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Integral type contraction and coupled fixed point theorems in ordered G-metric spaces

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Abstract. In this paper, we apply the idea of integral type contraction and prove some coupled fixed point theorems for such contractions in ordered G-metric space. Also, we support the main results by an illustrative example.

Keywords: Integral type contraction, Lebesgue integrals, ordered G-metric space, coupled fixed point.

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1. Introduction

The Banach contraction mapping principle is widely considered as the source of metric fixed point theory, and its significance is in its application in a number of branches of mathematics. Hence, there are many numerous generalizations of the Banach contraction principle. One of them is introduced by Branciari [7]. In 2002, Branciari investigated the idea of using Lebesgue integrals in metric fixed point theory and proved the existence and uniqueness of fixed points for integrally contractions whenever the metric space (X, d) is complete. After that many authors considered various versions of integral contractions and obtained fixed point results with respect to these contractions in various metric spaces in [4, 5, 16] and references contained therein.

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One of another extension of Banach contraction principle is considered by Ran and Reurings [15], and Nieto and Lopez [12]. They defined the Banach contraction principle in a metric space endowed with a partial order and proved the existence and uniqueness of fixed points for this contractive condition for the comparable elements of X. Further, the existence of fixed points in partially ordered sets has been applied for the proof of the existence of solutions to the ordinary and partial differential equations (see [1, 8, 12, 15]).

In 2006, Mustafa and Sims [11] introduced a new version of generalized metric spaces, which is called G-metric spaces and proved some well-known fixed point theorems in the framework of this space. After that, many authors continued the study of this space and obtained new trend (see [10, 17]). For having a good survey about G-metric spaces and its properties and applications, we refer to Agarwal et al.'s book [3].

In this paper, we combine three above mentioned concept and prove the existence and uniqueness of coupled fixed points in the kind of integrally contractions in partially ordered g-metric spaces. Also, we give a suitable example that support our main result. For this purpose, we start with some preliminary definitions and propositions which is needed in the sequel.

Definition 1.1 [11] Let X be a nonempty set and $G : X^3 \to \mathbb{R}^+$ be a function satisfying the following conditions:

(G1) $G(x, y, z) = 0$ if $x = y = z$ for all $x, y, z \in X$,
(G2) $0 < G(x, x, y)$ for all $x, y \in X$ with $x \neq y$,
(G3) $G(x, x, y) \leqslant G(x, y, z)$ for all $x, y, z \in X$ with $z \neq y$,
(G4) $G(x, y, z) = G(x, z, y) = G(y, z, x) = \dots$ for all $x, y, z \in X$,
(G5) $G(x, y, z) \leqslant G(x, a, a) + G(a, y, z)$ for all $x, y, z, a \in X$.

Then G is called a generalized metric or a G-metric on X and the pair (X, G) is a G-metric space.

***Example* 1.2** [11] Let (X, d) be a usual metric space, then (X, G_s) and (X, G_m) are G-metric space, where

$$G_s(x, y, z) = d(x, y) + d(y, z) + d(x, z), \quad x, y, z \in X,$$

$$G_m(x, y, z) = max\{d(x, y), d(y, z), d(x, z)\}, \quad x, y, z \in X.$$

Definition 1.3 [11] Let (X, G) be a G-metric space and $\{x_n\}$ be a sequence of points of X. We say that $\{x_n\}$ is convergent to x if $\lim\limits_{n,m\to\infty} G(x, x_n, x_m) = 0$; that is, for any $\epsilon > 0$, there exists $N \in \mathbb{N}$ such that $G(x, x_n, x_m) < \epsilon$ for all $n, m \geqslant N$. We refer to x as the limit of the sequence $\{x_n\}$ and write $x_n \to x$.

Definition 1.4 [11] Let (X, G) be a G-metric space. A sequence $\{x_n\}$ is called Cauchy if for given $\epsilon > 0$, there is $N \in \mathbb{N}$ such that $G(x_n, x_m, x_l) < \epsilon$ for all $n, m, l \geqslant N$; that is, if $G(x_n, x_m, x_l) \to 0$ as $n, m, l \to \infty$.

Definition 1.5 [11] A G-metric space(X, G) is said to be a complete G-metric space if every Cauchy sequence in (X, G) is convergent in (X, G).

