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A New Classes of Solutions of the Einstein-Maxwell Field Equations with Pressure Anisotropy

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Abstract. In this paper, we present a class of exact solutions to the Einstein-Maxwell system of equations for a spherically symmetric relativistic charged fluid sphere. The field equations are integrated by specifying the forms of the electric field, anisotropic factor, and one of the gravitational potentials which are physically reasonable. By reducing the condition of pressure isotropy to a linear, second order differential equation which can be solved in general, we show that it is possible to obtain closed-form solutions for a specific range of model parameters. The solution is regular, well behaved and complies with all the requirements of a realistic stellar model. An interesting feature of the new class of solutions is that one can easily switch off the electric and/or anisotropic effects in this formulation. Consequently, we regain some of the earlier solutions.

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1.Introduction

Solutions of the Einstein-Maxwell system of equations for static spherically symmetric interior spacetimes with anisotropic pressures are necessary to describe charged compact objects in relativistic astrophysics where the gravitational field is strong as in the case of neutron stars. The solutions to the field equations with pressure anisotropy generated have a number of different applications in relativistic stellar systems. It is for this reason that a number of investigations have been undertaken on the Einstein-Maxwell equations in recent times. A wide range of solutions to the Einstein-Maxwell system was compiled by Ivanov [1]. Bowers and Liang [2] have extensively analyzed the sources of anisotropy at the stellar interior. Subsequently, the origin and effects of local anisotropy on astrophysical objects have been reported by many authors [3–4] and Herrera and Santos [5] have reviewed and discussed possible causes for local anisotropy in self gravitating systems with examples of both Newtonian and general relativistic contexts. Current observations of mass-to-radius ratio of a wide range of pulsars have prompted many investigators to propose new theoretical models of

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compact stars by incorporating different types of matter composition. It is noteworthy that the effects of electric field and anisotropy plays an important role in constraining the mass radius relationship of a star as demonstrated by Komathiraj et .al [6] and Komathiraj and Sharma [7]. The role of quark star with charged and anisotropy was pursued by Malaver [8, 9, 10] who demonstrated exact analytical solutions. The treatment of Malaver [11, 12] deals with charged anisotropic matter with modified Tolman IV potential. From the above motivation it is clear that both anisotropy and the electromagnetic field are important in astrophysical processes.

In order to obtain the exact solutions of the field equations, various restrictions have been placed by investigators on the geometry of spacetime and the matter content. Mainly two distinct procedures have been adopted to solve these equations for spherically symmetric and static manifolds. Firstly, the coupled differential equations are solved by computation after choosing an equation of state. Secondly, the exact Einstein–Maxwell solution can be obtained by specifying the geometry and the forms of the anisotropy and electromagnetic field. The models obtained by the first method satisfy a barotropic equation of state, relating the radial pressure to the energy density are crucial important as demonstrated by Malaver [13] and Malaver and Kasmaei [14, 15, 16, 17]

In the recent past many new exact solutions have been developed by latter technique some of which are, in fact, generalizations of many of the well-known solutions. Most of the extensions have generally been done either by incorporating an electromagnetic field or anisotropy or both into the system. The generalized models allow us to study the impacts of charge and/or anisotropy on the gross physical behaviour of a compact star. A prime motivating factor for such a generalization in most previous works by authors was to fine-tune the stellar observables like mass and radius.

The objective of this paper is to generate new class of exact solutions corresponding to a static spherically anisotropic star possessing a net charge similar to the recent treatment of Komathiraj [18]. The idea is that once the anisotropy and/or charge are/is switched off we should be able to regain some of the well-behaved, physically interesting stellar solutions found earlier. The paper is organized as follows: In Section 2, we write down the Einstein-Maxwell field equations for a static, spherically symmetric, charged and anisotropic matter distribution and then we express the system as a new equivalent system of differential equations using a coordinate transformation due to Durgapal and Bannerji [19]. We choose particular forms for one of the gravitational potentials, the electric field intensity and the anisotropic factor in section 3. This enables us to obtain the master equation in the remaining gravitational potential which facilitates the integration process. In section 4, we consider the case $\alpha + \beta = \frac{a+b}{a}$ and solve the Einstein-Maxwell system in terms of elementary functions. Further, we treat the case $\alpha + \beta \neq \frac{a+b}{a}$ and generate the solution in a series form which yields recurrence relations, which we manage to solve from first principles. We generate two linearly independent classes of solutions by determining the specific restriction on the parameters for a terminating series; the general solution can be written explicitly in terms of elementary functions. In Section 5, the physical viability of the new class of solutions is examined. Finally, some concluding remarks are made in Section 6.

