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Numerical solution of a system of fuzzy polynomial equations by modified Adomian decomposition method

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Abstract. In this paper, we present some efficient numerical algorithm for solving system of fuzzy polynomial equations based on Newton's method. The modified Adomian decomposition method is applied to construct the numerical algorithms. Some numerical illustrations are given to show the efficiency of algorithms.

Keywords: Fuzzy numbers, system of polynomials, Adomian decomposition method.

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1. Introduction

Since the beginning of the 1980's. The Adomian decomposition method has been applied to a wide class of functional equations [10, 11]. Adomian gives the solution as of finite series usually converging to an accurate solution. Abbaoui and Cherruault [2] applied the standard Adomian decomposition on simple iteration method to solve the equation $f(x) = 0$, where $f(x)$ is a nonlinear function, and proved the convergence of the series solution.

Abbasbandy [3] improved Newton-Raphson method to solve the nonlinear equation $f(x) = 0$ based on modified Adomian's method, and in [4] he extended Newton's method for a system of nonlinear equation by modified Adomian decomposition method.

The concept of fuzzy numbers and arithmetic operation with these numbers were first introduce and investigated by [13, 15, 20]. One of the major applications of fuzzy number arithmetic is in nonlinear systems whose parameters are all or partially represented by fuzzy numbers [14, 17, 19].

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Abbasbandy and Asady [5], applied the Newton's method for solving fuzzy nonlinear equations, $f(x) = c$ and the numerical solution of a fuzzy nonlinear equation and system of fuzzy nonlinear equations was considered in [6, 7, 21]. Allahviranloo et al [12] applied the Fixed point method for solving fuzzy nonlinear equations. Tavassoli et al [23], applied the Newton's method for solving dual fuzzy nonlinear equations, $f(x) = g(x) + c$. The topic of numerical solution of fuzzy polynomials by fuzzy neural network investigated by Abbasbandy *et al.* [8], this method for finding solution to polynomials of the form $a_1x + a_2x^2 + \ldots + a_nx^n = a_0$ for $x \in \mathbb{R}$ (if exists) and $a_0, a_1, \ldots, a_n$ are fuzzy numbers and system of s fuzzy polynomial equations such as [9]:

$$\begin{aligned} f_1(x_1, x_2, \ldots, x_n) &= a_{10}, \\ &\vdots \\ f_l(x_1, x_2, \ldots, x_n) &= a_{l0}, \\ &\vdots \\ f_s(x_1, x_2, \ldots, x_n) &= a_{s0}, \end{aligned}$$

where $x_1, x_2, \ldots, x_n \in \mathbb{R}$ and all coefficients are fuzzy numbers. Otadi and Mosleh [22] applied the Adomian decomposition method for solving fuzzy polynomial equation of the form $a_1x + a_2x^2 + \ldots + a_nx^n = a_0$ where x, a_0 and all coefficients are fuzzy numbers. It is the purpose of this paper to introduce an efficient extension of Newton's method by modified Adomian decomposition method for solving (if it exists) system of fuzzy polynomials.

The structure of this paper is organized as follows: In Section 2, we recall some fundamental results on fuzzy numbers. The proposed algorithm for finding a fuzzy root (if it exists) of a system of fuzzy polynomials are discussed in Section 3. This leads us to conclude by giving a comparison with other methods in Section 4. Numerical examples are given in Section 5.

2. Preliminaries

Definition 2.1 [16, 24, 25] A fuzzy number is a fuzzy set like $u : R \to I = [0, 1]$ which satisfies

(1) u is upper semicontinuous,
(2) $u(x) = 0$ outside some interval $[c, d]$,
(3) There are real numbers a, b such that $c \leqslant a \leqslant b \leqslant d$ and
  3.1. $u(x)$ is monotonic increasing on $[c, a]$,
  3.2. $u(x)$ is monotonic decreasing on $[b, d]$,
  3.3. $u(x) = 1, a \leqslant x \leqslant b$.

The set of all these fuzzy numbers is denoted by E. An equivalent parametric is also given in [18] as follows:

Definition 2.2 A fuzzy number u is a pair $(\underline{u}, \overline{u})$ of functions $\underline{u}(r), \overline{u}(r); 0 \leqslant r \leqslant 1$ which satisfy the following requirements:

i. $\underline{u}(r)$ is a bounded monotonic increasing left continuous function on $(0, 1]$ and right continuous at 0.

ii. $\overline{u}(r)$ is a bounded monotonic decreasing left continuous function on $(0, 1]$ and right continuous at 0.

iii. $\underline{u}(r) \leqslant \overline{u}(r), 0 \leqslant r \leqslant 1$.

A popular fuzzy number is the trapezoidal fuzzy number $u = (x_0, y_0, \sigma, \beta)$ with interval defuzzifier $[x_0, y_0]$ and left fuzziness σ and right fuzziness β where the membership function is

$$u(x) = \begin{cases} \frac{x-x_0+\sigma}{\sigma}, & x_0 - \sigma \leq x \leqslant x_0, \\ 1 & x \in [x_0, y_0], \\ \frac{y_0-x+\beta}{\beta} & y_0 \leq x \leq y_0 + \beta, \\ 0 & otherwise. \end{cases}$$

Its parametric form is

$$\underline{u}(r) = x_0 - \sigma + \sigma r, \qquad \overline{u}(r) = y_0 + \beta - \beta r. \tag{1}$$

Let $u = (x_0, y_0, \sigma, \beta)$, be a trapezoidal fuzzy number and $x_0 = y_0$, then u is called a triangular fuzzy number and is denoted by $u = (x_0, \delta, \beta)$.

The addition and scaler multiplication of fuzzy numbers are defined by the extension principle and can be equivalently represented as follows.

