Compare Adomian Decomposition Method and Laplace Decomposition Method for Burger's-Huxley and Burger's-Fisher equations

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Abstract. In this paper, Adomian decomposition method (ADM) and Laplace decomposition method (LDM) used to obtain series solutions of Burgers-Huxley and Burgers-Fisher Equations. In ADM the algorithm is illustrated by studying an initial value problem and LDM is based on the application of Laplace transform to nonlinear partial differential equations. In ADM only few terms of the expansion are required to obtain the approximate solution which is found to be accurate and efficient and in LDM does not need linearization, weak nonlinearity assumptions, or perturbation theory. These methods are used to solve the examples and the results are presented in the tables.

Index to information contained in this paper

1. Introduction

It is shown that the decomposition method solve effectively, easily and accurately a large linear and nonlinear class of ordinary or partial differential equations. The approximation of nonlinear differential equations is important in solving physical

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problems [3, 11]. Adomian decomposition method has been used for a wide range of stochastic and deterministic problems in physics, biology and chemical reactions [9, 10, 14]. Adomian's method has a lot of strength and accuracy and can be used in applications of nonlinear evolution models. Adomian decomposition method is a powerful method with simple algorithm and is very simple and efficient for solving nonlinear differential equations that are created in physical applications[2, 7, 12]. The Laplace transform numerical scheme based on the decomposition method for solving nonlinear differential equations. The analysis will be adapted to the approximate solution of a class of nonlinear second-order initial-value problems, though the algorithm is well suited for a wide range of nonlinear problems. The Laplace decomposition method (LDM) was proved to be compatible with the versatile nature of the physical problems and was applied to a wide class of functional equations [4, 5, 8].

In Section 2, the governing equations are presented and in section 3 we will explain the ADM and will be explained in Section 4 of LDM. Finally, in Section 5, we solve and compare the Burger's-Huxley and Burger's-Fisher equations with these methods.

2. Burger's-Huxley and Burger's-Fisher equations

In this section, we introduce the governing equations in this paper [1]. Consider the generalized Burger's-Fisher equation

$$u_t + \alpha u^\delta u_x - u_{xx} = \beta u(1 - u^\delta) , \qquad \forall\, 0 \leqslant x \leqslant 1 , \; t \geqslant 0, \tag{1}$$

with the initial condition

$$u(x, 0) = f(x), \tag{2}$$

and exact solution is

$$u(x, t) = \left(\frac{1}{2} + \frac{1}{2} Tanh\left[\frac{-\alpha\delta}{2(\delta + 1)} \left(x - \left(\frac{\alpha}{\delta + 1} + \frac{\beta(\delta + 1)}{\alpha} \right) t \right) \right] \right)^{\frac{1}{\delta}}. \tag{3}$$

And consider the generalized Burger's-Huxley equation

$$u_t + \alpha u^\delta u_x - u_{xx} = \beta u(1 - u^\delta)(u^\delta - \gamma) , \qquad \forall\, 0 \leqslant x \leqslant 1 , \; t \geqslant 0, \tag{4}$$

with the initial condition

$$u(x, 0) = \left(\frac{\gamma}{2} + \frac{\gamma}{2} Tanh[A_1 x] \right)^{\frac{1}{\delta}}. \tag{5}$$

The exact solution of (4) is

$$u(x, t) = \left(\frac{\gamma}{2} + \frac{\gamma}{2} Tanh[A_1(x - A_2 t)] \right)^{\frac{1}{\delta}}, \tag{6}$$

where

$$A_1 = \frac{-\alpha\delta + \delta\sqrt{\alpha^2 + 4\beta(1 + \delta)}}{4(1 + \delta)} \gamma ,$$

$$A_2 = \frac{\gamma\alpha}{(1+\delta)} - \frac{(1+\delta-\gamma)\left(-\alpha + \sqrt{\alpha^2 + 4\beta(1+\delta)}\right)}{2(1+\delta)},$$

where α, β and δ are parameters, $\beta \geqslant 0, \delta > 0, \gamma \in (0,1)$.