2. Field equations

We assume a static spherically symmetric spacetime describing the interior of a compact relativistic star in the form

$$ds^2 = -e^{2\mu(r)}dt^2 + e^{2\lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (1)$$

in Schwarzchild coordinates $(x^a) = (t, r, \theta, \phi)$, where $\mu(r)$ and $\lambda(r)$ are yet to be determined. For an anisotropic imperfect fluid the energy momentum tensor reads

$$T_{ij} = \text{diag}\left(-\rho - \frac{1}{2}E^2, p_r - \frac{1}{2}E^2, p_t + \frac{1}{2}E^2, p_t + \frac{1}{2}E^2\right) \quad (2)$$

In the above ρ is the energy density, p_r is the radial pressure, p_t is the tangential pressure, E is the electric field intensity. The Einstein-Maxwell system of field equations corresponding to the line element (1) and the matter distribution (2) can be written in the form

$$\frac{1}{r^2}\left(1 - e^{-2\lambda}\right) + \frac{2\lambda'}{r}e^{-2\lambda} = \rho + \frac{1}{2}E^2 \quad (3)$$

$$-\frac{1}{r^2}\left(1 - e^{-2\lambda}\right) + \frac{2\mu'}{r}e^{-2\lambda} = p_r - \frac{1}{2}E^2 \quad (4)$$

$$e^{-2\lambda}\left(\mu'' + \mu'^2 + \frac{\mu'}{r} - \mu'\lambda' - \frac{\lambda'}{r}\right) = p_t + \frac{1}{2}E^2 \quad (5)$$

$$\frac{1}{r^2}e^{-\lambda}(r^2E)' = \sigma \quad (6)$$

In the above σ is the proper charge density, and a prime $(')$ denotes derivative with respect to the radial coordinate r.

A different but equivalent form of the field equations can be generated if we introduce a new independent variable x and introduce new functions y and Z proposed by Durgapal and Bannerji [19]:

$$A^2y^2(x) = e^{2\mu(r)}, \quad Z(x) = e^{-2\lambda(r)}, \quad x = Cr^2 \quad (7)$$

where A and C are arbitrary constants. Under the transformation (7), the system (3)-(6) becomes

$$\frac{1-Z}{x} - 2\dot{Z} = \frac{\rho}{C} + \frac{1}{2}E^2 \quad (8)$$

$$4Z\frac{\dot{y}}{y} + \frac{Z-1}{x} = \frac{p_r}{C} - \frac{1}{2C}E^2 \quad (9)$$

$$4Zx^2\ddot{y} + 2\dot{Z}x^2\dot{y} + \left(\dot{Z}x - Z + 1 - \frac{\Delta x}{C} - \frac{E^2x}{C}\right)y = 0 \quad (10)$$

$$\sigma^2 = \frac{4CZ}{x}\left(x\dot{E} + E\right)^2 \quad (11)$$

where a dot (.) denotes differentiation with respect to the variable x. The quantity $\Delta = p_t - p_r$ is defined as the measure of anisotropy which is required to vanish at the center.

3. Method of generating solutions

We must make physically reasonable choices for any three of the independent variables and then solve the system to generate exact models. In our approach, we plan to solve the Einstein-

Maxwell system by specifying physically reasonable forms of the gravitational potential Z, electric field E^2 and the measure of anisotropy Δ as:

$$Z(x) = \frac{(1+ax)^2}{1-bx} \tag{12}$$

$$\frac{\Delta}{C} = \frac{\alpha a(a+b)x}{(1-bx)^2} \tag{13}$$

$$\frac{E^2}{C} = \frac{\beta a(a+b)x}{(1-bx)^2} \tag{14}$$

In (12) – (14), a, b, α and β are real constants. A similar form of the gravitational potential in (12) and the electric field in (14) were studied previously by Komathiraj and Maharaj [20] and Thirukkanesh and Maharaj [21] in the case of isotropic pressure ($\Delta = 0$). In addition the choice for the anisotropic factor given in (13) is regular and well behaved for a wide range of the parameter α. Therefore, the choices made in (12)-(14) are physically reasonable. Our objective is to confirm that this type of potential is also consistent with anisotropic matter. Substituting (12)-(14) into (10) we obtain

$$\begin{aligned} 4(1+ax^2)(1-bx)\ddot{y} + 2(1+ax)[2(b+a) - b(1+ax)]\dot{y} \\ + (a+b)[a+b-(\alpha+\beta)a]y = 0 \end{aligned} \tag{15}$$