For arbitrary $u = (\underline{u}, \overline{u}), v = (\underline{v}, \overline{v})$ and $k > 0$ we define addition $(u+v)$, multiplication $(u.v)$ and multiplication by scalar k as

$$\begin{aligned} &(\underline{u+v})(r) = \underline{u}(r) + \underline{v}(r), \qquad (\overline{u+v})(r) = \overline{u}(r) + \overline{v}(r), \\ &(\underline{u.v})(r) = min\{\underline{u}(r).\underline{v}(r), \underline{u}(r).\overline{v}(r), \overline{u}(r).\underline{v}(r), \overline{u}(r).\overline{v}(r)\}, \\ &(\overline{u.v})(r) = max\{\underline{u}(r).\underline{v}(r), \underline{u}(r).\overline{v}(r), \overline{u}(r).\underline{v}(r), \overline{u}(r).\overline{v}(r)\}, \\ &(\underline{ku})(r) = k\underline{u}(r), \qquad (\overline{ku})(r) = k\overline{u}(r). \end{aligned} \tag{2}$$

Definition 2.3 Let u and v be fuzzy numbers with r-level set $[u]^r = [u_1(r), u_2(r)]$ and $[v]^r = [v_1(r), v_2(r)]$. We metricize the set of fuzzy numbers by the Hausdorff distance

$$D(u, v) = sup_{r\in[0,1]} max\{| u_1(r) - v_1(r) |, | u_2(r) - v_2(r) |\}. \tag{3}$$

i.e. $D(u, v)$ is the maximal distance between r level sets of u and v.

3. The Adomian decomposition method

Consider the system of s fuzzy polynomial equations

$$\begin{cases} f_1(x_1, x_2, \ldots, x_n) = c_1, \\ \qquad\vdots \\ f_l(x_1, x_2, \ldots, x_n) = c_l, \\ \qquad\vdots \\ f_s(x_1, x_2, \ldots, x_n) = c_s, \end{cases} \tag{4}$$

with

$$f_l(x_1,x_2,\ldots,x_n)=c_l=\sum_{i=1}^{n}a_{li}x_i+\sum_{i=1}^{n}\sum_{j=1}^{n}a_{lij}x_ix_j+ \\ \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}a_{lijk}x_ix_jx_k+\ldots,\qquad 1\leqslant l\leqslant s, \tag{5}$$

where $x_1,x_2,\ldots,x_n$ and all coefficients are fuzzy numbers.

This full form of mathematical description can be represented by a system of partial quadratic fuzzy polynomials consisting of only two variables in the form of

$$\begin{cases} P(x,y)=a_1x+a_2y+a_3xy+a_4x^2+a_5y^2=c_1,\\ Q(x,y)=b_1x+b_2y+b_3xy+b_4x^2+b_5y^2=c_2,\end{cases} \tag{6}$$

where x,y,c_1,c_2 and all coefficients are fuzzy numbers. Let

$$\begin{cases} P(x,y)=(\underline{P}(\underline{x},\overline{x},\underline{y},\overline{y};r),\overline{P}(\underline{x},\overline{x},\underline{y},\overline{y};r)),\\ Q(x,y)=(\underline{Q}(\underline{x},\overline{x},\underline{y},\overline{y};r),\overline{Q}(\underline{x},\overline{x},\underline{y},\overline{y};r)),\quad for\ \ r\in[0,1],\end{cases}$$

with

$$\begin{cases} \underline{P}(\underline{x},\overline{x},\underline{y},\overline{y};r)=min\{P(u,v)\mid u\in[\underline{x}(r),\overline{x}(r)],\\ \qquad\qquad v\in[\underline{y}(r),\overline{y}(r)],a_i\in[\underline{a}_i(r),\overline{a}_i(r)],i=1,...,5\},\\ \overline{P}(\underline{x},\overline{x},\underline{y},\overline{y};r)=max\{P(u,v)\mid u\in[\underline{x}(r),\overline{x}(r)],\\ \qquad\qquad v\in[\underline{y}(r),\overline{y}(r)],a_i\in[\underline{a}_i(r),\overline{a}_i(r)],i=1,...,5\},\\ \underline{Q}(\underline{x},\overline{x},\underline{y},\overline{y};r)=min\{Q(u,v)\mid u\in[\underline{x}(r),\overline{x}(r)],\\ \qquad\qquad v\in[\underline{y}(r),\overline{y}(r)],b_i\in[\underline{b}_i(r),\overline{b}_i(r)],i=1,...,5\},\\ \overline{Q}(\underline{x},\overline{x},\underline{y},\overline{y};r)=max\{Q(u,v)\mid u\in[\underline{x}(r),\overline{x}(r)],\\ \qquad\qquad v\in[\underline{y}(r),\overline{y}(r)],b_i\in[\underline{b}_i(r),\overline{b}_i(r)],i=1,...,5\}.\end{cases}$$

The parametric form for any $r\in[0,1]$, is as follows:

$$\begin{cases} \underline{P}(\underline{x},\overline{x},\underline{y},\overline{y};r)=\underline{c}_1(r),\\ \overline{P}(\underline{x},\overline{x},\underline{y},\overline{y};r)=\overline{c}_1(r),\\ \underline{Q}(\underline{x},\overline{x},\underline{y},\overline{y};r)=\underline{c}_2(r),\\ \overline{Q}(\underline{x},\overline{x},\underline{y},\overline{y};r)=\overline{c}_2(r),\end{cases} \tag{7}$$

where $c_1=(\underline{c}_1(r),\overline{c}_1(r))$ and $c_2=(\underline{c}_2(r),\overline{c}_2(r))$. The problem (7) can be reformulated in an equivalent form as

$$\begin{cases} \underline{F}(\underline{x},\overline{x},\underline{y},\overline{y};r)=0,\\ \overline{F}(\underline{x},\overline{x},\underline{y},\overline{y};r)=0,\\ \underline{G}(\underline{x},\overline{x},\underline{y},\overline{y};r)=0,\\ \overline{G}(\underline{x},\overline{x},\underline{y},\overline{y};r)=0,\end{cases} \tag{8}$$

where

$$\begin{cases} \underline{F}(\underline{x},\overline{x},\underline{y},\overline{y};r)=\underline{P}(\underline{x},\overline{x},\underline{y},\overline{y};r)-\underline{c}_1(r),\\ \overline{F}(\underline{x},\overline{x},\underline{y},\overline{y};r)=\overline{P}(\underline{x},\overline{x},\underline{y},\overline{y};r)-\overline{c}_1(r),\\ \underline{G}(\underline{x},\overline{x},\underline{y},\overline{y};r)=\underline{Q}(\underline{x},\overline{x},\underline{y},\overline{y};r)-\underline{c}_2(r),\\ \overline{G}(\underline{x},\overline{x},\underline{y},\overline{y};r)=\overline{Q}(\underline{x},\overline{x},\underline{y},\overline{y};r)-\overline{c}_2(r).\end{cases}$$