3. Adomian decomposition method

We begin with the equation [6]

$$Lu + R(u) + N(u) = g(t), \tag{7}$$

where L is the operator of the highest-ordered derivatives and R is the remainder of the linear operator. The nonlinear term is represented by $N(u)$. Thus we get

$$Lu = g(t) - R(u) - N(u), \tag{8}$$

The inverse

$$L^{-1} = \int_0^t (.)dt, \tag{9}$$

operating with the operator L^{-1} on both sides of (3) we have

$$u = f_0 + L^{-1}(g(t) - R(u) - N(u)), \tag{10}$$

where f_0 is the solution of homogeneous equation

$$Lu = 0, \tag{11}$$

involving the constants of integration. The integration constants involved in the solution of homogeneous equation (11) involve the constants of integration. The integration constants involved in the solution of homogeneous equation (11) are to be determined by the initial or boundary condition accordingly as the problem is initial-value problem or boundary-value problem.
The Adomian decomposition method assume that the unknown function $u(x,t)$ can be expressed by an infinite series of the form

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t), \tag{12}$$

and the nonlinear operator $N(u)$ can be decomposed by an infinite series of polynomials given by

$$N(u) = \sum_{n=0}^{\infty} A_n, \tag{13}$$

where $u_n(x,t)$ will be determined recurrently, and A_n are the so-called polynomials of $u_0, u_1, \cdots, u_n$ defined by

$$A_n = \frac{1}{n!} \frac{d^n}{d\lambda} \left[F\left(\sum_{i=0}^{\infty} \lambda^i u_i \right) \right]_{\lambda=0} , \quad n = 0, 1, 2, \cdots .$$ (14)

It is now well known in the literature that these polynomials can be constructed for all classes of non-linearity according to algorithms set by Adomian.

Theorem 3.1 The solution of the nonlinear PDEs in the form (7) with the initial $u(x,0) = f(x)$ can be determined by the series (12) with the iterative [6]

$$u_0(x,t) = f(x),$$

$$u_{n+1}(x,t) = f(x) + L^{-1}\left(R(u_n) - A_n \right) , \quad n \geqslant 0 .$$ (15)

4.   Laplace decomposition method

The aim of this section is to discuss the use of Laplace decomposition method for solving of partial differential equations written in an operator form [13]

$$L_t u + Ru + Nu = g,$$ (16)

with initial data

$$u(x,0) = f(x),$$ (17)

where L_t is considered a first-order partial differential operator, R and N are linear and nonlinear operators, respectively and g is source term. The method consists of first applying the Laplace transform to both sides of equation (16) and then by using initial conditions (17), we have

$$\mathcal{L}[L_t u] + \mathcal{L}[Ru] + \mathcal{L}[Nu] = \mathcal{L}[g],$$ (18)

using the differentiation property of Laplace transform, we get

$$\mathcal{L}[u] = \frac{f(x)}{s} + \frac{1}{s}\mathcal{L}[g] - \frac{1}{s}\mathcal{L}[Ru] - \frac{1}{s}\mathcal{L}[Nu].$$ (19)

The LDM defines the solutions $u(x,t)$ by the infinite series

$$u(x,t) = \sum_{n=0}^{\infty} u_n.$$ (20)

The nonlinear terms N is usually represented by an infinite series of the so-called Adomian polynomial, substituting (19) and (20) into (13) gives

$$\mathcal{L}\left[\sum_{n=0}^{\infty} u_n\right] = \frac{f(x)}{s} + \frac{1}{s}\mathcal{L}[g] - \frac{1}{s}\mathcal{L}\left[R\left(\sum_{n=0}^{\infty} u_n\right)\right] - \frac{1}{s}\mathcal{L}\left[\sum_{n=0}^{\infty} u_n\right], \qquad (21)$$

applying the linearity of the Laplace transform, we define the following recursively formula

$$\mathcal{L}[u_0] = \frac{f(x)}{s} + \frac{1}{s}\mathcal{L}[g], \qquad (22)$$

in general, for $k \geqslant 1$, the recursive relations are given by

$$\mathcal{L}[u_{k+1}] = -\frac{1}{s}\mathcal{L}[R(u_k)] - \frac{1}{s}\mathcal{L}[A_k], \qquad (23)$$

By applying the inverse Laplace transform, we can evaluate $u_k(k \geqslant 0)$.