Definition 1.6 [11] Let (X, G) and $(X^{'}, G^{'})$ be G-metric spaces and $f : (X, G) \to (X^{'}, G^{'})$ be a function. Then f is said to be continuous at a point $a \in X$ if for given $\epsilon > 0$, there exists $\delta > 0$ such that for $x, y \in X$; $G(a, x, y) < \delta$ implies $G(fa, fx, fy) < \epsilon$. A function f is continuous on X if and only if it is continuous at all $a \in X$.

Proposition 1.7 [3, 11] Let (X, G) be a G-metric space. Then the following statements are equivalent:

1. $\{x_n\}$ is G-convergent to x;
2. $G(x_n, x_n, x) \to 0$ as $n \to \infty$;
3. $G(x_n, x, x) \to 0$ as $n \to \infty$;
4. $G(x_n, x_m, x) \to 0$ as $n, m \to \infty$.

Proposition 1.8 [3, 11] Let (X, G) be a G-metric space. Then the following statements are equivalent:

1. $\{x_n\}$ is G-Cauchy;
2. for every $\epsilon > 0$, there is $k \in \mathbb{N}$ such that $G(x_n, x_m, x_m) < \epsilon$ for all $n, m \geqslant k$.

In 2006, Bhaskar and Lakshmikantham [12] defined coupled fixed point, proved some coupled fixed point theorems for a mixed monotone mapping in partially ordered matric spaces and studied the existence and uniqueness of a solution to a periodic boundary value problem (also, see [2, 9, 13, 14]).

Definition 1.9 [6] An element $(a, b) \in X^2$ is called a coupled fixed point of mapping $f : X^2 \to X$ if $f(a, b) = a$ and $f(b, a) = b$.

Definition 1.10 [6] Let $(X, \preceq)$ be a partially ordered set. The mapping $f : X^2 \to X$ is said to be have the mixed monotone property if f is monotone non-decreasing in its first argument and is monotone non-increasing in its second argument; that is, for all $x_1, x_2 \in X$, $x_1 \preceq x_2$ implies $f(x_1, y) \preceq f(x_2, y)$ for each $y \in X$, and for all $y_1, y_2 \in X$, $y_1 \preceq y_2$ implies $f(x, y_1) \succeq f(x, y_2)$ for each $x \in X$.

Definition 1.11 [9] Let $(X, \preceq)$ be an ordered partial metric space. If relation " $\sqsubseteq$ " is defined on X^2 by $(x, y) \sqsubseteq (u, v)$ iff $x \preceq u \wedge y \succeq v$, then $(X^2, \sqsubseteq)$ is an ordered partial metric space.

For more details on coupled fixed point and generally n-tupled fixed point theorems, we refer to Soleimani Rad et al.'s paper [18] and references contained therein.

2. Main results

Let $\phi, \omega : [0, +\infty) \to [0, +\infty)$ be two given functions. For convenience, we consider the following properties of these functions:

(ϕ_1) ϕ is non-increasing on $[0, \infty)$,
(ϕ_2) ϕ is Lebesgue integrable,
(ϕ_3) for any $\epsilon > 0$, $\int_0^\epsilon \phi(t)dt > 0$,
(ϕ_4) ϕ is continuous

and

(ω_1) ω is non-decreasing on $[0, \infty)$,
(ω_2) $\omega(t) \leqslant t$ for all $t > 0$,
(ω_3) ω is additive function,
(ω_4) $\sum\limits_{n=1}^{\infty} n\omega^n(t) < \infty$ for all $t > 0$.

The following is the main theorem of this work.

Theorem 2.1 Let $(X, G, \preceq)$ be a partially ordered complete G-metric space and $f : X^2 \to X$ be a continuous mapping having the mixed monotone property on X such that

$$\int_0^{G(f(x,y),f(u,v),f(w,z))} \phi(t)dt \leqslant \omega(\int_0^{G(x,u,w)+G(y,v,z)} \phi(t)dt) \tag{1}$$

for all $x, y, z, u, v, w \in X$ with $x \succeq u \succeq w$ and $y \preceq v \preceq z$ where either $u \neq w$ or $v \neq z$. If there exist $x_0, y_0 \in X$ such that $x_0 \preceq f(x_0, y_0)$ and $y_0 \succeq f(y_0, x_0)$, then f has a coupled fixed point in X.