Suppose that $(\alpha, \beta, \gamma, \theta)$ is the solution of (8), i.e.,

$$\begin{cases} \underline{F}(\alpha, \beta, \gamma, \theta; r) = 0, \\ \overline{F}(\alpha, \beta, \gamma, \theta; r) = 0, \\ \underline{G}(\alpha, \beta, \gamma, \theta; r) = 0, \\ \overline{G}(\alpha, \beta, \gamma, \theta; r) = 0. \end{cases}$$

Now, if we use the Taylor series of $\underline{F}, \overline{F}, \underline{G}, \overline{G}$ about $(\underline{x}, \overline{x}, \underline{y}, \overline{y})$, then for each $r \in [0,1]$,

$$\begin{cases} \underline{F}(\underline{x}-h, \overline{x}-k, \underline{y}-l, \overline{y}-d; r) = \underline{F}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r) - h\underline{F}_{\underline{x}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r) \\ \quad -k\underline{F}_{\overline{x}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r) - l\underline{F}_{\underline{y}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r) - d\underline{F}_{\overline{y}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r) \\ \quad +O(h^2+k^2+l^2+d^2+hk+hl+hd+kl+kd+ld) = 0, \\ \overline{F}(\underline{x}-h, \overline{x}-k, \underline{y}-l, \overline{y}-d; r) = \overline{F}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r) - h\overline{F}_{\underline{x}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r) \\ \quad -k\overline{F}_{\overline{x}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r) - l\overline{F}_{\underline{y}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r) - d\overline{F}_{\overline{y}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r) \\ \quad +O(h^2+k^2+l^2+d^2+hk+hl+hd+kl+kd+ld) = 0, \\ \underline{G}(\underline{x}-h, \overline{x}-k, \underline{y}-l, \overline{y}-d; r) = \underline{G}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r) - h\underline{G}_{\underline{x}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r) \\ \quad -k\underline{G}_{\overline{x}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r) - l\underline{G}_{\underline{y}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r) - d\underline{G}_{\overline{y}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r) \\ \quad +O(h^2+k^2+l^2+d^2+hk+hl+hd+kl+kd+ld) = 0, \\ \overline{G}(\underline{x}-h, \overline{x}-k, \underline{y}-l, \overline{y}-d; r) = \overline{G}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r) - h\overline{G}_{\underline{x}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r) \\ \quad -k\overline{G}_{\overline{x}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r) - l\overline{G}_{\underline{y}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r) - d\overline{G}_{\overline{y}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r) \\ \quad +O(h^2+k^2+l^2+d^2+hk+hl+hd+kl+kd+ld) = 0, \end{cases}$$

that $\underline{F}_{\underline{x}}$ means that, the derivative of $\underline{F}$ with respect to $\underline{x}$ and so on. We assume, of course, that all needed partial derivatives exist and are bounded. Therefore for sufficiently small $h(r), k(r), l(r)$ and $d(r)$ for each $r \in [0,1]$,

$$\begin{cases} \underline{F}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r) - h\underline{F}_{\underline{x}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r) - k\underline{F}_{\overline{x}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r) \\ \quad -l\underline{F}_{\underline{y}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r) - d\underline{F}_{\overline{y}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r) \simeq 0, \\ \overline{F}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r) - h\overline{F}_{\underline{x}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r) - k\overline{F}_{\overline{x}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r) \\ \quad -l\overline{F}_{\underline{y}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r) - d\overline{F}_{\overline{y}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r) \simeq 0, \\ \underline{G}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r) - h\underline{G}_{\underline{x}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r) - k\underline{G}_{\overline{x}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r) \\ \quad -l\underline{G}_{\underline{y}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r) - d\underline{G}_{\overline{y}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r) \simeq 0, \\ \overline{G}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r) - h\overline{G}_{\underline{x}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r) - k\overline{G}_{\overline{x}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r) \\ \quad -l\overline{G}_{\underline{y}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r) - d\overline{G}_{\overline{y}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r) \simeq 0, \end{cases}$$

and hence $h(r), k(r), l(r)$ and $d(r)$ are unknown quantities that can be obtained by solving the following equations, for each $r \in [0,1]$

$$J(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r) \begin{bmatrix} h(r) \\ k(r) \\ l(r) \\ d(r) \end{bmatrix} = \begin{bmatrix} \underline{F}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r) \\ \overline{F}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r) \\ \underline{G}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r) \\ \overline{G}(\underline{x}, \overline{x}, , \underline{y}, \overline{y}; r) \end{bmatrix}, \tag{9}$$

where

$$J(\underline{x},\overline{x},\underline{y},\overline{y};r)=\begin{bmatrix}\underline{F}_{\underline{x}} & \underline{F}_{\overline{x}} & \underline{F}_{\underline{y}} & \underline{F}_{\overline{y}}\\ \overline{F}_{\underline{x}} & \overline{F}_{\overline{x}} & \overline{F}_{\underline{y}} & \overline{F}_{\overline{y}}\\ \underline{G}_{\underline{x}} & \underline{G}_{\overline{x}} & \underline{G}_{\underline{y}} & \underline{F}_{\overline{y}}\\ \overline{G}_{\underline{x}} & \overline{G}_{\overline{x}} & \overline{G}_{\underline{y}} & \overline{F}_{\overline{y}}\end{bmatrix}(\underline{x},\overline{x},\underline{y},\overline{y};r).$$

The Newton's method is given by

$$\begin{cases}\underline{x}_{n+1}(r)=\underline{x}_n(r)+h_n(r),\\ \overline{x}_{n+1}(r)=\overline{x}_n(r)+k_n(r),\\ \underline{y}_{n+1}(r)=\underline{y}_n(r)+l_n(r),\\ \overline{y}_{n+1}(r)=\overline{y}_n(r)+d_n(r),\end{cases}\tag{10}$$

where $n=0,1,2,\dots$ and $h_n(r),k_n(r),l_n(r),d_n(r)$ are given by (9). For initial guess, one can use the trapezoidal fuzzy number

$$x_0=(\underline{x}(1),\overline{x}(1),\underline{x}(1)-\underline{x}(0),\overline{x}(0)-\overline{x}(1)),$$
$$y_0=(\underline{y}(1),\overline{y}(1),\underline{y}(1)-\underline{y}(0),\overline{y}(0)-\overline{y}(1)),$$

and in parametric form

$$\begin{aligned}\underline{x}_0(r)&=\underline{x}(1)+(\underline{x}(1)-\underline{x}(0))(r-1),\\ \overline{x}_0(r)&=\overline{x}(1)+(\overline{x}(0)-\overline{x}(1))(1-r),\\ \underline{y}_0(r)&=\underline{y}(1)+(\underline{y}(1)-\underline{y}(0))(r-1),\\ \overline{y}_0(r)&=\overline{y}(1)+(\overline{y}(0)-\overline{y}(1))(1-r).\end{aligned}$$

The iteration (10) will converge to $(\alpha,\beta,\gamma,\theta)$ if the starting point $(\underline{x}_0(r),\overline{x}_0(r)$ $,\underline{y}_0(r),\overline{y}_0(r))$ is close enough to $(\alpha,\beta,\gamma,\theta)$ for $0\leqslant r\leqslant 1$, local convergence property, see [11] for more details.

