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A new approach Spider's web initial solution and data envelopment analysis for solving an X-bar control chart

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Abstract. X-bar control charts are widely used to monitor and control business and manufacturing processes. Design of control charts refers to the selection of parameters, including sample size, control-limit width, and sampling frequency. Many researchers have worked on this issue and have also proposed various solutions. However, despite the numerous advantages, the proposed methods also have their own set of problems. The biggest challenge is the complexity of solving these issues. Due to the fact that optimal design of control charts can be formulated as a multi objective optimization problem, in this paper to solve this problem, we used initial solution Spider's web data envelopment analysis method. In previous methods used multiple algorithms to resolve the issue. But in the proposed method once using Data Envelopment Analysis method and without any other algorithm can solve multi objective problem and this method can yield desirable efficient. Lastly, we compare our method with others and demonstrate its application in a real industrial context.

Keywords: Data envelopment analysis (DEA), economical control chart design, multi objective optimization problem (MOOP), Spider's web initial solution (SWIS).

2010 AMS Subject Classification: 90C08

1. Introduction

Statistical process control (SPC) is one of the most effective continuous quality improvement strategies, which uses different statistical methods to improve quality and productivity in industrial processes. The primary tool of statistical process control is the

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statistical control chart. Engineering implementation of control charts require a number of technical and behavioral decision making processes. One important technical decision is the design of control chart, which refers to the selection of parameters, including sample size (n), control-limit width (k), and sampling frequency (h). Many researchers have worked on this issue and also have offered ways. Each method has some advantages and disadvantages such as complexity in implementation, statistical configurations, and cost effectiveness.

Duncan [7] developed the first model and applied it to an X-bar control chart. He proposed a single objective formulation for Shewhart's original X-bar control chart and considered a production process with a single assignable cause. Saniga et al.'s design [14] minimized the economic cost function and considered constraint that included upper and lower bounded respectively on the average time to signal and the power for some customer's specified shift sizes. Chung et al. [5] suggested an algorithm for computing the economically optimal X-bar control charts for a process with multi assignable causes. Chen and Liao [4] considered all possible combination of design parameters as a decision making unit. It is characterized by three attributes: hourly cost, the average run length of process being controlled and detection power of the chart designed with the selected parameters. Data envelopment analysis (DEA) is a method to measure the relative efficiency of decision making units (DMUs) performing similar tasks in a production system that consumes multiple inputs to produce multiple outputs. Li et al. [11] analyzed the design of the X-bar control chart problem using a DEA-based multi criteria branch and bound algorithm. Faraz et al. [9] used genetic algorithm optimized a two-objective economical statistical control chart design problem. The efficiency and fast convergence of the PSO in solving single objective has been extended to solve multiobjective problems Kennedy et al. [10]. Some extended version the MOPSO algorithm are presented by Durill et al. [8]. Mobin et al. [12] used the NSGA-II algorithm generated the efficient frontier an X-bar control chart problem. A new version of NSGA, called NSGAII and developed by Deb et al. (2000-2002), utilizes fast non-dominated sorting genetic algorithm. This method is computationally efficient, non-elitism preventing, and less dependent on a sharing parameter for diversity preservation. Recently, a reference-point based multi-objective NSGA-II algorithm (called NSGA-III) has been proposed by Deb and Jain [6], which is more efficient to solve problems with more than two objectives.

In general, it can be said that multi-criteria control chart design, generally divided into two parts. In first part, it is used most commonly and includes optimization algorithm to generate the optimal designs. In second part, tools such as DEA are used to find the efficient solution from of the optimal solutions generated by the optimization algorithm. To solve this problem, we used initial solution Spider's web and data envelopment analysis method introduced by Ranjbar et al. [13]. In the previous methods used multiple algorithms to resolve the issue, but in the proposed method once using data envelopment analysis method and without any other algorithm can solve multi objective problem and also, this method can yield desirable efficient frontier even in problems. Finally, an industrial application is presented to illustrate the solution procedure and we compare the proposed method with other methods.

2. Multi criteria X-bar control chart design

In model Duncan assumed one monitored the process to detect the occurrence of a single assignable causes a fixed shift in the process and define the relevant costs over a cycle. All notation used in describing the economical X-bar control cart design are presented in Table 1.