5. Application of methods in solving equations

In this section we will apply ADM and LDM for the two problems. The first is the generalized Burger's-Fisher equation (1) and the second is the generalized Burger's-Huxley equation (4).

5.1 *Adomian decomposition method for generalized Burger's-Fisher equation*

Applying the inverse operator L^{-1} on both sides of (1) and using the initial condition we find [6]

$$u(x,t) = f(x) - L^{-1}\left(\alpha u^\delta u_x - u_{xx} - \beta u(1 - u^\delta)\right), \qquad (24)$$

by using (9) and (10) into the functional equation (12) gives

$$\sum_{n=0}^{\infty} u_n(x,t) = f(x) - L^{-1}\left(\sum_{n=0}^{\infty} \alpha A_n - \left(\sum u_n\right)_{xx}\right). \qquad (25)$$

Identifying the zeros component $u_0(x,t)$ by $f(x)$, the remaining components $n \geqslant 1$ can be determined by using the recurrence relation

$$u_0(x,t) = f(x),$$

$$u_{n+1}(x,t) = -L^{-1}\left(\alpha A_n - (u_n)_{xx}\right), \qquad n \geqslant 0, \qquad (26)$$

where A_n are Adomian polynomials that represent the nonlinear term $(\alpha u^\delta u_x)$ and given by

$$A_0 = u_0^\delta u_{0x} + \beta u_0(1 - u_0^\delta),$$

$$A_1 = \delta u_0^{\delta-1} u_1 u_{0x} + u_0^{\delta} u_{1x} + \beta\Big[u_0(1 - u_1^{\delta}) + u_1(1 - u_0^{\delta})\Big].$$

Other polynomials can be generated in a similar way. The first few components of $u_n(x,t)$ follows immediately upon setting

$$u_0(x,t) = f(x),$$

$$u_1(x,t) = -L^{-1}\Big(\alpha A_0 - (u_0)_{xx}\Big),$$

$$u_2(x,t) = -L^{-1}\Big(\alpha A_1 - (u_1)_{xx}\Big), \tag{27}$$

$$u_3(x,t) = -L^{-1}\Big(\alpha A_2 - (u_2)_{xx}\Big),$$

$$u_4(x,t) = -L^{-1}\Big(\alpha A_3 - (u_3)_{xx}\Big).$$

We have taken $\alpha = 0.001, \beta = 0.001$ and $\delta = 1$. We now calculate $\sum_{n=0}^{4} u_n(x,t)$ and consider it as an approximation and we will present the comparison results in Table 1.

5.2   Laplace decomposition method for generalized Burger's-Fisher equation

Using the differentiation property of Laplace transform for (1) we get [13]

$$\mathcal{L}[u] = \frac{1}{s}\left(\frac{1}{2} - \frac{1}{2}Tanh[\frac{\alpha\delta}{2(\delta+1)}x]\right)^{\frac{1}{\delta}} - \alpha\frac{1}{s}\mathcal{L}[uu_x] + \frac{1}{s}\mathcal{L}[u_{xx}] + \beta\frac{1}{s}\mathcal{L}[u] - \beta\frac{1}{s}\mathcal{L}[u^{\delta}+1], \tag{28}$$

