Journal of
Linear and Topological Algebra
Vol. 05*, No.* 03*,* 2016*,* 175- 186

Fuzzy soft ideals of near-subraction semigroups

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Received 22 June 2016; Revised 10 August 2016; Accepted 25 August 2016.

Abstract. Our aim in this paper is to introduce the notion of fuzzy soft near-subtraction semigroups and fuzzy soft ideals of near-subtraction semigroups. We discuss some important properties of these new fuzzy algebraic structure and investigate some examples and counter examples.

Keywords: Near-subtraction semigroup, fuzzy soft set, fuzzy soft near-subtraction semigroup, fuzzy soft ideal.

2010 AMS Subject Classification: 03E72, 08A72, 06D72, 16D25.

1. Introduction

Schein[17] considered the system of the form $(\phi; \circ, \backslash)$ where ϕ is a set of functions closed under the composition $'\circ'$ together with the set theoretic subtraction and named this structure as subtraction semigroup. He established that every subtraction semigroup is isomorphic to a difference semigroup of invertible functions. The notion of atomic subtraction algebras, a special type of subtraction algebras was studied by Zelinka[18]. Jun et al.[10], introduced the concept of ideals in subtraction algebras and gave some characterizations. Dheena et al.[7–9] studied the structure near-subtraction semigroup, a generalization of subtraction semigroup and discussed some properties. A near-subtraction semigroup satisfies all axioms of subtraction semigroup, except one of the two distributive laws. Molodsov[14] initiated the study of soft sets as a mathematical tool to study some type of uncertainties. Several new operations on soft sets were defined by Maji et

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Print ISSN: 2252-0201
Online ISSN: 2345-5934

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al.[11, 12], who also gave an application of soft sets in decision making problem. Maji et al.[13] have also initiated the study of fuzzy soft sets. Recently, Prince Williams et al.[15, 16] have discussed fuzzy ideals and fuzzy soft ideals in subtraction algebras and gave some characterizations. Cheng-Fu Yang studied the notion of fuzzy soft semigroups and fuzzy soft ideals. The concept of fuzzy soft groups was first introduced by Aygunoglu et al.[4].

In this paper we introduce the notion of fuzzy soft near-subtraction semigroups and fuzzy soft ideals of near-subtraction semigroups. We give some examples and discuss some properties of these structures.

2. Preliminaries

In this section, some relevant definitions are reproduced based on [7, 11, 13, 14]. Throughout this paper X denotes right near-subtraction semigroup.

Definition 2.1 [7]*A nonempty set X together with a binary operation "$-$" is said to be a subtraction algebra if it satisfies the following conditions:*

(1) $x-(y-x)=x$,

(2) $x-(x-y)=y-(y-x)$,

(3) $(x-y)-z=(x-z)-y$ *for every* $x,y,z\in X$.

The last identity permits us to omit parenthesis in expressions of the form $(x-y)-z$.

Definition 2.2 [7]*A non-empty set X together with two binary operations "$-$" and "$\cdot$" is said to be a near-subtraction semigroup if it satisfies the following:*

(1) $(X,-)$ *is a subtraction algebra,*

(2) $(X,\cdot)$ *is a semigroup,*

(3) $(x-y)z=xz-yz$ *for all* $x,y,z\in X$.

It is clear that $0x=0$, for all $x\in X$. Similarly we can define a near-subtraction semigroup(left). In this paper a near-subtraction semigroup means a right near-subtraction semigroup only.

Example **2.3** Let $X=\{0,a,b,c\}$ in which "$-$" and "." are defined by:

$-$	0	a	b	c
0	0	0	0	0
a	a	0	a	a
b	b	b	0	b
c	c	c	c	0

$\cdot$	0	a	b	c
0	0	0	0	0
a	a	a	a	a
b	0	0	0	b
c	0	0	0	c

One can check that $(X,-,\cdot)$ is a near-subtraction semigroup.

Definition 2.4 [7]*A non-empty subset S of a subtraction algebra X is said to be a subalgebra of X, if $x-y\in S$, for all $x,y\in S$.*

Definition 2.5 [7]*A non-empty subset S of a near-subtraction semigroup X is said to be a near-subtraction subsemigroup of X, if $x-y, xy\in S$, for all $x,y\in S$.*

Definition 2.6 [7]*Let $(X,-,\cdot)$ be a near-subtraction semigroup. A non-empty subset I of X is called*

($I1$) *a left ideal if I is a subalgebra of $(X,-)$ and $xi-x(y-i)\in I$ for all $x,y\in X$ and $i\in I$.*

($I2$) *a right ideal if I is a subalgebra of $(X,-)$ and $IX\subseteq I$.*

$(I3)$ *an ideal if I is both a left and right ideal.*

Definition 2.7 [16]*A fuzzy subset μ of X is called a fuzzy ideal of X if it satisfies the following conditions:*
(1) $\mu(x-y) \geqslant \min\{\mu(x), \mu(y)\}$,
(2) $\mu(xi - x(y-i)) \geqslant \mu(i)$,
(3) $\mu(xy) \geqslant \mu(x)$, *for all* $x, y, i \in X$.

