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A fixed point method for proving the stability of ring (α, β, γ)-derivations in 2-Banach algebras

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Abstract. In this paper, we first present the new concept of 2-normed algebra. We investigate the structure of this algebra and give some examples. Then we apply a fixed point theorem to prove the stability and hyperstability of (α, β, γ)-derivations in 2-Banach algebras.

Keywords: Fixed point theorem, 2-normed algebras, (α, β, γ)-derivations, hyperstability.

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1. Introduction and preliminaries

One of the essential questions in the theory of functional equations giving the notion of stability is when is it true that the solution of an equation differing slightly from a given one, must be close to the solution of the given equation?

Such a problem was formulated by Ulam [1] in 1940 and solved in the next year for the Cauchy functional equation by Hyers [2]. Since Hyers, many authors have studied the stability theory (now named the Hyers-Ulam stability) for functional equations (see e.g. [3–17]).

S. Gähler [18, 19] in 1963-1964, introduced the concept of linear 2-normed spaces:

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Definition 1.1 Let X be a real linear space with dim $X > 1$ and let $\|\cdot,\cdot\| : X^2 \longrightarrow \mathbb{R}$ be a mapping. Then $(X, \|\cdot,\cdot\|)$ is called a linear 2-normed space if

(1) $\|x, z\| = 0$, if and only if x, z are linearly dependent,
(2) $\|x, z\| = \|z, x\|$,
(3) $\|\alpha x, z\| = |\alpha|\|x, z\|$, for any $\alpha \in \mathbb{R}$,
(4) $\|x + y, z\| \leqslant \|x, z\| + \|y, z\|$,

for all $x, y, z \in X$. Sometimes the condition (4) is called the triangle inequality.

In the recent years, 2-normed spaces have been considered by several authors (cf., e.g., [20, 21]). The concept of 2-normed algebra was introduced by Srivastava et al. [22, 23]. They also gave some examples satisfying their definition and showed that there exist 2-normed algebras (with or without unity) which are not normable and a 2-Banach algebra need not be a 2-Banach space.

Definition 1.2 (Srivastava et al. [22]) Let E be subalgebra of an algebra B with dim $E > 1$, $\|\cdot,\cdot\|$ be a 2-norm in B and $a_1, a_2 \in B$ be linearly independent, non-invertible and be such that for all $x, y \in E$, $\|xy, a_i\| \leqslant \|x, a_i\|\ \|y, a_i\|, i = 1, 2$. Then E is called a 2-normed algebra with respect to a_1, a_2.

The concept of 2-normed spaces was developed by Park [24]. Now, we are going to develop the concept of 2-normed algebra with the help of Park's definitions.

Lemma 1.3 Let $(X, \|\cdot,\cdot\|)$ be a linear 2-normed algebra. If $x \in X$ and $\|x, y\| = 0$ for all $y \in X$, then $x = 0$.

For a linear 2-normed algebra $(X, \|\cdot,\cdot\|)$, the functions $x \mapsto \|x, y\|$ are continuous functions of X into $\mathbb{R}$ for each fixed $y \in X$ as follows.

Remark 1 *Let $(X, \|\cdot,\cdot\|)$ be a linear 2-normed algebra. Then, by the conditions (2) and (4), we have*

$$\big|\, \|x, z\| - \|y, z\| \,\big| \leqslant \|x - y, z\| \tag{1}$$

for all $x, y, z \in X$. Hence the functions $x \mapsto \|x, y\|$ are continuous functions of X into $\mathbb{R}$ for each fixed $y \in X$.

Lemma 1.4 For a convergent sequence $\{x_n\}$ in a linear 2-normed algebra X,

$$\lim_{n\to\infty} \|x_n, y\| = \|\lim_{n\to\infty} x_n, y\|$$

for all $y \in X$.