If we use Taylor's expansion of $\underline{F}(\underline{x},\overline{x},\underline{y},\overline{y};r)$ and $\overline{F}(\underline{x},\overline{x},\underline{y},\overline{y};r)$ to a higher order and we are looking for $h(r),k(r),l(r)$ and $d(r)$ such as:

$$[\underline{F}-h\underline{F}_{\underline{x}}-k\underline{F}_{\overline{x}}-l\underline{F}_{\underline{y}}-d\underline{F}_{\overline{y}}+\frac{1}{2}(h^2\underline{F}_{\underline{x}\ \underline{x}}+k^2\underline{F}_{\overline{x}\ \overline{x}}+l^2\underline{F}_{\underline{y}\ \underline{y}}+d^2\underline{F}_{\overline{y}\ \overline{y}}$$
$$+2hk\underline{F}_{\underline{x}\ \overline{x}}+2ld\underline{F}_{\underline{y}\ \overline{y}}+2hl\underline{F}_{\underline{x}\ \underline{y}}++2hd\underline{F}_{\underline{x}\ \overline{y}}+2kl\underline{F}_{\overline{x}\ \underline{y}}++2kd\underline{F}_{\overline{x}\ \overline{y}})](\underline{x},\overline{x},\underline{y},\overline{y};r)\simeq 0,$$

$$[\overline{F}-h\overline{F}_{\underline{x}}-k\overline{F}_{\overline{x}}-l\overline{F}_{\underline{y}}-d\overline{F}_{\overline{y}}+\frac{1}{2}(h^2\overline{F}_{\underline{x}\ \underline{x}}+k^2\overline{F}_{\overline{x}\ \overline{x}}+l^2\overline{F}_{\underline{y}\ \underline{y}}+d^2\overline{F}_{\overline{y}\ \overline{y}}$$
$$+2hk\overline{F}_{\underline{x}\ \overline{x}}+2ld\overline{F}_{\underline{y}\ \overline{y}}+2hl\overline{F}_{\underline{x}\ \underline{y}}++2hd\overline{F}_{\underline{x}\ \overline{y}}+2kl\overline{F}_{\overline{x}\ \underline{y}}++2kd\overline{F}_{\overline{x}\ \overline{y}})](\underline{x},\overline{x},\underline{y},\overline{y};r)\simeq 0,$$

$$[\underline{G}-h\underline{G}_{\underline{x}}-k\underline{G}_{\overline{x}}-l\underline{G}_{\underline{y}}-d\underline{G}_{\overline{y}}+\frac{1}{2}(h^2\underline{G}_{\underline{x}\ \underline{x}}+k^2\underline{G}_{\overline{x}\ \overline{x}}+l^2\underline{G}_{\underline{y}\ \underline{y}}+d^2\underline{G}_{\overline{y}\ \overline{y}}$$
$$+2hk\underline{G}_{\underline{x}\ \overline{x}}+2ld\underline{G}_{\underline{y}\ \overline{y}}+2hl\underline{G}_{\underline{x}\ \underline{y}}++2hd\underline{G}_{\underline{x}\ \overline{y}}+2kl\underline{G}_{\overline{x}\ \underline{y}}++2kd\underline{G}_{\overline{x}\ \overline{y}})](\underline{x},\overline{x},\underline{y},\overline{y};r)\simeq 0,$$

$$[\overline{G}-h\overline{G}_{\underline{x}}-k\overline{G}_{\overline{x}}-l\overline{G}_{\underline{y}}-d\overline{G}_{\overline{y}}+\frac{1}{2}(h^2\overline{G}_{\underline{x}\ \underline{x}}+k^2\overline{G}_{\overline{x}\ \overline{x}}+l^2\overline{G}_{\underline{y}\ \underline{y}}+d^2\overline{G}_{\overline{y}\ \overline{y}}$$
$$+2hk\overline{G}_{\underline{x}\ \overline{x}}+2ld\overline{G}_{\underline{y}\ \overline{y}}+2hl\overline{G}_{\underline{x}\ \underline{y}}++2hd\overline{G}_{\underline{x}\ \overline{y}}+2kl\overline{G}_{\overline{x}\ \underline{y}}++2kd\overline{G}_{\overline{x}\ \overline{y}})](\underline{x},\overline{x},\underline{y},\overline{y};r)\simeq 0,$$