Molodsov[14] introduced the idea of soft set in the following way: Let U be the universe, E be the set of parameters, $A \subseteq E$ and $P(U)$ be the power set of U.

Definition 2.8 [14]*A soft set (F, A) of U is defined by a mapping $F : A \to P(U)$. A soft set of U can be represented by a set of ordered pairs $(F, A) = \{(e, F[e]) : e \in A, F[e] \in P(U)\}$. A soft set of U is a parameterized family of subsets of the universe U.*

Definition 2.9 [13]*Let U be the universe and E be a set of parameters. Let $FP(U)$ denote the set of all fuzzy subsets of U. Then $(\widetilde{F}, A)$ is called a fuzzy soft set of U defined by a mapping $\widetilde{F} : A \to FP(U)$.*

Definition 2.10 [15]*Let $(\widetilde{F}, A)$ be a fuzzy soft set of X and $t \in (0,1]$, the set $(\widetilde{F}, A)_t = \{x \in X \mid \widetilde{F}[e] \geqslant t\}$, where $e \in A$ is called a level soft set of X, with respect to the parameter e.*

Definition 2.11 [13]*Let $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ be two fuzzy soft sets of U. Then $(\widetilde{F}, A)$ is called a fuzzy soft subset of $(\widetilde{G}, B)$ is denoted by $(\widetilde{F}, A) \subseteq (\widetilde{G}, B)$, if*
(i) $A \subseteq B$,
(ii) for each $e \in A$, $\widetilde{F}[e] \leqslant \widetilde{G}[e]$.

Definition 2.12 [11] *Let $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ be any two fuzzy soft sets of a common universe U. The extended union of $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ is defined to be the fuzzy soft set $(\widetilde{H}, C)$ satisfying the following:*
(i) $C = A \cup B$,
(ii) *for all* $e \in C$

$$\widetilde{H}[e] = \begin{cases} \widetilde{F}[e] & \text{if } e \in A - B \\ \widetilde{G}[e] & \text{if } e \in B - A \\ \widetilde{F}[e] \cup \widetilde{G}[e] & \text{if } e \in A \cap B. \end{cases}$$

We write $(\widetilde{F}, A) \widetilde{\cup} (\widetilde{G}, B) = (\widetilde{H}, C)$.

Definition 2.13 [11]*Let $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ be any two fuzzy soft sets of a common universe U. The extended intersection of $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ is defined to be the fuzzy soft set $(\widetilde{H}, C)$ satisfying the following:*
(i) $C = A \cup B$,
(ii) *for all* $e \in C$

$$\widetilde{H}[e] = \begin{cases} \widetilde{F}[e] & \text{if } e \in A - B \\ \widetilde{G}[e] & \text{if } e \in B - A \\ \widetilde{F}[e] \cap \widetilde{G}[e] & \text{if } e \in A \cap B. \end{cases}$$

We write $(\widetilde{F}, A) \widetilde{\cap} (\widetilde{G}, B) = (\widetilde{H}, C)$.

Definition 2.14 [11]*Let* $(\widetilde{F}, A)$ *and* $(\widetilde{G}, B)$ *be any two fuzzy soft sets of a common universe* U*. The restricted intersection of* $(\widetilde{F}, A)$ *and* $(\widetilde{G}, B)$ *is a fuzzy soft set* $(\widetilde{H}, C)$ *of* U*, where* $C = A \cap B$ *and for each* $e \in C$*,* $\widetilde{H}[e] = \widetilde{F}[e] \cap \widetilde{G}[e]$*. We write* $(\widetilde{F}, A)\widetilde{\sqcap}(\widetilde{G}, B) = (\widetilde{H}, C)$*.*

Definition 2.15 [11] *Let* $(\widetilde{F}, A)$ *and* $(\widetilde{G}, B)$ *be any two fuzzy soft sets of a common universe* U*. Then* $(\widetilde{F}, A)$ *AND* $(\widetilde{G}, B)$ *denoted by* $(\widetilde{F}, A)\widetilde{\wedge}(\widetilde{G}, B)$ *is defined by* $(\widetilde{F}, A)\widetilde{\wedge}(\widetilde{G}, B) = (\widetilde{H}, A \times B)$ *where* $H[x, y] = \widetilde{F}[x] \cap \widetilde{G}[y]$ *for all* $(x, y) \in A \times B$*.*

Definition 2.16 [11]*Let* $(\widetilde{F}, A)$ *and* $(\widetilde{G}, B)$ *be any two fuzzy soft sets of a common universe* U*. Then* $(\widetilde{F}, A)$ *OR* $(\widetilde{G}, B)$ *denoted by* $(\widetilde{F}, A)\widetilde{\vee}(\widetilde{G}, B)$ *is defined by* $(\widetilde{F}, A)\widetilde{\vee}(\widetilde{G}, B) = (\widetilde{H}, A \times B)$ *where* $H[x, y] = \widetilde{F}[x] \cup \widetilde{G}[y]$ *for all* $(x, y) \in A \times B$*.*

3. Fuzzy soft ideals of near-subtraction semigroups

In this section, we introduce the notion of fuzzy soft ideals of near-subtraction semigroups and give some of its characterizations.