Definition 1.5 A sequence $\{x_n\}$ in a linear 2-normed algebra X is called a Cauchy sequence if there are two points $y, z \in X$ such that y and z are linearly independent and

$$\lim_{m,n\to\infty} \|x_n - x_m, y\| = 0 \quad , \quad \lim_{m,n\to\infty} \|x_n - x_m, z\| = 0.$$

Definition 1.6 Let $(X, \|\cdot,\cdot\|)$ be a linear 2-normed algebra. A sequence $\{x_n\}$ is said to be convergent in X, if there exists $x \in X$ such that

$$\lim_{n\to\infty} \|x_n - x, y\| = 0$$

for all $y \in X$. In this case, x is called the limit of the sequence $\{x_n\}$ and we denote it by $\lim_{n\to\infty} x_n = x$.

Every convergent sequence in 2-normed algebra is Cauchy. If each Cauchy sequence be convergent, then the 2-normed algebra is called 2-Banach algebra.

***Example* 1.7** (Srivastava et al. [22]). Let X be a finite dimensional ($dimX = n \geqslant 2$) algebra over the field $\mathbb{K}$, where $\mathbb{K}$ is the field of real or complex numbers and let $\{e_1, ..., e_n\}$ be a basis for X. Let a, b be two symbols (not in X) and define $B := \{x + \alpha a + \beta b : x \in X, \alpha, \beta \in \mathbb{K}\}$, with the agreement that $x + \alpha a + \beta b = 0$, if and only if $x = 0$, $\alpha = \beta = 0$, for $x \in X$, $\alpha, \beta \in \mathbb{K}$. For $y_i = x_i + \alpha_i a + \beta_i b \in B$, $i = 1, 2$, $\alpha \in \mathbb{K}$, define

$$\begin{aligned} y_1 + y_2 &= (x_1 + x_2) + (\alpha_1 + \alpha_2)a + (\beta_1 + \beta_2)b, \\ \alpha y_1 &= \alpha x_1 + (\alpha\alpha_1)a + (\alpha\beta_1)b, \\ y_1 y_2 &= x_1 x_2 + \alpha_1\alpha_2 a + \beta_1\beta_2 b. \end{aligned}$$

Then B is an algebra over $\mathbb{K}$ and if X has unit e, then $\tilde{e} = e + a + b$ is the unit of B. For $x = \sum_{i=1}^{n} \alpha_i e_i + s_1 a + s_2 b$, $y = \sum_{i=1}^{n} \beta_i e_i + t_1 a + t_2 b \in B$, define $\|x, y\|$ by

$$\|x, y\|^2 = \left(\sum_{i=1}^{n} |\alpha_i|^2 + |s_1|^2 + |s_2|^2\right)\left(\sum_{i=1}^{n} |\beta_i|^2 + |t_1|^2 + |t_2|^2\right) - \left|\sum_{i=1}^{n} \alpha_i \bar{\beta_i} + s_1 \bar{t_1} + s_2 \bar{t_2}\right|^2.$$

Then $\|\cdot, \cdot\|$ defines a 2-norm in B.

On X, define $\|\cdot\|_1$ by $\|x\|_1 = \|x, a\|$ for $x \in X$. Note that $\|\cdot\|_1$ is an $1-$norm on X. Let $\|\cdot\|$ be an $1-$norm on X so that $(X, \|\cdot\|)$ is an $1-$normed algebra. Then X becomes finite dimensional, both $1-$norms $\|\cdot\|_1$ and $\|\cdot\|$ on X become equivalent and hence there exist $k_1, k_2 > 0$ such that for every $x \in X$, $\|x\|_1 \leqslant k_1\|x\|$ and $\|x\| \leqslant k_2\|x\|_1$. For $x, y \in X$, we have

$$\|xy, a\| = \|x.y\|_1 \leqslant k_1\|x.y\| \leqslant k_1\|x\|\,\|y\| \leqslant k_1 k_2 \|x\|_1 k_2 \|y\|_1 = k_1 k_2^2 \|x, a\|\,\|y, a\|$$

and so for $a_1 = k_1 k_2^2 a$, we have $\|xy, a_1\| \leqslant \|x, a_1\|\,\|y, a_1\|$ for all $x, y \in X$. Similarly, for suitably chosen $k_3 > 0$, we have for $a_2 = k_3 b$, $\|xy, a_2\| \leqslant \|x, a_2\|\,\|y, a_2\|$ for all $x, y \in X$, and $(X, \|\cdot, \cdot\|)$ becomes a 2-normed algebra over $\mathbb{K}$ with respect to a_1, a_2.