The components of the cycle he considered are as follows. Assume the process starts in the in-control state, the time interval that the process remains in control is an exponential random variable with mean $\frac{1}{\lambda}$ hour, which represents the average in-control time. In other words, as Figure 1, the process going to out-of control state from in-control state is assumed to be a Poisson process with λ occurrences per hour.

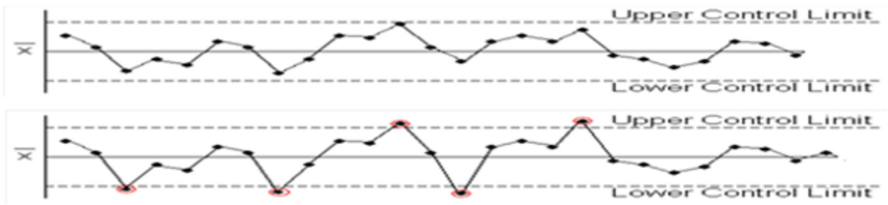

Figure 1. Upper and Lower control limits

Notation	Description
n	sample size
h	The time interval between successive samples (sampling frequency)
k	The control-limit width in terms of standard deviations σ
$s=(n,h,k)$	The DMU vector
$ARL_o(s)$	Average run length (ARL) with the process in control state ($\frac{1}{\alpha}$), affected by s
$p(s)$	The detection power of the economical control chart (after out-of-control status), affected by s
p_L	The lower bound of the economical control chart detection power
τ	The expected time of occurrence of the assignable cause within the interval between two samples
D	The time to search the assignable cause and make the process work at the in-control state again
λ	Assignable cause follows a Poisson process with rate λ
$\frac{1}{p}$	The expected number of samples taken before detecting a mean shift of the process
$\Phi(z)$	The probability density function of the standardized normal distribution
δ	The number of standard deviations σ in the shift of process mean μ_0
$\alpha(s)$	The probability of false alarm (type-I error), affected by $s=(n,h,k)$
α_U	The upper bound of the type-I error
$E_{CT}(s)$	The average cycle length
$E_{CC}(s)$	The expected cost per cycle
$E_{HC}(s)$	The expected cost per hour
a_1	The xed cost of sampling
a_2	The variable cost of sampling
a_3	The cost of searching for an assignable cause
a_4	The cost of investigating a false alarm
a_5	The hourly penalty cost associated with production in the out-of-control state
g	A constant used to estimate the average time of sampling, inspection, evaluation and plotting for sample gn

Table 1. Nomenclature used in the multi-objective X-bar economical control chart model [16]

When the process goes to out-of-control state, the probability that this state will be detected on any subsequent sampling is p, which represents the detection power of the chart and can be written as

$$p=\int_{-\infty}^{-k-\delta\sqrt{n}}\Phi(z)dz+\int_{k-\delta\sqrt{n}}^{+\infty}\Phi(z)dz, \tag{1}$$

where $\Phi(z)$ is the probability density function of standardized normal distribution, δ represents the number of standard deviations σ in the shift of process mean μ_0, and k is the control-limit width in terms of standard deviations σ. Accordingly, the expected number of sample taken before detecting a mean shift of the process is $\frac{1}{p}$. Moreover, the expected time of occurrence of the assignable cause within the interval between two samples is derived as

$$\tau=\frac{\int_{jh}^{(j+1)h}\lambda e^{-\lambda t}(t-jh)dt}{\int_{jh}^{(j+1)h}\lambda e^{-\lambda t}dt}=\frac{1-(1+\lambda h)e^{-\lambda h}}{-\lambda e^{-\lambda h}} \tag{2}$$

where h is the time interval between succeeding samples. The average time spent on sampling, inspection, evaluation, and plotting for each sample is a constant g proportion to the sample size n. Thus, the time delayed on this phase is gn. The time to search the assignable cause and make the process work at in-control state again is a constant D. The average cycle length of the process for a design $s = (n, h, k)$ can be expressed by $E_{CT}(s)$:

$$E_{CT}(s) = \frac{1}{\lambda} + (\frac{h}{p} - \tau) + gn + D. \tag{3}$$

Furthermore, the expected cost per cycle under a design $s = (n, h, k)$ can be expressed by $E_{CC}(s)$:

$$\begin{aligned} E_{CC}(s) &= \frac{(a_1 + a_2 n)E_{CT}(s)}{h} + a_3 + \frac{a_4 \alpha e^{-\lambda k}}{1 - e^{-\lambda k}} + a_5(\frac{h}{p} - \tau + gn + D), \\ \text{where} \quad \alpha &= \frac{1}{ARL_o} = 2\int_{-\infty}^{k} \Phi(z)dz. \end{aligned} \tag{4}$$