Proof. By the hypothesis of this theorem, there exist $x_0, y_0 \in X$ such that $x_0 \preceq f(x_0, y_0)$ and $y_0 \succeq f(y_0, x_0)$. We define $x_1, y_1 \in X$ as $x_1 = f(x_0, y_0) \succeq x_0$ and $y_1 = f(y_0, x_0) \preceq y_0$. Let $x_2 = f(x_1, y_1)$ and $y_2 = f(y_1, x_1)$. Then we obtain

$$\begin{aligned} f^2(x_0, y_0) &= f(f(x_0, y_0), f(y_0, x_0)) = f(x_1, y_1) = x_2, \\ f^2(y_0, x_0) &= f(f(y_0, x_0), f(x_0, y_0)) = f(y_1, x_1) = y_2. \end{aligned}$$

Now, the mixed monotone property of f implies that

$$\begin{aligned} x_2 &= f^2(x_0, y_0) = f(x_1, y_1) \succeq f(x_0, y_0) = x_1 \succeq x_0, \\ y_2 &= f^2(y_0, x_0) = f(y_1, x_1) \preceq f(y_0, x_0) = y_1 \preceq y_0. \end{aligned}$$

Continuing the above procedure, we have

$$x_0 \preceq x_1 \preceq x_2 \preceq \cdots \preceq x_{n+1} \preceq \cdots, \quad y_0 \succeq y_1 \succeq y_2 \succeq \cdots \succeq y_{n+1} \succeq \cdots$$

for all $n \geqslant 0$, where

$$\begin{aligned} x_{n+1} &= f^{n+1}(x_0, y_0) = f(f^n(x_0, y_0), f^n(y_0, x_0)), \\ y_{n+1} &= f^{n+1}(y_0, x_0) = f(f^n(y_0, x_0), f^n(x_0, y_0)). \end{aligned}$$

If $(x_{n+1}, y_{n+1}) = (x_n, y_n)$, then f has a coupled fixed point. Otherwise, let $(x_{n+1}, y_{n+1}) \neq (x_n, y_n)$ for all $n \geqslant 0$; that is, we assume that either $x_{n+1} = f(x_n, y_n) \neq x_n$ or $y_{n+1} = f(y_n, x_n) \neq y_n$. Now, by (1), we have

$$\begin{aligned} \int_0^{G(x_n,x_{n+1},x_{n+1})} \phi(t)dt &= \int_0^{G(f(x_{n-1},y_{n-1}),f(x_n,y_n),f(x_n,y_n))} \phi(t)dt \\ &\leqslant \omega(\int_0^{G(x_{n-1},x_n,x_n)+G(y_{n-1},y_n,y_n)} \phi(t)dt) \end{aligned} \tag{2}$$

and similarly,

$$\begin{aligned} \int_0^{G(y_n,y_{n+1},y_{n+1})} \phi(t)dt &= \int_0^{G(f(y_{n-1},x_{n-1}),f(y_n,x_n),f(y_n,x_n))} \phi(t)dt \\ &\leqslant \omega(\int_0^{G(y_{n-1},y_n,y_n)+G(x_{n-1},x_n,x_n)} \phi(t)dt) \end{aligned} \tag{3}$$

For all $n \geqslant 0$. Since ϕ is non-increasing, we obtain

$$\int_0^{a+b} \phi(t)dt \leqslant \int_0^a \phi(t)dt + \int_0^b \phi(t)dt \tag{4}$$

for all $a, b \geqslant 0$. Now, since ω is linear and non-decreasing, it follows from (1), (2), (3) and (4) that

$$\begin{aligned}
\int_0^{G(x_n,x_{n+1},x_{n+1})} \phi(t)dt &= \int_0^{G(f(x_{n-1},y_{n-1}),f(x_n,y_n),f(x_n,y_n))} \phi(t)dt \\
&\leqslant \omega(\int_0^{G(x_{n-1},x_n,x_n)+G(y_{n-1},y_n,y_n)} \phi(t)dt) \\
&\leqslant \omega(\int_0^{G(x_{n-1},x_n,x_n)} \phi(t)dt) + \omega(\int_0^{G(y_{n-1},y_n,y_n)} \phi(t)dt) \\
&\leqslant 2\omega^2(\int_0^{G(x_{n-2},x_{n-1},x_{n-1})+G(y_{n-2},y_{n-1},y_{n-1})} \phi(t)dt) \\
&\vdots \\
&\leqslant n\omega^n(\int_0^{G(x_0,x_1,x_1)+G(y_0,y_1,y_1)} \phi(t)dt).
\end{aligned} \tag{5}$$