Now, we show that $(X, \|\cdot, \cdot\|)$ is a 2-Banach algebra with respect to a_1, a_2. Let $\{x_n\}$ be a sequence in X so that $\lim_{n,m\to\infty} \|x_n - x_m, a_i\| = 0$, $i = 1, 2$. Then $\lim_{n,m\to\infty} \|x_n - x_m\|_1 = 0$. By finite dimensionality of X, $(X, \|\cdot\|_1)$ is a Banach space and hence there is an $x \in X$ such that $\lim_{n\to\infty} \|x_n - x\|_1 = 0$ or equivalently $\lim_{n\to\infty} \|x_n - x, a_1\|_1 = 0$. For $x \in X$, if we define $\|x\|_2 = \|x, a_2\|$, then $(X, \|\cdot\|_2)$ also becomes an $1-$normed space. Therefore, the norms $\|\cdot\|_1$ and $\|\cdot\|_2$ become equivalent on X and $\lim_{n\to\infty} \|x_n - x\|_2 = 0$ or equivalently $\lim_{n\to\infty} \|x_n - x, a_2\| = 0$. Hence, $(X, \|\cdot, \cdot\|)$ is a 2-Banach algebra with respect to a_1, a_2

In 1991, J. Baker [25] used the Banach fixed point theorem for prove the Hyers-Ulam stability. The method was generalized in [26]. We recall this fundamental result as follows.

Theorem 1.8 (Banach contraction principle) Let (X, m) be a complete generalized metric space and consider a mapping $T : X \longrightarrow X$ as a strictly contractive mapping, that is

$$m(Tx, Ty) \leqslant Lm(x, y)$$

for all $x, y \in X$ and for some (Lipschitz constant) $0 < L < 1$. Then

- T has one and only one fixed point $x^* = T(x^*)$;
- x^* is globally attractive, that is, $\lim_{n\to\infty} T^n x = x^*$ for any starting point $x \in X$;
- One has the following estimation inequalities for all $x \in X$ and $n \geqslant 0$
  $m(T^n x, x^*) \leqslant L^n m(x, x^*)$,
  $m(T^n x, x^*) \leqslant \frac{1}{1-L} L^n m(T^n x, T^{n+1} x)$,
  $m(x, x^*) \leqslant \frac{1}{1-L} m(x, Tx)$.

Theorem 1.9 (The Alternative of Fixed Point [27]) Suppose that we are given a complete generalized metric space (X, m) and a strictly contractive mapping $T : X \longrightarrow X$ with Lipschitz constant L. Then, for each given element $x \in X$, either $m(T^n x, T^{n+1} x) = +\infty$ for all nonnegative integers n or there exists a positive integer n_0 such that $m(T^n x, T^{n+1} x) < +\infty$ for all $n \geqslant n_0$. If the second alternative holds, then

⋆ The sequence $(T^n x)$ is convergent to a fixed point y^* of T;
⋆ y^* is the unique fixed point of T in the set $Y = \{y \in X, m(T^{n_0} x, y) < +\infty\}$;
⋆ $m(y, y^*) \leqslant \frac{1}{1-L} m(y, Ty)$, $y \in Y$.

Fixed point theorems have already been applied in the theory of the stability of functional equations by several authors (see for instance [8, 28–30]).