The cost per time unit $E_{HC}(s)$ under a design s can be obtained by dividing $E_{CC}(s)$, the expected cost per cycle, by $E_{CT}(s)$, the average cycle length [4].

In this paper, three objectives are derived based on the original economic design model of Duncan (1956). By considering two statistical constraints (the upper bond α_U of the type-1 error and the lower bound p_L of the detection power), which were integrated by Sangia (1989) into his economic model, the multi-objective X-bar economical control chart design can be formulated as follows:

$$\begin{aligned} &\max \quad ARL_o(s) \\ &\max \quad p(s) \\ &\min \quad E_{HC}(s) \\ &s.t. \quad \begin{array}{l} p(s) > p_L \\ \alpha(s) < \alpha_U \end{array} \end{aligned} \tag{5}$$

for all $s = (n, h, k)$

The decision variable in the multi-objective problem are the sample size n, the control limits k, with respect to a known process standard deviation σ, and the sampling frequency of two successive samples within the interval h. One possible design for the control chart consists of a combination of n, h and k [4]. After defining multi-objective design of the control chart problem and presenting its mathematical model, the modified NSGA-III and MOPSO algorithm and are utilized to generated on optimal design in Pareto frontier.

2.1 *NSGA-III*

NSGA-III is incorporated in the selection mechanism of NSGA-II. The idea is to use reference points which could be a set of predefined points, or one that are generated systematically. The pseudo-code of NSGA-III is shown in Table 2 (Deb & Jain, 2014).

The algorithm starts with N_{Pop} where P_0 denote the initial population. Notice that n, h and k are the parameters of the existing problem. Each solution represented by

1.	**Input**:
	P_0 (Initial Population),
	N_{Pop} size of population,
	t (iteration) $= 0$
	$It_{\max}$ (Maximum iteration).
2.	**While** $t < It_{\max}$ do
3.	Create offspring Q_t
4.	Mutation on Q_t
5.	Set $R_t = P_t \cup Q_t$
6.	Apply non-dominated sorting on R_t and find $F_1, F_2, \cdots$
7.	$S_t =$, $i = 1$;
8.	**While** $\|S_t\| \leqslant N_{Pop}$ do
9.	$S_t = S_t \cup F_i$
10.	$i = i + 1$
11.	**End**
12.	**If** $\|S_t\| = N_{Pop}$ do
13.	$P_{t+1} = S_t$; break
14.	**Else**
15.	$P_{t+1} = \bigcup_{j=1}^{l-1} F_j$
16.	Normalize St using min and intercept points of each objective
17.	Associate each member of St to a reference point
18.	Choose $N_{Pop} - \|P_{t+1}\|$ members from F_1 by niche-preserving operator
19.	**End**
20.	$t = t + 1$
21.	**End**
22.	Report P_t

Table 2. NSGA-III pseudo-code [16]

$s_i = (n_i, h_i, k_i)$ for $i = 1, 2, \cdots, N_{Pop}$. Note that individuals of the initial population are randomly generated such that $n_i \in \aleph^+$ and $h_i, k_i \in \Re^+$ for $i = 1, 2, \cdots, N_{Pop}$ as follows:

$$n_i = [n_{\min} + rand.(n_{\max} - n_{\min})],$$
$$h_i = [h_{\min} + rand.(h_{\max} - h_{\min})],$$
$$k_i = [k_{\min} + rand.(k_{\max} - k_{\min})],$$

where $x_{\min}$ and $x_{\max}$ represent lower and upper bounds for the variable x, respectively. Rand is a uniform number between 0 and 1, and $[x]$ represents the smallest integer greater than the real number x (M. Tavana et al. (2016)).