given

$$h(r) = [\underline{F} - k\underline{F}_{\overline{x}} - l\underline{F}_{\underline{y}} - d\underline{F}_{\overline{y}} + \frac{1}{2}(h^2\underline{F}_{\underline{x}\ \underline{x}} + k^2\underline{F}_{\overline{x}\ \overline{x}} + l^2\underline{F}_{\underline{y}\ \underline{y}} + d^2\underline{F}_{\overline{y}\ \overline{y}} + 2hk\underline{F}_{\underline{x}\ \overline{x}}$$
$$+ 2ld\underline{F}_{\underline{y}\ \overline{y}} + 2hl\underline{F}_{\underline{x}\ \underline{y}} + +2hd\underline{F}_{\underline{x}\ \overline{y}} + 2kl\underline{F}_{\overline{x}\ \underline{y}} + +2kd\underline{F}_{\overline{x}\ \overline{y}})/\underline{F}_{\underline{x}}](\underline{x}, \overline{x}, \underline{y}, \overline{y}; r),$$
$$k(r) = [\overline{F} - h\overline{F}_{\underline{x}} - l\overline{F}_{\underline{y}} - d\overline{F}_{\overline{y}} + \frac{1}{2}(h^2\overline{F}_{\underline{x}\ \underline{x}} + k^2\overline{F}_{\overline{x}\ \overline{x}} + l^2\overline{F}_{\underline{y}\ \underline{y}} + d^2\overline{F}_{\overline{y}\ \overline{y}} + 2hk\overline{F}_{\underline{x}\ \overline{x}}$$
$$+ 2ld\overline{F}_{\underline{y}\ \overline{y}} + 2hl\overline{F}_{\underline{x}\ \underline{y}} + +2hd\overline{F}_{\underline{x}\ \overline{y}} + 2kl\overline{F}_{\overline{x}\ \underline{y}} + +2kd\overline{F}_{\overline{x}\ \overline{y}})/\overline{F}_{\overline{x}}](\underline{x}, \overline{x}, \underline{y}, \overline{y}; r),$$
$$l(r) = [\underline{G} - h\underline{G}_{\underline{x}} - k\underline{G}_{\overline{x}} - d\underline{G}_{\overline{y}} + \frac{1}{2}(h^2\underline{G}_{\underline{x}\ \underline{x}} + k^2\underline{G}_{\overline{x}\ \overline{x}} + l^2\underline{G}_{\underline{y}\ \underline{y}} + d^2\underline{G}_{\overline{y}\ \overline{y}} + 2hk\underline{G}_{\underline{x}\ \overline{x}}$$
$$+ 2ld\underline{G}_{\underline{y}\ \overline{y}} + 2hl\underline{G}_{\underline{x}\ \underline{y}} + +2hd\underline{G}_{\underline{x}\ \overline{y}} + 2kl\underline{G}_{\overline{x}\ \underline{y}} + +2kd\underline{G}_{\overline{x}\ \overline{y}})/\underline{G}_{\underline{y}}](\underline{x}, \overline{x}, \underline{y}, \overline{y}; r),$$
$$d(r) = [\overline{G} - h\overline{G}_{\underline{x}} - k\overline{G}_{\overline{x}} - l\overline{G}_{\underline{y}} + \frac{1}{2}(h^2\overline{G}_{\underline{x}\ \underline{x}} + k^2\overline{G}_{\overline{x}\ \overline{x}} + l^2\overline{G}_{\underline{y}\ \underline{y}} + d^2\overline{G}_{\overline{y}\ \overline{y}} + 2hk\overline{G}_{\underline{x}\ \overline{x}}$$
$$+ 2ld\overline{G}_{\underline{y}\ \overline{y}} + 2hl\overline{G}_{\underline{x}\ \underline{y}} + +2hd\overline{G}_{\underline{x}\ \overline{y}} + 2kl\overline{G}_{\overline{x}\ \underline{y}} + +2kd\overline{G}_{\overline{x}\ \overline{y}})/\overline{G}_{\overline{y}}](\underline{x}, \overline{x}, \underline{y}, \overline{y}; r),$$

or

$$\begin{bmatrix} h(r) \\ k(r) \\ l(r) \\ d(r) \end{bmatrix} = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix} + N\left(\begin{bmatrix} h(r) \\ k(r) \\ l(r) \\ d(r) \end{bmatrix}\right) = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix} + \begin{bmatrix} N_1(h,k,l,d) \\ N_2(h,k,l,d) \\ N_3(h,k,l,d) \\ N_4(h,k,l,d) \end{bmatrix}, \tag{11}$$

where $e_1 = \frac{\underline{F}}{\underline{F}_{\underline{x}}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r)$, $e_2 = \frac{\overline{F}}{\overline{F}_{\overline{x}}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r)$, $e_3 = \frac{\underline{G}}{\underline{G}_{\underline{y}}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r)$ and $e_4 = \frac{\overline{G}}{\overline{G}_{\overline{y}}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r)$ are constants and N is a vector quadratic polynomial and for approximating $h(r)$, $k(r)$, $l(r)$ and $d(r)$, we can apply the multivariable Adomian decomposition method [1].

The Adomian decomposition technique considers representing the solution of (11) as a series

$$h = \sum_{n=0}^{\infty} h_n, \quad k = \sum_{n=0}^{\infty} k_n, \quad l = \sum_{n=0}^{\infty} l_n, \quad d = \sum_{n=0}^{\infty} d_n \tag{12}$$

and the nonlinear functions are decomposed as

$$N_i(h,k,l,d) = \sum_{n=0}^{\infty} A_{in}(h_0, \dots, h_n, k_0, \dots, k_n, l_0, \dots, l_n, , d_0, \dots, d_n), \quad i = 1, ..., 4. \tag{13}$$

where the A_{in}'s are Adomian's polynomials given by [3],

$$A_{in} = \frac{1}{n!}\frac{d^n}{d\lambda^n}[N_i(\sum_{j=0}^{\infty} \lambda^j h_j, \sum_{j=0}^{\infty} \lambda^j k_j, \sum_{j=0}^{\infty} \lambda^j l_j, \sum_{j=0}^{\infty} \lambda^j d_j)]_{\lambda=0}$$

for $i = 1, ..., 4, j = 0, 1, \dots$.

Upon substituting (12), (13) in the (11) yields

$$h_0 = e_1, \quad h_{n+1} = A_{1n}, \quad k_0 = e_2, \quad k_{n+1} = A_{2n},$$
$$l_0 = e_3, \quad l_{n+1} = A_{3n}, \quad d_0 = e_4, \quad d_{n+1} = A_{4n},$$

for $n = 0, 1, \ldots$, multivariable polynomials A_{in} are generated by practical formulae presented in [1], for $i = 1, 2, 3, 4$, we have

$$A_{i0} = N_i(h_0, k_0, l_0, d_0),$$
$$A_{in} = \sum_{\varphi} \frac{h_1^{p_1}}{p_1!} \cdots \frac{h_n^{p_n}}{p_n!} . \frac{k_1^{q_1}}{q_1!} \cdots \frac{k_n^{q_n}}{q_n!} . \frac{l_1^{s_1}}{s_1!} \cdots \frac{l_n^{s_n}}{s_n!} . \frac{d_1^{t_1}}{t_1!} \cdots \frac{d_n^{t_n}}{t_n!}$$
$$. \frac{\partial^{\varphi_1+\varphi_2+\varphi_3+\varphi_4}}{\partial h^{\varphi_1} \partial k^{\varphi_2} \partial d^{\varphi_3} \partial l^{\varphi_4}} N_i(h_0, k_0, d_0, l_0), \quad n \neq 0,$$