Definition 3.1 *Let* $(\widetilde{F}, A)$ *be a fuzzy soft set of* X*,* $(\widetilde{F}, A)$ *is called a fuzzy soft near-subtraction semigroup if and only if* $\widetilde{F}[e]$ *is a fuzzy near-subtraction subsemigroup of* X*, for each* $e \in A$*.*

Definition 3.2 *Let* $(\widetilde{F}, A)$ *be a fuzzy soft set of* X*,* $(\widetilde{F}, A)$ *is called a fuzzy soft ideal if and only if* $\widetilde{F}[e]$ *is a fuzzy ideal of* X *for each* $e \in A$*.*

***Example* 3.3** Let $X = \{0, a, b, c\}$ be the near-subtraction semigroup as in Example 2.3. Let X denote a set of houses and $E = \{$very big, big, small, medium$\}$ is a parameter space and $A = \{$very big, small, medium$\}$.

(1) Let $(\widetilde{F}, A)$ be a fuzzy soft set of X. Let $\widetilde{F}$[very big], $\widetilde{F}$[small] and $\widetilde{F}$[medium] be fuzzy sets defined as follows:

$\widetilde{F}$	0	a	b	c
very big	0.9	0.7	0.6	0.4
small	0.8	0.5	0.3	0.3
medium	0.7	0.3	0.6	0.3

Then $(\widetilde{F}, A)$ is a fuzzy soft ideal of X.

(2) Let $(\widetilde{G}, A)$ be a fuzzy soft set of X. Then $\widetilde{G}$[very big], $\widetilde{G}$[small] and $\widetilde{G}$[medium] be fuzzy sets defined as follows:

$\widetilde{G}$	0	a	b	c
very big	0.7	0.4	0.55	0.4
small	0.8	0.3	0.5	0.3
medium	0.9	0.6	0.3	0.6

Then $(\widetilde{G}, A)$ is not a fuzzy soft ideal of X, because $(\widetilde{G}, A)$ is not a fuzzy soft ideal with reference to the parameter 'medium' of X,

$$\begin{aligned}\widetilde{G}[\text{medium}](bc - b(0 - c)) &= \widetilde{G}[\text{medium}](b)\\ &= 0.3 \ngeq 0.6 = \widetilde{G}[\text{medium}](c).\end{aligned}$$

Remark 1 *Every fuzzy soft ideal of X is a fuzzy soft near-subtraction semigroup of X. But the converse is not true as shown in the following example.*

***Example* 3.4** Let $X = \{0, a, b, c\}$ be a near-subtraction semigroup with the following Tables:

$-$	0	a	b	c
0	0	0	0	0
a	a	0	a	0
b	b	b	0	0
c	c	b	a	0

$\cdot$	0	a	b	c
0	0	0	0	0
a	0	0	a	a
b	0	a	b	b
c	0	a	c	c

Here X is a set of sample design and $E = \{\text{red, green, blue, yellow}\}$ is the set of available colors for T-shirts and $A = \{\text{red, green, blue}\}$. Let $(\widetilde{F}, A)$ be a fuzzy soft set of X. Let $\widetilde{F}[\text{red}]$, $\widetilde{F}[\text{green}]$ and $\widetilde{F}[\text{blue}]$ be fuzzy sets defined as follows:

$\widetilde{F}$	0	a	b	c
red	0.8	0.6	0.3	0.2
green	0.9	0.7	0.5	0.5
blue	0.7	0.7	0.6	0.6

Then $(\widetilde{F}, A)$ is a fuzzy soft near-subtraction semigroup but not a fuzzy soft ideal of X, because $\widetilde{F}[\text{red}]$ is not a fuzzy ideal of X,

$$\widetilde{F}[\text{red}](cb - c(0 - b)) = \widetilde{F}[\text{red}](c) = 0.2 \ngeq 0.3 = \widetilde{F}[\text{red}](b).$$

Theorem 3.5 Let $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ be fuzzy soft ideals of X, then $(\widetilde{F}, A)\widetilde{\wedge}(\widetilde{G}, B) = (\widetilde{H}, C)$ is a fuzzy soft ideal of X.

Proof. Let $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ be fuzzy soft ideals of X. Let $(e_1, e_2) \in A \times B$ and $x, y, z \in X$. Then by Definition 2.15

$$\begin{aligned}\widetilde{H}[e_1, e_2](x - y) &= (\widetilde{F}[e_1] \cap \widetilde{G}[e_2])(x - y)\\ &= \min\{\widetilde{F}[e_1](x - y), \widetilde{G}[e_2](x - y)\}\\ &\geqslant \min\{\min\{\widetilde{F}[e_1](x), \widetilde{F}[e_1](y)\}, \min\{\widetilde{G}[e_2](x), \widetilde{G}[e_2](y)\}\}\\ &= \min\{\min\{\widetilde{F}[e_1](x), \widetilde{G}[e_2](x)\}, \min\{\widetilde{F}[e_1](y), \widetilde{G}[e_2](y)\}\}\\ &= \min\{(\widetilde{F}[e_1] \cap \widetilde{G}[e_2])(x), (\widetilde{F}[e_1] \cap \widetilde{G}[e_2])(y)\}\\ &= \min\{\widetilde{H}[e_1, e_2](x), \widetilde{H}[e_1, e_2](y)\}.\end{aligned}$$

