_h=20$, $\theta=0.01$, $C_d=100$, $a=7800$, $b=5875$

Optimum Solution:

The optimum equation is $11750T^3+7800T^2-9.52=0$.

The values of T are 0.0341, - 0. 6619, - 0.0359. Here, the T has one positive real root and two negatives real roots. Here, the positive real root T = 0.0341 is considered.
Therefore, $T = 0.0341$, $Q = 265.82$, Setup cost = 2934.36, Holding cost = 2726.39, Deteriorating cost = 136.32, Total cost = 5797.07.
The graphical representation is given below

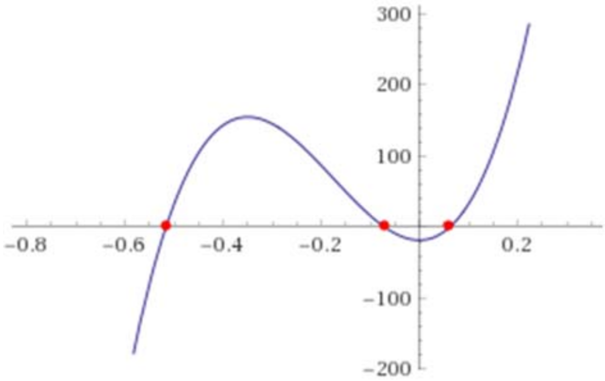

Figure 2. Graphical representation of linear demand for deteriorating items.

Table 1. Variation of rate of deteriorating items with inventory and total cost.

θ	T	Q	Setup cost	Holding cost	Deteriorating cost	Total cost
0.01	0.0341	272.48	2934.36	2726.39	136.32	5797.07
0.02	0.0331	266.52	3001.76	2663.67	266.37	5931.80
0.03	0.0326	260.64	3067.65	2605.09	390.76	6063.51
0.04	0.0319	255.15	3132.12	2550.21	510.04	6192.37
0.05	0.0313	249.98	3195.25	2498.66	624.66	6318.58
0.06	0.0307	245.12	3257.14	2450.12	735.03	6442.29
0.07	0.0301	240.53	3317.84	2404.29	841.50	6563.64
0.08	0.0296	236.19	3377.43	2360.95	944.38	6682.76
0.09	0.0291	232.07	3435.96	2319.87	1043.94	6799.77
0.10	0.0286	228.16	3493.49	2280.86	1140.43	6914.78

From the Table 1, a study of rate of deteriorative items with cycle time, optimum quantity, setup cost, holding cost, deteriorating cost and total cost is considered and it is concluded that when the rate of deteriorative items increases then the setup cost, deteriorating cost and total cost increases then it is positive relationship between themes. When the rate of deteriorating rate increases then the cycle time, optimum quantity and holding cost decreases then there is negative relationship between them.

Sensitivity Analysis: The total cost functions are the real solution in which the model parameters are assumed to be static values. It is reasonable to study the sensitivity i.e. the effect of making chances in the model parameters over a given optimum solution. It is important to find the effects on different system performance measures, such as cost function, inventory system, etc. For this purpose, sensitivity analysis of various system parameters for the models of this research are required to observe whether the current solutions remain unchanged, the current solutions become infeasible, etc.