where φ stands for $(p_1 + 2p_2 + \ldots + np_n) + (q_1 + 2q_2 + \ldots + nq_n) + (s_1 + 2s_2 + \ldots + ns_n) + (t_1 + 2t_2 + \ldots + nt_n) = n$, and $\varphi_1 = p_1 + p_2 + \ldots + p_n$, $\varphi_2 = q_1 + q_2 + \ldots + q_n$, $\varphi_3 = s_1 + s_2 + \ldots + s_n$, $\varphi_4 = t_1 + t_2 + \ldots + t_n$.
In practice, of course, the sum of the infinite series has to be truncated at some finite order M. The quantities $\sum_{n=0}^{M} h_n, \sum_{n=0}^{M} k_n, \sum_{n=0}^{M} l_n$ and $\sum_{n=0}^{M} d_n$, can thus be reasonable approximations of the exact solution of (8), provided M is sufficiently large. As $M \longrightarrow \infty$, the series converge smoothly toward the exact solution for $0 \leqslant r \leqslant 1$ [2]. Let

$$\begin{array}{l} H_M = h_0 + h_1 + \ldots + h_M = h_0 + A_{10} + A_{11} + \ldots + A_{1M-1}, \\ K_M = k_0 + k_1 + \ldots + k_M = k_0 + A_{20} + A_{21} + \ldots + A_{2M-1}, \\ L_M = l_0 + l_1 + \ldots + l_M = l_0 + A_{30} + A_{31} + \ldots + A_{3M-1}, \\ D_M = d_0 + d_1 + \ldots + d_M = d_0 + A_{40} + A_{41} + \ldots + A_{4M-1}, \end{array} \tag{14}$$

denote the $(M + 1)$-term approximations of h, k, l and d, respectively. Since the series converge very rapidly, then (14) can serve as a practical solution in each iteration.

We will show that the number of terms required to obtain an accurate computable solution is very small.
Case 1: For $M = 0$,

$$h \simeq H_0 = h_0 = \frac{\underline{F}}{\underline{F}_{\underline{x}}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r),$$

$$k \simeq K_0 = k_0 = \frac{\overline{F}}{\overline{F}_{\overline{x}}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r),$$

$$l \simeq L_0 = l_0 = \frac{\underline{G}}{\underline{G}_{\underline{y}}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r),$$

$$d \simeq D_0 = d_0 = \frac{\overline{G}}{\overline{G}_{\overline{y}}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r),$$

$$\alpha = \underline{x} - h \simeq \underline{x} - H_0 = \underline{x} - \frac{\underline{F}}{\underline{F}_{\underline{x}}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r),$$

$$\beta = \overline{x} - k \simeq \overline{x} - K_0 = \overline{x} - \frac{\overline{F}}{\overline{F}_{\overline{x}}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r),$$

$$\gamma = \underline{y} - l \simeq \underline{y} - L_0 = \underline{y} - \frac{\underline{G}}{\underline{G}_{\underline{y}}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r),$$

$$\theta = \overline{y} - d \simeq \overline{y} - D_0 = \overline{y} - \frac{\overline{G}}{\overline{G}_{\overline{y}}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r)$$

and

$$\begin{cases} \underline{x}_{n+1} = \underline{x}_n - \frac{\underline{F}}{\underline{F}_{\underline{x}}}(\underline{x}_n, \overline{x}_n, \underline{y}_n, \overline{y}_n; r), \\ \overline{x}_{n+1} = \overline{x}_n - \frac{\overline{F}}{\overline{F}_{\overline{x}}}(\underline{x}_n, \overline{x}_n, \underline{y}_n, \overline{y}_n; r), \\ \underline{y}_{n+1} = \underline{y}_n - \frac{\underline{G}}{\underline{G}_{\underline{y}}}(\underline{x}_n, \overline{x}_n, \underline{y}_n, \overline{y}_n; r), \\ \overline{y}_{n+1} = \overline{y}_n - \frac{\overline{G}}{\overline{G}_{\overline{y}}}(\underline{x}_n, \overline{x}_n, \underline{y}_n, \overline{y}_n; r), \end{cases}$$

for $n = 0, 1, \ldots$.

Case 2: For $M = 1$

$$h_1 = A_{1,0} = N_1(h_0, k_0, l_0, d_0) = [(\tfrac{h_0^2}{2}\underline{F}_{\underline{x}\ \underline{x}} + \tfrac{k_0^2}{2}\underline{F}_{\overline{x}\ \overline{x}} + \tfrac{l_0^2}{2}\underline{F}_{\underline{y}\ \underline{y}} + \tfrac{d_0^2}{2}\underline{F}_{\overline{y}\ \overline{y}}$$
$$+h_0k_0\underline{F}_{\underline{x}\ \overline{x}} + h_0l_0\underline{F}_{\underline{x}\ \underline{y}} + h_0d_0\underline{F}_{\underline{x}\ \overline{y}} + k_0l_0\underline{F}_{\overline{x}\ \underline{y}} + k_0d_0\underline{F}_{\underline{x}\ \overline{y}}$$
$$+l_0d_0\underline{F}_{\underline{y}\ \overline{y}})/\underline{F}_{\underline{x}}](\underline{x}, \overline{x}, \underline{y}, \overline{y}; r),$$

$$k_1 = A_{2,0} = N_2(h_0, k_0, l_0, d_0) = [(\tfrac{h_0^2}{2}\overline{F}_{\underline{x}\ \underline{x}} + \tfrac{k_0^2}{2}\overline{F}_{\overline{x}\ \overline{x}} + \tfrac{l_0^2}{2}\overline{F}_{\underline{y}\ \underline{y}} + \tfrac{d_0^2}{2}\overline{F}_{\overline{y}\ \overline{y}}$$
$$+h_0k_0\overline{F}_{\underline{x}\ \overline{x}} + h_0l_0\overline{F}_{\underline{x}\ \underline{y}} + h_0d_0\overline{F}_{\underline{x}\ \overline{y}} + k_0l_0\overline{F}_{\overline{x}\ \underline{y}} + k_0d_0\overline{F}_{\underline{x}\ \overline{y}}$$
$$+l_0d_0\overline{F}_{\underline{y}\ \overline{y}})/\overline{F}_{\overline{x}}](\underline{x}, \overline{x}, \underline{y}, \overline{y}; r),$$