Let $\mathcal{A}$ be a normed algebra and $\mathcal{B}$ be an $\mathcal{A}$-bimodule normed algebra. An additive mapping $f : \mathcal{A} \longrightarrow \mathcal{B}$ is called a ring (α, β, γ)-derivation if there exist $\alpha, \beta, \gamma \in \mathbb{C}$ such that

$$\alpha f(xy) = \beta f(x)y + \gamma x f(y) \tag{2}$$

for all $x, y \in \mathcal{A}$.

2. The main results

Hereafter, let $\mathcal{A}$ be a Banach algebra and $\mathcal{B}$ be an $\mathcal{A}$-bimodule unital 2-Banach algebra with unit $\mathbf{1}$ and with $\dim(\mathcal{B}) > 1$. Let $j \in \{-1, 1\}$, $\alpha, \beta, \gamma \in \mathbb{C}$ and let $\varphi, \phi : \mathcal{A}^3 \longrightarrow [0, \infty)$ be mappings such that

$$\lim_{n\to\infty} \frac{\varphi(2^{nj} x, 2^{nj} y, z)}{2^{nj}} = 0, \tag{3}$$

$$\lim_{n\to\infty} \frac{\phi(2^{nj} x, y, z)}{2^{nj}} = 0 \tag{4}$$

for all $x, y, z \in \mathcal{A}$.

Theorem 2.1 Suppose that $f : \mathcal{A} \longrightarrow \mathcal{B}$ is a mapping satisfying

$$\|f(x+y) - f(x) - f(y), z\mathbf{1}\| \leqslant \varphi(x, y, z), \tag{5}$$

$$\|\alpha f(xy) - \beta f(x)y - \gamma x f(y), z\mathbf{1}\| \leqslant \phi(x, y, z) \tag{6}$$

for all $x, y, z \in \mathcal{A}$. If there exists $0 < L = L(j) < 1$ such that

$$\varphi(x, y, z) \leqslant L 2^j \varphi\left(\frac{x}{2^j}, \frac{y}{2^j}, z\right) \tag{7}$$

for all $x, y, z \in \mathcal{A}$, then there exists a unique ring (α, β, γ)-derivation $d : \mathcal{A} \longrightarrow \mathcal{B}$ such that

$$\|f(x) - d(x), z\mathbf{1}\| \leqslant \frac{L^{\frac{1-j}{2}}}{2(1-L)}\varphi(x, x, z) \tag{8}$$

for all $x, z \in \mathcal{A}$.

Proof. Let $E = \{e : \mathcal{A} \longrightarrow \mathcal{B} \mid e(0) = 0\}$. For $x, z \in \mathcal{A}$, define $m : E \times E \longrightarrow [0, \infty]$ by

$$m(e_1, e_2) = inf\Big\{K \in \mathbb{R}_+ : \|e_1(x) - e_2(x), z\mathbf{1}\| \leqslant K\varphi(x, x, z)\Big\}.$$

It is easy to see that (E, m) is a complete generalized metric space. Let us consider the linear mapping $T : E \longrightarrow E,\ Te(x) = \frac{1}{2^j}e(2^j x)$ for all $x \in \mathcal{A}$. T is a strictly contractive mapping with the Lipschitz constant L. Indeed, for given e_1 and e_2 in E such that $m(e_1, e_2) < \infty$ and any $K > 0$ satisfying $m(e_1, e_2) < K$, we have

$$\begin{aligned}
&\|e_1(x) - e_2(x), z\mathbf{1}\| \leqslant K\varphi(x, x, z)\\
&\Rightarrow \left\|\frac{1}{2^j}e_1(2^j x) - \frac{1}{2^j}e_2(2^j x), z\mathbf{1}\right\| \leqslant \frac{1}{2^j}K\varphi(2^j x, 2^j x, z)\\
&\Rightarrow \left\|\frac{1}{2^j}e_1(2^j x) - \frac{1}{2^j}e_2(2^j x), z\mathbf{1}\right\| \leqslant LK\varphi(x, x, z)\\
&\Rightarrow m(Te_1, Te_2) \leqslant LK.
\end{aligned}$$

