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$b-(\varphi,\Gamma)-$graph contraction on metric space endowed with a graph

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Abstract. In this paper, we introduce the $b-(\varphi,\Gamma)-$graphic contraction on metric space endowed with a graph so that (M,δ) is a metric space, and $V(\Gamma)$ is the vertices of Γ coincides with M. We aim to obtain some new fixed-point results for such contractions. We give an example to show that our results generalize some known results.

Keywords: Metric space, fixed point, $b-(\varphi,\Gamma)-$graphic contraction.

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1. Introduction

Jachymski [13] offers some generalizations about the Banach contraction principle to map on a metric space with respect to a graph. Some recent articles give sufficient conditions for selfmap $f: M \to M$ to be a PO if (M,δ) is a metric space endowed with a graph. We give some conditions to show that $b-(\varphi,\Gamma)-$graphic contraction is PO. In order to study $b-(\varphi,\Gamma)-$graphic contraction, we need the following definitions (also, see [1, 2, 4–8, 10–12, 14–16, 18–22, 24]).

Let (M,δ) be a metric space, and Δ be the diagonal of $M\times M$. Let Γ be a directed graph so that the set $V(\Gamma)$ of its vertices coincides with M, and the set $S(\Gamma)$ of its edges contains all loops, i.e. $S(\Gamma)\supseteq\Delta$. Let Γ have no parallel edges, which is why one can identify Γ with the pair $(V(\Gamma),S(\Gamma))$. By Γ^{-1}, we denote the graph obtained from Γ by reversing the direction of edges, and call it the reverse of graph Γ. Thus,

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$S(\Gamma^{-1}) = \{(x,y) \in M \times M | (y,x) \in S(\Gamma)\}$. $\tilde{\Gamma}$ is the undirected graph obtained from Γ by removing the direction of the edges. Thus, we have $S(\tilde{\Gamma}) = S(\Gamma) \bigcup S(\Gamma^{-1})$.

A path from x to y of length $N (N \in \mathbf{N})$ is a sequence $(x_i)_{i=0}^{N}$ of $N+1$ vertices so that $x_0 = x, x_N = y$ and $(x_{n-1}, x_n) \in S(\Gamma)$ for $i = 1, ..., N$. Γ has a weak connection if $\tilde{\Gamma}$ is connected. $[x]_\Gamma$ is the equivalent class of relations $\Re$ defined on $V(\Gamma)$ by the rule: $z \Re y$ if there is a path in Γ from z to y. Γ_x is called the component of Γ, which comprises of all edges and vertices that are contained in some paths beginning at x.

If $f : M \to M$ is an operator, then $M^f := \{x \in M : (x, fx)\} \in S(\Gamma)\}$ and the set of all fixed points of f is denoted by $F_f := \{x \in M : f(x) = x\}$.

Definition 1.1 [3, 9] Let M be a set and $s \geqslant 1$ be a given real number. A function $\delta : M \times M \to \mathbf{R}^+$ is said to be a $b-$metric on M and the pair (M, δ) is called a $b-$metric space if, for all $x, y, z \in M$,

(δ_1) $\delta(x,y) = 0$ if and only if $x = y$,
(δ_2) $\delta(x,y) = \delta(y,x)$,
(δ_3) $\delta(x,z) \leqslant s[\delta(x,y) + \delta(y,z)]$.

Remark 1 *Set $s = 1$ in the Definition 1.1, then we obtain δ is a metric space on M.*

***Example* 1.2** [24] Let $M = l_p(\mathbf{R})$, where $l_p(\mathbf{R}) = \{x = \{x_n\} \subset \mathbf{R} : \sum_{n=1}^{\infty} \mid x_n \mid^p < \infty\}$ and $0 < p < 1$. Then $\delta(x,y) = (\sum_{n=1}^{\infty} \mid x_n - y_n \mid^p)^{\frac{1}{p}}$ is a $b-$metric on M with $s = 2^{\frac{1}{p}}$.

Definition 1.3 [8] A mapping $f : M \to M$ is called $\Gamma-$graphic contraction if

1. for all $x, y \in M$, if $(x,y) \in S(\Gamma)$ then $(f(x), f(y)) \in S(\Gamma)$;
2. there exists $a \in [0,1)$ so that $\delta(f(x), f^2(x)) \leqslant ad(x, f(x))$ for all $x \in M^f$.

Matkowski [17] defined class of $\varphi-$contractions in metric fixed-point theory.