2.2 *Multi-objective particle swarm optimization (MOPSO)*

Particle swarm optimization (PSO) is inspired by the social behavior of birds within a flack. A particle represents each potential solution of the problem and a swarm represents the population of solution. In PSO, each particle (solution) searcher the solution space based on its current position and velocity direction, where the search is affected by the history of the particle and other individuals. The efficiency and fast convergence of the PSO in solving single objective has been extended to solve multiobjective problems Kennedy et al. [10]. Some extended version of the MOPSO algorithm presented by Durill et al. [8]. (M. Tavana et al. (2016)). The peudo-code of the general MOPSO is presented in Table 3.

1.	**Input**: P_0 (Initial Population), N_{Pop} size of population, t (iteration) = 0 $It_{\max}$ (Maximum iteration).
2.	Record non-dominated particle in REP
3.	Generated the grid (hypercubes)
4.	Update p Best$_i^t$
5.	Update g Best^t
6.	**While** $t < It_{\max}$ **do**
7.	**For** each particle I **do**
8.	Update velocity v_i^t
9.	Update new position s_i^t
10.	Update p Best$_i^t$
11.	**End for**
12.	Update g Best$_i^t$
13.	Update REP
14.	$t = t + 1$
15.	**End While**

Table 3. Pseudo-code MOPSO [16]

Note that, for comparison, we use the results of MOPSO and NSGA-III algorithms of article (M. Tavana et al. [16]).

3. Multi-objective optimization problems by combined DEA model and GA algorithm

In this section, we describe a multi-objective optimization problem and the concept of Pareto optimal solution. Consider a multi-objective optimization problem as follows:

$$\begin{aligned} \min_{x} \quad & f(x) = (f_1(x), \cdots, f_m(x))^T, \\ s.t. \quad & x \in S = \{x \in \Re^n | g_j(x) \leqslant 0, j = 1, 2, \cdots, l\}, \end{aligned} \tag{6}$$

where $x = (x_1, \cdots, x_n)$ is a design variable and S is the set of all feasible solutions. Generally, unlike traditional optimization problem with a single objective function, an optimal solution in the meaning that minimize all objective function $f_i(x)$ for $i = 1, 2, \cdots, m$ simultaneously does not necessarily exist in the problem. Therefore, the concept of an optimal solution found on the relation of Pareto domination is given as follows [15]:

Definition 3.1 A point $\hat{x}$ is said to be a Pareto optimal solution to the MOOP if there exists no $x \in S$ such that $f(x) \leqslant f(\hat{x})$.

A final solution to the multi-objective problem may be found out from the set of Pareto optimal solution by eg_jxisting methods, such as lexicography method, aspiration level,... . For solving MOOP, the above method requires a lot of time, especially when these issues have either several aims or large number constraint. So we use the proposed method for solving multi-objective problems.

DEA which was initially proposed by Charnes-Cooper-Rhodes, is a method to measure the relative efficiency of Decision Making Units (DMUs) performing similar tasks in a production system that consumes multiple inputs to produce multiple outputs. There are CCR model [3], BCC model [2] and FDH model [17], as representative models. These models are classified by how to determine the production possibility set. In DEA, the efficiency θ of an individual x_k is given by solving the following linear programming problem:

$$\begin{array}{ll} \min\limits_{\theta,\lambda} & \theta, \\ s.t. & [f_1(x),\cdots,f_m(x)]\lambda - \theta f(x_k) \leqslant< 0, \\ & \lambda \in \Re^n \quad , \quad \lambda \geqslant 0. \end{array} \tag{7}$$

The degree of efficiency θ represents how far $f(x_k)$ is from DEA-efficient frontier. And only when $\theta = 1$, then $f(x_k)$ is located on DEA-efficient frontier. Arakawa et al. [1] suggested a method using DEA and genetic algorithm (GA) to find the answer efficient multi-objective problem.

In other words, this method investigates the relation of domination among individuals with respect to the shaded region (see Figure 2). In Figure 2, the solid curve represents the exact efficient frontier and the dotted line represents DEA-frontier at a generation. As the figure shows, individual C and G are removed fast, and then a good approximation of the exact efficient frontier can be obtained efficiently. Therefore, when the efficient frontier is convex, non-Pareto solution can be removed from a young generation. But when the efficient frontier is non-convex, the sunken part of it can't be generated according to Arakawa et al. method [1].