As the equation (15) is difficult to integrate in the above form, we introduce a transformation

$$1+ax = \frac{(a+b)}{b}X, \quad y(x) = Y(X) \tag{16}$$

to obtain a convenient form. Use of transformation (16), (15) becomes

$$4X^2(X-1)\frac{d^2Y}{dX^2} + 2X(X-2)\frac{dY}{dX} + \left(\alpha+\beta-\frac{a+b}{a}\right)Y = 0 \tag{17}$$

which is the master equation of the system and has to be integrated to find an exact model for a charged sphere with anisotropic pressures.

4. Classes of exact solutions

There are two categories of solutions to (17) in terms of different values of the parameters α and β. The two cases correspond to

$$\alpha+\beta = \frac{a+b}{a}, \qquad \alpha+\beta \neq \frac{a+b}{a}.$$

which generates new models.

4.1 *The case* $\boldsymbol{\alpha+\beta = \frac{a+b}{a}}$

In this case (17) becomes

$$2X(X-1)\frac{d^2Y}{dX^2}+(X-2)\frac{dY}{dX}=0 \tag{18}$$

Equation (18) is easily integrable and the solution can be written as

$$y(x)=c_1\left(\sqrt{\frac{a(bx-1)}{a+b}}-\tan^{-1}\sqrt{\frac{a(bx-1)}{a+b}}\right)+c_2 \tag{19}$$

in terms of the original variable $x = Cr^2$, where c_1 and c_2 are constants. The complete solution of the Einstein-Maxwell system can be obtained from (8) - (11) in terms of elementary functions using the solution (19).

4.2 *The case* $\boldsymbol{\alpha+\beta \neq \frac{a+b}{a}}$

With $\alpha+\beta \neq \frac{a+b}{a}$, equation (17) is difficult to solve. It is convenient at this point to introduce the transformation

$$Y(X)=X^d U(X), \tag{20}$$

where d is a constant. With the help of (20), the differential equation (17) can be written as

$$4X^2(X-1)\frac{d^2U}{dX^2}+2X[(4d+1)X-2(2d+1)]\frac{dU}{dX} + \left[2d(2d-1)X-\frac{a+b}{a}+\alpha+\beta-4d^2\right]U=0 \tag{21}$$

Note that there is substantial simplification if we take

$$\alpha+\beta-\frac{(a+b)}{a}=4d^2 \tag{22}$$

Then (21) becomes

$$2X[X-1]\frac{d^2U}{dX^2}+[(4d+1)X-2(2d+1)]\frac{dU}{dX}+d(2d-1)U=0 \tag{23}$$

The result (23) is a special case of hypergeometric differential equation which can be integrated using the method of Frobenius. When $d=0$ then $\alpha+\beta=\frac{(a+b)}{a}$ and we regain the result of Section 4.1. Therefore we take $d \neq 0$ in this section to ensure that $+\beta \neq \frac{a+b}{a}$.

As the point $X=1$ is a regular singular point of (23), there exist two linearly independent solutions of the form of a power series with centre $X=1$. These solutions can be generated using the method of Frobenius. Therefore we can write

$$U(X)=\sum_{i=0}^{\infty} c_i[X-1]^{i+r}, \quad c_0 \neq 0, \tag{24}$$

where c_i are the coefficients of the series and r is the constant. To complete the solution we need to find the coefficients c_i explicitly. On substituting (24) in (23) we obtain the indicial equation

$$c_0 r(2r-3)=0$$

and the recurrence relation

$$c_i = \frac{-[(i+r-1)(2i+2r+4d-3)+d(2d-1)]}{(i+r)(2i+2r-3)} c_{i-1}, \quad i \geq 1 \tag{25}$$

which governs the structure of the solution. As $c_0 \neq 0,$ we must have $r=0$ or $r=3/2$. Withe the help of (25) we can express the general form for the coefficient c_i in terms of the leading coefficient c_0 as:

$$c_i = (-1)^i \prod_{p=1}^{i} \frac{[(p+r-1)(2p+2r+4d-3)+d(2d-1)]}{(p+r)(2p+2r-3)} c_0 \tag{26}$$