Definition 1.4 [17] Let $\phi : \mathbf{R}^+ \to \mathbf{R}^+$. Consider the following properties:

$(i)_\varphi$ $t_1 \leqslant t_2 \Rightarrow \varphi(t_1) \leqslant \varphi(t_2)$ for all $t_1, t_2 \in \mathbf{R}^+$,
$(ii)_\varphi$ $\varphi(t) < t$ for $t > 0$,
$(iii)_\varphi$ $\varphi(0) = 0$,
$(iv)_\varphi$ $\lim_{n\to\infty} \varphi^n(t) = 0$ for all $t > 0$,
$(v)_\varphi$ $\sum_{n=0}^{\infty} \varphi^n(t)$ converges for all $t > 0$.

We state that a function φ satisfying $(i)_\varphi$ and $(iv)_\varphi$ is said to be a comparison function. Moreover, if a function φ satisfying $(i)_\varphi$ and $(v)_\varphi$ is said to be a $(c)-$comparison function

In Definition 1.4, $(i)_\varphi$ and $(iv)_\varphi$ imply $(ii)_\varphi$ and $(i)_\varphi$ and $(ii)_\varphi$ imply $(iii)_\varphi$.

Remark 2 *Any $(c)-$comparison function is a comparison function.*

Definition 1.5 [24] Let $s \geqslant 1$ be a fixed real number. A function $\varphi : \mathbf{R}^+ \to \mathbf{R}^+$ is known as $(b)-$comparison function if it satisfies $(i)_\varphi$ and the following holds:

$(vi)_\varphi$ $\sum_{n=0}^{\infty} s^n \varphi^n(t)$ converges for all $t \in \mathbf{R}^+$.

Remark 3 *By setting $s = 1$ in Definition 1.5, we obtain that the function φ is a comparison function.*

***Example* 1.6** [24] Let (M, δ) be a $b-$metric space with coefficient $s \geqslant 1$. Then $\varphi(t) = at$

for all $t \in \mathbf{R}^+$ with $0 < a < (\frac{1}{s})$ is a $(b)-$comparison function.

Definition 1.7 [24] A mapping $f : M \to M$ is called $b-(\varphi, \Gamma)-$contraction if

(i) for all $x, y \in M$, if $(x, y) \in S(\Gamma)$ then $(f(x), f(y)) \in S(\Gamma)$;
(ii) $\delta(f(x), f(y)) \leqslant \varphi(\delta(x, y))$ whenever $(x, y) \in S(\Gamma)$,

where $\phi : \mathbf{R}^+ \to \mathbf{R}^+$ is a comparison function.

Definition 1.8 [13] Let (M, δ) be a $b-$metric space and $f : M \to M$ a mapping. Two sequences $\{f^n x\}$ and $\{f^n y\}$ in M are said to be equivalent if $\lim_{n\to\infty} \delta(f^n x, f^n y) = 0$. Moreover, if each of them is a Cauchy sequence, they are called Cauchy equivalents.

In the next section, we state two fixed-point theorems for $b-(\varphi, \Gamma)-$graphic contraction.

2. Main results

In this section, we assume that (M, δ) is a $b-$metric space with coefficient $s \geqslant 1$ and Γ is a directed graph so that $V(\Gamma) = M$, $\Delta \subseteq S(\Gamma)$ and Γ has no parallel edges.

Definition 2.1 A mapping $f : M \to M$ is called $b-(\varphi, \Gamma)-$graphic contraction if

(i) for all $x, y \in M$, if $(x, y) \in S(\Gamma)$ then $(f(x), f(y)) \in S(\Gamma)$;
(ii) $\delta(f(x), f^2(x)) \leqslant \varphi(\delta(x, f(x)))$for all $x \in M^f$,

where $\phi : \mathbf{R}^+ \to \mathbf{R}^+$ is a comparison function.

Remark 4 *Any $\Gamma-$graphic contraction is a $b-(\varphi, \Gamma)-$graphic contraction.*

Lemma 2.2 Let (M, δ) be a $b-$metric space and $f : M \to M$ be a $b-(\varphi, \Gamma)-$graphic contraction, where $\phi : \mathbf{R}^+ \to \mathbf{R}^+$ is a $b-$comparison function. Then, for given $x \in M^f$ there exists $r(x) \geqslant 0$ so that $\delta(f^n x, f^{n+1} x) \leqslant \varphi^n(r(x))$ for all $n \in \mathbf{N}$.