Put $K = m(e_1, e_2) + \frac{1}{n}$ for positive integers n. Then $m(Te_1, Te_2) \leqslant L(m(e_1, e_2) + \frac{1}{n})$. Letting $n \to \infty$ gives

$$m(Te_1, Te_2) \leqslant Lm(e_1, e_2)$$

for all $e_1, e_2 \in E$. Putting $y := x$ in (5), we get

$$\|f(2x) - 2f(x), z\mathbf{1}\| \leqslant \varphi(x, x, z) \tag{9}$$

and so

$$\left\|f(x) - \frac{1}{2}f(2x), z\mathbf{1}\right\| \leqslant \frac{1}{2}\varphi(x, x, z)$$

for all $x, z \in \mathcal{A}$. Moreover, replacing x in (9) by $\frac{x}{2}$ implies the appropriate inequality for $j = -1$

$$\left\|f(x) - 2f\left(\frac{x}{2}\right), z\mathbf{1}\right\| \leqslant \varphi\left(\frac{x}{2}, \frac{x}{2}, z\right) \leqslant \frac{L}{2}\varphi(x, x, z)$$

for all $x, z \in \mathcal{A}$. Therefore $m(f, Tf) \leqslant \frac{L^{\frac{1-j}{2}}}{2}$. By Theorem 1.9, there exists a mapping $d : \mathcal{A} \longrightarrow \mathcal{B}$ which is the fixed point of T and satisfies

$$d(x) = \lim_{n\to\infty} \frac{f(2^{nj}x)}{2^{nj}}, \tag{10}$$

since $\lim_{n\to\infty} m(T^n f, d) = 0$. The mapping d is the unique fixed point of T in the set $M = \{e \in E : m(f, e) < \infty\}$. Using Theorem 1.9 we get

$$m(f, d) \leqslant \frac{1}{1-L} m(f, Tf)$$

which yields

$$\|f(x) - d(x), z\mathbf{1}\| \leqslant \frac{L^{\frac{1-j}{2}}}{2(1-L)} \varphi(x, x, z)$$

for all $x, z \in \mathcal{A}$. By Lemma 1.4 and inequality (5), we also have

$$\begin{aligned} \|d(x+y) - d(x) - d(y), z\mathbf{1}\| &= \lim_{n\to\infty} \left\| \frac{1}{2^{nj}} f\big(2^{nj}(x+y)\big) - \frac{1}{2^{nj}} f(2^{nj}x) - \frac{1}{2^{nj}} f(2^{nj}y), z\mathbf{1} \right\| \\ &= \lim_{n\to\infty} \frac{1}{2^{nj}} \left\| f(2^{nj}x + 2^{nj}y) - f(2^{nj}x) - f(2^{nj}y), z\mathbf{1} \right\| \\ &\leqslant \lim_{n\to\infty} \frac{1}{2^{nj}} \varphi(2^{nj}x, 2^{nj}y, z) = 0 \end{aligned}$$

for all $x, y, z \in \mathcal{A}$. By Lemma 1.3, we conclude that d is additive. Let $r(x, y) = \alpha f(xy) - \beta f(x)y - \gamma x f(y)$ for all $x, y \in \mathcal{A}$. Using inequality (6), we have

$$\begin{aligned} \left\| \frac{1}{2^{nj}} r(2^{nj}x, y), z\mathbf{1} \right\| &= \frac{1}{2^{nj}} \|\alpha f\left((2^{nj}x)y\right) - \beta f(2^{nj}x)y - \gamma 2^{nj} x f(y), z\mathbf{1}\| \\ &\leqslant \frac{1}{2^{nj}} \phi(2^{nj}x, y, z) \end{aligned}$$