Table 2. Effect of demand and cost parameters on optimal values

Parameters		Optimal Values					
		T	**Q**	**Setup cost**	**Holding cost**	**Deteriorative cost**	**Total cost**
θ	0.01	0.0341	292.64	2934.36	2726.39	136.32	5797.07
	0.02	0.0331	266.52	3001.76	2663.67	266.37	5931.80
	0.03	0.0326	260.64	3067.65	2605.09	390.76	6063.51
	0.04	0.0319	255.15	3132.12	2550.21	510.04	6192.37
	0.05	0.0313	249.98	3195.25	2498.66	624.66	6318.58
C_0	100	0.0341	272.48	2934.35	2726.39	136.32	5797.07
	110	0.0357	285.61	3081.15	2859.55	142.98	6083.68
	120	0.0372	298.34	3221.71	2986.79	149.34	6357.85
	130	0.0387	310.55	3356.80	3108.84	155.44	6621.09
	140	0.0401	322.29	3487.04	3226.29	161.31	6874.65
C_h	20	0.0341	272.48	2934.35	2726.39	136.31	5797.07
	25	0.0307	245.62	3257.14	3062.64	122.50	6442.29
	30	0.0282	224.79	3550.07	3365.63	112.18	7027.89
	35	0.0262	208.51	3820.16	3643.59	104.10	7567.86
	40	0.0245	195.33	4072.04	3901.86	97.54	8071.45
a	7400	0.0349	265.39	2864.71	2655.70	132.78	5652.19
	7500	0.0347	267.40	2881.52	2673.54	133.67	5688.75
	7600	0.0345	269.17	2899.23	2691.27	134.56	5725.07
	7700	0.0343	270.93	2916.84	2708.89	135.44	5761.18
	7800	0.0341	272.48	2934.35	2726.39	136.32	5797.07
b	5475	0.0341	272.62	2929.58	2726.29	136.31	5792.18
	5575	0.0341	272.63	2930.78	2726.31	136.32	5793.41
	5675	0.0341	272.64	2931.97	2726.34	136.32	5794.63
	5775	0.0341	272.64	2933.16	2726.36	136.32	5795.85
	5875	0.0341	272.48	2934.35	2726.39	136.32	5797.07
C_d	100	0.0341	272.48	2934.35	2726.39	136.32	5797.07
	110	0.0340	272.01	2941.16	2719.92	149.59	5810.69
	120	0.0339	271.36	2947.96	2713.49	162.81	5824.27
	130	0.0338	270.73	2954.74	2707.11	175.96	5837.82
	140	0.0337	270.14	2961.51	2700.77	189.05	5851.34

Managerial insights: A sensitivity analysis is performed to study the effects of change in the system parameters, deteriorative rate (θ), ordering cost (C_0), holding cost (C_h), cost of deteriorative items per unit (C_d), constant demand (a), and varying demand (b) and (c) on optimal values that is optimal cycle time (T), optimal quantity (Q), production time (T_1), maximum inventory (Q_1), setup cost, holding cost, deteriorative cost, shortage cost and total cost. The sensitivity analysis is performed by changing (increasing or decreasing) the parameter taking at a time, keeping the remaining parameters at their original values. The following influences can be obtained from sensitivity analysis based on Table 2.

1) with the increase in rate of deteriorating item (θ), setup cost, deteriorating cost, and total cost increases then there is positive relationship between them and Optimal cycle time T, optimal quantity (Q), Holding cost decreases then there is negative relationship between them.

2) with the increase in setup cost per unit (C_0), optimum quantity (Q^*), cycle time (T), setup cost, holding cost, deteriorative cost, and total cost increases then there is positive relationship between them.

3) with the increase in holding cost per unit per unit time (C_h), the setup cost, holding cost and total cost increases then there is positive relationship between them but optimal cycle time (T) and optimal lot size (Q), deteriorative cost decreases then there is negative relationship between, 4) Similarly, other parameters deteriorative cost per unit (C_d), initial demand "a", and time dependent demand "b" can also be observed from the Table 2.

3-2. Purchasing inventory model for linear demand with shortages

This model is developed a deteriorating inventory model in which demand is linear function of time that is $a+bt$, where $a>0,\ b\neq 0,$ at time t and "a" stands for the initial demand and "b" is positive trend. The inventory level decreases due to demand and deteriorating till it becomes zero in the interval $(0,T_1)$. The shortage interval keeps to the end of the current order cycle. The total process is repeated. The inventory level at different instants of time is shown in Figure 5.

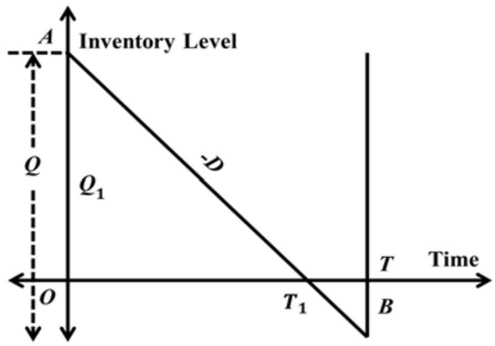

Figure 2. Purchasing inventory model with linear demand and shortages.