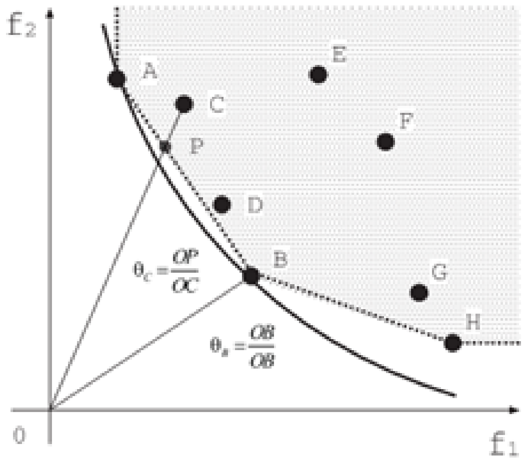

Figure 2. GA with DEA method [18]

$$\begin{array}{ll} \min\limits_{\Delta,\mu,\nu} & \Delta, \\ s.t. & \Delta \leqslant \tilde{d}_j + \alpha(\sum_{r=1}^{s} u_r(y_{rk}-y_{rj}) + \sum_{i=1}^{m} v_i(-x_{ik}+x_{ij}), \\ & \sum_{r=1}^{s} u_r + \sum_{i=1}^{m} v_i = 1, \\ & u_r, v_i \geqslant \varepsilon, \quad i=1,2,\cdots,m, \quad r=1,2,\cdots,s, \end{array} \tag{8}$$

Yun et al. [18] suggested a GDEA which includes existing DEA models. The efficiency based on generalized data envelopment analysis (GDEA) model is called GDEA-efficiency. Then, the GDEA-efficiency of DMU_k is judged by solving the following problem:

where α is a constant and ε is a sufficiently small positive number and

$$\tilde{d}_j = \max_{\substack{k=1,2,\cdots,n \\ j=1,2,\cdots,m}} \{u_r(y_{rk} - y_{rj}), v_i(-x_{ik} + x_{ij})\}$$

Various kinds of DEA-efficient frontier are obtained by changing the value of parameter α in problem (GDEA). To clarify we employ the example presented in Figure 3 consisting of six DMUs consume a single input to produce a single output. The figure indicates that GDEA-efficiency frontier with varying the value of α, and DMUs on the lines are α-efficient.

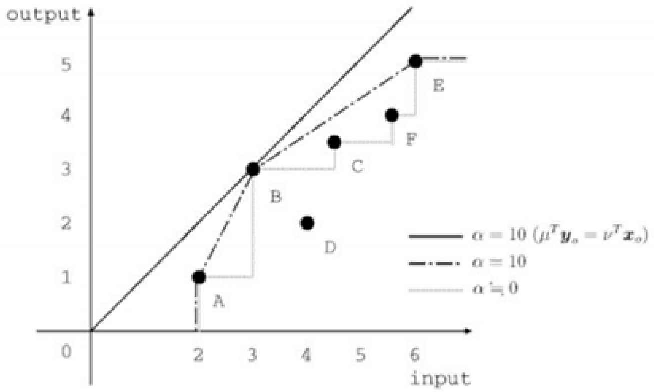

Figure 3. Efficiency frontier with various αin GDEA [18]

4. The Spider's web initial solution (SWIS) method

Ranjbar [13] introduced the SWIS method in 2020. In this method at first several feasible points on all the constraints of the problem are selected and then connected in the desired direction. From these points, and the resulting lines, we can extrapolate additional feasible points that are pertinent to the problems requirements. This expansion of points may yield a more extensive set of Pareto-optimal solutions. If we put more points, more Pareto answers will be gained.

Theorem 4.1 The SWIS method is feasible.