The second step in LDM is that we represent solution as an infinite series given (20), we will get

$$\mathcal{L}\left[\sum_{n=0}^{\infty} u_n\right] = \frac{1}{s}\left(\frac{1}{2} - \frac{1}{2}Tanh[\frac{\alpha\delta}{2(\delta+1)}x]\right)^{\frac{1}{\delta}} - \alpha\frac{1}{s}\mathcal{L}\left[\sum_{n=0}^{\infty} A_n\right]$$

$$+ \frac{1}{s}\mathcal{L}\left[\sum_{n=0}^{\infty}(u_n)_{xx}\right] + \beta\frac{1}{s}\mathcal{L}\left[\sum_{n=0}^{\infty} u_n\right] - \beta\frac{1}{s}\mathcal{L}\left[\sum_{n=0}^{\infty} B_n\right], \tag{29}$$

where A_n and B_n are Adomian polynomials by (14) that represent the nonlinear terms $u^{\delta}u_x, u^{1+\delta}$ respectively. We have the following relation:

$$\mathcal{L}[u_0] = \frac{1}{s}\left(\frac{1}{2} - \frac{1}{2}Tanh[\frac{\alpha\delta}{2(\delta+1)}x]\right)^{\frac{1}{\delta}},$$

$$\mathcal{L}[u_{k+1}] = -\alpha \frac{1}{s}\mathcal{L}[A_k] + \frac{1}{s}\mathcal{L}[(u_k)_{xx}] + \beta \frac{1}{s}\mathcal{L}[u_k] - \beta \frac{1}{s}\mathcal{L}[B_k], \quad k \geqslant 0,$$

taking the inverse Laplace transform of both sides of the it, we have

$$u_0(x,t) = \left(\frac{1}{2} - \frac{1}{2}Tanh[\frac{\alpha\delta}{2(\delta+1)}x]\right)^{\frac{1}{\delta}},$$

$$u_1(x,t) = \frac{1}{4}\left(\frac{1}{2} - \frac{1}{2}Tanh[\frac{\alpha\delta}{2(\delta+1)}x]\right)^{-2+\frac{1}{\delta}} t\Big[4\beta\frac{1}{2} - \frac{1}{2}Tanh[\frac{\alpha\delta}{2(\delta+1)}x]^2$$

$$+ \left(1 - \frac{1}{2}Tanh[\frac{\alpha\delta}{2(\delta+1)}x]\right)^2 \times \left(\frac{\left(\frac{\left(-1-\frac{1}{2}Tanh[\frac{\alpha\delta}{2(\delta+1)}x]\right)\alpha^2\delta^2\left(-1+\delta-Tanh[\frac{\alpha\delta}{2(\delta+1)}x](\delta+1)\right)}{2(\delta+1)^2}\right)+\cdots}{\delta^2}\right)\Big],$$

and so on for other components. Now consider $u = u_0 + u_1$ as an approximate solution, we have taken $\alpha = \beta = 0.001, \delta = 1$ and we present the results of computation and comparison in Table 1.

Table 1. Absolute errors ($\alpha = \beta = 0.001$, $\delta = 1$) for Burger's-Fisher equation

x	t	$ADM(5-terms)$	$LDM(2-terms)$
	0.005	9.68763×10^{-6}	3.19327×10^{-28}
0.1	0.001	1.93753×10^{-6}	5.19268×10^{-31}
	0.01	1.93752×10^{-5}	5.00494×10^{-27}
	0.005	9.68691×10^{-6}	1.62271×10^{-27}
0.5	0.001	1.93738×10^{-6}	2.60468×10^{-30}
	0.01	1.93738×10^{-5}	2.58591×10^{-27}
	0.005	9.68691×10^{-6}	2.92609×10^{-27}
0.9	0.001	1.93724×10^{-6}	4.69010×10^{-30}
	0.01	1.93724×10^{-5}	4.67132×10^{-26}