And

$$\begin{aligned}\widetilde{H}[e_1,e_2](xz-x(y-z)) &= (\widetilde{F}[e_1]\cap\widetilde{G}[e_2])(xz-x(y-z))\\ &= \min\{\widetilde{F}[e_1](xz-x(y-z)),\widetilde{G}[e_2](xz-x(y-z))\}\\ &\geqslant \min\{\widetilde{F}[e_1](z),\widetilde{G}[e_2](z)\} = (\widetilde{F}[e_1]\cap\widetilde{G}[e_2])(z)\\ &= \widetilde{H}[e_1,e_2](z).\end{aligned}$$

$$\begin{aligned}\widetilde{H}[e_1,e_2](xy) &= (\widetilde{F}[e_1]\cap\widetilde{G}[e_2])(xy)\\ &= \min\{\widetilde{F}[e_1](xy),\widetilde{G}[e_2](xy)\}\\ &\geqslant \min\{\widetilde{F}[e_1](x),\widetilde{G}[e_2](x)\} = (\widetilde{F}[e_1]\cap\widetilde{G}[e_2])(x) = \widetilde{H}[e_1,e_2](x).\end{aligned}$$

Therefore $(\widetilde{H},C)=(\widetilde{F},A)\widetilde{\wedge}(\widetilde{G},B)$ is a fuzzy soft ideal of X. ■

***Example* 3.6** Let $X=\{0,a,b,c\}$ be a near-subtraction semigroup with operations defined in the following Tables:

−	0	a	b	c
0	0	0	0	0
a	a	0	c	b
b	b	0	0	b
c	c	0	c	0

·	0	a	b	c
0	0	0	0	0
a	0	0	0	a
b	0	0	0	b
c	0	0	0	c

Let X be set of houses and $E=\{\text{expensive}(e_1),\ \text{beautiful}(e_2),\ \text{modern}(e_3),\ \text{wooden}(e_4)\}$ be the parameter space and $A=\{\text{expensive}(e_1),\ \text{beautiful}(e_2),\ \text{modern}(e_3)\}$, $B=\{\text{beautiful}(e_2),\ \text{wooden}(e_4)\}$. Consider the fuzzy soft sets $(\widetilde{F},A)$ and $(\widetilde{G},B)$ of X. Let $\widetilde{F}$[expensive], $\widetilde{F}$[beautiful], $\widetilde{F}$[modern], $\widetilde{G}$[beautiful] and $\widetilde{G}$[wooden] be the fuzzy sets defined as follows:

$\widetilde{F}$	0	a	b	c
expensive	0.9	0.4	0.6	0.4
beautiful	0.5	0.2	0.3	0.2
modern	0.7	0.3	0.5	0.3

$\widetilde{G}$	0	a	b	c
beautiful	0.8	0.6	0.2	0.2
wooden	0.7	0.4	0.5	0.4

Then clearly $(\widetilde{F},A)$ and $(\widetilde{G},B)$ are fuzzy soft ideals of X. Let $(\widetilde{F},A)\widetilde{\wedge}(\widetilde{G},B)=(\widetilde{H},C)$. Let $C=A\times B=\{(e_1,e_2),(e_1,e_4),(e_2,e_2),(e_2,e_4),(e_3,e_2),(e_3,e_4)\}$.

Consider the fuzzy soft set $(\widetilde{H},C)=(\widetilde{F},A)\widetilde{\wedge}(\widetilde{G},B)$.

$\widetilde{H}$	0	a	b	c
e_1, e_2	0.8	0.4	0.2	0.2
e_1, e_4	0.7	0.4	0.5	0.4
e_2, e_2	0.5	0.2	0.2	0.2
e_2, e_4	0.5	0.2	0.3	0.2
e_3, e_2	0.7	0.3	0.2	0.2
e_3, e_4	0.7	0.3	0.5	0.3

Then $(\widetilde{H}, C)$ is a fuzzy soft ideal of X.

In general, $(\widetilde{F}, A)\widetilde{\vee}(\widetilde{G}, B)$ is not a fuzzy soft ideal of X as shown in the following example.

***Example* 3.7** Let $X = \{0, a, b, c\}$ be a near-subtraction semigroup as in Example 3.6. In Example 3.6, $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ are fuzzy soft ideals of X. Let $(\widetilde{F}, A)\widetilde{\vee}(\widetilde{G}, B) = (\widetilde{H}, C)$.
Then $C = A \times B = \{(e_1, e_2), (e_1, e_4), (e_2, e_2), (e_2, e_4), (e_3, e_2), (e_3, e_4)\}$. The fuzzy soft set $(\widetilde{H}, C) = (\widetilde{F}, A)\widetilde{\vee}(\widetilde{G}, B)$.