Proof. The proof is similar representation Theorem in [5]. ■

5. Proposed method

We know that in DEA models that have several inputs and outputs, a unit is efficient which means it has the minimum input and maximum output. In this method, first, we guess an initial solution to approach feasible region and SWIS method. We put this

solution set in the function of objectives which are maximized as output, in continue we put the values of these feasible solutions in the function of minimum objectives as input. As it is known, DEA models find, as much as possible, units having minimum input and maximum output, and use the concept of dominant solution. Because they may have many objects in multi-objective problem, so the most important link between MOOP and DEA is that DEA is the only tool that can easily evaluate each multi-objective problem with a large number of objects. Finally, because our goal is to find a set of efficient solutions and not just some units, we imagine all the other units on the efficiency frontier. This will provide a Pareto solution set of the efficient solution.
The steps are summarized as follows:

(1) At first a SWIS of feasible region is selected, some points on the constraints are chosen and then connected in the desired direction.
(2) Again some feasible points on the resulting lines are selected.
(3) Next this solution set is put in the function of objectives which are maximum as output, Then the values of these feasible solutions are put in the function of minimum objectives as input.
(4) Efficiency of points by the DEA is obtained.
(5) We image all the other units of the problem on the efficiency frontier. Finally, we consider the solutions which are applied to the feasible region as the final solutions.

In what follows, we try to solve two examples. we solve an example that none of the existing methods have been able to solve. Finally, the proposed integrated optimization method is applied to the industrial case, borrowed from Tavana et al. [16] and Chen and Liao [4].

6. Examples

We consider the following examples with six objective functions to be compared with the proposed method.

$$
\begin{array}{rl}
\min & f_1 = -x_1 \\
\min & f_2 = -4x_1 - 3x_2 \\
\min & f_3 = x_1^2 + x_2^2 - 4 \\
\max & f_4 = x_2 \\
\max & f_5 = 8x_1 + 2x_2 \\
\max & f_6 = 5x_1 + 6x_2 \\
s.t. & \left\{ \begin{array}{c}
\frac{1}{2}x_1 - x_2 \leqslant \frac{3}{2}, \\
7x_1 + x_2 \leqslant 49, \\
x_1^2 + x_2^2 \geqslant 4, \\
-0.2222x_1^2 + x_1 - x_2 + 8 \geqslant 0, \\
x_1^3 - x_2 + 5 \geqslant 0, \\
0.8x_1 - x_2 + 6 \geqslant 0, \\
x_1 \geqslant 0, x_2 \geqslant 0
\end{array} \right.
\end{array}
$$

For solving Example at first, as shown in Figure 4, we choose 59 point by the SWIS into feasible region. We put the (SWIS) value the maximum objective function and we choose it as an output. Similarly we put SWIS value the minimum objective function and we choose it as an input. And we solve the problem in model output oriented BCC model. Then image all the other units on the efficient frontier. Finally, we consider the solutions that apply to the feasible region as the final solutions. We see the final result in Figure 5. As you can see, in the proposed method once using DEA method and without any other algorithm can solve most of the MOOP.