5.3 *Adomian decomposition method for generalized Burger's-Huxley equation*

Applying the inverse operator L^{-1} on both sides of (4) and using the initial condition we find [6]

$$u(x,t) = f(x) - L^{-1}\Big(\alpha u^\delta u_x - u_{xx} - \beta u(1-u^\delta)(u^\delta - \gamma)\Big), \tag{30}$$

by using (9) and (10) into the functional equation (12) gives

$$\sum_{n=0}^{\infty} u_n(x,t) = f(x) - L^{-1}\left(\sum_{n=0}^{\infty} \alpha A_n - \left(\sum u_n\right)_{xx}\right). \tag{31}$$

Identifying the zeros component $u_0(x,t)$ by $f(x)$, the remaining components $n \geqslant 1$ can be determined by using the recurrence relation

$$u_0(x,t) = f(x),$$

$$u_{n+1}(x,t) = -L^{-1}\left(\alpha A_n - (u_n)_{xx}\right), \qquad n \geqslant 0, \qquad (32)$$

where A_n are Adomian polynomials that represent the nonlinear term $(\alpha u^\delta u_x)$ and given by

$$A_0 = u_0^\delta u_{0x} + \beta u_0(1 - u_0^\delta)(u_0^\delta - \gamma),$$

$$A_1 = \delta u_0^{\delta-1} u_1 u_{0x} + u_0^\delta u_{1x} + \beta\left[u_0(1-u_0^\delta)(u_1^\delta-\gamma)+u_0(1-u_1^\delta)(u_0^\delta-\gamma)+u_1(1-u_0^\delta)(u_0^\delta-\gamma)\right].$$

Other polynomials can be generated in a similar way. The first few components of $u_n(x,t)$ follows immediately upon setting

$$u_0(x,t) = f(x),$$

$$u_1(x,t) = -L^{-1}\left(\alpha A_0 - (u_0)_{xx}\right),$$

$$u_2(x,t) = -L^{-1}\left(\alpha A_1 - (u_1)_{xx}\right), \qquad (33)$$

$$u_3(x,t) = -L^{-1}\left(\alpha A_2 - (u_2)_{xx}\right),$$

$$u_4(x,t) = -L^{-1}\left(\alpha A_3 - (u_3)_{xx}\right).$$

We have taken $\alpha = 1, \beta = 1, \gamma = 0.001$ and $\delta = 1$. We now calculate $\sum_{n=0}^{4} u_n(x,t)$ and consider it as an approximation, and we will present the comparison results in Table 2.

5.4 Laplace decomposition method for generalized Burger's-Huxley equation

Using the differentiation property of Laplace transform for (4) we get [13]

$$\mathcal{L}[u] = \frac{1}{s}\left(\frac{\gamma}{2} + \frac{\gamma}{2}Tanh[A_1 x]\right)^{\frac{1}{\delta}} - \alpha\frac{1}{s}\mathcal{L}[u^\delta u_x] + \frac{1}{s}\mathcal{L}[u_{xx}]$$

$$-\beta\gamma\frac{1}{s}\mathcal{L}[u] + \beta(1+\gamma)\frac{1}{s}\mathcal{L}[u^{1+\delta}] - \beta\frac{1}{s}\mathcal{L}[u^{2\delta}]. \qquad (34)$$

The second step in LDM is that we represent solution as an infinite series given (20), we will get

$$\mathcal{L}\left[\sum_{n=0}^{\infty} u_n\right] = \frac{1}{s}\left(\frac{\gamma}{2} + \frac{\gamma}{2}Tanh[A_1x]\right)^{\frac{1}{\delta}} - \alpha\frac{1}{s}\mathcal{L}\left[\sum_{n=0}^{\infty} A_n\right] + \frac{1}{s}\mathcal{L}\left[\sum_{n=0}^{\infty}(u_n)_{xx}\right]$$

$$-\beta\gamma\frac{1}{s}\mathcal{L}\left[\sum_{n=0}^{\infty} u_n\right] + \beta(1+\gamma)\frac{1}{s}\mathcal{L}\left[\sum_{n=0}^{\infty} B_n\right] - \beta\frac{1}{s}\mathcal{L}\left[\sum_{n=0}^{\infty} C_n\right], \qquad (35)$$