It is easy to establish that the result (26) holds for all positive integers p using the principle of mathematical induction. Now it is possible to generate two linearly independent solutions to (23) with the help of (24) and (26). For the parameter value $r=0$ we obtain the first solution

$$U_1(X) = c_0\left[1+\sum_{i=1}^{\infty}(-1)^i \prod_{p=1}^{i} \frac{[(p-1)(2p+4d-3)+d(2d-1)]}{p(2p-3)}(X-1)^i\right] \tag{27}$$

For the parameter value $r=3/2$ we obtain the second solution

$$U_2(X) = c_0(X-1)^{3/2}\left[1+\sum_{i=1}^{\infty}(-1)^i \prod_{p=1}^{i} \frac{[(2p+1)(p+2d)+d(2d-1)]}{p(2p+3)}(X-1)^i\right] \tag{28}$$

In terms of the variable $= Cr^2$, (27) and (28) become

$$y_1(x) = c_0\left[\frac{(b(1+ax))}{a+b}\right]^d \times\left[1+\sum_{i=1}^{\infty}(-1)^i \prod_{p=1}^{i} \frac{[(p-1)(2p+4d-3)+d(2d-1)]}{p(2p-3)}\left[\frac{(a(bx-1))}{a+b}\right]^i\right] \tag{29}$$

and

$$y_2(x) = c_0\left[\frac{(b(1+ax))}{a+b}\right]^d \left[\frac{(a(bx-1))}{a+b}\right]^{3/2} \times\left[1+\sum_{i=1}^{\infty}(-1)^i \prod_{p=1}^{i} \frac{[(2p+1)(p+2d)+d(2d-1)]}{p(2p+3)}\left[\frac{(a(bx-1))}{a+b}\right]^i\right] \tag{30}$$

where we have used (16). Thus we can write the general solution to the differential equation (23), for the choice of the anisotropic factor (13) and the electric field given in (14), as

$$y(x) = A_1 y_1(x) + A_2 y_2(x) \tag{31}$$

where y_1 and y_2 are given in (29) and (30) respectively, A_1, A_2 are constants. It is clear that the quantities y_1 and y_2 are linearly independent functions. From (8) - (11) with the help of (12) - (14), the general solution to the Einstein-Maxwell system can be written as

$$e^{2\lambda} = \frac{1 - bx}{(1 + ax)^2} \quad (32)$$

$$e^{2\nu} = A^2 y^2 \quad (33)$$

$$\frac{\rho}{C} = \frac{(bx - 3)(2a + b)}{(1 - bx)^2} - \frac{a^2 x(5 - 3bx)}{(1 - bx)^2} - \frac{\alpha a(a + b)x}{2(1 - bx)^2} \quad (34)$$

$$\frac{p_r}{C} = \frac{4(1 + ax)^2}{1 - bx}\frac{\dot{y}}{y} + \frac{a(2 + ax) + b}{1 - bx} + \frac{\alpha a(a + b)x}{2(1 - bx)^2} \quad (35)$$

$$\frac{p_t}{C} = \frac{4(1 + ax)^2}{1 - bx}\frac{\dot{y}}{y} + \frac{a(2 + ax) + b}{1 - bx} + \frac{\alpha a(b - a)x}{2(1 + bx)^2} + \frac{a(\alpha + 2\beta)(a + b)x}{2(1 - bx)^2} \quad (36)$$

$$\frac{E^2}{C} = \frac{\alpha a(a + b)x}{(1 - bx)^2} \quad (37)$$

$$\frac{\sigma^2}{c} = \frac{\alpha aC(a + b)(1 + ax)^2(3 - bx)^2}{(1 - bx)^5} \quad (38)$$

$$\frac{\Delta}{C} = \frac{\beta a(a + b)x}{(1 - bx)^2} \quad (39)$$

where y is given by (31). As the choice of the metric function (12) together with the anisotropic factor (13) and the electric field intensity (14) have not been considered earlier, to the best of our knowledge, the class of solutions (32) - (39) have not been reported previously. It should be stressed that the new class of solutions (32) - (39) holds good for isotropic as well as anisotropic; charged as well as uncharged cases. The form of the exact solution (32) - (39) is a generalisation of the general solution of Komathiraj and Maharaj [20] when $\beta = 0$ and $b = -1$ and Thirukkanesh and Maharaj [21] when $\beta = 0$ the anisotropy vanishes and we regain their models.