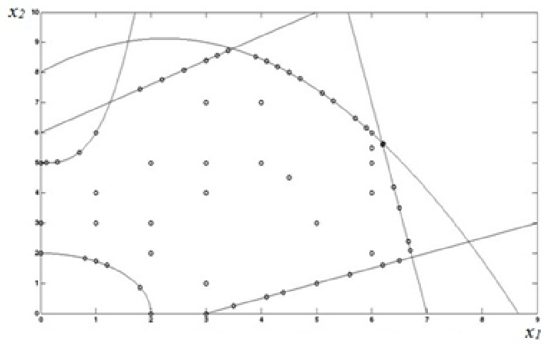

Figure 4. Feasible solution and the SWIS for Example 3

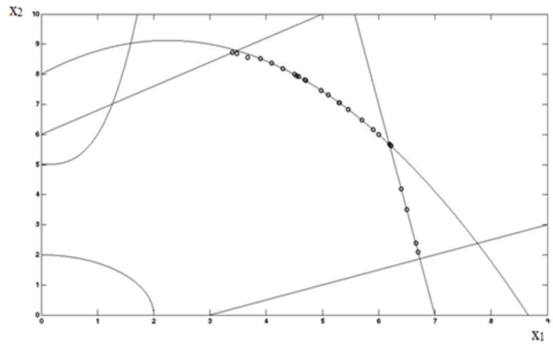

Figure 5. Efficiency frontier of Example

Example **6.1 (Case study)** In this section, the proposed integrated optimization method is applied to the industrial case, borrowed from Chen and Liao [2] and Tavana et al. [10]. The case study is about the process of producing electronic capacitors, where the target value of capacitance, for a particular model is set to 300 (in μF). The process shifts occur at random with a frequency of about 1 every 4 hours of operation ($\lambda = 0.25$). The fixed cost of sampling is estimated to be \$ 1.00 ($a_1 = 1$) and the variable cost is assumed to be \$ 0.1 per capacitor ($a_2 = 0.1$). The average time of sampling, measuring and recording the capacitance is estimated to be 0.01 h ($g = 0.01$). When the process goes out of control, the magnitude of the shift is approximately estimated to be one standard deviation ($\delta = 1.0$). The average time to search the assignable cause is 2 h ($D = 2$). The cost to search the assignable cause and also the measurable portion of the cost to investigate the false alarm are both \$ 50 ($a_3 = a_4 = 50$). The penalty cost associated with production in the OOC state is considered to be approximately \$ 200 per hour ($a_5 = 200$). Based on quality control experts' suggestions, the upper bound on the type-I error and the lower bound of the detection power are assumed to be 0.005 and 0.95, respectively ($\alpha_U = 0.005$, $p_L = 0.95$) [16].

The multi-objective X-bar economical control chart design formulation for this case study is as follow:

$$
\begin{aligned}
&\max \quad f_1 = ARL_o(s) = \max \frac{1}{2\int_{-\infty}^{-k} 2\Phi(z)dz} \\
&\max \quad f_2 = p = \int_{-\infty}^{-k-\delta\sqrt{n}} \Phi(z)dz + \int_{k-\delta\sqrt{n}}^{+\infty} \Phi(z)dz \\
&\min \quad f_3 = E_{HC}(s) = \frac{E_{CC}(s)}{E_{CT}(s)} = \frac{\frac{(a_1+a_2n)E_{CT}(s)}{h} + a_3 + \frac{a_4\alpha e^{-\lambda k}}{1-e^{-\lambda k}} + a_5(\frac{h}{p} - \tau + gn + D)}{\frac{1}{\lambda} + (\frac{h}{p} - \tau) + gn + D} \\
&s.t. \quad \begin{cases} p(s) \geqslant p_L, \\ \alpha(s) \leqslant \alpha_U \qquad \forall \text{ design } \; s = (n, h, k) \qquad \{n \in Z, 20 \leqslant n \leqslant 30\}, \\ \{h \in \Re, 0.4 \leqslant h \leqslant 0.5\} \text{ and } \{k \in \Re, 2.9 \leqslant k \leqslant 3.8\}, \end{cases}
\end{aligned} \tag{9}
$$

where $\Phi(z)$ can be obtained from the following formulation:

$$
\Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-\frac{u^2}{2}} du, \tau = \frac{1-(1+\lambda h)e^{-\lambda h}}{-\lambda e^{-\lambda h}}. \tag{10}
$$

Similarly, to solve Example 6.1, initially, as shown in Figure 6, we choose 55 point by the SWIS into feasible region. We solve the problem in model CCR model and image all the other units on the efficient frontier. As you can see, in the proposed method once using DEA method and without any other algorithm can solve the X-bar control charts problem. It should be noted that the points O show the frontier of efficiency without the image and points $*$ display the frontier of the efficiency of the points that are image on the efficiency frontier.