Proof. Assume that $x \in M^f$, then by induction, we have $(f^n x, f^{n+1} x) \in S(\Gamma)$ for each $n \in \mathbf{N}$. So

$$\delta(f^n x, f^{n+1} x) \leqslant \varphi(\delta(f^{n-1} x, f^n x)) \leqslant \cdots \leqslant \varphi^n(\delta(x, fx).$$

Set $r(x) = \delta(x, fx)$. ■

Lemma 2.3 Let (M, δ) be a $b-$metric space and $f : M \to M$ be a $b-(\varphi, \Gamma)-$graphic contraction, where $\phi : \mathbf{R}^+ \to \mathbf{R}^+$ is a $b-$comparison function. Furthermore, for each $x \in M^f$, there exists $x^*(x) \in M$ so that the sequence $(f^n x)_{n\in\mathbf{N}}$ converges to $x^*(x)$ as $n \to \infty$.

Proof. Let $x \in M^f$. By Lemma 2.2, $\delta(f^n x, f^{n+1} x) \leqslant \varphi^n(r(x))$ for all $n \in \mathbf{N}$. Hence, $\sum_{n=0}^{\infty} \delta(f^n x, f^{n+1} x) \leqslant \sum_{n=0}^{\infty} \varphi^n(r(x)) < \infty$. Thus, $\delta(f^n x, f^{n+1} x) \to 0$ as $n \to \infty$.
Therefore the sequence $(f^n x)_{n\in\mathbf{N}}$ is a Cauchy sequence. Since the space M is complete, there exists $x^*(x) \in X$ so that the sequence $(f^n x)_{n\in\mathbf{N}}$ converges to $x^*(x)$ as $n \to \infty$. ■

Definition 2.4 [23] Let $f : M \to M$, and let $y \in M$, and the sequence $\{f^n y\}$ in M so that $f^n y \to x^*$ with $(f^n y, f^{n+1} y) \in S(\Gamma)$ for all $n \in \mathbf{N}$. We say that a graph Γ is $(C_f)-$graph if there is a subsequence $\{f^{n_k} y\}$ and a natural number p so that $(f^{n_k} y, x^*) \in S(\Gamma)$ for all $k \geqslant p$.

Definition 2.5 [13] A mapping $f : M \to M$ is called orbitally $\Gamma-$ continuous if for all $x, y \in M$ and any sequence $(k_n)_{n\in\mathbf{N}}$ of positive integers, $f^{k_n}x \to y$ and $(f^{k_n}x, f^{k_{n+1}}x) \in S(\Gamma)$ imply $f(f^{k_n}x) \to fy$ as $n \to \infty$.

Theorem 2.6 Let (M, δ) be a complete $b-$metric space endowed with a graph Γ and Γ be $(C_f)-$graph. Let $f : M \to M$ be a $b-(\varphi, \Gamma)-$graphic contraction and f be orbitally $\Gamma-$continuous, where $\phi : \mathbf{R}^+ \to \mathbf{R}^+$ is a $b-$comparison function. Thus, the following statements hold.

(i) $F_f \neq \varnothing$ if and only if $M^f \neq \varnothing$.
(ii) If $M^f \neq \varnothing$ and Γ are weakly connected, then f is a weakly Picard operator.
(iii) For any $M^f \neq \varnothing$, $f\mid_{[x]_{\tilde{\Gamma}}}$ is a weak Picard operator.

Proof. First, we prove (iii). Let $x \in M^f$. By Lemma 2.3, there exists $x^* \in M$ so that $\lim_{n\to\infty} f^n x = x^*$. Since $x \in M^f$, then $f^n x \in M^f$ for every $n \in \mathbf{N}$. Now, we assume that $(x, fx) \in S(\Gamma)$. Since Γ is $(C_f)-$graph, there exists a subsequence $(f^{k_n}x)_{n\in\mathbf{N}}$ of $(f^n x)_{n\in\mathbf{N}}$ and $p \in \mathbf{N}$ so that $(f^{k_n}x, x^*) \in S(\Gamma)$ for each $k \geqslant p$. Now, we have a path in Γ by using the points $x, fx, \cdots, f^{k_l}x, x^*$ and hence, $x^* \in [x]_{\tilde{\Gamma}}$. On the other hand, since f is orbitally $\Gamma-$continuous, we have x^* as a fixed point for$f\mid_{[x]_{\tilde{\Gamma}}}$.
(i) is obtained using (iii) because $F_f \neq \varnothing$ if $M^f \neq \varnothing$. Now suppose that $F_f \neq \varnothing$. Using the assumption that $\triangle \subseteq S(\Gamma)$, we obtain $M^f \neq \varnothing$.
To prove (ii), let $x \in M^f$. Because Γ is weakly connected, we have $M = [x]_{\tilde{\Gamma}}$ and (iii) complete the proof. ■

In the next we study the case that $f : M \to M$ as a $b-(\varphi, \Gamma)-$graphic contraction can be a Picard operator. Thus, we need the following definition.