$$\frac{d}{dt}I(t)+\theta I(t)=-(a+bt);\ 0<t<T_1 \tag{10}$$

$$\frac{d}{dt}I(t)=-(a+bt);\ T_1<t<T \tag{11}$$

The boundary conditions are $I(0)=Q,\ I(T)=0$ (12)

From the equation (10)

$$I(t)=\left(\frac{a+bT}{\theta}-\frac{b}{\theta^2}\right)e^{\theta(T-t)}-\frac{a+bt}{\theta}+\frac{b}{\theta^2} \tag{13}$$

From the equation (11)

$$I(t)=a(t-T_1)+\frac{b}{2}\left(t^2-T_1^2\right) \tag{14}$$

From the equation (13) and boundary conditions

$$I(0)=Q_1 \Rightarrow Q_1=\left(\frac{a+bT_1}{\theta}-\frac{b}{\theta^2}\right)e^{\theta T_1}-\frac{a}{\theta}+\frac{b}{\theta^2} \tag{15}$$

Total cost: Total cost comprised of the sum of the setup cost + holding cost + deteriorating cost + shortage cost. They are grouped together after evaluating the above cost individually.

1. Setup cost = $\frac{C_0}{T}$ (16)

2. Holding cost $=\frac{C_h}{T}\left[\int_o^{T_1} I(t)dt\right]$

$$=\frac{C_h}{T}\int_o^{T}\left[\left(\frac{a+bT}{\theta}-\frac{b}{\theta^2}\right)e^{\theta(T_1-t)}-\frac{a+bt}{\theta}+\frac{b}{\theta^2}\right]dt$$

$$=\frac{C_h}{T}\left[\frac{-a}{\theta^2}-\frac{bT_1}{\theta^2}+\frac{b}{\theta^3}-\frac{aT_1}{\theta}-\frac{bT^2}{2\theta}+\frac{bT}{\theta^2}\right.$$
$$\left.+\left(\frac{a}{\theta^2}+\frac{bT_1}{\theta^2}-\frac{b}{\theta^3}\right)\left(1+\theta T_1+\frac{\theta^2 T_1{}^2}{2}\right)\right]$$

Higher power of θ is discarded since the value of higher power of θ is very small.

$$\text{Holding cost}=\frac{C_h}{T}\left[\frac{aT_1^2}{2}+\frac{bT_1^3}{2}\right]=\frac{C_h}{2T}\left[aT_1^2+bT_1^3\right] \tag{17}$$

3. Deteriorating cost = $\frac{\theta C_d}{T}\left[\int_o^{T_1} I(t)dt\right]=\frac{\theta C_d}{2T}\left[aT_1^2+bT_1^3\right]$ (18)

4. Shortage cost $=\frac{C_S}{T}\int_{T_1}^{T} I(t)dt$

$$=\frac{C_S}{T}\int_{T_1}^{T}\left[a(t-T_1)+\frac{b}{2}\left(t^2-T_1^2\right)\right]dt$$

$$=\frac{C_S}{T}\left[\frac{aT^2}{2}-aTT_1+\frac{bT^3}{6}-\frac{bT_1^2T}{2}+\frac{aT_1^2}{2}+\frac{bT_1^3}{3}\right]$$

$$=\frac{C_S}{T}\left[\frac{a}{2}(T-T_1)^2+\frac{b}{6}\left(T^3-3T_1^2T+2T_1^3\right)\right] \tag{19}$$

Therefore, Total cost = setup cost + holding cost + deteriorating cost + shortage cost

$$TC = \frac{C_0}{T} + \frac{(C_h + \theta C_d)\left(aT_1^2 + bT_1^3\right)}{2T} + \frac{C_S}{T}\left[\frac{a}{2}(T - T_1)^2 + \frac{b}{6}\left(T^3 - 3T_1^2 T + 2T_1^3\right)\right] \tag{20}$$

Optimality: It can be easily shown that $TC(T)$ is a convex function in *T*. Hence, an optimal cycle time *T* can be calculated from

$$\frac{\partial}{\partial T_1} TC(T) = 0 \text{ and } \frac{\partial^2}{\partial T_1^2} TC(T) > 0$$

$$\frac{\partial}{\partial T} TC(T) = 0 \text{ and } \frac{\partial^2}{\partial T^2} TC(T) > 0$$

Partially differentiate the total cost equation (27) with respect to T_1, then

$$\frac{\partial}{\partial T_1}(TC) = \frac{(C_h + \theta C_d)(2aT_1 + 3bT_1^2)}{2T} + \frac{C_S}{2T}\left[-2a(T - T_1) + \frac{b}{3}(-6TT_1 + 6T_1^2\right] = 0$$

and $\frac{\partial^2}{\partial T_1^2} TC(T) > 0$.