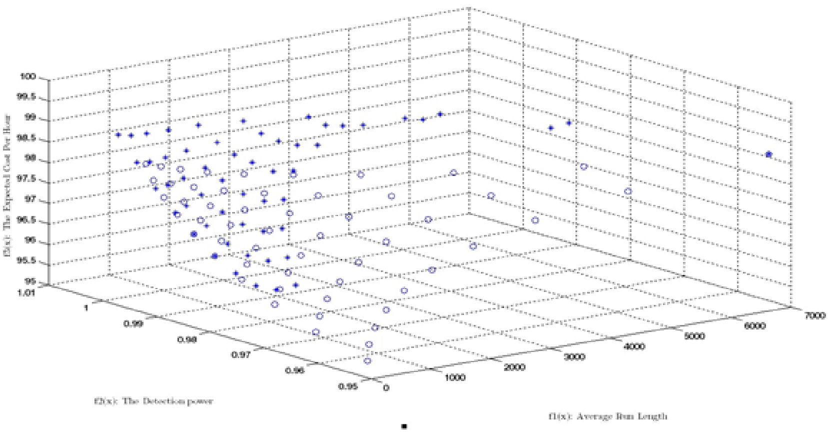

Figure 6. Efficiency frontier of Example 6.1

For comparison, the Pareto frontier obtained by NSGA-III and MOPSO plotted in Figure 7. Both Pareto frontier fall approximately in the same range, but the NSGA-III frontier were closed to the border area of the feasible solution and generated more solution at the edge of the Pareto frontier.

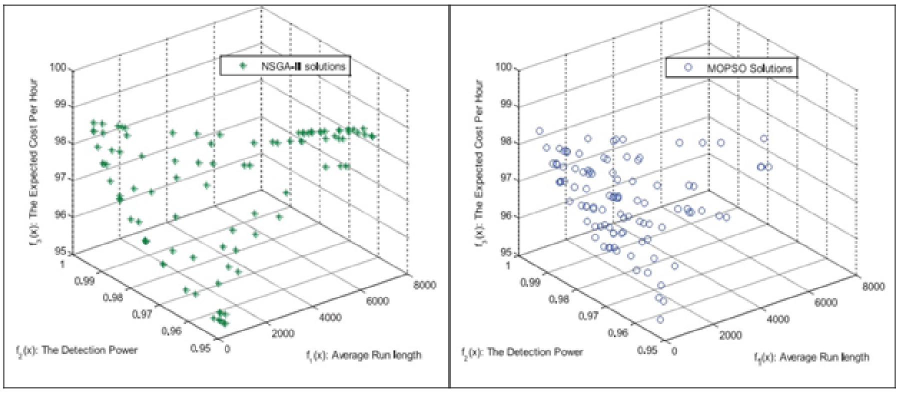

Figure 7. Efficiency frontier of NSGA-III and MOPSO method [16]

For better comparison, in addition to using methods NSGA-III and MOPSO, the results of Chen and Liao method, which is a data sensitivity analysis method, are also used in the following table. Table 4 presents a summary of the results obtained from the four optimization algorithms. As can be observed, all four algorithms generated solution such that the value of the decision variable and objective function fall approximately in the same ranges. As shown in Figures 6 and 7 and Table 4, the solutions obtained from the proposed method are very close to the standard methods. It is noteworthy that the proposed method solves problems with both the velocity and the accuracy of the calculations.

Parameters	NSGA-III			MOPSO			Chen and liao			Proposed method		
	Min	Max	Average	Min	Max	Average	Min	Max	Average	Min	Max	average
n	21	30	27.56	21	30	26.8	21	30	25	21	30	25
h	0.4	0.48639	0.438813	0.401441	0.499747	0.451195	0.4	0.4	0.4	0.4	0.5	0.4
k	2.9	3.8	3.343374	2.9	3.733528	3.215011	2.9	3.8	2.9	2.9	3.8	2.9
ARL_o(s)	267.9797	6911.037	2523.49	267.9797	5296.156	1090.571	267.98	6911.00	267.98	271.785	6911.00	268.007
p(s)	0.950003	0.99502	0.967355	0.950092	0.995019	0.972723	0.95377	0.95325	0.98214	0.96731	0.95325	0.98224
EHC(s)	95.32335	99.07514	97.88204	95.33462	98.85651	97.52637	95.236	98.632	96.706	96.5882	98.632	96.7157

Table 4. Summary solution

7. Conclusion

In this paper we applied a combined approach involving DEA method and SWIS to MOOP and X-bar control charts problem. The biggest challenge is the complexity of solving these issues. Due to the fact that optimal design of control charts can be formulated as a MOOP. To solve this problem, we used initial solution Spider's web data envelopment analysis method. In the proposed method without any algorithm could just one initial population standard (SWIS) with MATLAB easily obtained and image the answer on the efficient frontier. The proposed method solves problems with both the velocity and the accuracy of the calculations. This method works very well for convex problems. But for non-convex problems may solution the final number is a little lower than final solution are problems with convex feasible region.

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