In the same manner, we obtain

$$\int_0^{G(y_n,y_{n+1},y_{n+1})} \phi(t)dt \leqslant n\omega^n(\int_0^{G(y_0,y_1,y_1)+G(x_0,x_1,x_1)} \phi(t)dt). \tag{6}$$

Now, let $m, n \in \mathbb{N}$ with $m > n$. Then

$$\begin{aligned}
\int_0^{G(x_n,x_m,x_m)} \phi(t)dt \leqslant &\int_0^{G(x_n,x_{n+1},x_{n+1})} \phi(t)dt + \int_0^{G(x_{n+1},x_{n+2},x_{n+2})} \phi(t)dt \\
&+ \cdots + \int_0^{G(x_{m-1},x_m,x_m)} \phi(t)dt.
\end{aligned}$$

It follows from (5) that

$$\begin{aligned}
\int_0^{G(x_n,x_m,x_m)} \phi(t)dt &\leqslant \sum_{i=n}^{m-1} i\omega^i(\int_0^{G(x_0,x_1,x_1)+G(y_0,y_1,y_1)} \phi(t)dt) \\
&\leqslant \sum_{i=1}^{\infty} i\omega^i(\int_0^{G(x_0,x_1,x_1)+G(y_0,y_1,y_1)} \phi(t)dt).
\end{aligned}$$

Since $\sum_{i=1}^{\infty} i\omega^i(t) < \infty$ for all $t \in [0, +\infty)$, we have $\lim_{n,m\to\infty} G(x_n, x_m, x_m) = 0$ and the sequence $\{x_n\}$ is a Cauchy sequence in X. Similarly, $\{y_n\}$ is a Cauchy sequence in X. Since X is complete, there exist $x, y \in X$ such that $\lim_{n\to\infty} x_n = x$ and $\lim_{n\to\infty} y_n = y$. Since

f is continuous, it is obvious that $f(x,y)=x$ and $f(y,x)=y$; that is, (x,y) is a coupled fixed point of f. ■

Theorem 2.2 Suppose that the assumptions of Theorem 2.1 are hold and the assumption the continuity of f substitute by the following conditions:

(i) if a non-decreasing sequence $\{x_n\}$ convergent to $x\in X$, then $x_n \preceq x$ for all n;
(ii) if a non-increasing sequence $\{y_n\}$ convergent to $y\in X$, then $y_n \succeq y$ for all n.

Then f has a coupled fixed point.

Proof. Consider $\{x_n\}$ and $\{y_n\}$ similar to the proof of Theorem 2.1. Then, by hypothesis (i) and (ii), we have $x_n \preceq x$ and $y_n \succeq y$ for all $n \geqslant 0$. Let $x_n = x$ and $y_n = y$ for some n. Then, by construction, $x_{n+1} = x$ and $y_{n+1} = y$; that is, (x,y) is a coupled fixed point. Thus, we consider either $x_n \neq x$ or $y_n \neq y$. Then, by (1), we have

$$\begin{aligned}\int_0^{G(f(x,y),x,x)}\phi(t)dt &\leqslant \int_0^{G(f(x,y),f(x_n,y_n),f(x_n,y_n))+G(f(x_n,y_n),x,x)}\phi(t)dt\\ &\leqslant \int_0^{G(f(x,y),f(x_n,y_n),f(x_n,y_n))}\phi(t)dt+\int_0^{G(f(x_n,y_n),x,x)}\phi(t)dt\\ &\leqslant \omega(\int_0^{G(x,x_n,x_n)+G(y,y_n,y_n)}\phi(t)dt)+\int_0^{G(x_{n+1},x,x)}\phi(t)dt. \qquad (7)\end{aligned}$$

Now, take limit as $n\to\infty$. Then, by (7), we obtain $G(f(x,y),x,x)=0$, which implies that $f(x,y)=x$. Similarly, one can show that $f(y,x)=y$. This completes the proof of the theorem. ■

Theorem 2.3 Adding the following condition to the hypotheses of Theorem 2.1 (Theorem 2.2). Then the coupled fixed point of f is unique.

(H) for all $(x,y),(x_1,y_1)\in X^2$, there exists $(z_1,z_2)\in X^2$ such that is comparable with (x,y) and (x_1,y_1).