On simplification,

$$T_1 = \frac{aC_S T}{a(C_h + \theta C_d + C_S) - bC_S T} \tag{21}$$

Partially differentiating the total cost equation (27) with respect to *T*, then

$$\frac{\partial}{\partial T}(TC) = \frac{-C_0}{T^2} + \frac{(C_h + \theta C_d)(aT_1^2 + bT_1^3)}{2T^2} + \frac{C_S}{2T^2}\left[a(T - T_1)(T + T_1) + \frac{b}{3}\left(2T^3 - 2T_1^3\right)\right] = 0$$

and $\frac{\partial^2}{\partial T^2}(TC) = \frac{2C_0}{T^3} + \frac{(C_h + \theta C_d)a}{2} > 0$

On simplification,

$$bC_S\left[\frac{3(C_h + \theta C_d)C_S^2 - 2\left((C_h + \theta C_d + C_S)^3 - C_S^3\right)}{3(C_h + \theta C_d + C_S)^3}\right]T^3 + \frac{aC_S(C_h + \theta C_d)}{C_h + \theta C_d + C_S}T^2 - 2C_0 = 0 \tag{22}$$

which is the equation for optimum solution in third order equation.

Note: When $T^3 = 0$ and $a = D$ then $T = \sqrt{\frac{2C_0(C_h + \theta C_d + C_S)}{DC_S(C_h + \theta C_d)}}$ which is the standard inventory model.

Numerical Example:

$D = 8000$, $C_0 = 100$, $C_h = 20$, $\theta = 0.01$, $C_d = 100$, $a = 7700$, $b = 6063$, $C_S = 10$.

Optimum Solution:

The optimum equation is $-56677.49T^3 - 78878.04T^2 + 200 = 0$.

Therefore, $T = 0.0497$, $T_1 = 0.0248$, $Q = 397.76$, $Q_1 = 192.30$, Setup cost = 2011.25, Holding cost = 958.08, Deteriorating cost = 9.58, Shortage cost = 978.54, Total cost = 3957.45.

The graphical representation is given below

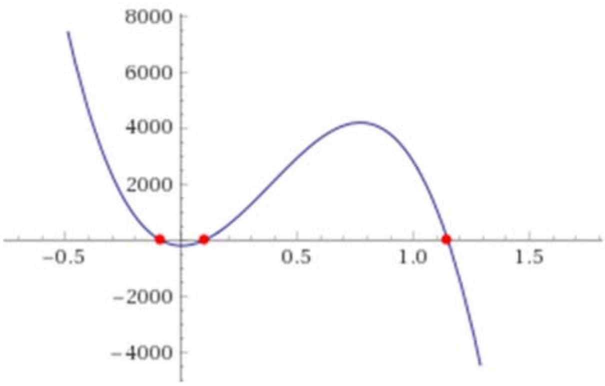

Figure 4. Graphical representation of linear demand with shortages.

Table 3. Variation of rate of deteriorating items with inventory and total cost.