Definition 2.7 Let (M, δ) be a metric space endowed with a graph Γ and $f : M \to M$ be a mapping. We state that the graph Γ has a $f-$path property, if for any path in Γ, $(x_i)_{i=0}^N$ from x to y so that $x_0 = x, x_N = y$ we have $fx_{i-1} = x_i$ for all $i = 1, \cdots, N$.

Proposition 2.8 Let (M, δ) be a $b-$metric space endowed with a graph Γ. Let $f : M \to M$ be a $b-(\varphi, \Gamma)-$graphic contraction, where $\phi : \mathbf{R}^+ \to \mathbf{R}^+$ is a $b-$comparison function. Thus, the following statements hold:

(i) f is a $b-(\varphi, \tilde{\Gamma})-$graphic contraction and a $b-(\varphi, \Gamma^{-1})-$graphic contraction;
(ii) $[x_0]_{\tilde{\Gamma}}$ is $f-$invariant, and $f\mid_{[x_0]_{\tilde{\Gamma}}}$ is a $b-(\varphi, \tilde{\Gamma}_{x_0})-$graphic contraction, and this means if $x_0 \in M$, then $fx_0 \in [x_0]_{\tilde{\Gamma}}$.

Proof. (i) is obtained using the symmetry of δ.
(ii) Let $x \in [x_0]_{\tilde{\Gamma}}$. Then there exists a path $(x_i)_{i=0}^l$ in $\tilde{\Gamma}$ from x to x_0 so that $x_0 = x, x_l = x_0$. Since f is a $b-(\varphi, \tilde{\Gamma})-$graphic contraction, then $(fx_{i-1}, fx_i) \in S(\Gamma)$ for each $i = 1, \cdots, l$. So $fx \in [fx_0]_{\tilde{\Gamma}} = [x_0]_{\tilde{\Gamma}}$. Now let $(x, y) \in S(\tilde{\Gamma}_{x_0})$. Thus, there exists a path from x to y passing through x, i.e., $(x_0, x_1, \cdots, x_{k-1} = x, x_k = y)$ in such a way that $(x_{i-1}, x_i) \in S(\tilde{\Gamma})$ for $i = 1, \cdots, k$. Since f is a $b-(\varphi, \tilde{\Gamma})-$graphic, $(fx_{i-1}, fx_i) \in S(\Gamma)$ for $i = 1, \cdots, k$. Let $(z_0, z_1, \cdots, z_{l-1}, z_l)$ be a path from x_0 to fx_0. So

$$(z_0 = x_0, z_1, \cdots, z_{l-1}, z_l = fx_0, fx_1, \cdots, fx_{k-1} = fx, fx_k = fy)$$

is a path in $\tilde{\Gamma}$ from x_0 to fy so that $(fx, fy) \in S(\tilde{\Gamma}_{x_0})$. Since f is a $\tilde{\Gamma}-$graphic contraction, and $S(\tilde{\Gamma}_{x_0}) \subset S(\tilde{\Gamma})$, then f is a $\tilde{\Gamma}_{x_0}-$graphic contraction. ■

Lemma 2.9 Let (M, δ) be a $b-$metric space endowed with a graph Γ. Let $f : M \to M$ be a $b-(\varphi, \Gamma)-$graphic contraction so that the graph Γ has the $f-$path property and

$\phi : \mathbf{R}^+ \to \mathbf{R}^+$ is a $b-$comparison function. Then for any $x \in M$ and $y \in [x]_{\tilde{\Gamma}}$ two sequences $(f^n x)_{n\in\mathbf{N}}$ and $(f^n y)_{n\in\mathbf{N}}$ are equivalent.