$\widetilde{H}$	0	a	b	c
e_1, e_2	0.9	0.6	0.6	0.4
e_1, e_4	0.9	0.4	0.6	0.4
e_2, e_2	0.8	0.6	0.3	0.2
e_2, e_4	0.7	0.4	0.5	0.4
e_3, e_2	0.8	0.6	0.5	0.3
e_3, e_4	0.7	0.4	0.5	0.4

Thus $(\widetilde{H}, C)$ is not a fuzzy soft ideal of X, because $\widetilde{H}[e_2, e_2]$ and $\widetilde{H}[e_3, e_2]$ are not fuzzy ideals of X,
$\widetilde{H}[e_2, e_2](a - b) = \widetilde{H}[e_2, e_2](c) = 0.2 \ngeq 0.3 = \min\{\widetilde{H}[e_2, e_2](a), \widetilde{H}[e_2, e_2](b)\}$ and
$\widetilde{H}[e_3, e_2](a - b) = \widetilde{H}[e_3, e_2](c) = 0.3 \ngeq 0.5 = \min\{\widetilde{H}[e_3, e_2](a), \widetilde{H}[e_3, e_2](b)\}$.

In the following theorem, we give a condition for $(\widetilde{H}, C) = (\widetilde{F}, A)\widetilde{\vee}(\widetilde{G}, B)$ to be a fuzzy soft ideal of X.

Theorem 3.8 Let $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ be fuzzy soft ideals of X. If $\widetilde{F}[e_1] \leqslant \widetilde{G}[e_2]$ or $\widetilde{G}[e_2] \leqslant \widetilde{F}[e_1]$ for all $(e_1, e_2) \in A \times B$, then $(\widetilde{F}, A)\widetilde{\vee}(\widetilde{G}, B)$ is a fuzzy soft ideal of X.

Proof. Let $(\widetilde{F}, A)\widetilde{\vee}(\widetilde{G}, B) = (\widetilde{H}, C = A \times B)$ where $\widetilde{H}(e_1, e_2) = \widetilde{F}(e_1) \cup \widetilde{G}(e_2)$, for all $(e_1, e_2) \in A \times B$, by Definition 2.16. By assumption, $\widetilde{F}(e_1) \leqslant \widetilde{G}(e_2)$ or $\widetilde{G}(e_2) \leqslant \widetilde{F}(e_1)$, for all $(e_1, e_2) \in A \times B$. If $\widetilde{F}(e_1) \leqslant \widetilde{G}(e_2)$, then $\widetilde{H}(e_1, e_2) = \widetilde{F}(e_1) \cup \widetilde{G}(e_2) = \widetilde{G}(e_2)$ is a fuzzy ideal of X, since $\widetilde{G}(e_2)$ is a fuzzy ideal of X. If $\widetilde{G}(e_2) \leqslant \widetilde{F}(e_1)$, then $\widetilde{H}(e_1, e_2) = \widetilde{F}(e_1) \cup \widetilde{G}(e_2) = \widetilde{F}(e_1)$ is a fuzzy ideal of X, since $\widetilde{F}(e_1)$ is a fuzzy ideal of X. In both cases, $\widetilde{H}(e_1, e_2)$ is a fuzzy ideal of X, for all $(e_1, e_2) \in C$. Therefore, $(\widetilde{H}, C) = (\widetilde{F}, A)\widetilde{\vee}(\widetilde{G}, B)$ is a fuzzy soft ideal of X. ■

Theorem 3.9 Let $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ be fuzzy soft ideals of a near-subtraction semigroup

X. If $A \cap B \neq \emptyset$, then $(\widetilde{F}, A)\widetilde{\sqcap}(\widetilde{G}, B) = (\widetilde{H}, C)$ is a fuzzy soft ideal of X.

Proof. Assume that $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ are fuzzy soft ideals of X. For each $e \in C = A \cap B$, $\widetilde{H}[e] = \widetilde{F}[e] \cap \widetilde{G}[e]$, by Definition 2.14. By hypothesis, $\widetilde{F}[e]$ and $\widetilde{G}[e]$ are fuzzy ideals of X. The intersection of any two fuzzy ideals is also a fuzzy ideal. Thus $\widetilde{H}[e]$ is a fuzzy ideal of X. Since e is arbitrary, it follows that $(\widetilde{F}, A)\widetilde{\sqcap}(\widetilde{G}, B) = (\widetilde{H}, C)$ is a fuzzy soft ideal of X. ■

Theorem 3.10 Let $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ be fuzzy soft ideals of X. Then $(\widetilde{F}, A)\widetilde{\cap}(\widetilde{G}, B) = (\widetilde{H}, C)$ is a fuzzy soft ideal of X.

Proof. Assume that $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ be fuzzy soft ideals of X. By Definition 2.13, for $e \in C$, we have

Case (1): If $e \in A - B$, then $\widetilde{H}[e] = \widetilde{F}[e]$ is a fuzzy ideal of X. Thus $\widetilde{H}[e]$ is a fuzzy ideal of X.