θ	T	T_1	Q	Q_1	**Setup cost**	**Holdi-ng cost**	**Deterio-rating cost**	**Shorta-ge cost**	**Total cost**
0.01	0.0494	0.0246	395.85	193.13	2020.92	960.36	9.60	987.96	3978.85
0.02	0.0489	0.0237	391.54	186.23	2043.17	903.40	18.06	1024.46	3989.10
0.03	0.0484	0.0229	387.41	179.83	2064.15	851.52	25.54	1059.59	4000.81
0.04	0.0478	0.0222	383.58	173.86	2083.97	804.12	32.16	1093.39	4013.66
0.05	0.0475	0.0214	380.02	168.31	2102.73	760.69	38.03	1125.95	4027.41
0.06	0.0471	0.0208	377.26	163.10	2120.51	720.77	43.24	1157.33	4041.86
0.07	0.0468	0.0202	373.63	158.22	2137.39	684.00	47.88	1187.58	4056.85
0.08	0.0464	0.0196	370.74	153.63	2153.43	650.04	52.00	1216.76	4072.23
0.09	0.0461	0.0191	368.03	149.31	2168.70	618.60	55.67	1244.92	4087.90
0.10	0.0458	0.0186	365.49	145.23	2183.26	589.44	58.94	1272.12	4103.76

From the Table 3, it is observed that a study of rate of deteriorative items with cycle time, purchasing time, maximum inventory, optimum quantity, setup cost, holding cost, deteriorating cost, shortage cost and total cost is considered and it is concluded that when the rate of deteriorative item increases then the setup cost, deteriorating cost and total cost increases then it is positive relationship between them. When the rate of deteriorating rate increases then the cycle time, purchasing cost, maximum inventory, optimum quantity and holding cost decreases then there is negative relationship between them.

Sensitivity Analysis:

Table 4. Effect of demand and cost parameters on optimal values.

Paramet ers		**Optimum values**								
		T	T_1	Q	Q_1	**Setup cost**	**Holding cost**	**Deterio-rating cost**	**Shortag e cost**	**Total cost**
θ	0.01	0.0494	0.0246	395.85	193.13	2020.92	960.36	9.60	987.96	3978.85
	0.02	0.0489	0.0237	391.54	186.23	2043.17	903.40	18.06	1024.46	3989.10
	0.03	0.0484	0.0229	387.41	179.83	2064.15	851.52	25.54	1059.59	4000.81
	0.04	0.0478	0.0222	383.58	173.86	2083.97	804.12	32.16	1093.39	4013.66
	0.05	0.0475	0.0214	380.02	168.31	2102.73	760.69	38.03	1125.95	4027.41
C_0	100	0.0494	0.0246	395.85	193.13	2020.92	960.36	9.60	987.96	3978.85
	110	0.0518	0.9257	414.37	202.02	2123.70	1002.96	10.03	1038.60	4175.29

	120	0.0540	0.0267	432.73	210.43	2222.35	1043.02	10.43	1087.37	4363.19
	130	0.0561	0.0277	450.31	218.42	2317.40	1080.84	10.81	1134.54	4543.60
	140	0.0581	0.0287	467.20	226.04	2409.25	1116.65	11.16	1180.29	4717.36
	20	0.0494	0.0246	395.85	193.13	2020.92	960.36	9.60	987.96	3978.85
	25	0.0471	0.0208	377.26	163.10	21.20	900.97	9.01	1157.33	4187.82
C_h	30	0.0455	0.0181	363.46	141.38	2197.15	843.49	8.43	1298.29	4347.47
	35	0.0442	0.0160	353.22	124.88	2258.06	790.32	7.90	1417.39	4473.68
	40	0.0433	0.0144	345.30	111.89	2307.68	742.00	7.42	1518.98	4576.09
	20	0.0494	0.0246	395.85	193.13	2020.92	960.36	9.60	987.96	3978.85
	25	0.0469	0.0260	375.25	204.42	2131.91	1133.72	11.34	918.85	4195.81
C_S	30	0.0451	0.0271	360.14	213.11	2217.03	1280.35	12.80	851.91	4362.09
	35	0.0438	0.0279	349.02	220.03	2284.54	1405.48	14.05	789.84	4493.91
	40	0.0427	0.0286	340.48	225.68	2339.45	1513.26	15.13	733.30	4601.14
	7300	0.0507	0.0252	385.69	188.38	1970.49	938.22	9.38	959.59	3877.70
a	7400	0.0504	0.0251	388.64	189.57	1983.21	943.79	9.44	966.77	3903.22
	7500	0.0501	0.0249	391.09	190.77	1995.85	949.34	9.49	973.89	3928.58
	7600	0.0498	0.0247	393.53	191.95	2008.43	954.86	9.55	980.96	3953.80
	7700	0.0494	0.0246	395.85	193.13	2020.92	960.36	9.60	987.96	3978.85
	5663	0.0495	0.0246	395.32	192.86	2018.67	957.81	9.58	989.89	3975.95
	5763	0.0495	0.0246	395.47	192.93	2019.23	958.45	9.58	989.41	3976.67
b	5863	0.0495	0.0246	395.60	192.99	2019.80	959.08	9.59	988.92	3977.40
	5963	0.4956	0.0246	395.73	193.06	2020.36	959.72	9.60	988.44	3978.12
	6063	0.0494	0.0246	395.85	193.13	2020.92	960.36	9.60	987.96	3978.85