It is interesting to observe that the series in (29) and (30) terminates for specific values of the parameters a, b, α and β. It is, therefore, possible to generate solutions in terms of elementary functions by imposing specific restrictions on the quantity $\alpha + b - \frac{a+b}{a}$. The solutions may be found in terms of polynomials and algebraic functions which is a more desirable form for the physical description of a charged anisotropic relativistic star. We can express the first category of solution to (23) as

$$y(x) = A\left[\frac{a+b}{b(1+ax)}\right]^{n}\sum_{i=0}^{n}(-1)^{i-1}\frac{(2i-1)}{(2i)!(2n-2i+1)!}\left[\frac{a(bx-1)}{a+b}\right]^{i}$$
$$+B\left[\frac{a+b}{b(1+ax)}\right]^{n}\left[\frac{a(bx-1)}{a+b}\right]^{\frac{3}{2}}\sum_{i=0}^{n-1}(-1)^{i-1}\frac{(i+1)}{(2i+3)!(2n-2i-2)!}\left[\frac{a(bx-1)}{a+b}\right]^{i} \quad (40)$$

where $\alpha+\beta-\frac{a+b}{a}=4n^2$. The second category of solution is given by

$$y(x) = A\left[\frac{a+b}{b(1+ax)}\right]^{n-1/2}\sum_{i=0}^{n}(-1)^{i-1}\frac{(2i-1)}{(2i)!(2n-2i)!}\left[\frac{a(bx-1)}{a+b}\right]^{i}$$
$$+B\left[\frac{a+b}{b(1+ax)}\right]^{n-1/2}\left[\frac{a(bx-1)}{a+b}\right]^{\frac{3}{2}}\sum_{i=0}^{n-2}(-1)^{i-1}\frac{(i+1)}{(2i+3)!(2n-2i-3)!}\left[\frac{a(bx-1)}{a+b}\right]^{i} \quad (41)$$

where $\alpha+\beta-\frac{a+b}{a}=4n(n-1)+1$.

In the above A and B are arbitrary constants. Thus we have extracted two classes of solutions (40) and (41) in terms of elementary functions from the infinite series solution (31). This class of solution helps in the study of the physical features of the model. It is possible to regain a number of physically reasonable isotropic ($\Delta = 0$) models from our general class solutions (40) and (41) for particular values of parameters. . If we set $\beta = 0$ and $b = -1$ then (40) - (41) are the same as the isotropic charged model of Komathiraj and Maharaj [20]. If we set $\beta = 0$, (40) – (41) become the isotropic charged model of Thirukkanesh and Maharaj [21]. Thus, we have regained the two previously reported isotropic charged models from our general class of solutions. .

5. New exact solutions and its physical features

It is interesting to observe that variety of new solutions can be obtained from the function (40) and (41) by substituting particular values for n. We illustrate one of such sample solution as example in this section.

If we set $b = (\alpha + \beta - 5)a$, then $n = 1$ and (40) becomes

$$y(x) = \frac{a_1[7 + \alpha + \beta + 3a(5 + \alpha + \beta)x] + b_1[1 + a(5 + \alpha + \beta)x]^{3/2}}{1 + ax} \quad (42)$$

where a_1 and b_1 are new constants. Subsequently, the general solution to the Einstein Maxwell system (8) - (11) can be expressed as

$$e^{2\lambda} = \frac{1 + a(5 - \alpha - \beta)x}{(1 + ax)^2} \quad (43)$$
$$e^{2\nu} = A^2 y^2 \quad (44)$$

$$\frac{\rho}{C} = \frac{a(3-\alpha-\beta-ax)}{1+a(5-\alpha-\beta)x} - \frac{2a(1+ax)[3-\alpha-\beta-a(5-\alpha-\beta)x]}{[1+a(5-\alpha-\beta)x]^2} + \frac{\alpha a^2(4-\alpha-\beta)x}{2[1+a(5-\alpha-\beta)x]^2} \quad (45)$$

$$\frac{p_r}{C} = \frac{4(1+ax)^2}{1+a(5-\alpha-\beta)x}\frac{\dot{y}}{y} + \frac{a(ax-3+\alpha+\beta)}{1+a(5-\alpha-\beta)x} - \frac{\alpha a^2(4-\alpha-\beta)x}{2[1+a(5-\alpha-\beta)x]^2} \quad (46)$$

$$\frac{p_t}{C} = \frac{4(1+ax)^2}{1+a(5-\alpha-\beta)x}\frac{\dot{y}}{y} + \frac{a(ax-3+\alpha+\beta)}{1+a(5-\alpha-\beta)x} - \frac{(\alpha+2\beta)a^2(4-\alpha-\beta)x}{2[1+a(5-\alpha-\beta)x]^2} \quad (47)$$