Proof. Assume that (x_1,y_1) is another coupled fixed point of f. We consider two cases:

Case 1. Suppose that (x,y) and (x_1,y_1) are comparable with respect to the partial ordering $\sqsubseteq$ in X^2. Without restriction to the generality, we can assume that $x\preceq x_1$ and $y\succeq y_1$. Applying the procedure of Theorem 2.1, we have

$$\int_0^{G(f^n(x,y),f^n(x,y),f^n(x_1,y_1))}\phi(t)dt \leqslant \sum_0^{\infty} n\omega^n(\int_0^{G(x,x,x_1)+G(y,y,y_1)}\phi(t)dt) \qquad (8)$$

Now, let $n\to\infty$. Then, (8) implies that $x=x_1$. Similarly, one can show that $y=y_1$.

Case 2. (x,y) and (x_1,y_1) are not comparable. From (H), there exists $(z_1,z_2)\in X^2$ such that is comparable to (x,y) and (x_1,y_1). Without restriction to the generality, we can suppose that $x\preceq z_1$, $y\succeq z_2$, $x_1\preceq z_1$ and $y_1\succeq z_2$. Now, using the procedure of Theorem 2.1, we have

$$\int_0^{G(f^n(x,y),f^n(x,y),f^n(z_1,z_2))}\phi(t)dt \leqslant \sum_0^{\infty} n\omega^n(\int_0^{G(x,x,z_1)+G(y,y,z_2)}\phi(t)dt) \qquad (9)$$

Now, Let $n \to \infty$. From (9), we have $G(f^n(x,y), f^n(x,y), f^n(z_1,z_2)) = 0$. Hence, $x = \lim_{n\to\infty} f^n(x,y) = \lim_{n\to\infty} f^n(z_1,z_2)$. Similar to above argument, we obtain $x_1 = \lim_{n\to\infty} f^n(x_1,y_1) = \lim_{n\to\infty} f^n(z_1,z_2)$. Hence, $x = x_1$. Similarly, we have $y = y_1$. Thus, $(x,y) = (x_1,y_1)$ in both cases; that is, the coupled fixed point of the mapping f is unique. ■

Theorem 2.4 In addition of the hypotheses of Theorem 2.1 (Theorem 2.2), suppose that every pair of elements of X has an upper or a lower bound in X. Then $x = y$.

Proof. Case 1. assume that x and y are comparable with respect to the partial ordering $\sqsubseteq$ in X^2. Without restriction to the generality, we can suppose that $x \preceq y$ and $y \succeq y$. Similar to Theorem 2.3, we have $x = y$.

Case 2. suppose that x is not comparable to y. Then there exists an upper bound or lower bound of x and y; that is, there exists $z \in X$ comparable with x and y. For example, we can suppose that $x \preceq z$ and $y \succeq z$. Similarly, from Theorem 2.3, we have $(x,y) = (z,z)$. Hence, we have $x = y$. ■

***Example* 2.5** Let $X = [0,1]$ and $G : X^3 \to \mathbb{R}^+$ be a mapping defined by $G(a,b,c) = |a-b| + |a-c| + |b-c|$ for all $a,b,c \in X$. Then (X,G) is a complete G-metric space (see [5]). Also, let $\omega(t) = \frac{t}{2}$ for all $t \in [0,+\infty)$ and $f : X^2 \to X$ be a mapping defined by $f(a,b) = \frac{1}{16}ab$. Since $|ab - pq| = |a-p| + |b-q|$ holds for all $a,b,p,q \in X$, the conditions of Theorem 2.1 holds. In fact, we have

$$\begin{aligned}
\int_0^{G(f(a,b),f(p,q),f(c,r))} \phi(t)dt &= \int_0^{|f(a,b)-f(p,q)|+|f(a,b)-f(c,r)|+|f(p,q)-f(c,r)|} \phi(t)dt \\
&= \int_0^{|\frac{1}{16}ab-\frac{1}{16}pq|+|\frac{1}{16}ab-\frac{1}{16}cr|+|\frac{1}{16}pq-\frac{1}{16}cr|} \phi(t)dt \\
&= \int_0^{\frac{1}{16}(|a-p|+|b-q|+|a-c|+|b-r|+|p-c|+|q-r|)} \phi(t)dt \\
&\leqslant \frac{1}{16}\int_0^{|a-p|+|b-q|+|a-c|+|b-r|+|p-c|+|q-r|} \phi(t)dt \\
&\leqslant \omega(\int_0^{G(a,p,c)+G(b,q,r)} \phi(t)dt)
\end{aligned}$$

for all $a,b,c,p,q,r \in X$. It is easy to see that f satisfies all the hypothesis of Theorem 2.1. Thus, f has a coupled fixed point.

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