$$l_1 = A_{3,0} = N_3(h_0, k_0, l_0, d_0) = [(\tfrac{h_0^2}{2}\underline{G}_{\underline{x}\ \underline{x}} + \tfrac{k_0^2}{2}\underline{G}_{\overline{x}\ \overline{x}} + \tfrac{l_0^2}{2}\underline{G}_{\underline{y}\ \underline{y}} + \tfrac{d_0^2}{2}\underline{G}_{\overline{y}\ \overline{y}}$$
$$+h_0k_0\underline{G}_{\underline{x}\ \overline{x}} + h_0l_0\underline{G}_{\underline{x}\ \underline{y}} + h_0d_0\underline{G}_{\underline{x}\ \overline{y}} + k_0l_0\underline{G}_{\overline{x}\ \underline{y}} + k_0d_0\underline{G}_{\underline{x}\ \overline{y}}$$
$$+l_0d_0\underline{G}_{\underline{y}\ \overline{y}})/\underline{G}_{\underline{y}}](\underline{x}, \overline{x}, \underline{y}, \overline{y}; r),$$

$$d_1 = A_{4,0} = N_4(h_0, k_0, l_0, d_0) = [(\tfrac{h_0^2}{2}\overline{G}_{\underline{x}\,\underline{x}} + \tfrac{k_0^2}{2}\overline{G}_{\overline{x}\,\overline{x}} + \tfrac{l_0^2}{2}\overline{G}_{\underline{y}\,\underline{y}} + \tfrac{d_0^2}{2}\overline{G}_{\overline{y}\,\overline{y}}$$

$$+h_0k_0\overline{G}_{\underline{x}\,\overline{x}} + h_0l_0\overline{G}_{\underline{x}\,\underline{y}} + h_0d_0\overline{F}_{\underline{x}\,\overline{y}} + k_0l_0\overline{G}_{\overline{x}\,\underline{y}} + k_0d_0\overline{G}_{\underline{x}\,\overline{y}}$$

$$+l_0d_0\overline{G}_{\underline{y}\,\overline{y}})/\overline{G}_{\overline{y}}](\underline{x}, \overline{x}, \underline{y}, \overline{y}; r),$$

where $h_0 = \frac{\underline{F}}{\underline{F}_{\underline{x}}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r)$, $k_0 = \frac{\overline{F}}{\overline{F}_{\overline{x}}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r)$, $l_0 = \frac{\underline{G}}{\underline{G}_{\underline{y}}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r)$ and $d_0 = \frac{\overline{G}}{\overline{G}_{\overline{y}}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r)$, then

$$\alpha = \underline{x} - h \simeq \underline{x} - H_1 = \underline{x} - h_0 - A_{1,0},$$
$$\beta = \overline{x} - k \simeq \overline{x} - K_1 = \overline{x} - k_0 - A_{2,0},$$
$$\gamma = \underline{y} - l \simeq \underline{y} - L_1 = \underline{y} - l_0 - A_{3,0},$$
$$\theta = \overline{y} - d \simeq \overline{y} - D_1 = \overline{y} - d_0 - A_{4,0}.$$

Hence, we have the following iterations:

$$\underline{x}_{n+1} = \underline{x}_n - H_1(\underline{x}_n, \overline{x}_n, \underline{y}_n, \overline{y}_n; r),$$
$$\overline{x}_{n+1} = \overline{x}_n - K_1(\underline{x}_n, \overline{x}_n, \underline{y}_n, \overline{y}_n; r),$$
$$\underline{y}_{n+1} = \underline{y}_n - L_1(\underline{x}_n, \overline{x}_n, \underline{y}_n, \overline{y}_n; r),$$
$$\overline{y}_{n+1} = \overline{y}_n - D_1(\underline{x}_n, \overline{x}_n, \underline{y}_n, \overline{y}_n; r)$$

for $n = 0, 1, \ldots$. We can also obtain similar relations for $M = 2, 3, \ldots$.

The Adomian decomposition method is simply generalized to more variables and upper degrees as well.

4. Comparison with other methods

This study would not be completed without comparing it with other existing methods. Some comparisons are as follows:

- In [5] and [6] researchers used the Newton's method for solving fuzzy nonlinear equations and systems of fuzzy nonlinear equations and in [12] researchers used the Fixed point method for solving fuzzy nonlinear equations. The Adomian decomposition method for $M = 0$ is the Newton's method. See examples $1, 2$ for more details.
- In [8, 9] a FNN_2 equivalent to the fuzzy polynomial equation and system of fuzzy polynomials F of s fuzzy polynomial equations such as

$$\begin{aligned} f_1(x_1, x_2, \ldots, x_n) &= A_{10}, \\ &\vdots \\ f_l(x_1, x_2, \ldots, x_n) &= A_{l0}, \\ &\vdots \\ f_s(x_1, x_2, \ldots, x_n) &= A_{s0} \end{aligned} \tag{15}$$

where, $x_1, x_2, \ldots, x_n \in \mathbb{R}$ and all coefficients are fuzzy numbers were built. In this paper, Adomian decomposition method for solving system of fuzzy polynomials where $x_1, x_2, \ldots, x_n$ and all coefficients are fuzzy numbers was proposed. See examples 1, 2 for more details.

- In [22] researchers used the Adomian decomposition method for solving fuzzy polynomial equations of the form $\sum_{i=1}^{n} a_i x^i = c$ where x, c and all coefficients are fuzzy numbers. In this paper, Adomian decomposition method for solving system of fuzzy polynomial equations was proposed.

5. Numerical examples

We consider some examples for the Adomian decomposition method. In the computer simulation of this examples, we use the following specifications of the Adomian decomposition method. For each fuzzy numbers, we use $r = 0, 0.1, \ldots, 1$, where we calculate the total error of each iteration by

$$e_i = max\{D(x_i, x_{i-1}), D(y_i, y_{i-1})\}.$$

***Example* 5.1** Consider the system of fuzzy polynomial equations

$$\begin{cases} x^2 + y = (3, 1, 1.75), \\ x + y^2 = (5, 1.4375, 2.75), \end{cases}$$

assume that x and y are positive, then the parametric form of this equation is as follows:

$$\begin{cases} \underline{x}^2(r) + \underline{y}(r) = 2 + r, \\ \overline{x}^2(r) + \overline{y}(r) = 4.75 - 1.75r, \\ \underline{x}(r) + \underline{y}^2(r) = 3.5625 + 1.4375r, \\ \overline{x}(r) + \overline{y}^2(r) = 7.75 - 2.75r. \end{cases}$$