Case (2): If $e \in B - A$, then $\widetilde{H}[e] = \widetilde{G}[e]$ is a fuzzy ideal of X. Thus $\widetilde{H}[e]$ is a fuzzy ideal of X.

Case (3): If $e \in A \cap B$, then $\widetilde{H}[e] = \widetilde{F}[e] \cap \widetilde{G}[e]$. Since $\widetilde{F}[e]$ and $\widetilde{G}[e]$ are fuzzy ideals of X, we have for all $x, y, z \in X$,

$$\begin{aligned}
\widetilde{H}[e](x-y) &= \widetilde{F}[e](x-y) \cap \widetilde{G}[e](x-y) \\
&\geqslant \min\{\widetilde{F}[e](x), \widetilde{F}[e](y)\} \cap \min\{\widetilde{G}[e](x), \widetilde{G}[e](y)\} \\
&= \min\{(\widetilde{F}[e](x) \cap \widetilde{G}[e](x)), (\widetilde{F}[e](y) \cap \widetilde{G}[e](y))\} \\
&= \min\{(\widetilde{F}[e] \cap \widetilde{G}[e])(x), (\widetilde{F}[e] \cap \widetilde{G}[e])(y)\} \\
&= \min\{(\widetilde{H}[e](x), \widetilde{H}[e](y)\}.
\end{aligned}$$

$$\begin{aligned}
\widetilde{H}[e](xz - x(y-z)) &= \widetilde{F}[e](xz - x(y-z)) \cap \widetilde{G}[e](xz - x(y-z)) \\
&\geqslant \widetilde{F}[e](z) \cap \widetilde{G}[e](z) \\
&= \widetilde{H}[e](z). \\
\widetilde{H}[e](xy) &= \widetilde{F}[e](xy) \cap \widetilde{G}[e](xy) \\
&\geqslant \widetilde{F}[e](x) \cap \widetilde{G}[e](x) \\
&= \widetilde{H}[e](x).
\end{aligned}$$

Thus $(\widetilde{F}, A)\widetilde{\cap}(\widetilde{G}, B) = (\widetilde{H}, C)$ is a fuzzy soft ideal of X. ■

In general, the union of two fuzzy soft ideals is not a fuzzy soft ideal, as shown in the following example.

***Example* 3.11** Let $X = \{0, a, b, c\}$ be a near-subtraction semigroup as in Example 3.6, $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ are fuzzy soft ideals of X. Let $(\widetilde{F}, A)\widetilde{\cup}(\widetilde{G}, B) = (\widetilde{H}, C)$, by Definition 2.12. Let $C = A \cup B = \{\text{expensive, beautiful, modern, wooden}\}, A \cap B = \{\text{beautiful}\}$ and $\widetilde{H}[\text{beautiful}]=\widetilde{F}[\text{beautiful}]\cup\widetilde{G}[\text{beautiful}]$.

$(\widetilde{H}, C)$	0	a	b	c
expensive	0.9	0.4	0.6	0.4
beautiful	0.8	0.6	0.3	0.2
modern	0.7	0.3	0.5	0.3
wooden	0.7	0.4	0.5	0.4

Then $(\widetilde{H}, C)$ is not a fuzzy soft ideal of X, since

$$\begin{aligned}\widetilde{H}[\text{beautiful}](a-b) &= \widetilde{H}[\text{beautiful}](c)\\ &= 0.2 \ngeqslant 0.3\\ &= \min\{0.6, 0.3\}\\ &= \min\{\widetilde{H}[\text{beautiful}](a), \widetilde{H}[\text{beautiful}](b)\}.\end{aligned}$$

In the next theorem we give a condition for the union of two fuzzy soft ideals to be a fuzzy soft ideal.

Theorem 3.12 Let $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ be fuzzy soft sets of X. If $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ are fuzzy soft ideals of X with $A \cap B = \emptyset$, then $(\widetilde{F}, A)\widetilde{\cup}(\widetilde{G}, B) = (\widetilde{H}, C)$ is a fuzzy soft ideal of X.

Proof. By Definition 2.12, we can write $(\widetilde{F}, A)\widetilde{\cup}(\widetilde{G}, B) = (\widetilde{H}, C)$. Since $A \cap B = \emptyset$, it follows that either $e \in A - B$ or $e \in B - A$. If $e \in A - B$, then $\widetilde{H}[e] = \widetilde{F}[e]$ is a fuzzy ideal of X. If $e \in B - A$, then $\widetilde{H}[e] = \widetilde{G}[e]$ is a fuzzy ideal of X. Thus $(\widetilde{H}, C)$ is a fuzzy soft ideal of X. ■

Theorem 3.13 Let $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ be fuzzy soft sets of X and $(\widetilde{F}, A)\widetilde{\subseteq}(\widetilde{G}, B)$ with $\widetilde{F}[e](x) \leqslant \widetilde{G}[e](x)$ for all $x \in X$. If $(\widetilde{G}, B)$ is a fuzzy soft ideal of X, then
(1) $\widetilde{G}[e](x-y) \geqslant \min\{\widetilde{F}[e](x), \widetilde{F}[e](y)\}$,
(2) $\widetilde{G}[e](xz - x(y-z)) \geqslant \widetilde{F}[e](z)$,
(3) $\widetilde{G}[e](xy) \geqslant \widetilde{F}[e](x)$, for all $x, y, z \in X$ and $e \in A$.