Managerial insights: A sensitivity analysis is performed to study the effects of change in the system parameters, rate of deteriorative (θ). Ordering cost (C_0), shortage cost (C_S), holding cost (C_h), constant demand (a), and varying demand (b) on optimal values that is optimal cycle time (T), optimal quantity (Q), production time (T_1), maximum inventory (Q_1), setup cost, holding cost, deteriorative cost, shortage cost and total cost. The sensitivity analysis is performed by changing (increasing or decreasing) the parameter taking at a time, keeping the remaining parameters at their original values. The following influences can be obtained from sensitivity analysis based on table 6.

1) with the increase in rate of deteriorating item (θ), Setup cost, Deteriorating cost, shortage cost and total cost increases then there is positive relationship between them and Optimal cycle time *T*, Maximum inventory level Q_1, Optimal quantity Q, Holding cost decreases but there is negative relationship between them.

2) with the increase in setup cost per unit (C_0), optimum quantity (Q^*), cycle time (T), production time (T_1), maximum inventory (Q_1), setup cost, holding cost, deteriorative cost, shortage cost and total cost increases then there is positive relationship between them,

3) with the increase in holding cost per unit per unit time (C_h), the setup cost, shortage cost and total cost increases then there is positive relationship between them but optimal cycle time (T) and optimal lot size (Q), production time (T_1), maximum inventory (Q_1), deteriorative cost decreases, holding cost then there is negative relationship between,

4) with the increase in shortage cost per unit, setup cost , holding cost , deteriorative cost, and total cost increases then there is positive relationship between them and cycle time (T), optimum quantity (Q), and shortage cost decreases then there is negative relationship between them.

4) Similarly, other parameters deteriorative cost per unit (C_d), initial demand "a", and time dependent demand "b" can also be observed from the Table 4.

4. Conclusion

This research considers inventory systems for purchasing inventory models where the objective is to find the optimal cycle time and optimal lot size which minimizes the total cost. Several aspects such as time value of money, linear demand, constant deteriorating and shortages are considered in developing different models. Two models are developed. In two models, it is observed that a study of rate of deteriorative items with cycle time, purchasing time, maximum inventory, optimum quantity, setup cost, holding cost, deteriorating cost, shortage cost and total cost is considered and it is concluded that when the rate of deteriorative item increases then the setup cost, deteriorating cost and total cost increases then it is positive relationship between them. When the rate of deteriorating rate increases then the cycle time, purchasing cost, maximum inventory, optimum quantity and holding cost decreases then there is negative relationship between them. This research can be extended as follows:

a) Most of the production systems today are multi-stage systems and in a multi-stage system the defective items and scrap can be produced in each stage. Again, the defectives and scrap proportion for a multi-stage system can be different in different stages. Taking these factors into consideration this research can be extended for a multi-stage production process.
b) Traditionally, inspection procedures incurring cost is an important factor to identify the defectives and scrap and remove them for the finished goods inventory. For better production, the placement and effectiveness of inspection procedures are required which is ignored for this research, so inspection cost can be included in developing the future models.
c) The demand of a product may decrease with time owing to the introduction of a new product which is either technically superior or more attractive and cheaper than the old one. On the other hand the demand of new product will increase. Thus, demand rate can be varied with time, so variable demand rate can be used to develop the model.

The proposed model can assist the manufacturer and retailer in accurately determining the optimal quantity, cycle time and inventory total cost. Moreover, the proposed inventory model can be used in inventory control of certain items such as food items, fashionable commodities, stationary stores and others.

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