where A_n, B_n and C_n are Adomian polynomials by (14) that represent the nonlinear terms $u^\delta u_x, u^{1+\delta}$ and $u^{2\delta}$ respectively. We have the following relation:

$$\mathcal{L}[u_0] = \frac{1}{s}\left(\frac{\gamma}{2} + \frac{\gamma}{2}Tanh[A_1x]\right)^{\frac{1}{\delta}},$$

$$\mathcal{L}[u_1] = \frac{1}{s}\mathcal{L}\left[-\alpha A_0 + (u_0)_{xx} - \beta\gamma u_0 + \beta(1+\gamma)B_0 - \beta C_0\right],$$

in general the recursive relation is given by

$$\mathcal{L}[u_{k+1}] = -\alpha\frac{1}{s}\mathcal{L}[A_k] + \frac{1}{s}\mathcal{L}[(u_k)_{xx}] - \beta\gamma\frac{1}{s}\mathcal{L}[u_k] + \beta(1+\gamma)\frac{1}{s}\mathcal{L}[B_k] - \beta\frac{1}{s}\mathcal{L}[C_k],$$

using the recurrence relation above and by using the inverse Laplace transform of both sides of the it, we have

$$u_0(x,t) = \left(\frac{\gamma}{2} + \frac{\gamma}{2}Tanh[A_1x]\right)^{\frac{1}{\delta}},$$

$$u_1(x,t) = \frac{1}{2}t\left[-2\beta\left(\frac{\gamma}{2}(1 + Tanh[A_1x])^{\frac{1}{\delta}}\right)^{2\delta+1}\right.$$

$$+\frac{2\left(\frac{\gamma}{2}(1 + Tanh[A_1x])^{\frac{1}{\delta}}\right)^{\delta+1}\left(A_1\alpha(Tanh[A_1x] - 1) + \beta\delta(1+\gamma)\right)}{\delta}$$

$$\left.+\frac{\gamma\left(1 + Tanh[A_1x]\right)^{\frac{1}{\delta}}\left(A_1^2(Tanh[A_1x] - 1)(-1 + \delta + Tanh[A_1x](\delta + 1)) - \beta\gamma\delta^2\right)}{\delta^2}\right],$$

and so on for other components. Now consider $u = u_0 + u_1$ as an approximate solution, we have taken $\alpha = 1, \beta = 1, \gamma = 0.001$ and $\delta = 1$ and we present the results of computation and comparison in Table 2.

6. Conclusion

In this study, LDM and ADM are used to obtain an approximate solution of the Burger's-Huxley equation in general and the Burger's-Fisher equation. The results

Table 2.   Absolute errors ($\alpha = \beta = 1$, $\gamma = 0.001$, $\delta = 1$) for Burger's-Huxley equation

x	t	$ADM(5 - terms)$	$LDM(2 - terms)$
	0.005	1.93715×10^{-7}	1.87406×10^{-8}
0.1	0.001	3.87434×10^{-7}	3.74812×10^{-8}
	0.01	3.87501×10^{-6}	3.74812×10^{-7}
	0.005	1.93730×10^{-7}	1.87406×10^{-8}
0.5	0.001	3.87464×10^{-7}	3.74812×10^{-8}
	0.01	3.87531×10^{-6}	3.74812×10^{-7}
	0.005	1.93745×10^{-7}	1.87406×10^{-8}
0.9	0.001	3.87494×10^{-7}	3.74812×10^{-8}
	0.01	3.87561×10^{-6}	3.74812×10^{-7}

show that LDM is more effective and more accurate than ADM. It also shows that the Lapl's breakdown technique (LDM) is a powerful tool for searching various nonlinear solutions. The proposed scheme can be used for other nonlinear equations of physics applications.

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