for all $x, y, z \in \mathcal{A}$ and $n \in \mathbb{N}$. By Lemma 1.3, Lemma 1.4 and (4) we obtain

$$\lim_{n\to\infty} \frac{1}{2^{nj}} r(2^{nj}x, y) = 0$$

for all $x, y \in \mathcal{A}$. Applying (10), we get

$$\alpha d(xy) = \beta d(x)y + \gamma x f(y) \tag{11}$$

for all $x, y \in \mathcal{A}$. Indeed

$$\begin{aligned} \alpha d(xy) &= \lim_{n\to\infty} \frac{1}{2^{nj}} \alpha f\big(2^{nj}(xy)\big) = \lim_{n\to\infty} \frac{1}{2^{nj}} \alpha f\big((2^{nj}x)y\big) \\ &= \lim_{n\to\infty} \frac{\beta f(2^{nj}x)y + 2^{nj}\gamma x f(y) + r(2^{nj}x, y)}{2^{nj}} \\ &= \lim_{n\to\infty} \left(\frac{\beta f(2^{nj}x)y}{2^{nj}} + \gamma x f(y) + \frac{r(2^{nj}x, y)}{2^{nj}} \right) \\ &= \beta d(x)y + \gamma x f(y) \end{aligned}$$

for all $x, y \in \mathcal{A}$. Using (11) and the additivity of d, we have

$$\begin{aligned}2^{nj}\beta d(x)y + \gamma x f(2^{nj}y) &= \beta d(x)2^{nj}y + \gamma x f(2^{nj}y)\\ &= \alpha d\big(x(2^{nj}y)\big) = \alpha d\big((2^{nj}x)y\big)\\ &= \beta d(2^{nj}x)y + 2^{nj}\gamma x f(y)\\ &= 2^{nj}\beta d(x)y + 2^{nj}\gamma x f(y)\end{aligned}$$

for all $x, y \in \mathcal{A}$ and $n \in \mathbb{N}$. Hence,

$$x f(y) = x\frac{f(2^{nj}y)}{2^{nj}} \tag{12}$$

for all $x, y \in \mathcal{A}$ and $n \in \mathbb{N}$. Tending n to infinity, we see that

$$x f(y) = x d(y) \tag{13}$$

for all $x, y \in \mathcal{A}$. Combining (11) and (13), we conclude that d satisfies (2). ∎

Next, we are going to establish the hyperstability of ring (α, β, γ)-derivations.

Theorem 2.2 Assume that $f : \mathcal{A} \longrightarrow \mathcal{B}$ is a mapping satisfying

$$\|f(x+y) - f(x) - f(y), z\mathbf{1}\| \leqslant \varphi(x, y, z)$$

$$\|\alpha f(xy) - \beta f(x)y - \gamma x f(y), z\mathbf{1}\| \leqslant \phi(x, y, z)$$

for all $x, y, z \in \mathcal{A}$. Let $\mathcal{B}$ be an $\mathcal{A}$-bimodule unital 2-Banach algebra without order, i.e. if $b \in \mathcal{B}$, then $\mathcal{A}b = 0$ or $b\mathcal{A} = 0$ implies that $b = 0$. If there exists $0 < L = L(j) < 1$ such that the mapping φ has the property

$$\varphi(x, y, z) \leqslant L2^j\varphi\left(\frac{x}{2^j}, \frac{y}{2^j}, z\right),$$

then f is a ring (α, β, γ)-derivation.

Proof. According to Theorem 2.1, there exists a unique ring (α, β, γ)-derivation $d : \mathcal{A} \longrightarrow \mathcal{B}$ such that

$$d(x) = \lim_{n\to\infty}\frac{f(2^{nj}x)}{2^{nj}} \tag{14}$$

for all $x \in \mathcal{A}$. By applying (14) in (12), we conclude that $x\big(f(y) - d(y)\big) = 0$ for all $x, y \in \mathcal{A}$. Therefore $f = d$. ∎

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