$$\frac{E^2}{C} = -\frac{\alpha a^2(4-\alpha-\beta)x}{2[1+a(5-\alpha-\beta)x]^2} \quad (48)$$

$$\frac{\Delta}{C} = -\frac{\beta a^2(4-\alpha-\beta)x}{2[1+a(5-\alpha-\beta)x]^2} \quad (49)$$

where y is given in (42). It is remarkable that the solution (43) - (49) completely expressed in terms of elementary function. For a physically viable model [22], the class of solutions obtained by our approach should satisfy certain regularity and physical requirements. We analyse the features of our solutions (43) - (49) and examine whether the solutions can be used for describing realistic stars.

In (43) and (44), we note that $e^{2\lambda}(r=0)=1$, $\left(e^{2\lambda}\right)'(r=0)=0$ and $e^{2\nu}(r=0)=A^2y^2(r=0)$, $(e^{2\nu})'(r=0)=(A^2y^2)'(r=0)$. Thus the gravitational potentials (43) and (44) are regular at the centre of the star $r=0$.

Using equation (45), we obtain the central density $\rho_0=\rho(r=0)=4aC(3-\alpha-\beta)$, which implies that we must have $a(3-\alpha-\beta)>0$. Using equation (46) at the center of the star($r=0$), we must have

$$p_r(r=0)=p_t(r=0)=4C\left(\frac{\dot{y}}{y}\right)(r=0)-aC(3-\alpha-\beta)>0$$

The radial pressure and the tangential pressure will be positive if we choose our model parameters in such a manner that the above condition is satisfied.

At the boundary of the star ($r=R$r), we impose the condition that the radial pressure vanishes, i.e., $p_R=p(x=CR^2)=0$, which yields

$$a_1=-b_1[1+aCR^2(5-\alpha-\beta)]^{\frac{3}{2}}\times\frac{f_1}{f_2},$$

where f_1 and f_2 are functions of a, C, R, α and β. Essentially, this equation places a restriction on the constants a_1 and b_1.

The solution of the Einstein-Maxwell system for $r > R$ is given by the Reissner-Nordstrom metric

$$ds^2 = -\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)dt^2 + \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2),$$

where $M = m(R)$ and $Q = E(R)R^2$ are the total mass and charge of the star. Matching the line element (1) with this equation across the boundary R, we have

$$A^2[y(CR^2)\,]^2 = \left(1 - \frac{2M}{R} + \frac{Q^2}{R^2}\right)$$

$$\frac{1 + a(5 - \alpha - \beta)CR^2}{(1 + aCR^2)^2} = \left(1 - \frac{2M}{R} + \frac{Q^2}{R^2}\right)^{-1}$$

where y is given by (42). These matching conditions determines the constants A as:

$$A = \frac{(1\ +\ aCR^2)^3}{1\ +\ aCR^2(5 - \alpha - \beta)} \times f_3$$

where f_3 is a function of a, C, R, α and β.

Thus the simple closed-form nature of the above new solution (43) - (49) facilitates its physical analysis as discussed above. Utilizing the above results, we can analyze physical viability of the solution (43) - (49) graphically for a given set of choices of model parameters as in the recent work by Komathiraj and Sharma [7, 23]. This is under preparation.

6. Conclusion

In this paper, we have found new solutions (32) – (39) to the Einstein–Maxwell system (8) – (11) by utilizing the method of Frobenius for an infinite series; a particular form for one of the gravitational potentials was assumed and the anisotropic factor with electric field intensity was specified. These solutions are given in terms of special functions. Moreover, we have demonstrated that for the specific set of model parameters, it is possible to obtain closed-form solutions from the general series solution. We used this feature to find two classes of exact solutions (40) and (41) to the Einstein–Maxwell system in terms of polynomials and product of polynomials and algebraic functions. These solutions may be useful in studying the physical behaviour of dense charged objects in relativity which will be the objective in future work. The advantage of the new class of solutions is that the general form of the closed-form solutions can be used to study all possible compositions (isotropic and uncharged, isotropic and charged, anisotropic and uncharged and anisotropic and charged). This facilitates an analysis of the impacts of electric field and anisotropy of compact stars. The most interesting feature of the class of solutions is that many well known stellar solutions can be regained simply by switching off the parameters specifying the anisotropy and/or charge distribution in this formulation. It will be interesting to probe what other combinations of the model parameters can provide new solutions in simple analytic forms. This will be taken up elsewhere.

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