Proof. Let $(\widetilde{G}, B)$ be fuzzy soft ideal of X and $(\widetilde{F}, A)\widetilde{\subseteq}(\widetilde{G}, B)$. Let $x, y, z \in X$ and $e \in A$. Then

$$\widetilde{G}[e](x-y) \geqslant \min\{\widetilde{G}[e](x), \widetilde{G}[e](y)\} \geqslant \min\{\widetilde{F}[e](x), \widetilde{F}[e](y)\}.$$

And

$$\widetilde{G}[e](xz - x(y-z)) \geqslant \widetilde{G}[e](z) \geqslant \widetilde{F}[e](z).$$
$$\widetilde{G}[e](xy) \geqslant \widetilde{G}[e](x) \geqslant \widetilde{F}[e](x).$$

Hence the theorem is proved. ■

In the following example we prove that, if $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ are fuzzy soft sets of X with $(\widetilde{F}, A)\widetilde{\subseteq}(\widetilde{G}, B)$ and $(\widetilde{G}, B)$ is a fuzzy ideal of X, then $(\widetilde{F}, A)$ need not be a fuzzy soft ideal of X.

***Example* 3.14** Let $X = \{0, 1, 2\}$ be a near-subtraction semigroup with the following operations:

$-$	0	1	2
0	0	0	0
1	1	0	1
2	2	2	0

$\cdot$	0	1	2
0	0	0	0
1	0	1	1
2	0	2	2

Let X denote a set of students and $E = \{$brilliant, average, healthy$\}$ be the set of parameters. Consider, $A = \{$brilliant$\}$ and $B = \{$brilliant, average$\}$. Clearly, $A \subset B$. Let $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ be fuzzy soft sets of X defined as:

$\widetilde{F}$	0	1	2
brilliant	0.6	0.2	0.3

$\widetilde{G}$	0	1	2
brilliant	0.7	0.5	0.5
average	0.9	0.3	0.3

Obviously, $\widetilde{F}[\text{brilliant}] = \{0.6, 0.2, 0.3\} \subseteq \{0.7, 0.5, 0.5\} = \widetilde{G}[\text{brilliant}]$. It is easy to check that $(\widetilde{G}, B)$ is a fuzzy soft ideal of X. Now $\widetilde{F}[\text{brilliant}](12 - 1(0 - 2)) = \widetilde{F}[\text{brilliant}](1) = 0.2 \ngeqslant 0.3 = \widetilde{F}[\text{brilliant}](2)$. Thus $(\widetilde{F}, A)$ is not a fuzzy soft ideal of X.

Definition 3.15 Let $(\widetilde{F}, A)$ be a fuzzy soft set of X and for $t \in [0, 1]$ let $(\widetilde{F}, A)_t = \{x \in X | \widetilde{F}[e](x) \geqslant t\}$ for some $e \in A$. $(\widetilde{F}, A)_t$ is called a level soft set of X with respect to the parameter $e \in A$.

Theorem 3.16 Let $(\widetilde{F}, A)$ be a fuzzy soft set of X. Then $(\widetilde{F}, A)$ is a fuzzy soft ideal of X if and only if the level soft set $(\widetilde{F}, A)_t$ is an ideal of X, for each $t \in (0, 1]$.

Proof. Assume that $(\widetilde{F}, A)$ is a fuzzy soft ideal of X. Let $t \in (0, 1]$, $e \in A$ and $x, y \in (\widetilde{F}, A)_t$. This means that $\widetilde{F}[e](x) \geqslant t$ and $\widetilde{F}[e](y) \geqslant t$. By hypothesis $\widetilde{F}[e]$ is a fuzzy ideal of X, hence $\widetilde{F}[e](x - y) \geqslant \min\{\widetilde{F}[e](x), \widetilde{F}[e](y)\} \geqslant t$, that is, $x - y \in (\widetilde{F}, A)_t$. For $t \in (0, 1]$ let $z \in (\widetilde{F}, A)_t, e \in A$ and $x, y \in X$. Then $\widetilde{F}[e](xz - x(y - z)) \geqslant \widetilde{F}[e](z) \geqslant t$, which implies that $xz - x(y - z) \in (\widetilde{F}, A)_t$. Similarly, we can prove that $xy \in (\widetilde{F}, A)_t$ for all $x \in (\widetilde{F}, A)_t$ and $y \in X$. Hence $(\widetilde{F}, A)_t$ is an ideal of X.