Initial guess is $x_0 = (1.25, 0.5, 0.25)$ and $y_0 = (1.75, 0.25, 0.5)$. For $M = 0$

$$h \simeq H_0 = h_0 = \frac{\underline{F}}{\underline{F}_{\underline{x}}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r), \;\; k \simeq K_0 = k_0 = \frac{\overline{F}}{\overline{F}_{\overline{x}}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r),$$

$$l \simeq L_0 = l_0 = \frac{\underline{G}}{\underline{G}_{\underline{y}}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r), \;\; d \simeq D_0 = d_0 = \frac{\overline{G}}{\overline{G}_{\overline{y}}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r),$$

and

$$\alpha = \underline{x} - h \simeq \underline{x} - H_0 = \underline{x} - \frac{\underline{F}}{\underline{F}_{\underline{x}}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r),$$

$$\beta = \overline{x} - k \simeq \overline{x} - K_0 = \overline{x} - \frac{\overline{F}}{\overline{F}_{\overline{x}}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r),$$

$$\gamma = \underline{y} - l \simeq \underline{y} - L_0 = \underline{y} - \frac{\underline{G}}{\underline{G}_{\underline{y}}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r),$$

$$\theta = \overline{y} - d \simeq \overline{y} - D_0 = \overline{y} - \frac{\overline{G}}{\overline{G}_{\overline{y}}}(\underline{x}, \overline{x}, \underline{y}, \overline{y}; r),$$

then

M	Iter 1	Iter 2	Iter 3	Iter 4	Iter 5	Iter 6
0	0.2639	0.1336	0.0395	0.0357	0.0103	0.0100
1	0.1831	0.0422	0.0131	0.0092	2.6131×10^{-3}	2.4532×10^{-4}

Table 1. The error of Adomian decomposition method.

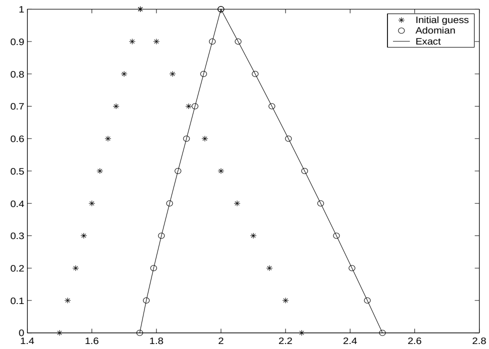

Fig. 2. Approximate and analytical solution of example 1 for y.

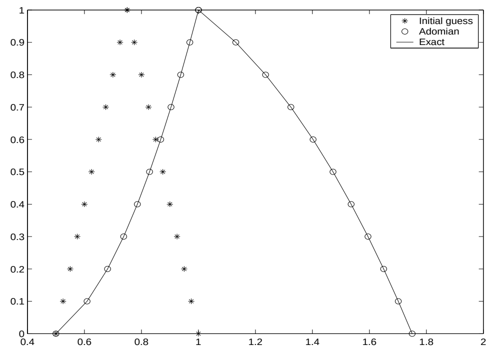

Fig. 3. Approximate and analytical solution of example 2 for x.

$$\begin{cases} \underline{x}_{n+1} = \underline{x}_n - \frac{\underline{x}_n^2 + \underline{y}_n - (2+r)}{2\underline{x}_n}, \\ \overline{x}_{n+1} = \overline{x}_n - \frac{\overline{x}_n^2 + \overline{y}_n - (4.75 - 1.75r)}{2\overline{x}_n}, \\ \underline{y}_{n+1} = \underline{y}_n - \frac{\underline{x}_n + \underline{y}_n^2 - (3.5625 + 1.4375r)}{2\underline{y}_n}, \\ \overline{y}_{n+1} = \overline{y}_n - \frac{\overline{x}_n + \overline{y}_n^2 - (7.75 - 2.75r)}{2\overline{y}_n}, \end{cases}$$

for $n = 0, 1, \ldots, 6$.
By Adomian decomposition method, we obtain the numerical results for $M = 0, 1$. See figures 1,2 and table 1 for more details.

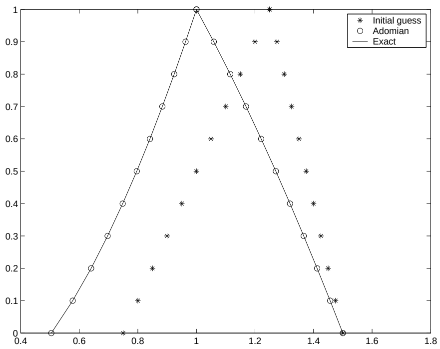

Fig. 1. Approximate and analytical solution of example 1 for x.

***Example* 5.2** Consider the system of fuzzy polynomial equations

$$\begin{cases} x^3 + y = (2.5, 1.375, 4.859375), \\ x + y^2 = (3.25, 1.75, 2.5), \end{cases}$$

assume that x and y are positive, then the parametric form of this equation is as follows:

$$\begin{cases} \underline{x}^3(r) + \underline{y}(r) = 1.125 + 1.375r, \\ \overline{x}^3(r) + \overline{y}(r) = 7.359375 - 4.859375r, \\ \underline{x}(r) + \underline{y}^2(r) = 1.5 + 1.75r, \\ \overline{x}(r) + \overline{y}^2(r) = 5.75 - 2.5r. \end{cases}$$

M	Iter 1	Iter 2	Iter 3	Iter 4	Iter 5	Iter 6
0	1.4531	0.5312	0.1432	0.0339	0.0198	0.0114
1	0.4436	0.1253	0.0635	0.0092	1.5131×10^{-3}	3.464×10^{-4}

Table 2. The error of Adomian decomposition method.

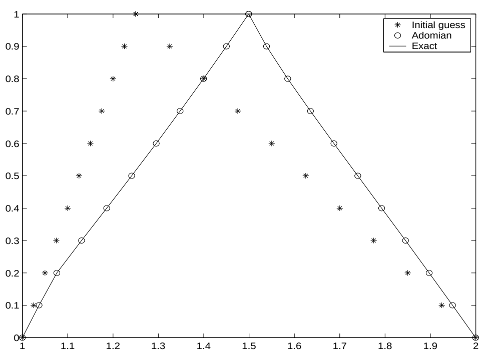

Fig. 4. Approximate and analytical solution of example 2 for y.

Initial guess is $x_0 = (0.75, 0.25, 0.25)$ and $y_0 = (1.25, 0.25, 0.75)$.
By Adomian decomposition method, we obtain the numerical results for $M = 0, 1$. See figures 3,4 and table 2 for more details.

6. Conclusion

In this paper, we proposed numerical method for solving a system of fuzzy polynomial equations. Initially we wrote fuzzy polynomials in a parametric form and then solve it by Adomian decomposition method.

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