Proof. Let $x \in M$ and $y \in [M]_{\tilde{\Gamma}}$. Then there exists a path $(x_i)_{i=0}^l$ in $\tilde{\Gamma}$ from x to y so that $x_0 = x, x_l = y$ with $(x_{i-1}, x_i) \in S(\Gamma)$ and $fx_{i-1} = x_i$ for all $i = 1, \cdots, l$. From Proposition 2.8, f is a $b-(\varphi, \tilde{\Gamma})-$graphic contraction. Thus, $(f^{n+1}x_{i-1}, f^{n+1}x_i) \in S(\tilde{\Gamma})$ for all $n \in N$. So

$$\delta(f^{n+1}x_{i-1}, f^{n+1}x_i) = \delta(f^n x_i, f^{n+1}x_i) \leqslant \varphi(\delta(f^{n-1}x_i, f^n x_i)).$$

Hence,

$$\delta(f^n x_{i-1}, f^n x_i) \leqslant \varphi^{n-1}\delta(x_i, fx_i) = \varphi^{n-1}\delta(x_i, x_{i+1}). \tag{1}$$

We know that $(f^n x_i)_{i=0}^l$ is a path in $\tilde{\Gamma}$ from $f^n x$ to $f^n y$. Using Definition 1.1(d_3) and (1), we have

$$\delta(f^n x, f^n y) \leqslant \sum_{i=1}^{l} s^i \delta(f^n x_{i-1}, f^n x_i) \leqslant a^n \sum_{i=1}^{l} s^i \varphi^{n-1}(\delta(x_i, x_{i+1})).$$

Assuming $n \to \infty$, we get $\delta(f^n x, f^n y) \to 0$. ■

Theorem 2.10 Let (M, δ) be a complete $b-$metric space endowed with a graph Γ, so that Γ is $(C_f)-$graph, and has a $f-$path property. Let $f : M \to M$ be a $b-(\varphi, \Gamma)-$graphic contraction, and f be orbitally $\Gamma-$continuous, where $\phi : \mathbf{R}^+ \to \mathbf{R}^+$ is a $b-$comparison function. Let $z \in M$ so that $z \in M^f$, and thus the following statements hold:

(1) $f \mid_{[z]_{\tilde{\Gamma}}}$ is a Picard operator;
(2) if Γ is weakly connected, then f is a Picard operator.

Proof. (1) Using (iii) Theorem 2.6, there exists $x^*(z) \in [z]_{\tilde{\Gamma}}$ so that $\lim_{n\to\infty} f^n(z) = x^*(z)$, and $x^*(z)$ is a fixed point of f. Now, let $y \in [z]_{\tilde{\Gamma}}$ and $\lim_{n\to\infty} f^n(y) = x^*(y)$. Then, by Lemma 2.9, two sequences $(f^n z)_{n\in\mathbf{N}}$ and $(f^n y)_{n\in\mathbf{N}}$ are equivalent. Since both are convergent sequences, they are Cauchy sequences. Hence, they are Cauchy equivalent. This means $x^*(y) = x^*(z)$.
(2) Since $z \in M^f$ and Γ is weakly connected, we have $M = [z]_{\tilde{\Gamma}}$. Then we only need to apply (1). ■

The following example shows that $b-(\varphi, \Gamma)-$graphic contraction is a generalization of $b-(\varphi, \Gamma)-$ contraction.

***Example* 2.11** Let $M = [0, 1]$ and $\delta(x, y) = \mid x - y \mid^2$. Define the graph Γ by $S(\Gamma) = \{(0, 0)\} \bigcup \{(0, x), x \geqslant \frac{1}{2}\} \bigcup \{(x, y) : x, y \in (0, 1]\}$. and $f : M \to M$ by

$$fx = \begin{cases} \frac{x}{2}, & x \in (0, 1); \\ \frac{3}{4}, & x = 0; \\ 1, & x = 1. \end{cases}$$

So if $\varphi(t) = \frac{t}{3}$, then δ is a $b-$metric on M with $s = 2$, and f is a $b-(\varphi, \Gamma)-$graphic contraction. But f is not $b-(\varphi, \Gamma)-$ contraction, because

$$\delta(f(0), f(\frac{1}{2}) \nleqslant \frac{\delta(0, \frac{1}{2})}{3}.$$

Definition 2.12 A mapping $f : M \to M$ is called $b-(\varphi, \Gamma)-$almost contraction if:

(i) for all $x, y \in M$, if $(x, y) \in S(\Gamma)$ then $(f(x), f(y)) \in S(\Gamma)$;
(ii) there exists $L \geqslant 0$ so that $\delta(f(x), f(y)) \leqslant \varphi(\delta(x, y)) + L\delta(y, f(x))$ whenever $(x, y) \in S(\Gamma)$,

where $\phi : \mathbf{R}^+ \to \mathbf{R}^+$ is a comparison function.

Remark 5 *Note that if $f : M \to M$ is a $b-(\varphi, \Gamma)-$almost contraction, then f is a $b-(\varphi, \Gamma)-$graphic contraction with $L = 0$ and $y = f(x)$.*

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