Conversely, assume that $(\widetilde{F}, A)_t$ is an ideal of X for each $t \in (0, 1]$. Let $x, y \in X$ and $e \in A$. Suppose that $\widetilde{F}[e](x - y) < \min\{\widetilde{F}[e](x), \widetilde{F}[e](y)\}$. Then there exist t such that $\widetilde{F}[e](x - y) < t < \min\{\widetilde{F}[e](x), \widetilde{F}[e](y)\}$. This implies that $x, y \in (\widetilde{F}, A)_t$ but $x - y \notin (\widetilde{F}, A)_t$, which is a contradiction, since $(\widetilde{F}, A)_t$ is an ideal of X, we have, $x - y \in (\widetilde{F}, A)_t$. Thus $\widetilde{F}[e](x - y) \geqslant \min\{\widetilde{F}[e](x), \widetilde{F}[e](y)\}$. If there exist $x, y, z \in X$ such that $\widetilde{F}[e](xz - x(y - z)) < \widetilde{F}[e](z)$. Select $t \in (0, 1]$ such that $\widetilde{F}[e](xz - x(y - z)) < t < \widetilde{F}[e](z)$. Then $z \in (\widetilde{F}, A)_t$ but $xz - x(y - z) \notin (\widetilde{F}, A)_t$, which is a contradiction. Hence $\widetilde{F}[e](xz - s(y - z)) \geqslant \widetilde{F}[e](z)$. Similarly, we can prove the $\widetilde{F}[e](xy) \geqslant \widetilde{F}[e](x)$. Therefore $\widetilde{F}[e]$ is a fuzzy ideal of X. By Definition 3.2, $(\widetilde{F}, A)$ is a fuzzy soft ideal of X. ■

Theorem 3.17 Let I be an ideal of X. For any $t \in (0, 1]$ there exist a fuzzy soft ideal $(\widetilde{F}, A)$ such that $(\widetilde{F}, A)_t = I$.

Proof. Let I be an ideal of X. Let $(\widetilde{F}, A)$ be a fuzzy soft set of X defined by

$$\widetilde{F}[e](x) = \begin{cases} t & \text{if } x \in I \\ 0 & \text{otherwise} \end{cases}$$

where $t \in (0,1]$ and $e \in A$. Obviously, $(\widetilde{F}, A)_t = I$. Let $x, y, z \in X$ and $e \in A$. If there exist $x, y \in X$ such that $\widetilde{F}[e](x-y) < \min\{\widetilde{F}[e](x), \widetilde{F}[e](y)\}$, then $\widetilde{F}[e](x-y) = 0$ and $\min\{\widetilde{F}[e](x), \widetilde{F}[e](y)\} = t$ implies $x, y \in I$, but $x-y \notin I$ a contradiction to our assumption that I is an ideal of X. Thus $\widetilde{F}[e](x-y) \geqslant \min\{\widetilde{F}[e](x), \widetilde{F}[e](y)\}$. Suppose that $\widetilde{F}[e](xz-x(y-z)) < \widetilde{F}[e](z)$. Then $\widetilde{F}[e](xz-x(y-z)) = 0$ and $\widetilde{F}[e](z) = t$. This implies that $z \in I$ but $xz - x(y-z) \notin I$, this leads to a contradiction. So, $\widetilde{F}[e](xz - x(y-z)) \geqslant \widetilde{F}[e](z)$. Assume that $\widetilde{F}[e](xy) < \widetilde{F}[e](x)$. Then $\widetilde{F}[e](xy) = 0$ and $\widetilde{F}[e](x) = t$. Consequantly, $x \in I$ but $xy \notin I$, which is not possible. Thus, $\widetilde{F}[e](xy) \geqslant \widetilde{F}[e](x)$. Therefore $(\widetilde{F}, A)$ is a fuzzy soft ideal of X. ■

Theorem 3.18 Let $(\widetilde{F}, A)$ be a fuzzy soft ideal of X and $B \subset A$, then $(\widetilde{F}, B)$ is a fuzzy soft ideal of X.

Proof. Straightforward. ■

Converse of Theorem 3.18 is not true, this means there exist a fuzzy soft set $(\widetilde{F}, A)$ of X such that

(1) $(\widetilde{F}, A)$ is not a fuzzy soft ideal of X.

(2) There exist $B \subset A$ such that $(\widetilde{F}, B)$ is a fuzzy soft ideal of X.

***Example* 3.19** Consider Example 3.3(2), $(\widetilde{G}, A)$ is not a fuzzy soft ideal of X. But $B = \{\text{very big, small}\} \subset A = \{\text{very big, small, medium}\}$. Clearly, $(\widetilde{G}, B)$ is a fuzzy soft ideal of X.

4. Conclusion

In this paper we have presented some algebraic properties of fuzzy soft ideals of near-subtraction semigroups and illustrate with some examples. This work can be extended in the following directions.

(1) soft set theory can be applied to other types of fuzzy ideals of near-subtraction semigroups.

(2) soft set theory can be applied to different fuzzy ideals of subtraction semigroups.

(3) The results can be generalized to other types of fuzzy sets, namely, interval valued fuzzy sets, intuitionistic fuzzy sets and interval valued intuitionistic fuzzy sets.

Acknowledgements

The second author is thankful to the University Grant Commission, New Delhi-110021, India, for providing BSR fellowship under grant # F4-1/2006(BSR)/7-254/2009(BSR).

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